

Territorial heterogeneity and educational efficiency in the Brazilian Amazon: a regional dynamic and spatial analysis

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Abstract

This study examines whether municipal educational efficiency in the Brazilian Amazon exhibits distinct static, dynamic, and spatial regional patterns. Using municipal data for Pará from 2012 to 2020, we combine an output-oriented BCC data envelopment analysis, the Malmquist productivity index, and univariate and bivariate spatial autocorrelation statistics. Enrollment is interpreted as the realized territorial reach of the municipal school network and is assessed jointly with student-progression indicators. The results reveal substantial territorial heterogeneity. Carajás records the highest average static efficiency but the weakest regional productivity trajectory, whereas Xingu combines relatively low static efficiency with the only regional total factor productivity point estimate above unity. Global Moran statistics provide no evidence that technical efficiency or technological change forms systematic spatial clusters across the twelve Integration Regions. These findings show that static resource conversion, intertemporal productivity, and spatial dependence capture different dimensions of regional public-service performance. They also indicate that administrative proximity does not necessarily define a common educational-performance environment, supporting territorially differentiated analysis in geographically dispersed education systems.

Keywords: regional efficiency; educational services; spatial dependence; Malmquist productivity; data envelopment analysis; Brazilian Amazon

Highlights

- Static efficiency and productivity dynamics reveal different regional patterns.
- Territorial heterogeneity persists across Pará's Integration Regions.
- Geographic contiguity does not imply common educational-efficiency trajectories.
- Enrollment is modeled as the realized territorial reach of educational provision.

JEL classification: C61; H75; I22; R12; R58

1. Introduction

Regional disparities in public-service provision arise not only from differences in resources but also from the spatial conditions under which services are delivered. Distance, settlement patterns, transport networks, and peripheral location can alter the effective reach and cost of local government activity (Kopczewska 2013). Education is especially sensitive to these conditions because access depends on the spatial distribution of schools and students; geographical accessibility can therefore become an important source of unequal educational opportunity (Bruno et al. 2022). The regional question is consequently not simply how much municipalities spend, but how effectively resources are converted into accessible educational services across heterogeneous territories.

This problem is particularly salient in the Brazilian Amazon. Pará combines very large municipal territories, dispersed rural and Indigenous settlements, river-dependent mobility, and pronounced inequalities in access to public services. Such conditions create a form of territorial peripherality in which geographic constraints interact with institutional and infrastructural differences (Barbero and Rodríguez-Crespo 2022). Municipal school systems must therefore organize finance, staffing, transport,

and facilities while ensuring that students in remote communities can enter and remain in elementary education.

Educational-efficiency studies show that higher expenditure does not automatically generate stronger outcomes and that comparable local governments may occupy very different positions relative to an efficiency frontier (Worthington 2001; De Witte and López-Torres 2017; Povedano et al. 2021). Brazilian evidence likewise points to substantial municipal heterogeneity (Savian and Bezerra 2013; Xavier and Silva 2020). Yet static efficiency alone does not show whether productivity is improving over time, while statewide or regional averages can conceal whether neighboring territories share common performance dynamics. The resulting evidence gap concerns whether static resource conversion, intertemporal productivity, and spatial dependence describe the same or different regional patterns.

We address this gap using municipal elementary education in Pará from 2012 to 2020. Municipalities are the production units, while the state's twelve Integration Regions provide the territorial framework for interpreting aggregated patterns. An output-oriented BCC model measures contemporaneous technical efficiency; the Malmquist index separates productivity change into efficiency catch-up and frontier change; and univariate and bivariate Moran statistics assess whether regional efficiency and technological-change patterns are spatially associated. This design treats static efficiency, dynamic productivity, and geographic dependence as complementary rather than interchangeable dimensions of regional performance.

A distinctive feature of the production model is the treatment of municipal enrollment as an output. In conventional school-production models, the enrolled student population is often treated as an input or scale variable. In geographically dispersed systems, however, enrollment can also represent the realized territorial reach of the public school network: maintaining access may require river transport, dispersed facilities, small schools, and additional coordination. Enrollment is therefore evaluated jointly with approval and inverse age-grade distortion so that coverage is not confused with educational quality or progression.

This interpretation is consistent with studies that treat student numbers or enrollment rates as outputs when public authorities are responsible for extending educational access (Bogetoft et al. 2015; Melo-Becerra et al. 2020; Dutta 2012). It is also substantively consistent with the Amazonian context, where transport and infrastructure can condition students' effective access to schooling (Carmo et al. 2020; Ferreira et al. 2023). The production specification therefore connects efficiency measurement to a regional-science concern with accessibility and territorial service reach.

The paper asks whether the territorial geography of educational efficiency is stable across analytical perspectives. Specifically, do regions that are relatively close to the static efficiency frontier also display stronger productivity trajectories, and do neighboring regions exhibit similar efficiency or technological-change patterns? By addressing these questions, the study contributes to regional science in three ways: it documents territorial heterogeneity in a geographically constrained public service; it distinguishes static frontier position from productivity change; and it tests whether administrative planning regions correspond to coherent spatial patterns of educational performance.

2. Regional educational provision and spatial heterogeneity

Regional analysis of educational provision requires linking three literatures that are often treated separately: territorial accessibility to public services, frontier-based educational efficiency, and spatial patterns of productivity. Together they clarify why the same fiscal input may produce different service

outcomes across territories and why administrative regions need not coincide with functional performance regions.

2.1. Territorial accessibility and public-service provision

Public services are produced within spatially differentiated environments. Local governments operating in peripheral or weakly connected territories may face higher coordination and access costs, while users experience different effective distances to public facilities (Kopczewska 2013). In education, the spatial distribution of schools and students matters directly for service accessibility; regional disparities in school access can therefore generate inequalities even before learning outcomes are considered (Bruno et al. 2022).

This perspective is important for the Amazon because territorial reach can be resource intensive. River routes, dispersed settlement, small schools, and limited infrastructure may raise the cost of maintaining access without necessarily indicating poor management. Regional disadvantage should therefore be interpreted as multidimensional: geographical and institutional forms of peripherality can overlap and shape the conditions under which public services are delivered (Barbero and Rodríguez-Crespo 2022).

2.2. Educational efficiency and regional heterogeneity

Data envelopment analysis (DEA) is well suited to education because it accommodates multiple inputs and outputs without imposing a predefined production function (Worthington 2001; Johnes 2006; De Witte and López-Torres 2017). However, efficiency scores are relative to the selected variables, comparison set, and operating environment. They should therefore be interpreted as measures of resource conversion rather than comprehensive measures of educational quality.

Brazilian applications consistently reveal marked heterogeneity in municipal educational efficiency (Savian and Bezerra 2013; Povedano et al. 2021; Xavier and Silva 2020). This heterogeneity is especially relevant in Pará, where differences in accessibility, settlement structure, and teacher availability coexist with a common national financing and regulatory framework. The regional-science issue is not only which municipalities are closer to the frontier, but whether these efficiency differences form persistent territorial patterns.

2.3. Dynamic productivity and spatial dependence

Static efficiency and productivity change answer different questions. A municipality may be close to the contemporaneous frontier while experiencing declining productivity, or remain relatively inefficient while improving over time. Regional productivity research similarly emphasizes that productivity trajectories are heterogeneous across space and can evolve differently across territories (Aresu et al. 2026; Gómez-Tello et al. 2025). The Malmquist framework is therefore useful for separating catch-up from movement of the frontier itself.

A spatial perspective adds a further distinction. Neighboring regions may share infrastructure, labor markets, transport systems, or institutional practices, but adjacency does not guarantee similar performance. Spatial autocorrelation analysis allows this proposition to be tested directly rather than assumed. For Pará, this is particularly relevant because the Integration Regions are administrative planning territories; whether they also constitute coherent educational-efficiency regions is an empirical question.

3. Data and methods

Municipalities constitute the primary decision-making units (DMUs) throughout the efficiency and productivity analyses. The Integration Regions are not treated as DMUs; rather, municipal results are

aggregated at the regional level to support territorial comparison and spatial analysis. Three complementary empirical perspectives are combined. First, an output-oriented BCC model measures annual pure technical efficiency under variable returns to scale. Second, the Malmquist index evaluates intertemporal productivity change and separates efficiency change from frontier change. Third, univariate and bivariate Moran statistics examine whether the regional patterns of technical efficiency and technological change are spatially associated across Pará's Integration Regions. Together, these procedures distinguish contemporaneous resource conversion, intertemporal productivity dynamics, and geographic dependence rather than compressing the assessment into a single efficiency ranking.

3.1. Data and variables

The state of Pará comprises 144 municipalities. The effective static DEA sample contains 104 municipalities after excluding units with insufficient information for the selected variables. The analysis covers 2012–2020. Educational data were obtained from INEP, including School Census indicators, while fiscal information was obtained from public transparency sources and demographic information from IBGE (Instituto Nacional de Estudos e Pesquisas Educacionais Anísio Teixeira (INEP), 2023; Instituto Brasileiro de Geografia e Estatística (IBGE), 2023). Regional summaries are presented for the twelve Integration Regions of Pará. The dynamic Malmquist analysis uses the subset of municipalities with valid adjacent-year observations, yielding 95 municipalities and 749 usable transitions.

The regional structure follows the configuration applicable to the study period. Pará's twelve Integration Regions were established as territorial units for regional planning under Decree 1,066/2008 (Pará 2008). Because Oeiras do Pará was transferred from Tocantins to Marajó only in January 2022, it is retained in Tocantins for the 2012–2020 analysis (Pará 2022). Regional contiguity was checked against official FAPESPA cartography of the Integration Regions, which is based on the state's municipal territorial structure (FAPESPA 2024).

The DEA input is FUNDEB-related educational expenditure per student, expressed in Brazilian reais (BRL). The outputs are the approval rate in the initial years, the approval rate in the final years, inverse age-grade distortion in the initial years, inverse age-grade distortion in the final years, and total municipal enrollment in elementary education. Teacher counts and teacher-to-student ratios are used only descriptively to characterize municipal school systems; they are not included in the efficiency frontier and therefore do not directly determine the BCC scores.

Table 1. Variables used in the efficiency assessment

Role	Variable	Interpretation	Source
Input	FUNDEB-related educational expenditure per student (BRL)	Public educational expenditure per enrolled student used as the DEA input.	Public finance / MEC
Output	Approval, initial years	Percentage of approved students in grades 1–5.	INEP
Output	Approval, final years	Percentage of approved students in grades 6–9.	INEP
Output	Inverse age-grade distortion, initial years	1 –age-grade distortion rate; higher values indicate better age-grade adequacy.	INEP
Output	Inverse age-grade distortion, final years	1 –age-grade distortion rate; higher values indicate better age-grade adequacy.	INEP
Output	Total enrollment	Number of students enrolled in municipal elementary education.	INEP

Notes: Brazilian Ensino Fundamental comprises grades 1–9. Teacher counts and teacher-to-student indicators are reported descriptively and are not included in the final frontier specification. Enrollment

captures coverage and administrative reach; approval and inverse age-grade distortion capture progression-related outcomes.

Enrollment is treated as a coverage output because municipalities are responsible for organizing school networks that reach the school-age population. This differs from school-production models that treat the enrolled population as predetermined. Prior studies likewise use student numbers or enrollment rates as outputs when access is part of the public authority's mandate (Bogetoft et al. 2015; Melo-Becerra et al. 2020; Dutta 2012; Condez 2024; Aikins et al. 2026; Gupta and Verhoeven 2001; Herrera and Pang 2005). Enrollment is therefore interpreted as service reach, not as a measure of learning quality.

The territorial context makes this choice especially relevant. Riverine, rural, and Indigenous populations may require transport, dispersed facilities, and additional coordination to make schooling accessible (Carmo et al. 2020; Félix and Zoia 2022; Ferreira et al. 2023). Because absolute enrollment also reflects municipality size, it enters the BCC model jointly with approval and inverse age-grade distortion under variable returns to scale; coverage is therefore evaluated together with student progression rather than as a stand-alone performance measure.

The descriptive statistics reinforce the complexity of this production environment. Mean FUNDEB expenditure per student is approximately BRL 4,243.54, with substantial dispersion. Municipalities have an average of 6,833 enrolled students. The mean number of teachers is 151.8 in the initial years and 178.3 in the final years. Approval averages 86.12% in the initial years and 81.31% in the final years. The inverse age-grade distortion indicator averages 0.717 in the initial years and 0.508 in the final years, indicating greater age-grade mismatch in grades 6–9. These differences suggest that the final years of elementary education constitute a particularly important point of attention for local education policy.

3.2. BCC model

Let i index municipalities, x inputs, and y outputs. For municipality i , let x_{io} and y_{ro} represent the observed input and output levels. The output-oriented BCC model is (Banker et al. 1984; Cooper et al. 2007)

$$\max_{\phi, \lambda} \phi_o \quad (1)$$

$$\text{s.t.} \quad \sum_{j=1}^n \lambda_j x_{ij} \leq x_{io}, \quad i = 1, \dots, m \quad (2)$$

$$\sum_{j=1}^n \lambda_j y_{rj} \geq \phi_o y_{ro}, \quad r = 1, \dots, s \quad (3)$$

$$\sum_{j=1}^n \lambda_j = 1 \quad (4)$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n \quad (5)$$

The convexity condition $\sum_j \lambda_j = 1$ imposes variable returns to scale. Since $\phi_o^{\text{opt}} \geq 1$ under output orientation, the normalized efficiency score is

$$E_o^{\text{VRS}} = \frac{1}{\phi_o^{\text{opt}}}, \quad 0 < E_o^{\text{VRS}} \leq 1. \quad (6)$$

A score equal to one indicates that the municipality lies on the contemporaneous VRS frontier. Values below one indicate the proportional output expansion that would be required, given the observed input level, for the municipality to reach the frontier before considering slacks.

The output orientation is consistent with the study objective. Public managers face limits on available resources and are interested in increasing school coverage and educational outcomes without necessarily increasing expenditure. In the Amazonian context, this orientation is also preferable to mechanically interpreting efficiency as a mandate to reduce teachers, schools, or classes.

3.3. Malmquist index

The dynamic analysis uses the output-oriented Malmquist productivity index. Let $D_o^t(x^t, y^t)$ denote the output distance between an observation and the technology in period t . The index between t and $t+1$ is (Färe et al. 1994; Coelli et al. 2005)

$$M_o^{t,t+1} = \left[\frac{D_o^t(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t)} \frac{D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^{t+1}(x^t, y^t)} \right]^{1/2}. \quad (7)$$

The decomposition is

$$M_o^{t,t+1} = \text{TECH}_o^{t,t+1} \times \text{TECCH}_o^{t,t+1}, \quad (8)$$

where TECH represents efficiency change and TECCH frontier change. Values greater than one indicate improvement, values equal to one indicate stability, and values below one indicate deterioration. Regional and statewide results are aggregated using geometric means.

Seven municipality-period rows with nonnumeric placeholders were excluded from the dynamic database, leaving 749 usable transitions from 95 municipalities.

3.4. Spatial analysis

We evaluate spatial dependence at the Integration Region level to test whether regional educational performance is geographically structured rather than randomly distributed across Pará. Technical efficiency (TE) is represented by the 2012–2020 mean BCC score for each region, whereas technological change (TC) is represented by the regional geometric mean of the Malmquist TECCH component. This produces one TE and one TC value for each of the twelve regions and permits both same-variable and cross-variable spatial comparisons. At the municipality-year level, a BCC score of one identifies a technically efficient unit and a TECCH value of one indicates no frontier change. In the Moran analysis, by contrast, “high” and “low” denote standardized regional values above or below the twelve-region mean; these labels describe relative spatial positions rather than absolute thresholds of one.

A first-order binary contiguity matrix was constructed from the shared-border topology of the Integration Regions and row-standardized so that the weights of each region sum to one. Let z_i denote the standardized value for region i and w_{ij} the spatial weight linking regions i and j . The global univariate Moran statistic is (Moran 1950)

$$I = \frac{n}{S_0} \frac{\sum_i \sum_j w_{ij} z_i z_j}{\sum_i z_i^2}, \quad S_0 = \sum_i \sum_j w_{ij}. \quad (9)$$

It is calculated separately for TE and TC to test whether similar values are spatially clustered.

Cross-spatial association is evaluated in both directions. The bivariate Moran statistic for a focal variable x and the neighboring values of y is

$$I_{x,y} = \frac{n}{S_0} \frac{\sum_i \sum_j w_{ij} z_{x,i} z_{y,j}}{\sum_i z_{x,i}^2}. \quad (10)$$

Accordingly, assesses whether regions with relatively high technical efficiency tend to be adjacent to regions with relatively high technological change, while evaluates the reverse spatial relationship. Bivariate spatial association is directional, so the two statistics need not be identical (Wartenberg 1985; Lee 2001).

Global significance is evaluated using 99,999 random permutations and two-sided pseudo- values. Local Moran statistics are also calculated to identify high–high, low–low, high–low, and low–high spatial configurations (Anselin 1995). Because there are only twelve regions, local significance is obtained by exact conditional enumeration of feasible neighbor sets rather than asymptotic approximation. Local results are treated as exploratory and are additionally checked using the Benjamini–Hochberg false-discovery-rate correction. The spatial figures are shown as geographic-style choropleth maps for the historical configuration of the Integration Regions of Pará, preserving the territorial interpretation of the regional results.

3.5. Diagnostics

Regional uncertainty around Malmquist TFP change is summarized with municipality-level bootstrap intervals based on 10,000 resamples within each Integration Region. These are descriptive dispersion intervals rather than bias-corrected DEA confidence intervals because the frontier is not re-estimated in each draw. Pearson and Spearman correlations across the twelve regions compare static efficiency with dynamic productivity.

4. Results

The results reveal three distinct dimensions of regional educational performance. Static BCC efficiency varies substantially across the twelve Integration Regions. The Malmquist decomposition shows that efficiency catch-up and frontier movement frequently proceed in opposite directions, so regions that perform comparatively well in levels do not necessarily improve over time. The spatial tests add a third dimension: neither technical efficiency nor technological change forms a significant global cluster, although Xingu presents exploratory local configurations. These findings are reported separately below and interpreted jointly in the Discussion.

4.1. BCC efficiency

The BCC results indicate substantial differences among the Integration Regions of Pará. Figure 1 presents the annual regional averages. The unweighted mean of the twelve regional period averages is 0.3968. Carajás has the highest average score (0.5546), followed by Rio Caeté (0.5302) and Araguaia (0.4967). Tocantins has the lowest average (0.2725), followed by Tapajós (0.3135), Rio Capim (0.3193), Xingu (0.3256), and Guajará (0.3276).

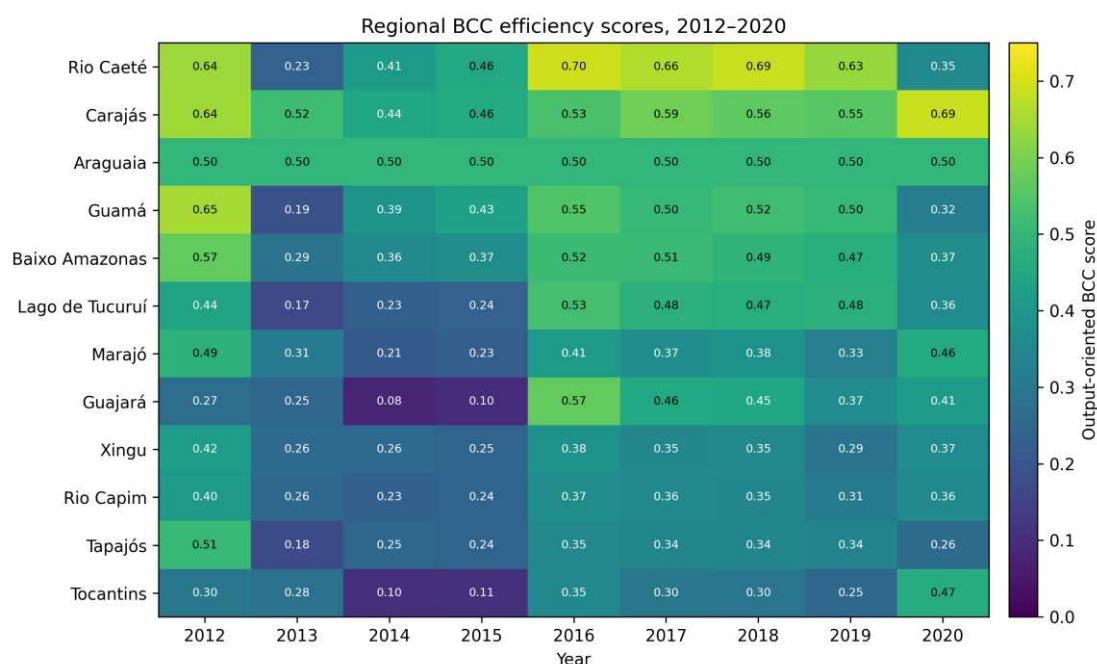


Fig. 1 Regional BCC efficiency scores, 2012–2020

Table 2. Mean regional BCC efficiency and Malmquist productivity

Integration Region	BCC mean	TFPCH	TECH	TECCH	N
Rio Caeté	0.530	0.981	1.045	0.939	12
Araguaia	0.497	0.968	1.025	0.944	6
Guamá	0.450	0.973	1.065	0.913	12
Baixo Amazonas	0.439	0.997	1.029	0.969	7
Carajás	0.555	0.948	1.113	0.852	6
Lago de Tucuruí	0.377	0.977	1.019	0.959	6
Guajará	0.328	0.977	1.089	0.897	3
Tapajós	0.313	0.991	1.065	0.930	4
Tocantins	0.273	0.965	1.034	0.933	7
Xingu	0.326	1.005	1.055	0.952	6
Rio Capim	0.319	0.990	1.097	0.903	12
Marajó	0.355	0.984	1.060	0.928	14
Unweighted regional BCC mean Statewide Malmquist geometric mean	0.397	0.980	1.058	0.926	95

Table 2. Mean regional BCC efficiency and Malmquist productivity. Note: BCC regional means are based on the 104-municipality static sample; N refers to municipalities included in the 95-municipality Malmquist sample

The concentration of regional averages between approximately 0.30 and 0.40 indicates substantial scope for investigating differences in the conversion of educational resources into municipal outputs. The result suggests that expenditure alone does not account for the observed variation in relative efficiency. The regional gaps therefore warrant closer examination of planning, coordination, resource

allocation, and monitoring as possible managerial factors, without treating the DEA scores as causal evidence about any specific practice.

The annual pattern also shows considerable instability. Rio Caeté records 0.6389 in 2012, falls to 0.2349 in 2013, and later reaches 0.6961 in 2016. Guajará falls to 0.0809 in 2014 and 0.0963 in 2015 before increasing to 0.5690 in 2016. Carajás is comparatively more stable and reaches 0.6863 in 2020. The heat map therefore reinforces that the efficiency problem is not uniform across space or time.

Tocantins warrants particular attention because it presents the lowest regional average. The score of 0.2725 indicates that, within the production system and comparison set considered, educational resources are converted into coverage and progression outcomes less efficiently than in the higher-scoring regional benchmarks. This does not imply that resources should be reduced; rather, it identifies a relative efficiency gap that merits closer investigation of the organization and use of the existing resource base.

The result is consistent with previous evidence from Brazil. Savian and Bezerra (2013) also identify widespread inefficiency in elementary-education spending and argue that the public sector should reconsider how resources are allocated. Giacomello and Oliveira (2014) emphasize the usefulness of DEA benchmarks and targets for identifying units whose practices can guide less efficient organizations. In the present context, benchmark municipalities may offer information on school organization, resource management, and educational progression, provided that comparisons respect territorial differences.

The descriptive data show a larger number of teachers in the final years, which is consistent with subject specialization at that stage, while approval rates and age-grade adequacy are simultaneously worse in grades 6–9. Because teacher variables are not included in the DEA frontier, these descriptive differences should not be interpreted as determinants of the BCC scores. They instead suggest areas for subsequent managerial investigation, including teacher deployment across schools and subjects, retention of qualified teachers in remote municipalities, and the relationship between staffing arrangements and student progression (Teixeira and Nunes 2019; Simielli 2023).

Finally, the enrollment output adds an important regional interpretation to the BCC scores. A municipality can improve relative efficiency by increasing the joint output vector, coverage and progression-related outcomes, given the observed fiscal input. In densely populated urban systems, enrollment may be comparatively less sensitive to territorial barriers. In remote Amazonian municipalities, however, maintaining enrollment often depends on transportation, school infrastructure, and geographically dispersed services. Consequently, the BCC score reflects the joint conversion of educational resources into territorial coverage and student progression, not educational quality in a comprehensive sense.

4.2. Productivity change

The Malmquist results provide a dynamic perspective on the 2012–2020 period. Across 749 usable municipality-period transitions, statewide geometric-mean TFP change is 0.980, corresponding to an average decline of about 1.98% per adjacent-year transition. Efficiency change is above one (TECH = 1.058), whereas frontier change is below one (TECCH = 0.926). Municipalities therefore tended to improve their relative position vis-à-vis the frontier, but the frontier itself shifted sufficiently unfavorably to generate a small net productivity decline.

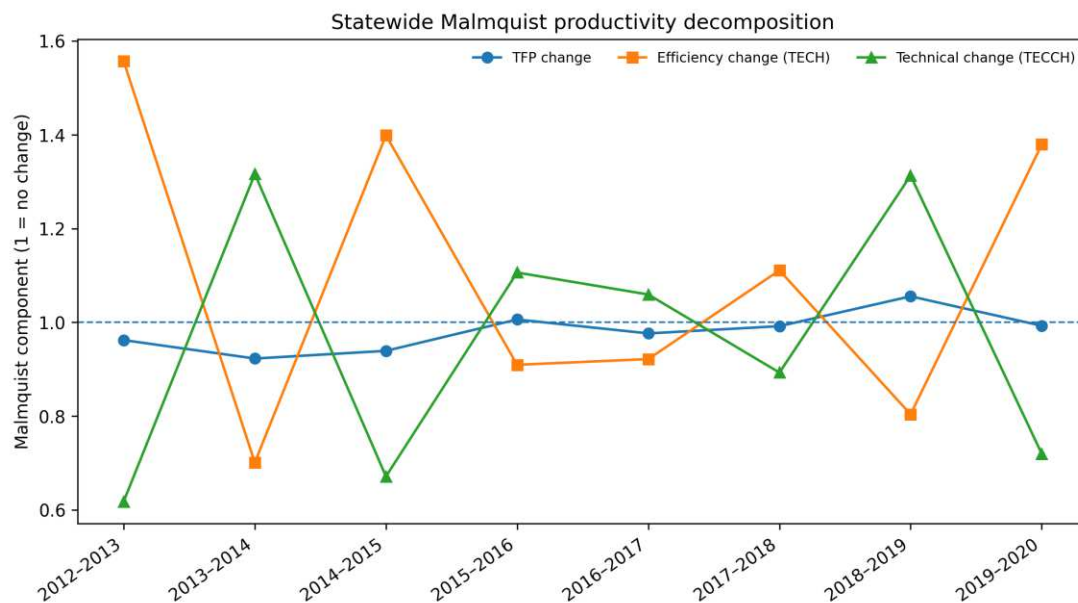


Fig. 2 Malmquist productivity decomposition by adjacent-year period

Table 3. Malmquist decomposition by period

Period	TFPCH	TECH	TECCH	Observations
2012–2013	0.962	1.557	0.618	91
2013–2014	0.923	0.701	1.316	94
2014–2015	0.939	1.399	0.672	94
2015–2016	1.006	0.910	1.106	92
2016–2017	0.976	0.922	1.059	94
2017–2018	0.992	1.111	0.893	94
2018–2019	1.056	0.804	1.313	94
2019–2020	0.993	1.379	0.720	96

The decomposition shows that efficiency change and frontier change frequently move in opposite directions. This pattern indicates that improvements in relative municipal positioning do not necessarily imply a generalized improvement in educational productivity. The 2019–2020 transition coincides with the first year of the COVID-19 pandemic, when school access, approval procedures, and the continuity of educational services were disrupted; the component values for this interval therefore reflect an exceptional institutional environment in addition to ordinary changes in municipal educational production.

At the regional level, Xingu is the only region with a TFP point estimate above one (1.005). Baixo Amazonas (0.997), Tapajós (0.991), and Rio Capim (0.990) are close to stability. Carajás presents the lowest regional TFP estimate (0.948) even though it has the highest BCC average. Thus, strong static efficiency does not necessarily imply favorable productivity dynamics.

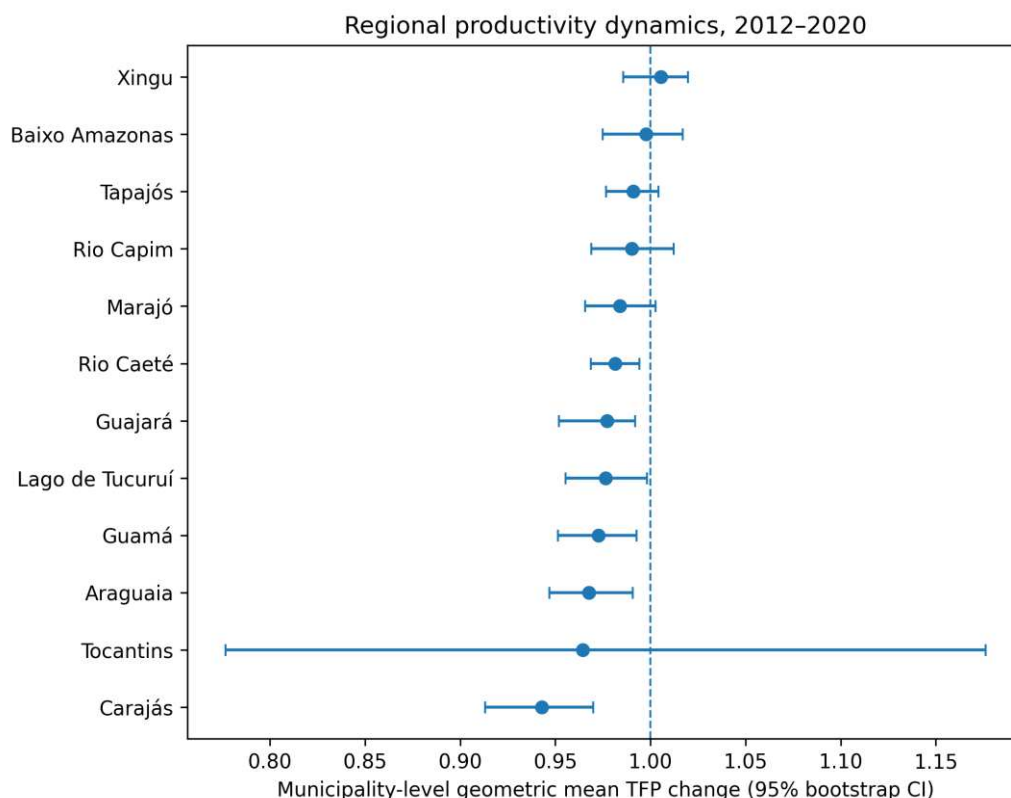


Fig. 3 Regional Malmquist TFP change with 95% descriptive bootstrap intervals

The regional intervals indicate substantial heterogeneity within several Integration Regions. In particular, Tocantins displays a wide interval, suggesting that the regional average conceals very different municipal trajectories. This result reinforces the need for policies that go beyond regional rankings and examine municipal conditions individually.

4.3. Static-dynamic comparison

The comparison between mean BCC efficiency and Malmquist TFP change is shown in Figure 4. The figure reports a Pearson correlation of approximately $r = -0.45$ ($p = 0.146$). Using the rounded regional values reported in Table 2, the corresponding Spearman rank correlation is approximately $\rho = -0.37$ ($p = 0.235$). Neither association is statistically significant at conventional levels.

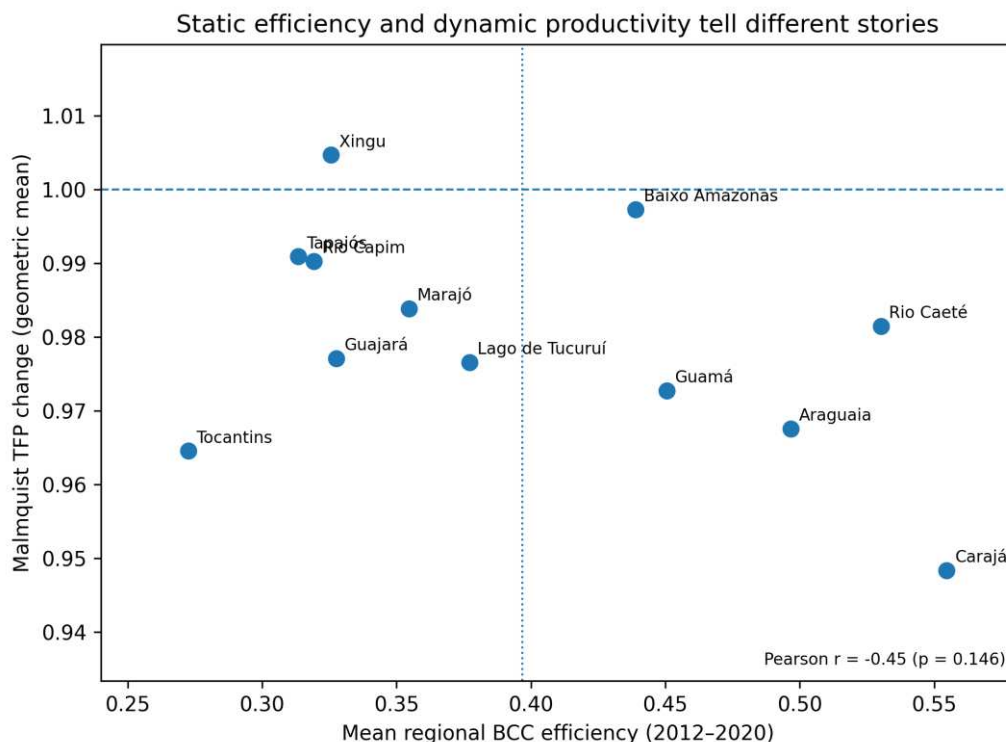


Fig. 4 Regional BCC efficiency and Malmquist productivity

The comparison illustrates why a dynamic assessment is important. Carajás presents the highest static BCC average but the weakest TFP trajectory. Xingu, by contrast, combines a relatively low static BCC score with the only regional TFP point estimate above one. Tocantins performs weakly in both dimensions. These differences show that distance to the contemporaneous frontier and changes in productivity answer distinct management questions.

4.4. Spatial dependence

Technical efficiency and technological change show no significant global spatial concentration across the twelve Integration Regions. The univariate Moran statistic for regional BCC efficiency is positive but small ($I = 0.082$, $p = 0.363$), while technological change is close to the absence of spatial association ($I = -0.041$, $p = 0.778$). In practical terms, neighboring regions do not systematically share similar levels of static resource conversion or similar rates of frontier change. Geographic contiguity alone therefore does not appear to generate a common performance environment across the Integration Regions.

Table 4. Global univariate and bivariate Moran statistics

Spatial relationship	Moran's I	Two-sided permutation p	Interpretation
TE $\rightarrow$ neighboring TE	0.082	0.363	No global clustering
TC $\rightarrow$ neighboring TC	-0.041	0.778	No global clustering
TE $\rightarrow$ neighboring TC	-0.210	0.162	Negative, not significant
TC $\rightarrow$ neighboring TE	-0.104	0.537	Negative, not significant

Notes: TE is the regional mean BCC technical-efficiency score; TC is the regional geometric mean of the Malmquist technological-change component (TECCH). Results use row-standardized first-order contiguity weights and 99,999 permutations.

The bivariate results are also nonsignificant. The association between focal-region TE and neighboring TC is negative ($I = -0.210$, $p = 0.162$), and the reverse association is likewise negative ($I = -0.104$, $p = 0.537$). Thus, the data do not support a general spatial relationship in which higher static

technical efficiency is surrounded by stronger technological change, or vice versa. As a robustness check, the spatial tests were repeated using the Malmquist efficiency-change component (TECH) in place of the mean BCC score; the substantive conclusion remained unchanged, with no evidence of broad spatial association. Because the supporting robustness statistics are not tabulated in the current manuscript, they are interpreted qualitatively rather than reported with incomplete numerical notation.

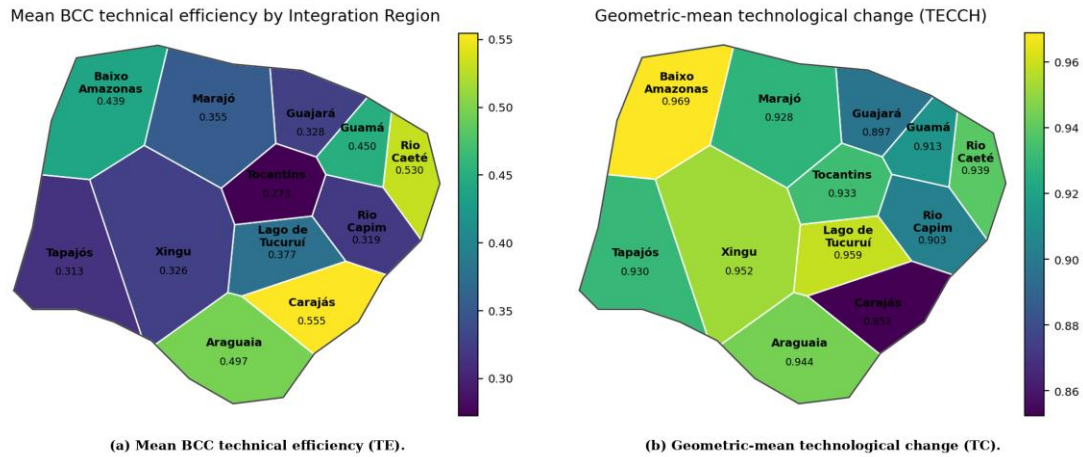


Fig. 5 Spatial distribution of technical efficiency and technological change across the Integration Regions of Pará

No Integration Region forms a statistically significant local cluster for BCC technical efficiency at the 5% level, but technological change produces an exploratory local configuration in Xingu. Xingu's regional TECCH geometric mean is 0.952, above the twelve-region mean of approximately 0.927, and the local pattern is classified as high-high relative to the standardized regional distribution. Because Xingu's value remains below one, this classification does not imply technological progress in an absolute Malmquist sense; it indicates that frontier deterioration is less pronounced in Xingu and its contiguous regional system than in many other regions. The local result is treated as exploratory rather than as evidence of a stable spillover process.

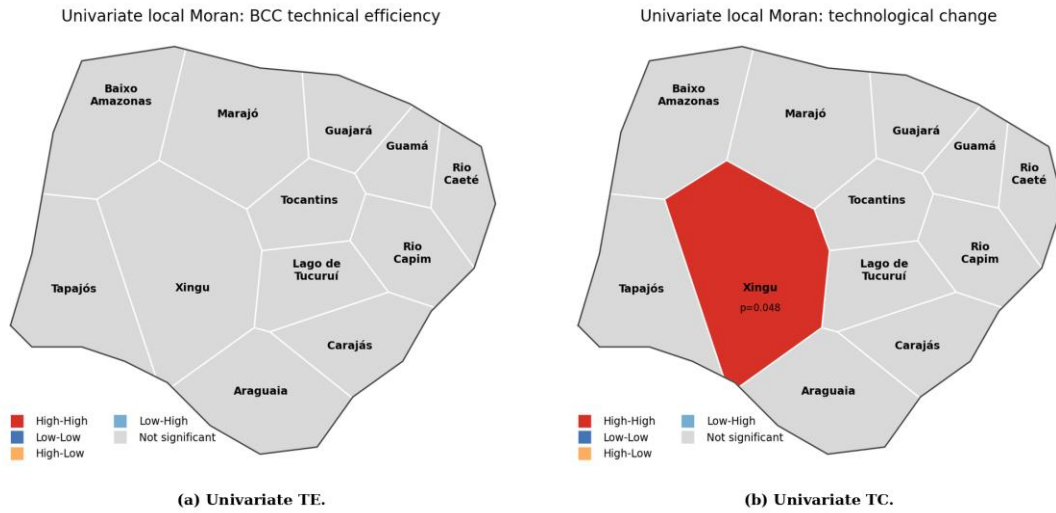


Fig. 6 Local Moran cluster maps for technical efficiency and technological change. High and low are defined relative to the standardized regional means

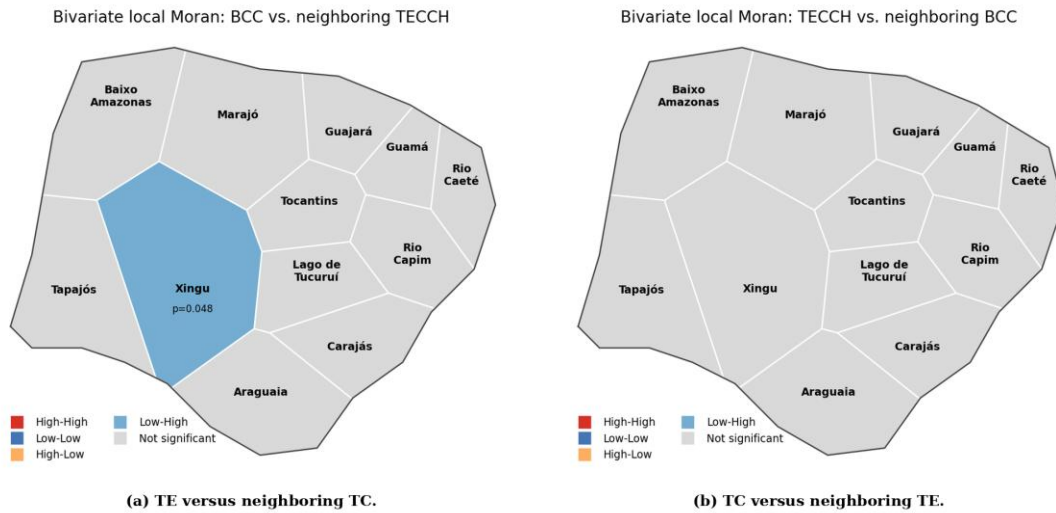


Fig. 7 Bivariate local Moran cluster maps. Only unadjusted local associations with exact $p < 0.05$ are colored as significant

The bivariate TE-to-neighboring-TC result makes the Xingu configuration relevant for educational management. Xingu's mean BCC efficiency is 0.326, below the twelve-region mean of 0.397, while its neighboring regions display comparatively more favorable technological-change values. This produces an exploratory low–high configuration: comparatively weak static resource conversion in Xingu coexists with a neighboring regional environment experiencing less adverse frontier change. The pattern identifies a mismatch between Xingu's distance from the BCC frontier and the relative dynamics observed around it, rather than a simple high-performance cluster.

For regional educational policy, this low–high configuration raises a concrete comparative question: which practices or constraints help explain why the neighboring regional system experiences comparatively more favorable frontier dynamics while Xingu remains below the average static efficiency level? Relevant areas for investigation include school access in remote communities, river and road transportation, teacher deployment and retention, the organization of dispersed school networks, and progression in the final years of elementary education. The local associations are significant only before multiple-testing correction and do not survive the Benjamini–Hochberg

adjustment; they are therefore used as diagnostic evidence for policy comparison rather than as confirmation of causal or statistically stable spillovers.

Geographic proximity alone does not appear sufficient to generate common efficiency or technological-change patterns, which strengthens the case for deliberate interregional coordination. The Integration Regions were conceived as planning territories with physical and socioeconomic links, but the spatial results indicate that educational improvements remain heterogeneous within that territorial structure. Mechanisms for transferring managerial practices, coordinating teacher supply, planning riverine and road transport, and sharing solutions for dispersed school networks may therefore need to be institutionally organized rather than expected to arise from proximity alone.

5. Discussion

The central finding is that territorial educational performance depends on the analytical dimension considered. Static efficiency, productivity change, and spatial association do not produce a single regional hierarchy. This matters for regional analysis because a planning territory that appears comparatively efficient at one point in time may follow an unfavorable productivity trajectory, while a lower-efficiency region may display more favorable change. The divergence is consistent with broader regional-productivity research showing that spatially heterogeneous trajectories cannot be inferred from cross-sectional performance alone (Aresu et al. 2026; Gómez-Tello et al. 2025).

5.1. Territorial heterogeneity and educational efficiency

The efficiency differences observed across Pará should be read in the context of territorial accessibility. In remote municipalities, maintaining enrollment may depend on river transport, small and dispersed schools, and costly service networks. These arrangements can perform an equity function even when they raise the fiscal cost of provision. The result reinforces the distinction between low relative efficiency and poor educational policy: frontier scores identify resource-conversion gaps for investigation, but they do not reveal whether territorial costs are avoidable. This interpretation is consistent with evidence that geographic accessibility itself can generate educational disparities (Bruno et al. 2022) and that peripheral conditions may combine geographic and institutional constraints (Barbero and Rodríguez-Crespo 2022).

5.2. Static efficiency and dynamic productivity capture different regional processes

Carajás illustrates why static and dynamic measures should not be treated as substitutes: it has the highest regional BCC average but the weakest TFP trajectory, whereas Xingu combines relatively low static efficiency with the only regional TFP point estimate above one. For regional policy, a static benchmark can identify distance from current best practice, while Malmquist change indicates whether that position is improving or deteriorating. Using both measures therefore reduces the risk of directing attention only to high- or low-scoring regions at a single point in time.

5.3. Administrative regions are not necessarily functional efficiency regions

The absence of significant global Moran statistics indicates that geographic contiguity alone does not create a common educational-performance environment. This is substantively relevant because the Integration Regions are administrative planning units, yet their boundaries do not automatically map onto coherent regimes of technical efficiency or technological change. The exploratory Xingu pattern is useful as a spatial diagnostic, but its local significance does not survive multiple-testing correction and should not be interpreted as evidence of a stable spillover process. The broader implication is that functional relationships in educational performance must be demonstrated empirically rather than inferred from administrative adjacency.

5.4. Regional policy implications for geographically dispersed education systems

The combined results support territorially differentiated diagnosis rather than uniform resource-allocation rules. Regional and municipal planners can use static efficiency to identify potential resource-conversion gaps, dynamic productivity to distinguish improving from deteriorating trajectories, and spatial diagnostics to test whether neighboring territories form meaningful comparison groups. For remote Amazonian systems, subsequent investigation should focus on access costs, transport organization, teacher deployment and retention, school-network configuration, and progression in grades 6–9. These are priorities for comparative investigation rather than causal prescriptions because the DEA and spatial analyses do not directly estimate the effects of specific management practices.

6. Conclusion

This study examines the regional geography of municipal educational efficiency in Pará by combining static, dynamic, and spatial perspectives. The evidence shows substantial territorial heterogeneity, but the regional ordering depends on the measure considered. Carajás and Rio Caeté are closest to the static frontier on average, while dynamic productivity trajectories tell a different story and Xingu is the only region with a TFP point estimate above unity.

Conceptually, treating enrollment as the realized territorial reach of the school network connects efficiency measurement to accessibility. In geographically dispersed areas, serving students is not simply a scale burden: it may require costly transport, dispersed facilities, and additional coordination. Evaluating enrollment jointly with progression indicators therefore provides a broader representation of public educational service provision without treating coverage as a measure of learning quality.

The main regional-science contribution is the finding that static resource conversion, intertemporal productivity, and spatial dependence capture different territorial processes. The absence of significant global spatial autocorrelation further indicates that administrative proximity does not necessarily define a functional educational-performance region. Regional policy analysis should therefore avoid relying on a single static ranking or assuming that neighboring territories share common productivity conditions.

The findings are limited by the relative nature of DEA, the smaller Malmquist sample, only twelve units in the regional spatial analysis, and the exclusion of socioeconomic and family conditions from the frontier. The 2019–2020 transition also coincides with the onset of COVID-19. Future research should test municipal-level spatial structures, alternative spatial-weight matrices, conditional or robust frontiers, and sensitivity to excluding the pandemic transition.

Statements and Declarations

Data availability: The study uses publicly available educational, fiscal, and demographic data from INEP, government transparency systems, and IBGE. Analytical materials used to produce the tables and figures can be provided with the submission or deposited as supplementary material.

Competing Interests

The authors declare that they have no financial or non-financial interests that are directly or indirectly related to the work submitted for publication.

Ethics approval: Not applicable. The study uses publicly available, aggregated administrative data and did not involve recruitment, intervention, or interaction with human participants.

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