

Supplementary Information

A copula-based spatial justice index for pedestrian–vehicle interaction on school streets

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Supplementary Note 1: Attenuation of Kendall’s tau under jittering

Let (P_1, V_1) and (P_2, V_2) be independent copies of the pair (pedestrian count, cautionary speed), and let U_1, U_2 be independent $\mathcal{U}(0, 1)$ variables, independent of the pairs, with $P_i^* = P_i + U_i$. Denote by:

$$p_c = \Pr\{(P_1 - P_2)(V_1 - V_2) > 0\}, \quad p_d = \Pr\{(P_1 - P_2)(V_1 - V_2) < 0\}$$

the probabilities of concordance and discordance, and by $t_P = \Pr(P_1 = P_2) = \sum_k \pi_k^2$ and $t_V = \Pr(V_1 = V_2) = \sum_l \omega_l^2$ the probabilities of ties, where $\pi_k = \Pr(P = k)$ and ω_l are the probabilities of the atoms of V . The population version of the tie-corrected coefficient is:

$$\tau_b(P, V) = \frac{p_c - p_d}{\sqrt{(1 - t_P)(1 - t_V)}}.$$

If $P_1 \neq P_2$, then $|P_1 - P_2| \geq 1 > |U_1 - U_2|$ almost surely, so that $\text{sign}(P_1^* - P_2^*) = \text{sign}(P_1 - P_2)$. If $P_1 = P_2$, then $\text{sign}(P_1^* - P_2^*) = \text{sign}(U_1 - U_2)$, which equals $+1$ or -1 with probability $1/2$ each, independently of (V_1, V_2) ; such pairs therefore contribute equally to concordance and discordance. Hence the probabilities of concordance and discordance of the jittered pair, p_c^* and p_d^* , satisfy $p_c^* - p_d^* = p_c - p_d$, and:

$$\tau(P^*, V) := p_c^* - p_d^* = \tau_b(P, V) \sqrt{(1 - t_P)(1 - t_V)},$$

which is equation (4) of the main text. If the tie-corrected coefficient is also used for the jittered pair, whose first component has no ties, the denominator becomes $\sqrt{1 - t_V}$ and $\tau_b(P^*, V) = \tau_b(P, V) \sqrt{1 - t_P}$.

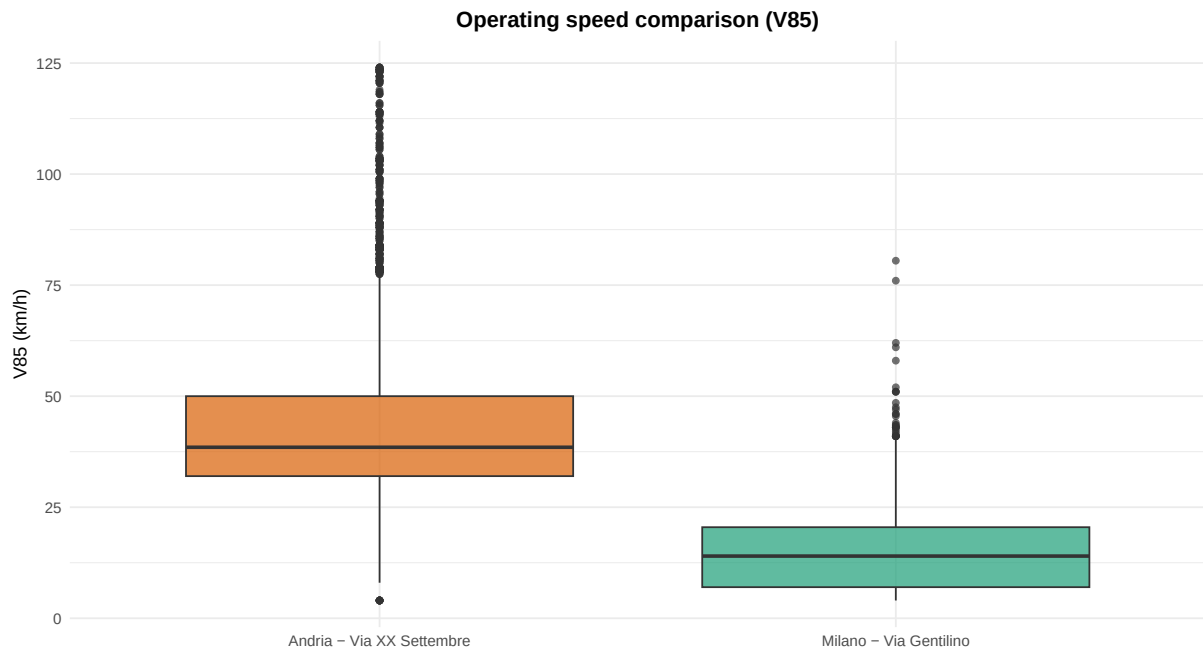
In either case the attenuation factor is at most $\sqrt{1 - t_P}$. At Andria, 52.7% of free-flow intervals record no pedestrians, so $t_P \geq 0.527^2 = 0.278$ and the factor is at most 0.85; the observed ratio between the jittered and the raw coefficients, $0.168/0.203 \approx 0.83$, is consistent with this bound.

Supplementary Table S1

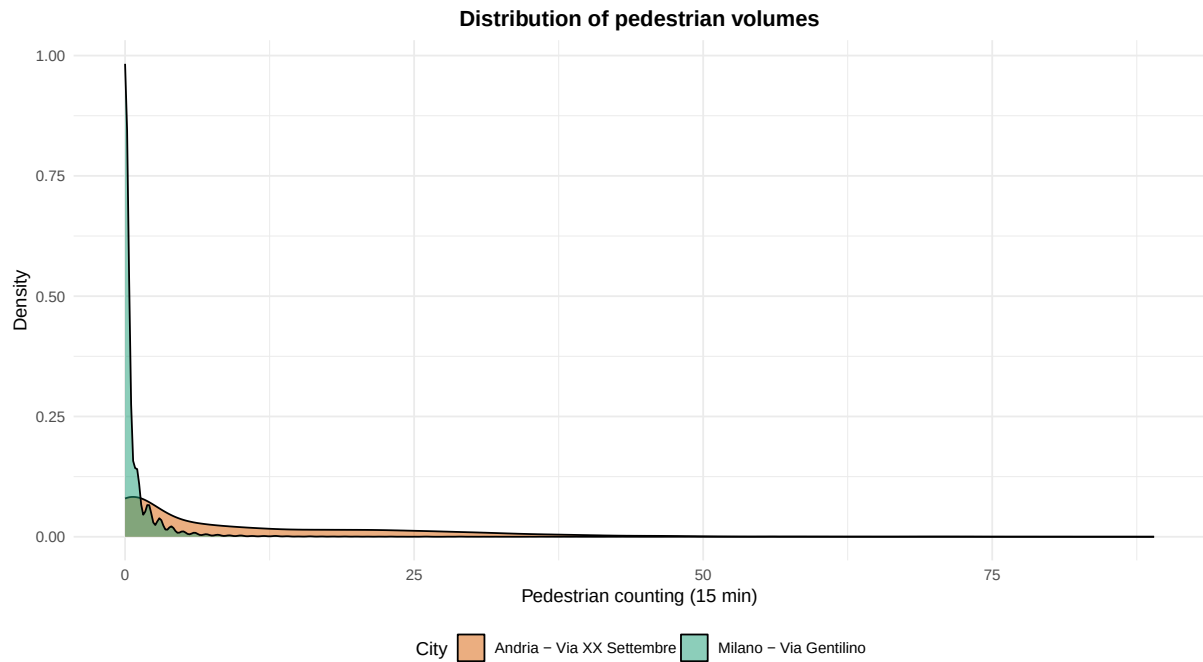
Supplementary Table S1: Summary of the literature on pedestrian-vehicle interaction and of the gaps addressed by this study.

Research area	Key references	Methodological approach	Identified limitations
Urban space and side friction	[1–4]	Shift from “capacity impedance” to spatial justice and the “visual wall” effect.	Lacks quantitative, scalable frameworks to measure spatial equity from empirical data.
Microscopic modelling	[5–7]	High-fidelity trajectory analysis and behavioural models of crossing decisions.	Computationally expensive; requires rare high-resolution data; difficult to scale.
Macroscopic traffic analysis	[3, 8–10]	Aggregated variables (mean speed, flow) and linear regression of speed on side-friction factors.	Ignores localised decelerations; does not separate side friction from congestion.
Safety and copula models	[11–16]	Joint probabilistic modelling of non-linear dependence.	Mixed discrete–continuous data introduce ties, so the copula is not uniquely identified.

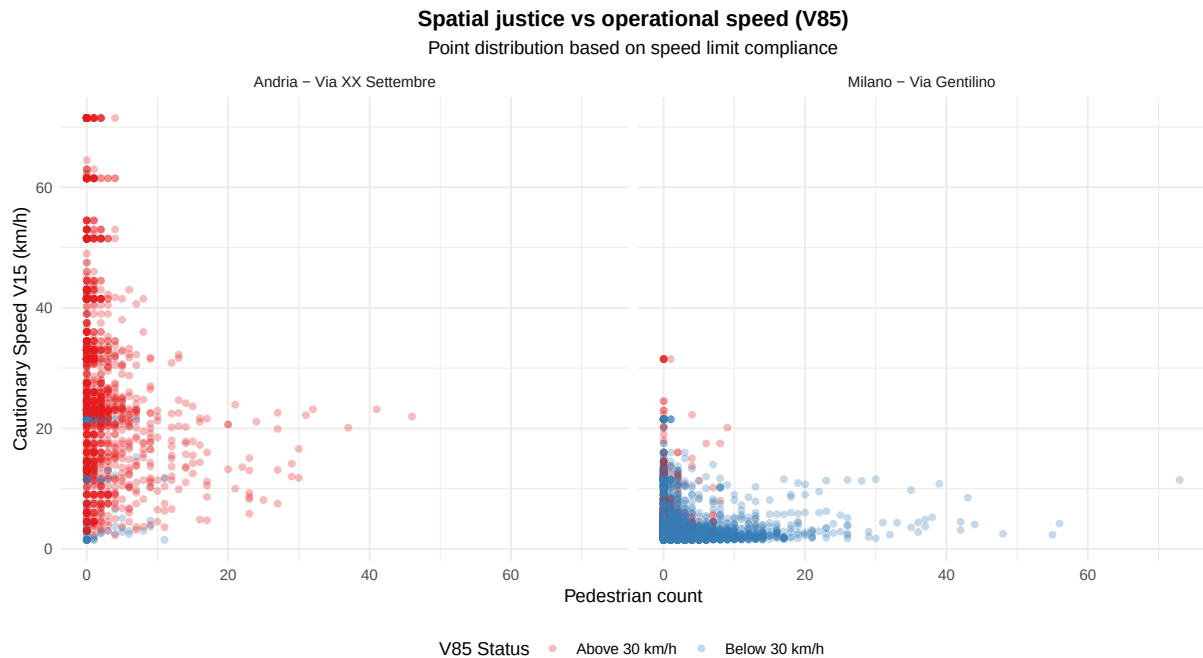
Supplementary Figures



Supplementary Figure S1: Operating speed at the two study streets. Distribution of the 85th percentile operating speed (V_{85}) per 15-minute interval at Andria (Via XX Settembre) and Milan (Via Gentilino), raw data before regime filtering. Boxes show the median and interquartile range; whiskers extend to 1.5 times the interquartile range; points are observations beyond the whiskers. Values above 100 km/h at Andria are attributed to detection errors in low-volume intervals.



Supplementary Figure S2: Pedestrian activity at the two study streets. Kernel density estimates of the number of pedestrians per 15-minute interval at Andria and Milan, raw data before regime filtering.



Supplementary Figure S3: Cautionary speed and operating speed. Cautionary speed (V_{15}) against pedestrian count under the free-flow regime at Andria and Milan, with each 15-minute interval classified by its operating speed: red, V_{85} above 30 km/h; blue, V_{85} at or below 30 km/h. Low cautionary speeds occur in both classes.

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