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Relative Orbit Reconfiguration and Rephasing under Illumination and Observability Constraints

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Abstract

Optical inspection of a resident space object requires the observing spacecraft to fly over the regions of interest while they are properly illuminated. When the chaser is confined to a passive relative orbit, and the target attitude is fixed with respect to the local orbital frame, the same regions are repeatedly flown over under the same illumination, so coverage saturates unless the phase along the relative orbit is actively changed. This paper derives the conditions that satisfy the illumination constraints and the observation geometry, so that the chaser reaches the designated relative orbit at the time that maximizes its observation capability. Three such conditions are obtained: instantaneous alignment with the illumination direction, alignment averaged over a revolution, and maximization of the observable surface. Because the commanded phase is tied to a rotating direction, the terminal state is itself a function of the arrival time, so that choosing when to arrive and choosing where to arrive become the same decision. The benefit of the proposed guidance is assessed in terms of the amount of surface information the resulting geometry makes available, and a Monte Carlo campaign is used to verify the behaviour of the controller and to compare the closed-form solution of the relaxed problem against the full non-convex one.

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1 Introduction

Among space operations, proximity operations between two or more spacecraft are one of the most complex and safety-critical activities. At the same time, the growing New Space economy is driving an increasing need for In-Orbit Servicing (IOS) and other forms of Rendezvous and Proximity Operations (RPOs), aimed at extending spacecraft lifetime and supporting critical operations. One particularly relevant application is the observation of a target spacecraft for damage assessment, anomaly detection, or operational support. Such inspection activities can also provide valuable situational awareness during critical operations, including rendezvous and servicing performed by third-party spacecraft [1–3]. Several recent and upcoming missions have demonstrated, or are expected to demonstrate, the capability to perform close-proximity inspection and observation tasks [4, 5].

Safety and collision avoidance are indeed one of the primary concerns during these operations, as the observing spacecraft must maintain a safe separation from the target throughout the entire mission. For this reason, in near-Earth space, Passive Relative Orbits (PROs) are often considered a suitable alternative to continuously generated or actively controlled relative trajectories [6, 7]. The passive nature of these orbits is also convenient at mission level, since it allows the onboard resources to be scheduled across several tasks [8]. PROs exploit the natural relative dynamics of the spacecraft and provide bounded, periodic trajectories around the target, which can be designed to maintain a safe separation without requiring continuous thrust. Simulation results suggest that the approach remains effective when the assumptions underlying the linearised model are relaxed, namely in the presence of orbital perturbations and for slightly eccentric target orbits [9]. As a result, they can significantly reduce propellant consumption and operational complexity while providing inherently safe relative motion.

Safety, however, is not the only constraint governing an inspection operation. The observing spacecraft, according to the mission goals, must reach the appropriate relative position and orientation to observe the desired region of the target at the required time. In addition, illumination conditions play a fundamental role in determining the quality and feasibility of optical observations. Consequently, the relative geometry between the observer, the target, and the Sun must be considered simultaneously.

The problem of adjusting the phase of a spacecraft along a given orbit is a classical one in astrodynamics, extensively investigated in the Keplerian two-body framework in the context of rendezvous, constellation deployment and station acquisition [10, 11]. The baseline solution is the two-impulse coplanar phasing maneuver, which shares the same structure of the Hohmann transfer, from which it differs only in that the initial and final orbits coincide, and is bounded in practice by the minimum admissible periapsis altitude of the phasing orbit. When large phase angles must be recovered within a limited number of revolutions, three-impulse bi-elliptic strategies become competitive. All these solutions can be regarded as particular cases of the more general Lambert problem, which allow the transfer to be executed at a prescribed arrival epoch [12, 13]. In the close-proximity regime, the same logic is applied to the linearized relative dynamics, where two-impulse targeting based on the Clohessy–Wiltshire [14],

or equivalently on relative orbital elements [15, 16], provides closed-form phasing solutions.

Impulsive strategies, however, rest on the assumption that the velocity change is delivered instantaneously, and is an open-loop strategy, that does not account for disturbances in the thrusters, navigation and so on. Therefore, the rephasing problem is more properly cast as a continuous-acceleration closed-loop optimal control problem. Classical results for low-thrust transfers, such as the Edelbaum solution [17] and its extensions, while shape-based and Lyapunov-driven feedback laws, most notably the Q-law [18, 19], offer computationally inexpensive guidance suitable for onboard implementation. Within the relative-motion framework, continuous control laws formulated on relative orbital elements [20] and model predictive control schemes [21, 22] allow the phase to be steered while explicitly enforcing passive safety along the whole reconfiguration arc.

In this work we address spacecraft reconfiguration maneuvers for rephasing within the same Passive Relative Orbit (PRO) or transferring to a new one, subject to illumination and observation constraints. The relative motion is parameterised through Relative Orbital Elements derived from the Clohessy–Wiltshire dynamics [23], in which the orbital phase appears explicitly as a state variable; the maneuver itself is obtained from the classical fixed-endpoint minimum-energy solution, leaving the terminal time as the only free decision variable. A further crucial point arises from the requirement to maintain a prescribed minimum separation from the target throughout the reconfiguration, and such a Keep-Out Zone (KOZ) constraint makes the problem intrinsically non-convex. Similar non-convex collision-avoidance constraints have commonly been addressed through sequential convexification techniques [24, 25]. In this work, the KOZ constraint is incorporated into the rephasing problem through a suitable convexification procedure, allowing the resulting optimization to retain the computational structure of a convex subproblem. On this basis, we make two contributions. First, we derive the motion of the illumination direction in the target’s local frame analytically and use it to formulate the phasing constraint in closed form, both for simple Sun-relative alignment and for the observation of a prescribed region of the target under suitable lighting; given a set of points of interest, this yields the phase that maximizes the number of them observed under acceptable illumination. Second, we show that the residual phasing error along a PRO is irreducible and depends only on the shape of the relative orbit, and we derive both this bound and the terminal phase that centres the error about it. The paper is organised as follows: in Section 2 we introduce the proximity operations problem with a non-cooperative target, together with the reference frames and the relative dynamics; Section 3 describes the fixed-endpoint controller that drives the chaser to the target relative orbit with the commanded phase; in Section 4 we derive the commanded phase from the desired illumination condition; the results are presented in Section 5, testing both the phasing conditions and, through a Monte Carlo campaign, the reliability of the controller in satisfying the operational constraints; finally, Section 6 summarises the conclusions and the limitations of this work.

2 Frames and Dynamics

Let $\mathcal{I} = \{O, \hat{\mathbf{e}}_x^I, \hat{\mathbf{e}}_y^I, \hat{\mathbf{e}}_z^I\}$ denote the Earth-centered inertial (ECI) frame. The object to be observed is referred to as the target and identified by the subscript T , while the observing spacecraft is the chaser, identified by the subscript C . The Sun is denoted by S , and $\hat{\mathbf{n}}_S^I$ denotes the unit vector pointing from the target towards the Sun in the ECI frame. The Sun lies on the ecliptic, with 23.44° with respect to the ECI frame, and its apparent direction changes by roughly 1° per day; for RPO tasks lasting a few hours, this variation is negligible, and $\hat{\mathbf{n}}_S^I$ is therefore assumed constant.

We assume purely Keplerian dynamics and a target in a circular orbit of radius a , period T_s , and mean motion $n = \sqrt{\mu/a^3}$, where μ is Earth's gravitational parameter. The local Hill frame centered at the target is $\mathcal{H} = \{T, \hat{\mathbf{e}}_x^H, \hat{\mathbf{e}}_y^H, \hat{\mathbf{e}}_z^H\}$, where the unit vectors represent, respectively: the radial direction, pointing from Earth's center to the target; the along-track direction, aligned with the target's orbital velocity; and the cross-track direction, parallel to the orbital angular momentum. The frame $\mathcal{H}$ rotates with respect to $\mathcal{I}$ with a constant angular velocity $\boldsymbol{\omega} = n \hat{\mathbf{e}}_z^H$.

The position and velocity of the chaser relative to the target, expressed in $\mathcal{H}$, are $\mathbf{r} = [x, y, z]^T \in \mathbb{R}^3$ and $\mathbf{v} = [\dot{x}, \dot{y}, \dot{z}]^T \in \mathbb{R}^3$, respectively. Assuming the chaser stays sufficiently close to the target ($\|\mathbf{r}\| \ll a$), the relative dynamics are well described by the linearized Clohessy–Wiltshire (CW) equations, which form the linear time-invariant system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}, \quad \mathbf{x} = \begin{bmatrix} \mathbf{r} \\ \mathbf{v} \end{bmatrix} \in \mathbb{R}^6, \quad \mathbf{u} \in \mathbb{R}^3, \quad (1)$$

with

$$\mathbf{A} = \begin{bmatrix} \mathbf{0}_{3 \times 3} & \mathbf{I}_3 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -n^2 & 0 & 0 & 0 \end{bmatrix} \in \mathbb{R}^{6 \times 6}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0}_{3 \times 3} \\ \mathbf{I}_3 \end{bmatrix} \in \mathbb{R}^{6 \times 3}, \quad (2)$$

where n is the mean motion of the target orbit and $\mathbf{u} = [u_x, u_y, u_z]^T$ is the control acceleration expressed in $\mathcal{H}$. The in-plane motion (x – y) is coupled through the Coriolis terms and admits a secular drift along $\hat{\mathbf{e}}_y^H$, whereas the out-of-plane motion along z is decoupled and purely harmonic at frequency n .

In the Hill frame, the sunlight direction $\hat{\mathbf{n}}_S^H$ is not constant: because $\mathcal{H}$ rotates about $\hat{\mathbf{e}}_z^H$ at rate n while $\hat{\mathbf{n}}_S^I$ is fixed, the Sun exhibits an apparent rotation about the cross-track axis at the same rate but in the opposite direction,

$$\hat{\mathbf{n}}_S^H(t) = R_z(-nt) \hat{\mathbf{n}}_S^H(t_0), \quad \hat{\mathbf{n}}_S^H(t_0) = R_{H \leftarrow I}(t_0) \hat{\mathbf{n}}_S^I, \quad (3)$$

with $R_z(\theta)$ the elementary rotation matrix about the z -axis and $R_{H \leftarrow I}(t)$ the rotation from ECI to Hill at time t . Figure 1 illustrates the geometry of the problem at hand.

With no acceleration input, the Clohessy–Wiltshire equations admit a closed-form solution that can be parameterised in terms of the relative orbital elements (ROEs)

introduced by [23]. The six ROEs are

$$\boldsymbol{\rho} = [x_r, \quad y_r, \quad a_r, \quad E_r, \quad A_z, \quad \gamma]$$

where x_r and y_r define the centre of the relative motion in the orbital plane at epoch, a_r is the semi-major axis of the in-plane relative ellipse, and $E_r(t) = E_{r,0} + n(t - t_0)$ is a parameter analogous to the eccentric anomaly, describing the phase of the in-plane relative motion. The corresponding relative position is

$$\begin{aligned} x(t) &= x_r - \frac{1}{2}a_r \cos E_r, \\ y(t) &= y_r - \frac{3}{2}n x_r (t - t_0) + a_r \sin E_r, \\ z(t) &= A_z \sin(E_r + \gamma), \end{aligned} \tag{4}$$

and the corresponding velocity

$$\begin{aligned} \dot{x}(t) &= \frac{1}{2}n a_r \sin E_r, \\ \dot{y}(t) &= -\frac{3}{2}n x_r + n a_r \cos E_r, \\ \dot{z}(t) &= n A_z \cos(E_r + \gamma). \end{aligned} \tag{5}$$

All ROEs are constant, with the phase E_r advancing uniformly at the mean motion, while the secular along-track drift is captured by the term $-\frac{3}{2}n x_r(t - t_0)$. Bounded relative motion is therefore obtained for $x_r = 0$, defining the class of Passive Relative Orbits (PROs) considered throughout this work. We further restrict the analysis to PROs satisfying $|y_r| < a_r$; this condition is assumed throughout all subsequent derivations.

3 Reconfiguration Guidance

In this section, we describe the control law that drives the chaser to a prescribed terminal relative state, chosen to satisfy the phasing requirement derived in Section 4.

3.1 Discrete-time model

The control acceleration is applied as a zero-order hold of timestep h , so that $\mathbf{u}(t) = \mathbf{u}_k$ for $t \in [t_k, t_k + h)$. Under this actuation, the discretized relative dynamics representation is:

$$\mathbf{x}_{k+1} = \boldsymbol{\Phi} \mathbf{x}_k + \boldsymbol{\Gamma} \mathbf{u}_k, \quad \boldsymbol{\Phi} = e^{\mathbf{A}h}, \quad \boldsymbol{\Gamma} = \left(\int_0^h e^{\mathbf{A}s} ds \right) \mathbf{B}. \tag{6}$$

Propagating Eq. (6) over N steps gives

$$\mathbf{x}_N = \boldsymbol{\Phi}^N \mathbf{x}_0 + \sum_{k=0}^{N-1} \mathbf{M}_k \mathbf{u}_k, \quad \mathbf{M}_k = \boldsymbol{\Phi}^{N-1-k} \boldsymbol{\Gamma} \in \mathbb{R}^{6 \times 3}. \tag{7}$$

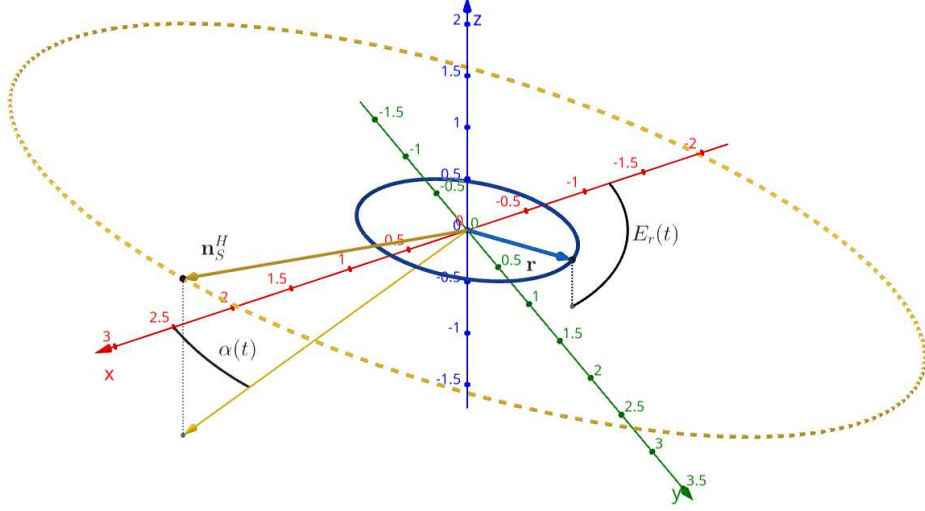


Fig. 1 Geometry of the problem in the Hill frame.

3.2 Fixed-endpoint minimum-energy steering

Requiring the terminal state to coincide with the last controller step, $\mathbf{x}_N = \mathbf{x}_f$, imposes a single linear constraint on the whole input sequence:

$$\sum_{k=0}^{N-1} \mathbf{M}_k \mathbf{u}_k = \mathbf{c}, \quad \mathbf{c} = \mathbf{x}_f - \Phi^N \mathbf{x}_0. \quad (8)$$

The commanded acceleration is bounded by the maximum allowed thrust, and the chaser must remain outside a keep-out zone (KOZ) modeled as a sphere of radius R_{KOZ} centered on the target. The optimal input sequence $\{\mathbf{u}_k\}$ is then obtained by solving the following:

$$\begin{aligned} \underset{\{\mathbf{u}_k\}}{\text{minimize}} \quad & J = \sum_{k=0}^{N-1} \frac{\|\mathbf{u}_k\|^2}{w_k} \\ \text{s.t.} \quad & \sum_{k=0}^{N-1} \mathbf{M}_k \mathbf{u}_k = \mathbf{c}, \\ & \|\mathbf{u}_k\|_\infty \leq u_{\max}, \quad k = 0, \dots, N-1, \\ & \|\mathbf{r}_k\| \geq R_{KOZ}, \quad k = 1, \dots, N, \end{aligned} \quad (9)$$

where the input weights w_k encode the shape of the control effort along the horizon as in [26, 27]. A uniform profile $w_k = 1$ recovers the unweighted minimum-energy solution, in which thrust is switched on and off within a single step at both ends of

the maneuver. A raised-sine profile is of the form

$$w_k = \sin^2\left(\frac{\pi(\varepsilon + k + 1/2)}{D}\right), \quad D = \varepsilon + N, \quad (10)$$

where N is the number of remaining steps and ε is the number of steps already elapsed in the current maneuver. This profile tapers the thrust smoothly to nearly zero at both ends of the whole maneuver. The half-step offset keeps every w_k strictly positive, so that the cost in Eq. (9) remains finite. Anchoring the profile to ε , rather than re-deriving it from the remaining horizon at each re-plan, preserves a single ramp across the maneuver.

Because of the KOZ constraint, the problem in Eq. (9) is non-convex, and no closed-form solution exists. A standard approach is successive convexification. Given a reference trajectory $\{\bar{\mathbf{r}}_k\}$, the keep-out constraint is replaced by its supporting hyperplane at $\bar{\mathbf{r}}_k$,

$$\hat{\mathbf{m}}_k^\top \mathbf{r}_k \geq R_{KOZ}, \quad \hat{\mathbf{m}}_k = \frac{\bar{\mathbf{r}}_k}{\|\bar{\mathbf{r}}_k\|}, \quad (11)$$

Since Eq. (7) is affine in the inputs, Eq. (11) is a linear inequality, and the resulting subproblem is a convex quadratic program. This problem can be solved via Sequence Convex Programming, using the solution of the relaxed problem, with only the terminal constraint, as a warm start. This relaxed problem can be solved in closed form via Lagrange multipliers [28]:

$$\mathbf{u}_k = w_k \mathbf{M}_k^\top \mathbf{W}^{-1} \mathbf{c}, \quad J^* = \mathbf{c}^\top \mathbf{W}^{-1} \mathbf{c}, \quad (12)$$

where $\mathbf{W} \in \mathbb{R}^{6 \times 6}$ is the weighted reachability Gramian [29]:

$$\mathbf{W} = \sum_{k=0}^{N-1} w_k \mathbf{M}_k \mathbf{M}_k^\top.$$

The relaxation is solved in any case, so its solution is first propagated and checked against the two inequality constraints of Eq. (9). If neither is violated, the relaxed solution is already the global optimum of the full problem, and it is retained without solving the full problem via SCP.

3.3 Selection of the terminal time

To fully determine the maneuver, the terminal state $\mathbf{x}_f$ must still be fixed. Since the arrival PRO is prescribed, its relative orbital elements

$$\boldsymbol{\rho}^* = [\bar{x}_r, \bar{y}_r, \bar{a}_r, E_r^*, \bar{A}_z, \bar{\gamma}]^\top \quad (13)$$

are known except for the phase E_r^* , the remaining five elements being constant along the orbit. Using Eq. (4) the terminal state then follows from the mapping:

$$\mathbf{x}_f = \mathcal{M}(\boldsymbol{\rho}^*), \quad (14)$$

so that $\mathbf{x}_f$ is entirely determined by the single scalar E_r^* . However, the phasing condition fixes it, but at a given arrival time: because the chaser is chasing a rotating reference direction, the phase that satisfies the observation requirement is itself a function of t_f . The condition therefore does not return a point, but a one-parameter family of admissible terminal states,

$$E_r^* = E_r^*(t_f) \quad \implies \quad \mathbf{x}_f = \mathbf{x}_f(t_f), \quad (15)$$

and choosing when to arrive and choosing where to arrive become the same decision. This constitutes the main difference with respect to most reconfiguration problems in which the target state is assigned a priori and the transfer time only determines how it is reached. Defining the terminal time as:

$$t_f = t_0 + Nh \quad (16)$$

the remaining design problem is a one-dimensional minimization over the horizon length N .

We now augment the cost function from Eq. (12) with a penalty term on transfer duration:

$$\mathcal{C}(t_f) = J^*(t_f)h + \frac{\lambda_t(t_f - t_0)}{T_s} = \mathbf{c}(t_f)^\top \mathbf{W}(t_f)^{-1} \mathbf{c}(t_f)h + \frac{\lambda_t(t_f - t_0)}{T_s}, \quad (17)$$

The time penalty term, with $\lambda_t \geq 0$, prices maneuver time against control effort, and is monotonically increasing. The arrival phase E_r^* advances at the mean motion, and, from Eq. (14), so does $\mathbf{x}_f(t_f)$. When the departure state lies on a PRO, the free response $\Phi^N \mathbf{x}_0$ is periodic as well, and the residual $\mathbf{c}(t_f)$ is therefore periodic in t_f with period T . For a rephasing on the same PRO, the two states share every element but the phase, whose difference $\sigma^* - \sigma_0$ is constant. The constant terms then cancel and $\mathbf{c}$ reduces to a pure first harmonic, so that $\mathbf{c}(t_f + T/2) = -\mathbf{c}(t_f)$ and any fixed quadratic form in $\mathbf{c}$ is periodic with period $T/2$. For a reconfiguration towards a different PRO this anti-symmetry is lost, and only the period T survives. In either case, $J^*(t_f)$ is an oscillating residual weighted by a horizon-dependent matrix, and it is not itself periodic. Adding the monotonic time penalty yields a cost $\mathcal{C}(t_f)$ that is in general multimodal, with local minima recurring roughly once per half orbital period for a rephasing and once per period for a reconfiguration, so that a single gradient-based search is not guaranteed to reach the global one.

Therefore, the minimisation is carried out globally over the admissible range of t_f , either by initializing a gradient-based method once per half orbital period, or by a coarse sweep over discrete horizons followed by local refinement about each sampled minimum. From Eq. (16), a search over a continuous t_f requires the resulting optimum to be rounded onto the grid, whereas searching directly over N avoids this step without any loss — the executed sequence must in any case lie on the grid, this being where the exact terminal condition originates.

Finally, an actuator bound $u_{\max}$ is enforced at this stage by rejecting candidate horizons whose peak commanded acceleration exceeds it, rather than by saturating

the input sequence: saturation would violate Eq. (8) and destroy the exact terminal condition, whereas rejection drives the search towards the longer, gentler horizons that satisfy the bound. A remark on optimality is in order. The horizon returned by this search is the exact minimiser of Eq. (17) for the relaxed problem, which admits the closed-form solution of Eq. (12), but not necessarily for the full non-convex problem of Eq. (9): evaluating the latter at every candidate horizon would require a convexification loop per candidate, at a cost incompatible with an onboard implementation. However, this choice is well founded, since dropping the keep-out constraint enlarges the feasible set and J^* can therefore never exceed the cost of the full problem at the same horizon. Whenever the relaxed trajectory already satisfies the constraints, the two costs coincide, and the selected horizon is then the global optimum of the full problem; as shown in Section 5.4, this is the case for the large majority of the maneuvers considered.

Finally, Figure 2 summarizes the complete workflow of the proposed guidance, in which the terminal state is selected either by phasing the chaser with the illumination direction or by maximizing the number of observable points on the target surface; both criteria are derived in Section 4.

4 Terminal State for Phasing

This section constructs the terminal state $\mathbf{x}_f$ for the guidance law. Two conditions are explored, the first one is to be in exact phase with the illumination source, the second is to phase with the observable points on the surface of the target.

4.1 Motion of the illumination direction

Given the apparent motion of the Sun in the Hill frame described in Eq. (3), its representation in $\mathcal{H}$ is

$$\hat{\mathbf{n}}_S^H(t) = \begin{bmatrix} \cos \beta \cos \alpha(t) \\ \cos \beta \sin \alpha(t) \\ \sin \beta \end{bmatrix}, \quad \alpha(t) = \alpha_0 - n(t - t_0), \quad (18)$$

where β is the constant solar elevation above the target orbital plane and α the in-plane solar phase, sweeping backwards at exactly the mean motion, as shown in Figure 1. The phasing requirement is therefore purely in-plane, and can be stated in terms of the in-plane position angle of the chaser, using Eq. (4),

$$\nu_r(t) = \text{atan2}(y_r + a_r \sin E_r(t), -\frac{1}{2}a_r \cos E_r(t)), \quad (19)$$

as the phasing error

$$\Delta(t) = \alpha(t) - \nu_r(t). \quad (20)$$

Although the relative anomaly advances uniformly at $\dot{E}_r = n$, the position angle, which may be regarded as analogous to a true anomaly, does not evolve at a constant

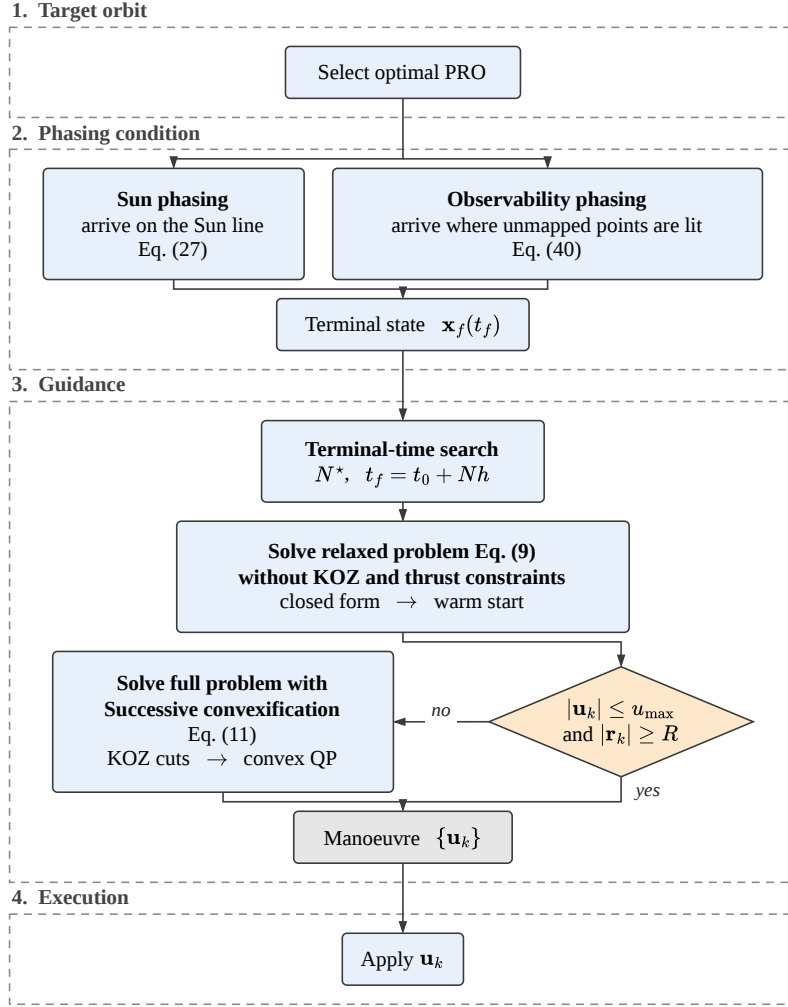


Fig. 2 Flowchart of the proposed reconfiguration guidance.

rate. Deriving Eq. (19)

$$\dot{\nu}_r(t) = -\frac{2a_r n [a_r + y_r \sin E_r(t)]}{a_r^2 \cos^2 E_r(t) + 4[y_r + a_r \sin E_r(t)]^2}, \quad (21)$$

whose mean value over a revolution is exactly $-n$ for any $|y_r| < a_r$, but its instantaneous rate therefore differs from that of the solar phase, so an alignment established at one instant cannot be maintained. Moreover, the excursion of Δ over a revolution

is a property of the PRO, and only its offset can be chosen. This follows by noting that, since α decreases and E_r increases at the same rate, the quantity

$$\sigma \triangleq E_r(t) + \alpha(t) \quad (22)$$

is an invariant of the free relative motion with illumination dynamics. Eliminating α from (20) then gives

$$\Delta = \sigma - f(E_r), \quad f(E) \triangleq \nu_r(E) + E, \quad (23)$$

so that the phasing error is determined by the single scalar σ : f fixes the shape of the error over a revolution, and σ shifts it rigidly. Being invariant, σ characterizes the observation geometry and can be altered only through a maneuver, so that rephasing is equivalent to resetting it. The terminal condition takes the form

$$E_r^*(t_f) = \sigma^* - \alpha(t_f), \quad \frac{dE_r^*}{dt_f} = n, \quad (24)$$

once the commanded value σ^* has been specified. Two choices of σ^* are considered below.

4.1.1 Instantaneous phasing

Requiring the chaser to lie on the illumination direction at the arrival instant, from Eq. (20), the condition is $\nu_r(t_f) = \alpha(t_f)$. Written using the in-plane cross product, $g \triangleq x n_y - y n_x = 0$ with $(n_x, n_y) = (\cos \alpha(t_f), \sin \alpha(t_f))$, and substituting Eq. (4), it collapses to a single sinusoid in the unknown phase,

$$A \cos E_r + B \sin E_r = C, \quad (25)$$

with

$$A = -\frac{1}{2}a_r n_y, \quad B = -a_r n_x, \quad C = y_r n_x,$$

and whose solutions, as a function of the arrival time, are

$$E_r^*(t_f) = \delta \pm \arccos(C/R), \quad (26)$$

given

$$R = \sqrt{A^2 + B^2}, \quad \delta = \text{atan2}(B, A),$$

and provided $|C| \leq R$. The two roots lie on opposite sides of the target, and the admissible one is selected by the sign test $\mathbf{r}_{xy} \cdot \mathbf{n}_{xy} > 0$, the other corresponding to the eclipse-side geometry. Note that for a centered PRO one has $C = 0$, so that the condition is solvable at every arrival time and admits exactly one admissible root, while for an offset PRO the feasibility condition $|C| \leq R$ fails over some intervals of t_f .

4.1.2 Centering the phasing error

The previous condition only guarantees an instantaneous alignment, but as discussed previously, there is an excursion of Δ that depends on the PRO. In order to mitigate this effect we might want to place σ at the centre of the range of f . Note that, from (19), the map $E \mapsto \pi - E$ sends $\nu_r \mapsto \pi - \nu_r$ and hence

$$f(\pi - E) = 2\pi - f(E),$$

irrespective of a_r and y_r . The range of f is therefore centered on π for every PRO, and centering the pointing error requires

$$\sigma^* = \pi \quad \Longleftrightarrow \quad E_r^*(t_f) = \pi - \alpha(t_f). \quad (27)$$

The residual excursion that no choice of σ can remove follows from Eq. (23) as

$$\Delta_{1/2} = \max_E |f(E) - \pi|. \quad (28)$$

4.2 Phasing for surface observability

During an inspection task on a PRO, the not-yet-inspected regions of the target must be flown over while they are lit. For a nadir-pointing target the surface orientation is fixed in $\mathcal{H}$, so that the same faces are repeatedly overflown under the same illumination, and only a rephasing maneuver can change which of them are lit at fly-over. In this section the phasing condition is therefore derived directly from the observability of the target surface.

4.2.1 Fly-over geometry

Let the target surface be represented by M points observable from the assigned PRO, with the j -th being best observed along the direction $\hat{\mathbf{n}}_j$, expressed in $\mathcal{H}$. This set may be obtained by any suitable method, such as the sampling-based or graph-based algorithms in [9]. Point j is observed when the chaser lies along that direction. Imposing the in-plane alignment between the relative position of Eq. (4) and $\hat{\mathbf{n}}_j$ gives the same sinusoidal condition as Eq. (25), with the point normal in place of the solar direction, whose admissible root is the fly-over anomaly E_j . For a centered PRO the condition reduces to

$$E_j = \text{atan2}(n_{j,y}, -2n_{j,x}). \quad (29)$$

Being determined by the relative orbit and by the point normal alone, E_j is computed once for each point and is unaffected by the commanded shift. The forward angular distance from the current phase to that fly-over is

$$\delta E_j = (E_j - E_r(t_0)) \bmod 2\pi, \quad (30)$$

and the corresponding arrival epoch, after a rephasing maneuver commanding a phase shift φ , is

$$t_{\text{arr},j} = t_0 + \frac{\varphi + \delta E_j}{n}. \quad (31)$$

Since φ advances or delays the chaser along the same relative orbit, the same points are overflown in the same order and from the same relative positions; what it changes is the epoch of each fly-over, and hence, through Eq. (3), the solar direction at that epoch.

4.2.2 Self-shadow condition

Evaluating the solar phase of Eq. (18) at the shifted arrival (31) gives

$$\alpha(t_{\text{arr},j}(\varphi)) = \alpha_0 - \varphi - \delta E_j, \quad (32)$$

Defining the illumination quality threshold q , the lit condition is translated as:

$$\Gamma_j(\varphi) = [g_j(\varphi) > q], \quad g_j(\varphi) \triangleq \hat{\mathbf{n}}_j \cdot \hat{\mathbf{n}}_S^H(t_{\text{arr},j}(\varphi)). \quad (33)$$

Writing the unit normal in spherical coordinates

$$\hat{\mathbf{n}}_j = \begin{bmatrix} \cos \delta_j \cos \eta_j \\ \cos \delta_j \sin \eta_j \\ \sin \delta_j \end{bmatrix},$$

with $\delta_j \in [-\pi/2, \pi/2]$ its elevation above the target orbital plane and η_j its in-plane azimuth. Then the lit condition is a pure cosine function in the commanded phase shift,

$$g_j(\varphi) = \rho_j \cos(\varphi - c_j) + C_j \quad (34)$$

with

$$\rho_j = \cos \delta_j \cos \beta, \quad C_j = \sin \delta_j \sin \beta, \quad c_j = \alpha_0 - \eta_j - \delta E_j. \quad (35)$$

For each point the set of phase shifts at which it is observed, $\mathcal{A}_j = \{\varphi : g_j(\varphi) > q\}$, is an arc of the circle. With the half-width

$$\psi_j = \arccos\left(\frac{q - C_j}{\rho_j}\right), \quad (36)$$

the three cases listed in Table 1 exhaust the possibilities.

4.2.3 Eclipse condition

During its orbit the target may pass behind the Earth, entering full eclipse, and this possibility must be accounted for. This second constraint shifts with φ for the same reason the first does. Let $\theta_T(t) = \theta_{T,0} + n(t - t_0)$ denote the target argument of

Table 1 Classification of the admissible set $\mathcal{A}_j$ of a surface point according to the amplitude ρ_j and offset C_j of its observability margin.

Case	Condition	$\mathcal{A}_j$
Always observable	$C_j - \rho_j \geq q$	$[0, 2\pi)$
Never observable	$C_j + \rho_j \leq q$	$\emptyset$
Partially observable	otherwise	$(c_j - \psi_j, c_j + \psi_j) \bmod 2\pi$

latitude and $[\theta_{\text{in}}, \theta_{\text{out}}]$ the eclipse arc, of forward width Δ_{ecl} and midpoint θ_{m} . The target argument of latitude at the arrival of point j is

$$\theta_T(t_{\text{arr},j}(\varphi)) = \theta_{T,0} + \varphi + \delta E_j, \quad (37)$$

so the shifts placing that arrival inside the shadow form an arc in φ of half-width $\Delta_{\text{ecl}}/2$. Its complement, which on a circle is again an arc, is the admissible set

$$\mathcal{E}_j : \text{centre } \theta_{\text{m}} - \theta_{T,0} - \delta E_j + \pi, \quad \text{half-width } \pi - \frac{1}{2}\Delta_{\text{ecl}}. \quad (38)$$

The admissible set of point j is therefore $\mathcal{A}_j \cap \mathcal{E}_j$, the intersection of two arcs, which consists of at most two intervals.

4.2.4 Observability score

The phase shift is selected by maximizing the number of points observed at fly-over,

$$\varphi^* \in \arg \max_{\varphi \in [0, 2\pi)} S(\varphi), \quad S(\varphi) = \sum_{j: \varphi \in \mathcal{A}_j \cap \mathcal{E}_j} w_j. \quad (39)$$

where $w_j \in [0, 1]$ is the observation priority of point j , ranging from 0 for points of no interest because already mapped, or unreachable from the current relative orbit, to 1 for those most in need of observation. S is piecewise constant, with at most $4m + 2$ breakpoints, and its global maximizer set is a finite union of closed intervals. Problem (39) is thus an instance of maximum weighted arc overlap. Since S changes value only at the endpoints of the admissible sets, its exact global maximum is obtained by a sweep with cost $\mathcal{O}(M \log M)$: accumulate $+w_j$ at each interval start and $-w_j$ at each end over the sorted list of endpoints, and retain the segments of maximal accumulated value. The maximizer is a segment rather than a point, and often several disjoint ones. The representative φ^* is selected within it either as the phase maximizing the smallest active margin, which is robust to navigation error, or as the one of smallest $|\varphi|$, which is the cheapest maneuver.

In order to obtain the terminal condition in the form established in Eq. (24) we define $\sigma_0 = E_r(t_0) + \alpha(t_0)$ as the current value of the invariant Eq. (22), which free motion preserves. A shift φ^* decreases the relative phase by φ^* while leaving the solar

phase untouched. The commanded invariant is therefore

$$\sigma^* = \sigma_0 - \varphi^* \quad (40)$$

and the terminal condition follows from Eq. (24) unchanged,

$$E_r^*(t_f) = \sigma^* - \alpha(t_f), \quad \frac{dE_r^*}{dt_f} = n, \quad (41)$$

from which $\mathbf{x}_f(t_f)$ is obtained through the mapping Eq. (14). The observability condition therefore has the same structure as the centering condition of Section 4.1.2, and differs from it only in the commanded value of σ^* , the latter being the particular case $\sigma^* = \pi$. Both generate the one-parameter family (15) with the same constant sensitivity, so the terminal-time search of Section 3.3 applies unchanged. Two remarks are in order. First, the transfer arc is not itself a PRO: the minimum-energy solution leaves the bounded family in transit and rejoins it at t_f . Passive safety is therefore not continuous, but is recovered for the whole of the observation phase, with the maneuver occupying a small fraction of the mission duration.

Second, a single rephasing does not in general cover the whole surface: points left out because of the current phase, rather than because they are never lit at fly-over, become observable at other values of σ^* . The condition is therefore re-evaluated as the inspection proceeds, each maneuver unlocking a further portion of the same PRO's reachable surface.

5 Results

In this section we validate the proposed guidance for both the illumination phasing and the observability condition for an inspection task. We then compare the closed-form relaxation against the convexified solution over a Monte Carlo campaign.

5.1 Simulation setup

The reference scenario considers a target on a circular low Earth orbit at 630 km of altitude, whose inclination and right ascension of the ascending node place the Sun at a constant elevation below the target orbital plane. As a consequence, roughly 9.47% of the target surface is never illuminated during the observation window and cannot be recovered by any rephasing maneuver. The target is modeled as a sphere, and the keep-out zone as a larger concentric sphere, so that each sampled surface point is best observed along its outward normal. The chaser is initially placed on a centered passive relative orbit, with no along-track offset and a cross-track oscillation phased with respect to the in-plane motion. The guidance runs at a fixed timestep with a bound on the commanded acceleration, and the terminal-time search explores horizons between half and one and a half orbital periods, weighting the maneuver duration against control effort through a constant time penalty. All parameters of the scenario are collected in Table 2. The convexified subproblems are solved with OSQP [30], initialised with the closed-form relaxed solution. The SCP loop stops when

the trajectory satisfies the KOZ constraint and its maximum change between two iterations falls below 0.1 mm, or after at most 10 iterations.

Table 2 Parameters of the reference scenario.

Parameter	Symbol	Value
<i>Target orbit</i>		
Semi-major axis	a	7000 km
Mean motion	n	1.078×10^{-3} rad/s
Orbital period	T_s	5828 s
Inclination	i	30°
Right ascension of the ascending node	Ω	270°
Solar elevation	β	-35°
Permanently dark surface fraction	—	9.47%
Target radius	r_T	3 m
Keep-out zone radius	R	4.5 m
<i>Initial relative orbit</i>		
In-plane semi-major axis	a_r	20 m
Centre of motion	x_r, y_r	0 m, 0 m
Cross-track amplitude	A_z	10 m
Cross-track phase	γ	30°
<i>Controller</i>		
Control timestep	h	10 s
Acceleration bound	$u_{\max}$	1×10^{-4} m/s ²
Time penalty	λ_t	$0.5 \text{ m}^2/\text{s}^3$
Terminal-time search range	$t_f - t_0$	$[0.5, 1.5] T_s$

5.2 Rephasing to the centered illumination phase

The chaser starts with an invariant $\sigma_0 = 0.14$ rad, away from the value $\sigma^* = \pi$ that centres the phasing error, and is commanded to rephase, on the same PRO, with the terminal condition of Eq. (27). Figure 3 shows the resulting trajectory in the Hill frame. The search for the optimal t_f returns a maneuver lasting $0.51 T_s$, along which the transfer arc leaves the bounded family and rejoins it slightly behind the departure point, almost at the same location on the relative orbit. The orange dotted line is the initial guess obtained by solving the relaxed problem: its minimum distance to the target along the transfer is 1.55 m, so the keep-out constraint is violated and the SCP loop is entered, yielding a refined trajectory whose minimum distance is 4.76 m and which therefore satisfies the KOZ constraint. The thrust bound is not violated by the initial guess, and the refined solution requires a maximum input of 3.79×10^{-5} m/s².

Figure 4 shows the acceleration input along the three axes. Note that, although the rephasing acts on the phase of the chaser relative to the solar angle $\alpha(t)$, both of which are in-plane angles, the cross-track component of the input is not zero. This is only partially an effect of the KOZ constraint pushing the trajectory out of the plane, since the component is already present in the closed-form warm start. The reason is that the PRO phase is carried by E_r in-plane and by $E_r + \gamma$ across it, so reaching the

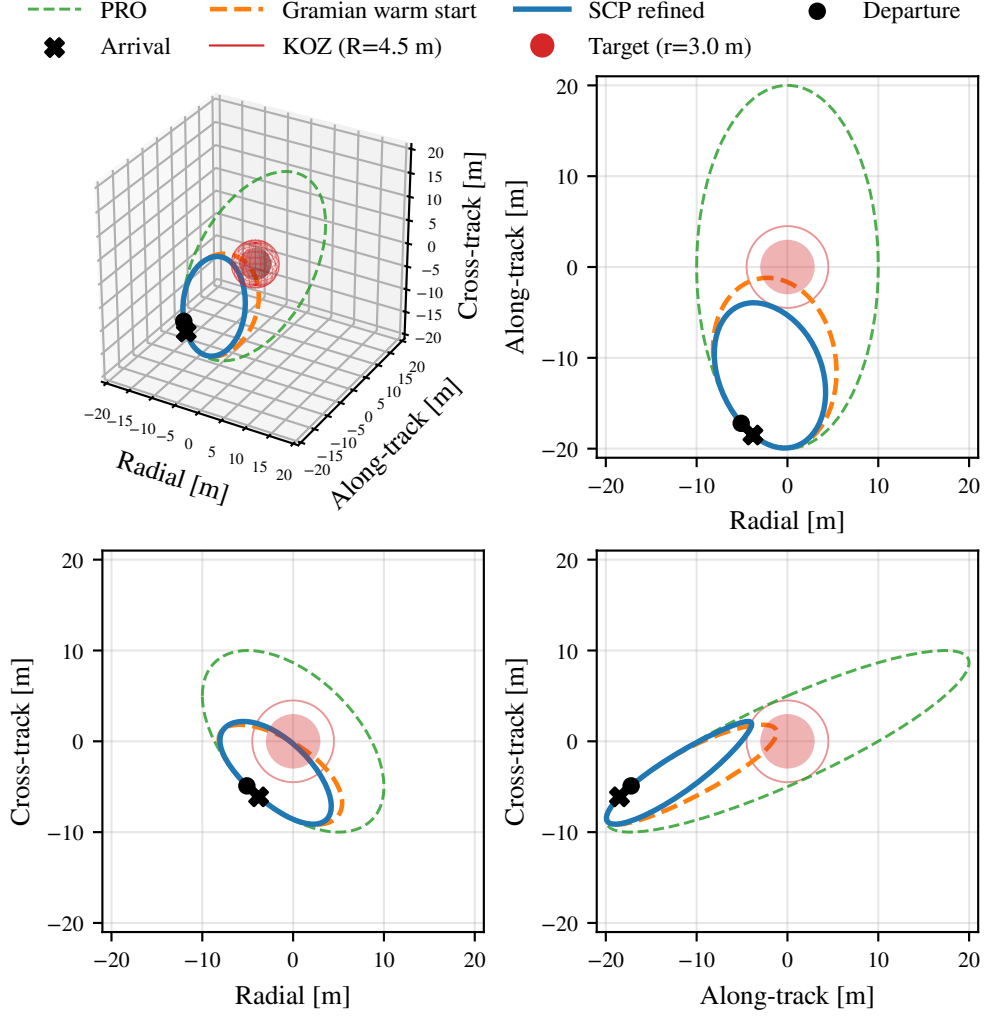


Fig. 3 Rephasing maneuver within the same PRO, targeting the terminal phase that centres the pointing error about the solar direction ($\sigma^* = \pi$).

same orbit at a phase shifted by δ , with $\bar{A}_z$ and $\bar{\gamma}$ preserved, requires the cross-track oscillator to be re-timed by the same amount. In Figure 5 the ROEs during the whole simulation time are shown. The transfer takes approximately 48 min, after which every element is constant, apart from E_r , returning to their initial values. So the chaser lands on the same PRO it left; only E_r has been changed, which is the purpose of the maneuver: on a passive orbit E_r is an exact sawtooth, advancing at the mean motion and wrapping every orbital period; here, between 0 and 48 min the slope is no longer constant, peaking at 6.2 times the mean motion, and it then returns with an offset by $+172^\circ$. As a result, as shown in Figure 6, the in-plane angle difference Δ is reduced

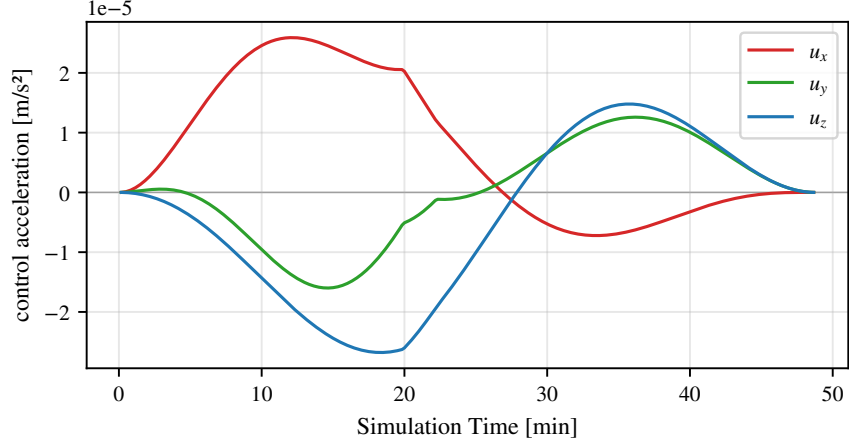


Fig. 4 Commanded acceleration along the three axes of the Hill frame over the transfer horizon.

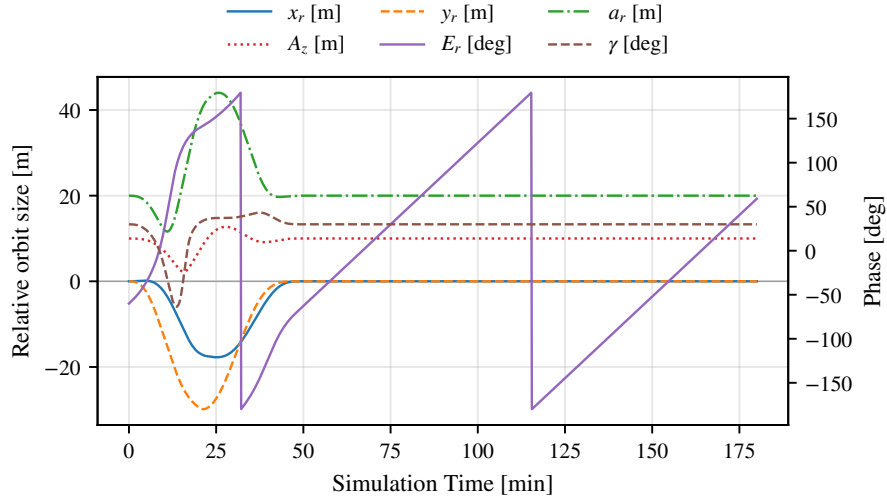


Fig. 5 Variation of the Relative Orbital Elements during the maneuver time. On the left ordinate the length elements (x_r , y_r , a_r , A_z) in metres, on the right the phase angles (E_r , γ) in degrees.

from an initial value near π to a periodic oscillation about zero with an excursion of $\Delta_{1/2} = 19.47^\circ$, as predicted by Eq. (28) for a PRO with these characteristics. The total angle between chaser and Sun (dotted line) shows the same oscillation with an offset given on average by the solar elevation β . This does not mean the total angle is always larger: whenever the cross-track motion carries the chaser to the same side of the plane as the Sun, the two are closer than their projections suggest — especially when they are nearly opposite in plane, as at the start of the run. This residual offset cannot be removed by any rephasing maneuver, since β is fixed by the orientation of

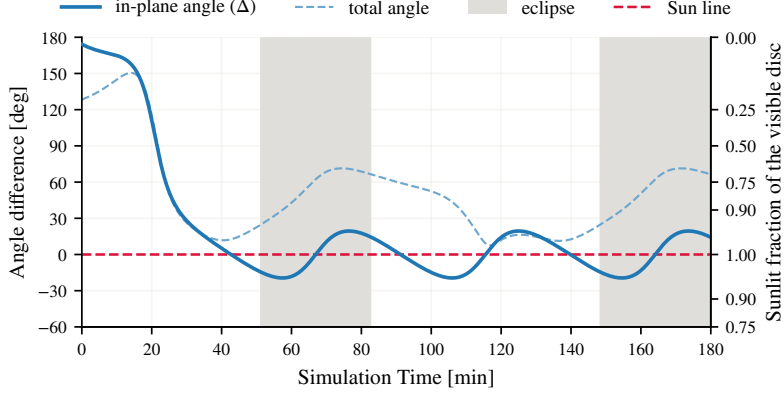


Fig. 6 Phasing error between the chaser and the sun line over the simulation time. The solid line is the in-plane angle phase $\Delta(t)$, while the dotted angle is the total angle between the chaser and the sun direction.

the target orbit with respect to the Sun and is therefore not a degree of freedom of the guidance. What matters in practice is only that the resulting phase angle remains within a range compatible with optical observation.

5.3 Target surface observability

During an in-orbit inspection mission, being on a PRO that is in phase with the solar direction does not by itself guarantee that every point of the target surface can be observed under a suitable illumination angle. As already discussed, restricting the chaser to PROs is convenient because it minimises the time spent on unbounded, and therefore passively unsafe, relative orbits; the price is that observing the points that are not lit at fly-over under the current phase requires one or more rephasing maneuvers. Figure 7 shows how the set of observable points changes with the phase shift commanded to the chaser. The lower panel is read by horizontal lines: each row is a surface point, identified by the anomaly at which the chaser flies over it, and the colour along that row indicates, for every commanded shift on the horizontal axis, the condition met at fly-over — target in eclipse, point not lit because the Sun illuminates a different portion of the surface, or point lit and therefore observable. The upper panel is the resulting number of observed points, obtained by counting the observable rows at each shift.

Of the 215 points of the target surface that are geometrically observable from the assigned PRO, the phase that aligns the chaser with the solar direction yields only 123: illumination phasing centres the observation geometry, but it is not the condition that maximizes coverage. The maximum is 149 points, attained at a phase shift of $\varphi = 123^\circ$. The points whose fly-over anomaly lies approximately between 60° and 180° are excluded at this shift, not because of their orientation with respect to the Sun but because the target is in eclipse when the chaser flies over them; they become observable at a shift of about 250° . Neither of the two useful shifts coincides with the value that would place the chaser on the illumination direction: the coverage peaks

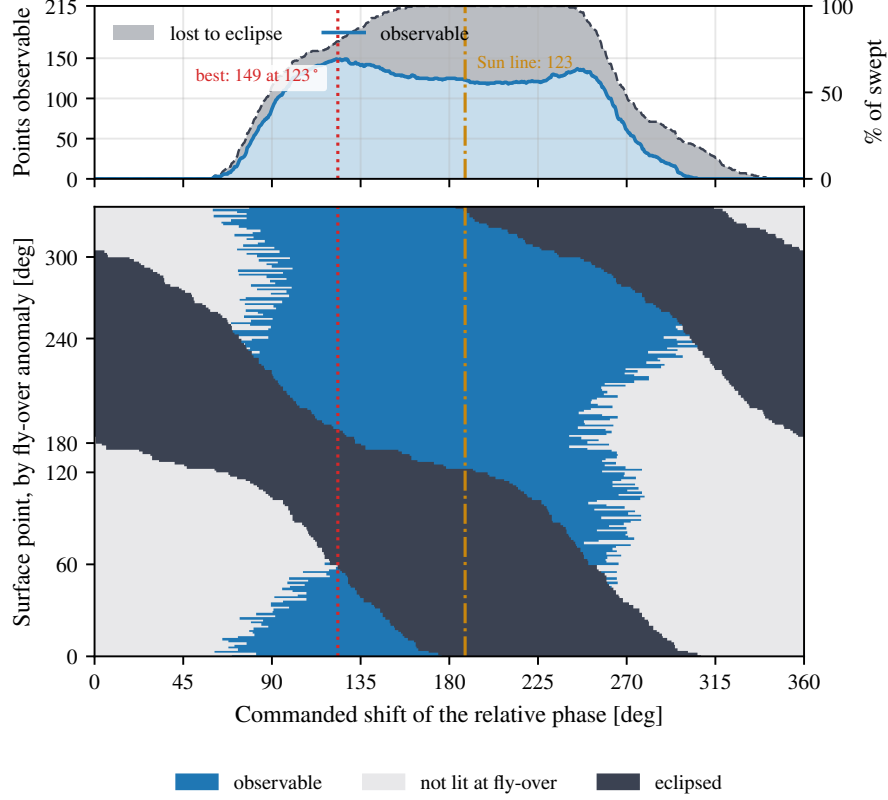


Fig. 7 Target points observability depending on the applied phase with respect to the Sun.

on either side of it, so that observing the largest number of points requires leading or lagging the Sun rather than following it exactly. Starting from a relative state different from the assigned PRO, the resulting strategy is therefore to reach it with the phase shift of 123° , which maximizes the immediate return, and to complete the coverage with a second reconfiguration at the shift that unlocks the remaining points. The same logic applies when the inspection requires a change of relative orbit: each new PRO is entered at the phase shift that maximizes the number of points observed from it.

5.4 Relaxed solution versus successive convexification

To assess how often the closed-form relaxation is sufficient, and the associated cost when it is not, a Monte Carlo campaign was run over two families of maneuvers: a rephasing on the same PRO, and a reconfiguration from the initial PRO to a different one entered at a prescribed phase. For each family the grid spans 5 values of y_r ($-6, -3, 0, 3, 6$ m) and 10 values of each of $A_z \in [0, 20]$ m, $\gamma \in [0, 2\pi)$, $E_r \in [0, 2\pi)$ and $\Delta\sigma \in [0, 2\pi)$. The relative orbit size a_r and the keep-out zone radius R are held fixed, and with therefore their ratio; a broader campaign would sample this ratio as well, but the value adopted here is the one representative of the proximity operations

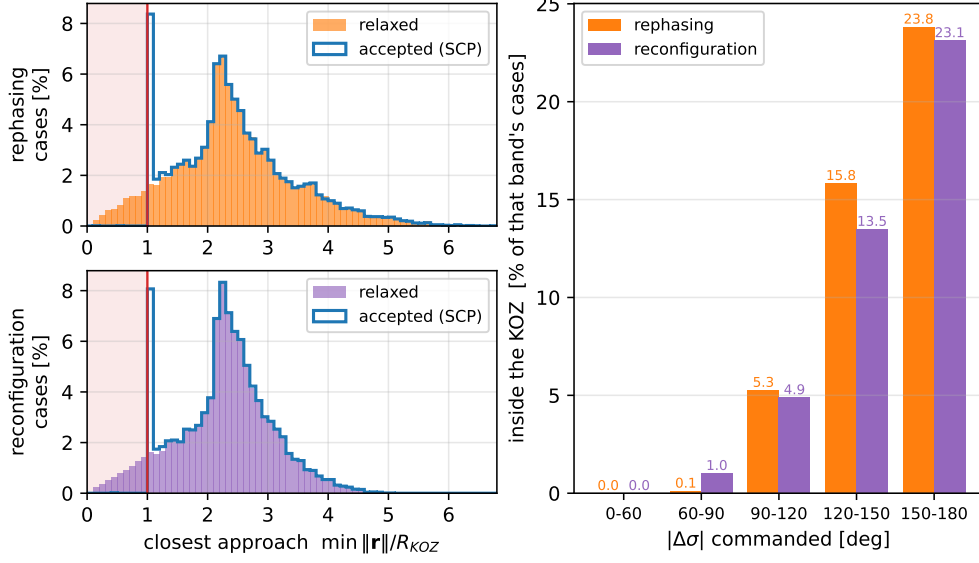


Fig. 8 Monte Carlo campaign over the rephasing and reconfiguration maneuvers. On the left, the distribution of the minimum distance to the target along the transfer, normalized by the KOZ radius, for the relaxed and the convexified solution. On the right, the fraction of cases in which the relaxed solution violates the KOZ constraint, per band of commanded phase shift; each bar is normalized by the number of cases in its own band.

scenario under study, and increasing the separation can only reduce the likelihood of activating the keep-out constraint.

The results of Figure 8 show that the rephasing and reconfiguration cases are statistically indistinguishable. In both families the minimum distance along the transfer peaks at $\min \|\mathbf{r}\|/R \approx 2.2$ for either solver. The relaxed solution enters the keep-out zone, and is therefore rejected, in 7.51% of the rephasing cases and 7.19% of the reconfiguration cases; these require the convexified problem to be solved by SCP, which produces the second peak just above unity, satisfying the constraint in nearly all of them. Only two cases are found in which the SCP solver fails to converge to a feasible trajectory. For the remaining 92.5% of the campaign the closed-form relaxation is already the global optimum and no convexification is needed. The rejected cases are moreover essentially independent of all sampled parameters except the magnitude of the commanded phase shift $|\Delta\sigma|$. The right panel of Figure 8 reports, for each band of $|\Delta\sigma|$, the fraction of the cases in that band for which the relaxed solution enters the keep-out zone. No violation occurs below 60° , and only a marginal fraction of cases is affected between 60° and 90° (0.1% for the rephasing and 1.0% for the reconfiguration). Beyond 90° the rate grows monotonically, reaching 23.8% and 23.1%, respectively, in the band between 150° and 180° .

6 Conclusions

In this paper we addressed the problem of phasing a chaser along a passive relative orbit so that the observation geometry it realises matches an illumination requirement, either alignment with the solar direction or maximum coverage of the target surface, for a target whose attitude is fixed with respect to the local orbital frame. The same regions of the surface are repeatedly flown over under the same illumination, so the information returned by an inspection saturates after the first revolution unless the phase along the relative orbit is actively changed. A rephasing maneuver is therefore needed to extend coverage while remaining on a bounded, and hence passively safe, relative orbit.

We identified an invariant quantity, formed by the relative phase and the solar phase, that fully characterizes the observation geometry of a given relative orbit, and every phasing requirement considered here reduces to the choice of a single commanded value for it. Three such requirements were derived on this basis. The first places the chaser on the illumination direction at the arrival instant. The second centres the phasing error over a revolution, and was shown to correspond to the same commanded value for every passive relative orbit, so that the residual oscillation of the pointing error is a property of the relative orbit alone and cannot be removed by any maneuver. The third is derived from the observability of the target surface: the illumination margin of each surface point turns out to be a simple harmonic function of the commanded phase shift, so that each point contributes an interval of admissible shifts, the eclipse constraint contributes a second one, and the optimal shift follows from the maximum weighted overlap of these intervals.

A guidance law was then formulated to realise the commanded phase shift, selecting the optimal arrival time and the corresponding minimum-energy input sequence. Because the commanded phase is tied to a rotating reference direction, the terminal state is not assigned in advance but is itself a function of the arrival time, so that choosing when to arrive and choosing where to arrive become the same decision, unlike most reconfiguration problems in which the transfer time only determines how a prescribed state is reached. The design problem reduces to a one-dimensional search over the maneuver duration, the cost of each candidate being evaluated in closed form; the same closed-form solution then provides the initial guess for the successive convexification when the keep-out constraint is active, and is retained as the optimal solution whenever it already satisfies the keep-out and thrust bounds.

We ran a Monte Carlo campaign to test the guidance law spanning both rephasing maneuvers on the same relative orbit and reconfigurations towards a different one. The closed-form solution was found to be admissible, and therefore optimal, in more than ninety per cent of the cases, with the remaining ones concentrated at the largest commanded shifts and entirely absent below a threshold. The inspection scenario showed that the phase aligned with the Sun observes appreciably fewer surface points than the optimal one, with coverage peaking on either side of the solar direction rather than on it, and a further group of points is recoverable only through a second maneuver because the target is in eclipse when they are flown over.

Some limitations remain. If the rationale for confining the chaser to passive relative orbits is to guarantee that a loss of control leaves it on a bounded trajectory, then

the rephasing strategy proposed here weakens that guarantee: the transfer arc is not itself a passive relative orbit, so the chaser temporarily leaves the bounded family and passive safety is suspended for the duration of the maneuver, being recovered only at arrival. The exposure is limited, since the maneuver occupies a small fraction of the mission and the observation phases on either side of it are fully safe, but it is not eliminated, and enforcing bounded motion throughout the transfer would require a different formulation of the terminal condition. Finally, the Monte Carlo campaign kept the ratio between the size of the relative orbit and the keep-out zone fixed; sampling it would characterize the regime in which the closed-form solution ceases to be sufficient.

Declarations

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