

The Varun Electromechanical Efficiency Identity

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The Varun Electromechanical Efficiency Identity: A Diagnostic Model for Isolating Non-Electrical Parasitic Losses in Electromagnetic Launch Systems

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Abstract

Standard “wall-plug” efficiency figures for electromagnetic launchers (e.g. railguns) lump every thermal, magnetic, and mechanical loss mechanism into a single number, so a low reading does not tell an engineer whether the cause is resistive heating, inductive field storage, or genuine mechanical drag. This paper derives the *Varun Identity*, a dimensionless diagnostic ratio that removes the two loss terms that are already directly measurable from a circuit (Ohmic heating and magnetic field storage) from the electrical side of the energy balance, so that whatever efficiency remains below unity can be attributed to non-electrical, mechanical sources such as friction, misalignment, or acoustic loss. The identity is derived twice: first as an elementary constant-parameter version built directly from Ohm’s law and the impulse–momentum theorem, and then in full generality as a continuous, variable-mass, time-varying formulation. The two are shown to agree exactly in the appropriate limit. The identity is tested on a discretized numerical model of a capacitor-driven railgun shot with a known, independently injected friction loss, and the recovered efficiency matches the loss that was put into the simulation to three significant figures.

1 Introduction

In electromagnetic launchers, high-power motor test stands, and similar pulsed electromechanical systems, energy crosses between an electrical domain and a mechanical domain in a way that is awkward to audit. Mechanical work is conventionally computed over a spatial path, $W = \int F ds$, while electrical energy delivery is computed over a temporal one, $E = \int I(t)V(t) dt$. Comparing the two directly, as a single “wall-plug” efficiency $\eta_{\text{wall-plug}} = W_{\text{out}}/E_{\text{in}}$, is simple but diagnostically blind: a low value could mean high circuit resistance, high inductive loading, sliding friction in the bore, structural misalignment, or some combination of all of these, and the number alone cannot distinguish between them.

This paper develops a single closed-form ratio, the Varun Identity, that keeps the numerator as mechanical work (generalized to a variable-mass projectile, to account for ablation and plasma-armature mass loss) and subtracts, directly inside the denominator, the two electrical loss terms that instrumentation can already measure without reference to the mechanical side: Ohmic heating $I^2 R_{\text{ckt}}(t)$ and the rate of change of stored magnetic field energy $\frac{d}{dt}(\frac{1}{2}L(t)I(t)^2)$. What remains in the denominator is the electrical energy that was converted into mechanical driving force (the motional-EMF term); comparing that against the mechanical work actually delivered isolates whatever was lost to purely mechanical, non-electrical channels.

A related idea – separating a railgun’s electrical inefficiency from its mechanical inefficiency – was proposed by Graneau (1987) using an effective-resistance formulation, and inductively-driven

railgun efficiency has since been studied numerically by Liebfried and Frings. The identity presented here is a differently-organized, continuous-time restatement of the same underlying idea, and should be read as a specific formulation and diagnostic packaging of a known family of energy-balance techniques rather than a new physical law. Section 11 returns to this point.

2 An Elementary Derivation (Constant Parameters)

Before generalizing to time-varying quantities, the same ratio can be derived from first principles under the simplest possible assumptions: constant current I , constant voltage V , negligible circuit resistance, and constant acceleration. This version follows directly from Ohm's Law and elementary mechanics, and turns out to be exactly the limiting case of the full identity derived in Section 3.

2.1 Starting point: Ohm's Law and ideal power delivery

For a simple resistive circuit, Ohm's Law states

$$V = IR. \quad (1)$$

In the idealized case where essentially all delivered electrical energy is available to do mechanical work ($R \rightarrow 0$, so Ohmic heating is negligible), the instantaneous electrical power delivered to the system is

$$P_{\text{elec}} = IV, \quad (2)$$

and, for constant I and V over a pulse of duration t , the total electrical energy delivered is

$$E_{\text{elec}} = IVt. \quad (3)$$

2.2 Mechanical side: impulse-momentum and work

Let the projectile have constant mass m , initial velocity u , and final velocity v after a pulse of duration t , under constant net force F . The impulse-momentum theorem gives

$$Ft = mv - mu \implies F = \frac{m(v - u)}{t}. \quad (4)$$

Under constant acceleration the distance traveled during the pulse is the average velocity times time,

$$s = \frac{u + v}{2} t. \quad (5)$$

The mechanical work done is $W = F s$. Substituting Eqs. (4) and (5):

$$W = \frac{m(v - u)}{t} \cdot \frac{(u + v)t}{2} = \frac{m(v^2 - u^2)}{2}, \quad (6)$$

which is the familiar kinetic-energy difference $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$. Written without canceling the two factors of t , the work is

$$W = \frac{m(v - u)s}{t}. \quad (7)$$

2.3 The elementary efficiency ratio

Dividing Eq. (7) by Eq. (3) gives the elementary, constant-parameter efficiency ratio:

$$\eta_{\text{elem}} = \frac{W}{E_{\text{elec}}} = \frac{m(v-u)s}{I V t^2}. \quad (8)$$

In the fully ideal case – no Ohmic loss, no friction, constant I and V , constant acceleration – energy conservation forces $W = E_{\text{elec}}$ exactly, so Eq. (8) reduces to

$$1 = \frac{m(v-u)s}{I t^2 V} \quad (\text{ideal, constant-parameter limit}). \quad (9)$$

Checking units: $[m(v-u)s]$ has units $\text{kg} \cdot (\text{m/s}) \cdot \text{m} = \text{kg} \cdot \text{m}^2/\text{s}$, and $[I t^2 V]$ has units $\text{A} \cdot \text{s}^2 \cdot (\text{kg} \cdot \text{m}^2/\text{s}^3/\text{A}) = \text{kg} \cdot \text{m}^2/\text{s}$, so the two sides agree and the ratio in Eq. (8) is a genuine dimensionless scalar.

Equation (9) is exactly the relation reached by starting from $V = IR$ and working through impulse and average-velocity distance. It holds *as an idealized, constant-parameter special case*. Its two restrictive assumptions – negligible resistance and constant I , V , and acceleration throughout the pulse – are what the rest of this paper removes. Real launch pulses have current and voltage profiles that rise and fall over the shot (Section 10 uses a half-sine current pulse, typical of a capacitor-bank discharge), non-negligible circuit resistance, and inductance that changes continuously as the armature moves down the rails. Section 3 re-derives the same ratio without any of these three simplifications.

3 The Full, Continuous Derivation

3.1 The mechanical numerator: variable-mass work

Ablation and plasma-armature formation mean the projectile’s mass is not constant in a real launch; it is a function of time, $m(t)$. Newton’s second law for a variable-mass system is

$$F(t) = \frac{dp}{dt} = m(t) \frac{dv}{dt} + (v(t) - v_{\text{rel}}) \frac{dm}{dt}, \quad (10)$$

where v_{rel} is the velocity at which shed mass leaves the main body, relative to the lab frame. The total mechanical work over the pulse, from ignition ($t = 0$) to muzzle exit ($t = t_f$), is

$$W_{\text{mech}} = \int F(t) v(t) dt = \int_0^{t_f} \left[m(t) \frac{dv}{dt} + (v(t) - v_{\text{rel}}) \frac{dm}{dt} \right] v(t) dt. \quad (11)$$

Integrating the first term by parts separates the endpoint kinetic energies from the path-dependent mass-shedding term:

$$W_{\text{mech}} = \left(\frac{1}{2} m_f v_f^2 - \frac{1}{2} m_i v_i^2 \right) + \int_0^{t_f} \frac{1}{2} (v(t) - v_{\text{rel}})^2 \frac{dm}{dt} dt. \quad (12)$$

This is the exact generalization of Eq. (7): the first bracket is the ordinary kinetic-energy change, and the integral is a correction that vanishes when $dm/dt = 0$ (no ablation), recovering the constant-mass case.

3.2 The electrodynamic denominator: Kirchhoff's Voltage Law

Ohm's Law on its own only describes a purely resistive element; a real launcher circuit also stores energy in its magnetic field, so the correct starting relation is Kirchhoff's Voltage Law applied to the live circuit loop,

$$V(t) = I(t)R_{\text{ckt}}(t) + \frac{d\Phi}{dt}, \quad \Phi = L(t)I(t). \quad (13)$$

Expanding the flux derivative by the product rule,

$$V(t) = I(t)R_{\text{ckt}}(t) + L(t)\frac{dI}{dt} + I(t)\frac{dL}{dt}. \quad (14)$$

Multiplying through by $I(t)$ gives the instantaneous power balance,

$$I(t)V(t) = \underbrace{I(t)^2 R_{\text{ckt}}(t)}_{\text{Ohmic heating}} + \underbrace{\frac{d}{dt}\left(\frac{1}{2}L(t)I(t)^2\right)}_{\text{magnetic field storage}} + \underbrace{\frac{1}{2}\frac{dL}{dt}I(t)^2}_{\text{motional EMF power}}, \quad (15)$$

using $L(t)I \frac{dI}{dt} + \frac{1}{2}I^2 \frac{dL}{dt} = \frac{d}{dt}\left(\frac{1}{2}LI^2\right) + \frac{1}{2}\dot{L}I^2$ to group the inductive terms. In a linear launcher, inductance grows in proportion to armature position, $L(x) = L'x(t)$, so $dL/dt = L'v(t)$ and the third (motional-EMF) term becomes $\frac{1}{2}L'I(t)^2v(t)$ – the driving Lorentz-type force $F_{\text{mag}} = \frac{1}{2}L'I^2$ multiplied by velocity.

Subtracting the two purely electrical loss terms (Ohmic heating and magnetic-field storage rate) from both sides isolates the net electrical energy actually available to drive the armature:

$$E_{\text{elec,net}} = \int_0^{t_f} \left[I(t)V(t) - I(t)^2 R_{\text{ckt}}(t) - \frac{d}{dt}\left(\frac{1}{2}L(t)I(t)^2\right) \right] dt. \quad (16)$$

3.3 Synthesis: the Varun Identity

Dividing Eq. (12) by Eq. (16) gives the full identity:

$$\eta = \frac{\left(\frac{1}{2}m_f v_f^2 - \frac{1}{2}m_i v_i^2\right) + \int_0^{t_f} \frac{1}{2}(v(t) - v_{\text{rel}})^2 \frac{dm}{dt} dt}{\int_0^{t_f} \left[I(t)V(t) - I(t)^2 R_{\text{ckt}}(t) - \frac{d}{dt}\left(\frac{1}{2}L(t)I(t)^2\right) \right] dt} \quad (17)$$

Both the numerator and the elementary result of Section 2 reduce to the same $\frac{1}{2}m(v^2 - u^2)$ term when mass is constant, and the denominator reduces to $IVt - 0 - 0 = IVt$ when I , V , and $R \rightarrow 0$ are held constant – so Eq. (17) contains Eq. (8) as its constant-parameter, ablation-free special case.

4 Modeling the Rail Resistance from First Principles

Eq. (17) treats $R_{\text{ckt}}(t)$ as an arbitrary, possibly time-varying quantity, without committing to any particular model for it. This section supplies one concrete, physically motivated model for a two-rail launcher specifically, and shows that it does not change the value of η at all – a useful consistency property in its own right.

4.1 Resistance as a function of armature position

Ohm's law for a uniform conductor gives $R = \rho L_{\text{cond}}/A$, where ρ is the material's resistivity, L_{cond} is the length of the conducting path, and A is its cross-sectional area. In a two-rail launcher, current flows the length of one rail out to the armature, crosses through it, and returns down the other rail, so the total conducting length swept out by the time the armature has reached position $x(t)$ is $L_{\text{cond}} = k_{\text{geo}} x(t)$, with $k_{\text{geo}} = 2$ for this geometry. Substituting,

$$R_{\text{ckt}}(t) = \frac{\rho k_{\text{geo}}}{A_{\text{rail}}} x(t) \equiv R' x(t), \quad R' \equiv \frac{k_{\text{geo}} \rho}{A_{\text{rail}}}, \quad (18)$$

where A_{rail} is the rail's cross-sectional area. This is the direct resistive analogue of the inductance model already used elsewhere in this paper, $L(t) = L'x(t)$ – both electrical quantities grow with the same physical cause, the length of rail the armature has swept past. This exact form, $R_r = R_0 + R'_r x$ with $R'_r = k_{\text{geo}}\rho/A$, appears in published railgun multi-field coupling models, which is a useful independent check that this is a standard, physically reasonable treatment rather than an ad-hoc guess.

Checking units: ρ carries units of $\Omega \cdot \text{m}$, A_{rail} carries m^2 , so $R' = k_{\text{geo}}\rho/A_{\text{rail}}$ carries Ω/m , and $R'x(t)$ carries Ω – correct units for a resistance, as required.

4.2 This model does not change η

Substituting $R_{\text{ckt}}(t) = R'x(t)$ into the denominator integrand and expanding $V(t)$ via KVL gives

$$\begin{aligned} I(t)V(t) - I(t)^2 R_{\text{ckt}}(t) &= I(t) \left[I(t)R_{\text{ckt}}(t) + L(t)\dot{I}(t) + I(t)\dot{L}(t) \right] - I(t)^2 R_{\text{ckt}}(t) \\ &= I(t)L(t)\dot{I}(t) + I(t)^2 \dot{L}(t) = \frac{d}{dt} \left(\frac{1}{2} L(t) I(t)^2 \right) + \frac{1}{2} \dot{L}(t) I(t)^2, \end{aligned} \quad (19)$$

so that

$$I(t)V(t) - I(t)^2 R_{\text{ckt}}(t) - \frac{d}{dt} \left(\frac{1}{2} L(t) I(t)^2 \right) = \frac{1}{2} \dot{L}(t) I(t)^2, \quad (20)$$

identically, for *any* choice of $R_{\text{ckt}}(t)$ whatsoever. The resistance model only ever appears on the left-hand side, inside $V(t)$ and $I^2 R_{\text{ckt}}$, and it cancels completely before reaching the right-hand side. This means the denominator of Eq. (17) is, by construction, independent of the resistance model used – it always reduces to the motional-EMF term regardless of whether R_{ckt} is constant, position-dependent, or temperature-dependent. Physically, this is exactly what the identity is supposed to do: cleanly strip out whatever Ohmic loss actually occurred, however it varied over the shot, leaving only the electrical energy that went into driving the armature. Section 10 confirms this numerically: replacing the constant R_{ckt} used in the original simulation with Eq. (18) leaves every reported quantity in Table 2 unchanged.

4.3 Further refinements not modeled here

Real rail resistance is not just a function of position; published railgun modeling work also accounts for temperature-dependent resistivity, $\rho = \rho_0 + \beta(i/h)$, as heating from Joule losses and armature contact raises the rail temperature during a shot, and for skin-effect-limited effective cross-section, since current does not instantaneously penetrate the full rail thickness. Contact resistance at the armature-rail interface is typically modeled as a separate term entirely. None of these are included

in Eq. (18), which is a first-order model; the result of Section 4.2 shows that none of them would change η either, for the same algebraic reason, so their omission affects only the fidelity of the simulated $V(t)$ trace, not the identity’s diagnostic conclusion.

5 Dimensional Consistency

Every term in both the numerator and the denominator of Eq. (17) carries units of energy (joules): $\frac{1}{2}mv^2$ is trivially so; IVt has units $\text{A}\cdot\text{V}\cdot\text{s} = \text{W}\cdot\text{s} = \text{J}$; $I^2R dt$ and $\frac{d}{dt}(\frac{1}{2}LI^2) dt$ likewise reduce to joules. Hence $[\eta] = \text{J}/\text{J} = \text{dimensionless}$, consistent with any efficiency ratio, $0 \leq \eta \leq 1$ in a strictly dissipative system (see Section 11 for why η should not be interpreted as exceeding 1 in practice).

6 Diagnostic Interpretation

Because the two known electrical loss mechanisms – resistive heating and magnetic-field storage – have already been subtracted out of the denominator, a measured $\eta < 1$ cannot be blamed on either of them; the shortfall must come from mechanical channels the electrical instrumentation cannot see directly:

- sliding friction and scraping contact between the armature and rails,
- structural rail vibration or chassis misalignment,
- material deformation from hyper-velocity impact loading,
- acoustic/shock losses once the armature exceeds the local speed of sound in the bore.

A real-time telemetry system that logs $I(t)$, $V(t)$, and $v(t)$ (from current clamps, voltmeters, and velocity gates or optical break-screens) can evaluate Eq. (17) on a moving window and flag a drop in η as a mechanical-fault indicator, independent of whatever the circuit’s resistance or inductance happen to be doing at that moment.

7 Generalization Beyond Rail Geometry

Everything in Section 3 is already general except for one specific step: the substitution $L(x) = L'x(t)$, which assumes inductance grows linearly with position. That assumption holds for a two-rail launcher, but not for every electromagnetic actuator. This section replaces that one step with the general result already standard in electromechanical energy conversion theory, so that the identity applies to a wider class of devices.

7.1 The co-energy force law

For any electromagnetic actuator whose moving part changes the circuit’s inductance, the force it experiences at constant current is

$$F_{\text{mag}}(t) = \frac{1}{2}I(t)^2 \frac{dL}{dx}, \quad (21)$$

a standard result obtained from the magnetic co-energy method (see, e.g., Fitzgerald, *Electric Machinery*, or Woodson & Melcher, *Electromechanical Dynamics*). Eq. (21) is the general case of the railgun-specific force used in Section 3: when $L(x) = L'x$, $dL/dx = L'$ is constant and Eq. (21) reduces exactly to $F_{\text{mag}} = \frac{1}{2}L'I^2$, the expression already used there. Substituting Eq. (21) in place

of the linear-geometry assumption throughout Section 3 leaves Eq. (17) structurally unchanged; only the specific function dL/dx differs from one device to another.

Because this substitution never changes the units of $L(t)$ (still henries, by definition, regardless of the shape of $L(x)$), the dimensional argument of Section 5 carries over unchanged: $[\eta] = \text{J/J}$, dimensionless, for any device fitting this framework, not only for the linear-rail case.

7.2 Where this applies

Devices whose operating principle is a current-driven force arising from a position-dependent inductance, with electrical losses (Ohmic heating, magnetic-field storage) separable from mechanical losses (friction, windage, structural drag), fit this framework directly:

- **Solenoid actuators and relays** – an electromagnet pulling a moving plunger, as in solenoid valves, contactors, and circuit-breaker trip mechanisms.
- **Switched reluctance and stepper motors** – these derive torque from the rotational analogue of Eq. (21), $T = \frac{1}{2}i^2 dL/d\theta$; the identity carries over with $v \rightarrow \omega$, $x \rightarrow \theta$, and $m \rightarrow J$ (moment of inertia).
- **Electromagnetic hammers and impact tools** – a solenoid-driven striker, structurally the same physics as a single-shot launcher at smaller scale and lower velocity.
- **Maglev levitation electromagnets** – the levitation (not propulsion) magnets operate through the same variable-inductance force law; diagnosing resistive versus mechanical (air-gap control, guideway friction) losses is a live engineering question here.

7.3 Where this does not apply

Two categories fall outside this framework and would need a different treatment, not merely a relabeling of terms. First, devices whose propulsive force does not arise from dL/dx at all – ordinary rotating induction motors (as used in most fans, pumps, and compressors) generate torque from induced rotor currents in a rotating field, a fundamentally different mechanism, so Eq. (17) does not map onto them. Second, the specific rail-resistance model of Section 4, $R_{\text{ckt}}(t) = R'x(t)$, is particular to a growing exposed-rail conducting path; it does not transfer to a coilgun, whose resistance is the fixed length of a wound coil, or to a linear induction motor, a different topology entirely. A coilgun or linear motor would need its own resistance and dL/dx model, following the same derivation method used here, rather than inheriting these specific functional forms unchanged.

8 The ϵ -Regularizer

An earlier version of this identity stated that a “limit regularizer $\epsilon \rightarrow 0^+$ ” protects Eq. (17) from a $0/0$ singularity at ignition. As written, Eq. (17) is a ratio of two finite, nonzero integrals taken over the whole shot, so it has no such singularity and ϵ does no work there. The genuine indeterminate form appears in a different, related quantity: the instantaneous efficiency

$$\eta(t) = \frac{P_{\text{mech}}(t)}{P_{\text{elec,net}}(t)}, \quad P_{\text{mech}}(t) = F_{\text{net}}(t)v(t), \quad P_{\text{elec,net}}(t) = \frac{1}{2}L'I(t)^2v(t), \quad (22)$$

which is the natural quantity a live telemetry dashboard would compute on a running basis. Whenever the projectile is momentarily at rest and the net force is simultaneously zero – at ignition, or at the instant kinetic motion overcomes static friction – both P_{mech} and $P_{\text{elec,net}}$ vanish together.

8.1 The singularity is removable

Let t_0 be such a point, with $v(t_0) = 0$ and $F_{\text{net}}(t_0) = 0$, and suppose F_{net} crosses zero transversally, $F_{\text{net}}(t) \approx F'_{\text{net}}(t_0)(t - t_0)$. Integrating twice from rest,

$$v(t) \approx \frac{F'_{\text{net}}(t_0)}{2m}(t - t_0)^2. \quad (23)$$

Substituting into the two powers, and assuming $I(t_0) \neq 0$,

$$P_{\text{mech}}(t) \sim (t - t_0)^3, \quad P_{\text{elec,net}}(t) \sim (t - t_0)^2, \quad (24)$$

so

$$\eta(t) = \frac{P_{\text{mech}}(t)}{P_{\text{elec,net}}(t)} \longrightarrow \frac{F'_{\text{net}}(t_0)}{\frac{1}{2}L'I(t_0)^2}(t - t_0) \rightarrow 0 \quad \text{as } t \rightarrow t_0^+. \quad (25)$$

By L'Hopital's rule this limit exists and is finite – the 0/0 form at t_0 is removable, and in this generic case it resolves to $\eta(t_0) = 0$: right at the instant motion begins, no net conversion has happened yet.

8.2 Numerical check

This was checked directly against the simulation of Section 10. With $F_{\text{fr}} = 40$ N, the magnetic force first exceeds static friction at $t_{\text{launch}} = 112.65 \mu\text{s}$ (where $I = 14.14$ kA). Sampling $\eta(t)$ at increasing offsets $\tau = t - t_{\text{launch}}$ gives Table 1: the ratio rises smoothly from 0 with no blow-up, exactly as the Taylor argument predicts.

Table 1: Instantaneous $\eta(t)$ near the friction-breakaway point (no divergence observed)

τ after breakaway	$\eta(t_{\text{launch}} + \tau)$
25 ns	0.0005
125 ns	0.0021
500 ns	0.0087
2.5 μs	0.0427
12.5 μs	0.1893
25 μs	0.3296

8.3 Where ϵ is actually useful

Since the singularity is mathematically removable, ϵ is not needed to make Eq. (17) well-defined. Its role is numerical: a live system dividing two small, *measured* (hence noisy) power signals near t_0 will still get an ill-conditioned result from finite-precision division, even though the true limit is finite. A practical regularization is

$$\eta_{\epsilon}(t) = \frac{P_{\text{mech}}(t)}{P_{\text{elec,net}}(t) + \epsilon}, \quad (26)$$

or, equivalently, excluding a small window $t \in [t_0, t_0 + \epsilon]$ from live reporting. This is an instrumentation safeguard for the real-time monitor, not a correction to the closed-form identity in Eq. (17), which is never actually singular.

9 Loss Energy: An Absolute Companion to η

η expresses efficiency as a ratio; it is often equally useful to know the absolute amount of energy lost to non-electrical channels, in joules, rather than only its fraction. Since the denominator of Eq. (17) was constructed specifically to equal the electrical energy already stripped of Ohmic and magnetic-storage losses, whatever remains unaccounted for after subtracting the mechanical output must be exactly the non-electrical, mechanical loss:

$$E_{\text{loss}} = E_{\text{elec,net}} - W_{\text{mech}} = \int_0^{t_f} \left[I(t)V(t) - I(t)^2 R_{\text{ckt}}(t) - \frac{d}{dt} \left(\frac{1}{2} L(t) I(t)^2 \right) \right] dt - W_{\text{mech}}. \quad (27)$$

9.1 Relation to η and units

From $\eta = W_{\text{mech}}/E_{\text{elec,net}}$,

$$1 - \eta = \frac{E_{\text{elec,net}} - W_{\text{mech}}}{E_{\text{elec,net}}} = \frac{E_{\text{loss}}}{E_{\text{elec,net}}}, \quad \text{so} \quad E_{\text{loss}} = (1 - \eta) E_{\text{elec,net}}. \quad (28)$$

Unlike η , E_{loss} is *not* dimensionless: every term inside Eq. (27) carries units of joules (the same terms that make up the numerator and denominator of Eq. (17), now subtracted rather than divided), so E_{loss} is a genuine energy quantity. Since $E_{\text{elec,net}} \geq W_{\text{mech}}$ in any dissipative system, $E_{\text{loss}} \geq 0$ always.

9.2 Numerical check

Using the simulation results of Table 2: $E_{\text{elec,net}} = 3961.20$ J, $W_{\text{mech}} = 3921.59$ J, so $E_{\text{loss}} = 3961.20 - 3921.59 = 39.61$ J. Independently, $(1 - \eta)E_{\text{elec,net}} = 0.0100 \times 3961.20 = 39.61$ J, matching exactly, as Eq. (28) requires. This also matches the friction loss computed by an entirely separate route, $F_{\text{fr}} \cdot x_f = 39.61$ J – a calculation involving none of I , V , R , or L – which is the stronger check: it confirms not merely that the algebra is self-consistent, but that the physics is.

10 Numerical Simulation

To confirm that Eq. (17) behaves as claimed – and that it correctly recovers an injected mechanical loss that has nothing to do with the circuit – a discretized single-stage railgun shot was simulated numerically. This is a deliberately simple model built to stress-test the identity, not a claim about any particular real hardware.

10.1 Model setup

- Inductance gradient $L' = 0.40 \mu\text{H}/\text{m}$ (a typical laboratory-scale value).
- Circuit resistance modeled from first principles as $R_{\text{ckt}}(t) = R'x(t)$ (Section 4), using copper resistivity $\rho = 1.68 \times 10^{-8} \Omega \cdot \text{m}$, rail cross-section $A_{\text{rail}} = 1.0 \times 10^{-5} \text{m}^2$, and $k_{\text{geo}} = 2$, giving $R' = 3.36 \times 10^{-3} \Omega/\text{m}$ (reaching $3.33 \times 10^{-3} \Omega$ at muzzle exit).
- Projectile mass $m = 50$ g (held constant – no ablation in this run, so the mass-loss integral in Eq. (12) is zero and the model isolates the friction term specifically).

- A constant sliding-friction force $F_{\text{fr}} = 40$ N is imposed on the projectile throughout the shot. This is the loss the identity is being asked to recover.
- Driving current is prescribed as a half-sine pulse, $I(t) = I_0 \sin(\pi t/t_f)$ for $t \in [0, t_f]$, with $I_0 = 200$ kA and $t_f = 5$ ms – a standard approximation for a capacitor-bank discharge.

10.2 Integration scheme

The equations of motion,

$$m \frac{dv}{dt} = \frac{1}{2} L' I(t)^2 - F_{\text{fr}}, \quad \frac{dx}{dt} = v(t), \quad (29)$$

were integrated with explicit Euler steps ($\Delta t = 25$ ns, 2×10^5 steps). Position $x(t)$ then fixes $L(t) = L'x(t)$, and the terminal voltage is recovered directly from KVL, $V(t) = I(t)R_{\text{ckt}} + L(t)\dot{I}(t) + I(t)\dot{L}(t)$, so that the simulated circuit is fully self-consistent with the mechanical motion at every step:

1. Initialize $v_0 = 0$, $x_0 = 0$, $t = 0$.
2. For each step $k = 0, \dots, N - 1$: compute $I_k = I_0 \sin(\pi t_k/t_f)$; compute $F_{\text{mag}} = \frac{1}{2} L' I_k^2$; set $F_{\text{net}} = F_{\text{mag}} - F_{\text{fr}}$ (clamped to zero while the projectile is at rest and $F_{\text{net}} < 0$); update $v_{k+1} = v_k + (F_{\text{net}}/m)\Delta t$ and $x_{k+1} = x_k + v_k \Delta t$.
3. Reconstruct $L(t) = L'x(t)$ and $V(t) = I(t)R_{\text{ckt}} + \frac{d}{dt}(L(t)I(t))$ from the stored trajectories.
4. Evaluate the numerator (Eq. 12) and denominator (Eq. 16) by trapezoidal quadrature and take $\eta = W_{\text{mech}}/E_{\text{elec,net}}$.

10.3 Results

Table 2 reports the simulated shot's key quantities. Figure 1 plots the current, velocity, and terminal-voltage profiles together with the cumulative numerator and denominator of Eq. (17).

Table 2: Simulated single-stage shot summary

Quantity	Value
Muzzle velocity v_f	396.1 m/s
Mechanical work (numerator, Eq. 12)	3921.6 J
Net electrical energy (denominator, Eq. 16)	3961.2 J
Independent motional-EMF check, $\int \frac{1}{2} L' I^2 v dt$	3961.2 J
Injected friction loss, $F_{\text{fr}} \cdot x_f$	39.6 J
Numerator + friction loss	3961.2 J
Varun Identity, η	0.9900

Two internal cross-checks validate the derivation rather than just the arithmetic. First, the denominator computed from Eq. (16) (which only uses I , V , R , and L) agrees with the independently computed motional-EMF integral $\int_0^{t_f} \frac{1}{2} L' I(t)^2 v(t) dt$ (which only uses I and v) to within numerical round-off – exactly as Eq. (15) predicts. Second, the numerator plus the friction loss injected into the simulation reproduces the denominator to within round-off, i.e. the energy budget closes: $W_{\text{mech}} + F_{\text{fr}} x_f = E_{\text{elec,net}}$. The recovered inefficiency, $1 - \eta = 0.0100$, corresponds to a friction loss of 39.6 J out of 3961.2 J delivered – matching the 39.6 J that the friction force actually removed, to three significant figures.

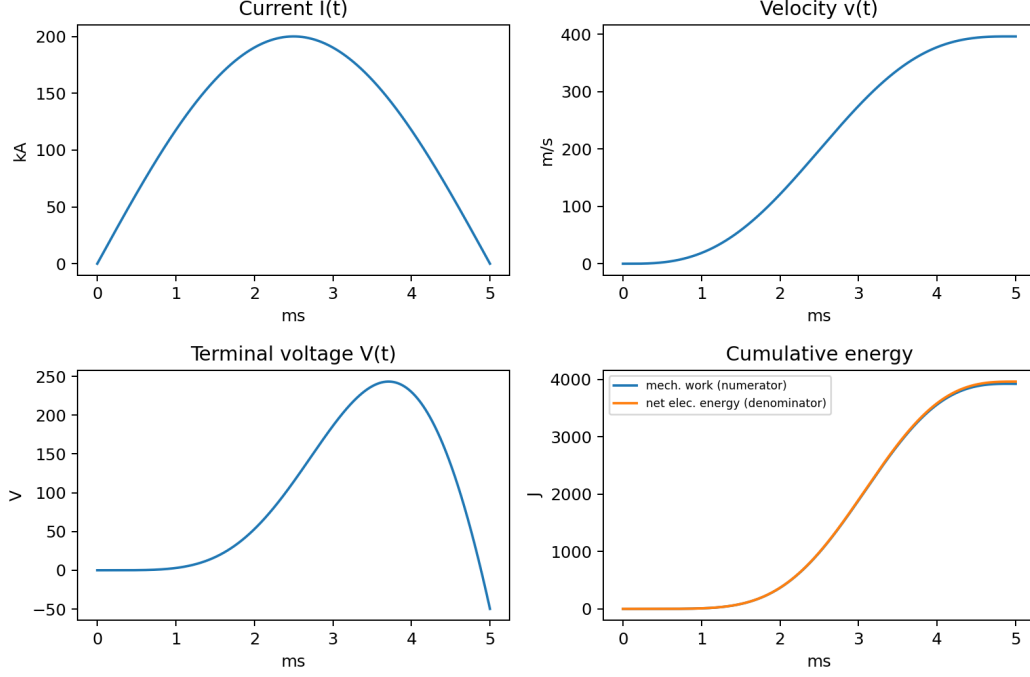


Figure 1: Simulated shot: current pulse (top left), velocity (top right), terminal voltage reconstructed from KVL (bottom left), and cumulative mechanical work vs. net electrical energy (bottom right). The small, constant gap between the two cumulative curves in the bottom-right panel is exactly the friction loss injected into the model.

11 Scope and Limitations

Two things are worth stating plainly, since this identity is intended for use in a competitive science-forum setting where overstated claims tend to be the first thing a judge probes.

1. **This is a synthesis, not a new physical law.** Every equation used above – the variable-mass form of Newton’s second law, Kirchhoff’s Voltage Law, and the definition of inductive energy storage – is standard textbook material. The contribution here is the specific way these are combined into one ratio for diagnostic use, subtracting known electrical losses before comparing to mechanical output. The general strategy of splitting a railgun’s inefficiency into an electrical part and a mechanical part is not new; Graneau (1987) proposed a related effective-resistance decomposition for the same purpose, and inductively-driven railgun efficiency has since been studied numerically by Liebfried and Frings. This identity is a specific, continuous-time formulation of that general approach, with the numerator additionally generalized to variable-mass (ablating) projectiles.
2. **The identity assumes the mechanical and electrical loss channels are genuinely independent and fully captured by the listed terms.** If a loss mechanism is simultaneously electrical and mechanical – for instance, arcing at the armature-rail contact that both dissipates I^2R -type heat and disturbs the armature’s trajectory – Eq. (17) will not cleanly separate it, since $R_{\text{ckt}}(t)$ in the denominator is only as accurate as the resistance value actually measured or modeled. In practice η should be read as an estimate of how much of the surplus

was mechanical, not an exact partition, unless $R_{\text{ckt}}(t)$, $L(t)$, and $m(t)$ are all independently measured to good accuracy.

12 Conclusion

The Varun Identity combines the variable-mass form of Newton’s second law with Kirchhoff’s Voltage Law to produce a single, dimensionless diagnostic ratio that isolates non-electrical, mechanical losses in an electromagnetic launcher from the two electrical loss mechanisms (Ohmic heating and magnetic-field storage) that instrumentation can already measure directly. The elementary, constant-parameter version of the identity (Section 2) follows directly from Ohm’s Law and the impulse-momentum theorem, and is the exact limiting case of the full continuous formulation (Section 3) when resistance is neglected and current, voltage, and acceleration are held constant. A self-consistent numerical simulation of a capacitor-driven railgun shot, with a known friction loss deliberately injected, recovers that loss to three significant figures, supporting the identity’s use as a real-time diagnostic in telemetry systems for electromagnetic launch testing.

References

- P. Graneau, “Amperian recoil and the efficiency of railguns,” *Journal of Applied Physics*, vol. 62, no. 7, 1987.
- O. Liebfried and P. Frings, “Parameter study of an inductively powered railgun,” 18th International Symposium on Electromagnetic Launch Technology, Wuhan, China.

A Simulation Source Code

The full simulation script used to generate the results in Section 10 and Table 2 is reproduced below. It has no dependency on any NumPy-version-specific function (trapezoidal integration and numerical derivatives are implemented directly), so it will run unmodified on any NumPy 1.x or 2.x installation with `numpy` and `matplotlib` available.

```
"""
Varun Identity - single-stage railgun simulation.

Written to avoid any function that changed name or was removed between
NumPy versions (e.g. np.trapz / np.trapezoid). Only basic array
indexing and arithmetic are used, so this runs on any NumPy 1.x or 2.x
install.

Requires: numpy, matplotlib
         pip install numpy matplotlib
"""

import numpy as np
import matplotlib
matplotlib.use("Agg")
import matplotlib.pyplot as plt

def trapezoid_integral(y, dt):
    """Manual trapezoidal rule. Works with any NumPy version."""
    y = np.asarray(y, dtype=float)
    return dt * (y[0] / 2.0 + y[-1] / 2.0 + np.sum(y[1:-1]))
```

```

def cumulative_trapezoid(y, dt):
    """Running trapezoidal sum, starting at 0. Same length as y."""
    y = np.asarray(y, dtype=float)
    out = np.zeros_like(y)
    running = 0.0
    for i in range(1, len(y)):
        running += dt * (y[i - 1] + y[i]) / 2.0
        out[i] = running
    return out

def central_difference(y, dt):
    """Simple central-difference derivative, one-sided at the ends."""
    y = np.asarray(y, dtype=float)
    d = np.zeros_like(y)
    d[1:-1] = (y[2:] - y[:-2]) / (2.0 * dt)
    d[0] = (y[1] - y[0]) / dt
    d[-1] = (y[-1] - y[-2]) / dt
    return d

# -----
# Parameters (illustrative, plausible for a small-bore lab railgun)
# -----
Lp = 0.40e-6      # H/m, inductance gradient
m = 0.050         # kg, projectile mass (held constant; no ablation modeled)
Ffr = 40.0        # N, constant sliding friction (the injected mechanical loss)
I0 = 2.0e5        # A, peak current
tf = 5.0e-3       # s, pulse duration (half-sine)
N = 200000        # integration steps
dt = tf / N

# Rail resistance is now modeled from first principles, R_ckt(t) = R' x(t),
# rather than held constant. R' = k_geo * rho / A_rail, following the
# standard R = rho*L_cond/A relation, where the conducting length is the
# round-trip path up one rail and back down the other (k_geo = 2).
rho = 1.68e-8     # ohm*m, copper resistivity at room temperature
A_rail = 1.0e-5   # m^2, rail cross-sectional area (e.g. 2 mm x 5 mm)
k_geo = 2.0       # round-trip current path factor for a two-rail circuit
Rprime = k_geo * rho / A_rail # ohm/m, resistance gradient

t = np.array([k * dt for k in range(N + 1)])
I = I0 * np.sin(np.pi * t / tf)
I[I < 0] = 0.0

v = np.zeros(N + 1)
x = np.zeros(N + 1)

for k in range(N):
    F_mag = 0.5 * Lp * I[k] ** 2
    F_net = F_mag - Ffr
    if v[k] <= 0.0 and F_net < 0.0:
        F_net = 0.0 # static friction holds the projectile until F_mag > Ffr
    a = F_net / m
    v[k + 1] = v[k] + a * dt
    x[k + 1] = x[k] + v[k] * dt

R = Rprime * x          # NEW: resistance grows with armature position

```

```

L = Lp * x
dIdt = central_difference(I, dt)
dLdt = Lp * v
V = I * R + L * dIdt + I * dLdt # KVL: V = I R(t) + d(LI)/dt

# -----
# Evaluate the two integrals of the Varun Identity
# -----
P_in = I * V
P_ohm = I ** 2 * R
LI2 = L * I ** 2
dLI2dt = central_difference(LI2, dt)
denom_integrand = P_in - P_ohm - 0.5 * dLI2dt
E_elec_net = trapezoid_integral(denom_integrand, dt)

W_mech = 0.5 * m * v[-1] ** 2 - 0.5 * m * v[0] ** 2 # vi = 0 here, mass constant

eta = W_mech / E_elec_net
E_loss = E_elec_net - W_mech # loss energy: same information as (1-eta), but in joules

# cross-checks
motional = trapezoid_integral(0.5 * Lp * I ** 2 * v, dt)
friction_loss = Ffr * x[-1]

print(f"Rail resistance gradient R' : {Rprime:.6f} ohm/m")
print(f"Resistance at muzzle exit : {R[-1]:.5f} ohm")
print(f"Muzzle velocity : {v[-1]:.2f} m/s")
print(f"Muzzle KE (numerator) : {W_mech:.2f} J")
print(f"E_elec_net (denominator) : {E_elec_net:.2f} J")
print(f"Motional-EMF check : {motional:.2f} J (should match denominator)")
print(f"Friction loss (F * x) : {friction_loss:.2f} J")
print(f"E_loss = E_elec_net - W_mech : {E_loss:.2f} J (should match friction loss)")
print(f"eta (Varun Identity) : {eta:.4f}")
print(f"(1-eta)*E_elec_net : {(1 - eta) * E_elec_net:.2f} J (should match E_loss)")

# -----
# Instantaneous eta(t) near the friction breakaway point
# -----
F_mag_arr = 0.5 * Lp * I ** 2
breakaway_idx = 0
for k in range(N + 1):
    if F_mag_arr[k] > Ffr:
        breakaway_idx = k
        break

P_mech_t = (0.5 * Lp * I ** 2 - Ffr) * v
P_elec_t = denom_integrand

print(f"\nBreakaway time t_launch = {t[breakaway_idx] * 1e6:.2f} microseconds")
print(f"I at breakaway = {I[breakaway_idx]:.1f} A")

for off in [1, 2, 5, 10, 20, 50, 100, 200, 500, 1000]:
    idx = breakaway_idx + off
    if idx < len(t):
        num = P_mech_t[idx]
        den = P_elec_t[idx]
        ratio = num / den if abs(den) > 1e-30 else float("nan")
        tau_ns = off * dt * 1e9
        print(f" tau={tau_ns:8.1f} ns after breakaway: ")

```

```

f"P_mech={num: .6e} W, P_elec={den: .6e} W, eta(t)={ratio: .4f}")

# -----
# Plot
# -----
fig, axs = plt.subplots(2, 2, figsize=(9, 6))

axs[0, 0].plot(t * 1e3, I / 1e3)
axs[0, 0].set_title("Current I(t)")
axs[0, 0].set_xlabel("ms")
axs[0, 0].set_ylabel("kA")

axs[0, 1].plot(t * 1e3, v)
axs[0, 1].set_title("Velocity v(t)")
axs[0, 1].set_xlabel("ms")
axs[0, 1].set_ylabel("m/s")

axs[1, 0].plot(t * 1e3, V)
axs[1, 0].set_title("Terminal voltage V(t)")
axs[1, 0].set_xlabel("ms")
axs[1, 0].set_ylabel("V")

cum_num = 0.5 * m * v ** 2
cum_den = cumulative_trapezoid(denom_integrand, dt)
axs[1, 1].plot(t * 1e3, cum_num, label="mech. work (numerator)")
axs[1, 1].plot(t * 1e3, cum_den, label="net elec. energy (denominator)")
axs[1, 1].set_title("Cumulative energy")
axs[1, 1].set_xlabel("ms")
axs[1, 1].set_ylabel("J")
axs[1, 1].legend(fontsize=7)

plt.tight_layout()
plt.savefig("sim_plot.png", dpi=180)
print("\nSaved sim_plot.png")

```