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## Article

**Keywords:** Shield tunnel, fault dip angle, face stability, ultimate support pressure, calculation method

**Posted Date:** October 30th, 2026

**DOI:** <https://doi.org/10.21203/rs.3.rs-11141213/v1>

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**Additional Declarations:** No competing interests reported.

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# A calculation method for the ultimate support pressure of a shield tunnel excavation face accounting for fault dip angle

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**Abstract:** This study investigated the influence of fault dip angle on tunnel excavation-face stability. Starting from the conventional wedge instability model for homogeneous strata, an associated flow rule was introduced to improve the wedge model, and a modified wedge calculation model that accounts for the fault dip angle is proposed. Then, by establishing the compatibility relationship between the rupture angles of the upper and lower strata, an analytical formula for the ultimate support pressure of the excavation face under inclined fault conditions is derived using the limit equilibrium method, along with the corresponding calculation procedure. Three-dimensional numerical simulations are performed to evaluate the theoretical model, and the results reveal that the trends in the theoretical predictions and numerical simulations are consistent, with relative errors ranging from 3.2% to 15% across all working conditions, indicating good agreement between the theoretical and numerical results. The calculation results reveal that stratum strength parameters and fault dip angle are the key factors controlling ultimate support pressure. The proposed calculation method for the ultimate support force of the excavation face considering fault dip angle can quantitatively estimate ultimate support pressure in faulted strata, providing a theoretical basis for assessing support parameters and evaluating excavation-face stability during slurry-shield tunneling through fault fracture zones.

**Keywords:** Shield tunnel; fault dip angle; face stability; ultimate support pressure; calculation method

## 1 Introduction

Maintaining the excavation-face stability is crucial for safe shield tunneling. A slurry shield stabilizes the tunnel face by applying slurry support pressure to balance the earth and water pressures acting ahead of the face <sup>1,2</sup>. For construction safety, the face support pressure acting on the excavation face should be appropriately controlled; otherwise, excessively low support pressure may induce collapse of the excavation face, whereas an excessively high support pressure may cause surface heave or blow-out <sup>1,2</sup>. Therefore, determining the ultimate support pressure at the excavation face is essential for ensuring

31 construction safety<sup>3,4</sup>.

32 Theoretical approaches for analyzing active failure of the ground ahead of the excavation face mainly  
33 include the limit analysis and limit equilibrium methods<sup>5,6</sup>. Owing to its simple theoretical basis and  
34 calculation procedure, the limit equilibrium method has been widely adopted<sup>7-9</sup>. For homogeneous strata,  
35 the classical wedge model determines the ultimate support pressure by establishing force equilibrium  
36 between the wedge in front of the excavation face and the overlying prism<sup>8,9</sup>. Recent studies have further  
37 refined this model by incorporating layered strata, pore water pressure, soil arching, seepage, and other  
38 factors<sup>7-12</sup>. However, the core assumption of these models is that the rupture angle of the failure surface  
39 is  $45^\circ + \varphi/2$ ; this assumption has been widely used for sands and normally consolidated clays<sup>9,10</sup>, whereas  
40 fault fracture zones are characterized by pronounced heterogeneity and complex fractured structures,  
41 determining that the rupture angle becomes considerably more complicated<sup>13-15</sup>.

42 The presence of a fault fracture zone poses two major difficulties for the stability analysis of the  
43 excavation face. First, the mechanical parameters (cohesion and internal friction angle) of the rock mass  
44 within the fracture zone are significantly lower than those of the intact rock mass and vary strongly in  
45 space<sup>13</sup>. Second, a fault usually intersects the tunnel axis at a certain dip angle, resulting in  
46 heterogeneous and direction-dependent ground conditions ahead of the excavation face<sup>16-18</sup>. To address  
47 this issue, some researchers have idealized the fault as a homogeneous stratum with weakened strength  
48 parameters, while others have introduced a non-homogeneity coefficient to describe the linear variation  
49 of the parameters within the fracture zone<sup>13,16</sup>. Han et al.<sup>13</sup> considered the influence of the interface  
50 between the intact rock mass and the fault fracture zone under high hydraulic pressure, improved the  
51 wedge-prism model, and found that the ultimate support pressure increased significantly when the  
52 excavation face approaches the interface. Tu et al.<sup>16</sup> analyzed the face stability of shield tunnels in  
53 inclined layered soils using the limit analysis method and revealed the effect of interface inclination on  
54 limit support pressure. Li et al.<sup>17</sup> further studied the face stability of shield tunnels excavated along  
55 inclined strata and clarified how stratum inclination affects the failure mechanism and the critical support  
56 pressure. Man et al.<sup>19</sup> analyzed the face stability of circular tunnels in layered rock masses using the  
57 upper bound theorem and established a face stability database covering layered rock conditions; they  
58 further developed an analytical model for the face stability of longitudinally inclined layered rock masses  
59 with a weak interlayer<sup>20</sup>, which provides a useful basis for analyzing tunneling through fault-affected

60 ground. Ding et al. <sup>21</sup> analyzed the face stability of large shield-driven tunnels in rock-soil interface  
61 composite formations and highlighted the dominant role of the interface location in the failure mode.  
62 Wang et al. <sup>22</sup> investigated the stability of the surrounding rock and the structural response of shield-  
63 driven twin tunnels crossing a fault fracture zone through three-dimensional numerical simulation. Huang  
64 et al. <sup>23</sup> studied the progressive failure mechanism of shield tunnel faces crossing fault fracture zones  
65 under high hydraulic pressure, revealing the evolution of the seepage field and the limiting support force  
66 as the tunnel face approaches the fracture zone. Di et al. <sup>24</sup> experimentally investigated face stability of  
67 shield tunnels in sandy cobble strata with different densities and examined the influence of the stratum  
68 density on the failure mode and the support pressure. Chen et al. <sup>25</sup> assessed the face stability of a  
69 longitudinally inclined tunnel considering pore water pressure by combining FLAC3D simulation and an  
70 improved rotational failure mechanism. Li et al. <sup>26</sup> developed an intelligent prediction model for the limit  
71 support pressure of river-crossing shield tunnels based on extensive numerical simulations. Li et al. <sup>27</sup>  
72 investigated the face stability of shallow shield tunnels in inclined sand-clay strata by combining  
73 centrifuge tests and discrete element modeling, and systematically examined the effects of the dip angle  
74 and interface elevation on the critical support pressure. These studies have contributed substantially to  
75 the understanding of face stability theory for faulted strata. Nevertheless, most of these studies adopted a  
76 traditional approach to determining the rupture angle within the wedge model, and the influence of the  
77 distinctive mechanical characteristics of fault fracture zones on the rupture angle has not yet been  
78 explored.

79 In summary, most existing theoretical models of excavation face instability have been developed for  
80 homogeneous strata, and the influence of stratum-dependent rupture angles on the ultimate support  
81 pressure has largely been neglected for layered and faulted strata. Therefore, a wedge-type theoretical  
82 model for the active failure of the excavation face in faulted strata is needed.

83 Accordingly, this study focused on determining the ultimate support pressure of the excavation face  
84 when a slurry shield traverses a fault fracture zone. Based on theoretical analysis of the traditional wedge  
85 model for homogeneous strata, an associated flow rule was introduced to improve the estimation of the  
86 rupture angle in the wedge failure model during shield tunneling across the fault. The validity of the  
87 improved model was evaluated through numerical simulations. Finally, the influence of stratum strength  
88 parameters and fault dip angle on the ultimate support pressure and the excavation-face failure

89 mechanism was analyzed.

## 90 2 Instability Mode of Tunnel Excavation Face

91 To clarify the instability characteristics of the excavation face of shield tunnels in inclined fault strata,  
92 three-dimensional numerical simulations were performed using FLAC3D. A refined three-dimensional  
93 numerical model was established with fixed displacement boundaries for the tunnel lining to simulate  
94 practical lining restraint effects. The model boundary conditions were defined as follows: the top surface  
95 was set as a free boundary, allowing vertical displacement, while the lateral and bottom boundaries were  
96 fixed to constrain horizontal and vertical displacements, respectively, ensuring accurate simulation of  
97 stratum stress and deformation. The numerical model is presented in Fig. 1.

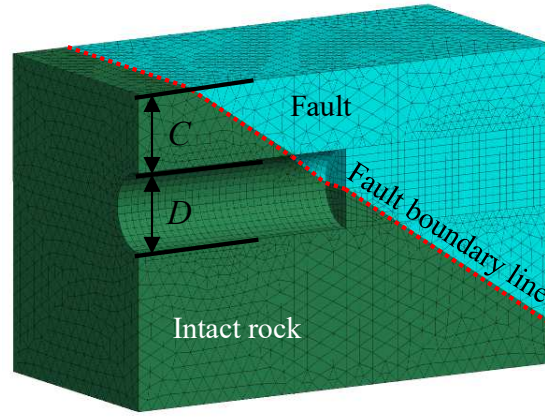


Fig.1 Schematic diagram of numerical simulation.

98 To investigate the influence of faults on the instability modes of the excavation face in shield  
99 tunneling, the mechanical parameters of the faulted stratum were selected to represent a relatively weak  
100 condition prone to failure. Accordingly, the cohesion and internal friction angle of the upper rock layer  
101 or fault zone were set to  $c=0$  and  $\varphi = 15^\circ$ , respectively.. While the mechanical parameters of the lower  
102 rock layer were varied in numerical simulations. Figs. 2 and 3 present the resulting displacement contours  
103 of the shield tunnel excavation face. Fig. 2 displays the case in which only the cohesion of the lower layer  
104 rock was varied, while Fig. 3 depicts the case in which only the internal friction angle of the intact rock  
105 was varied.

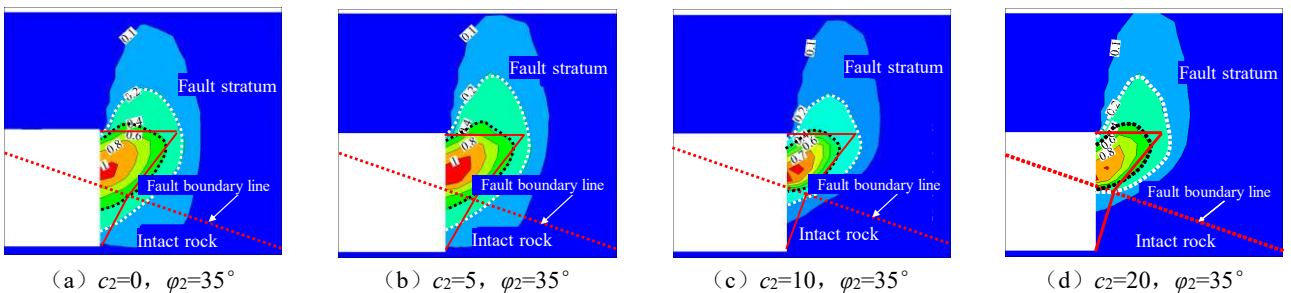


Fig.2 Displacement contours at the limiting state in inclined fault strata with varying cohesion of the lower stratum.

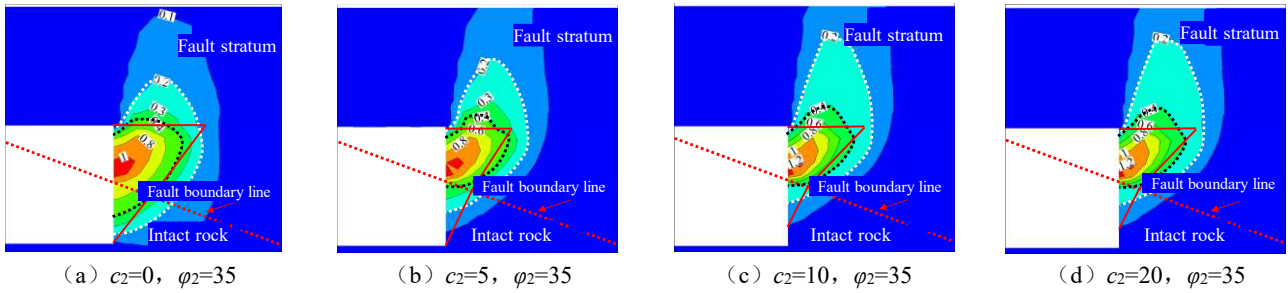


Fig.3 Displacement contours at the limiting state in inclined fault strata with varying internal friction angle of the lower stratum.

Figs. 2 and 3 reveal that as the mechanical parameters of the lower strata vary, the cohesion of the lower strata increases and the unstable contour in front of the excavation face gradually decreases. The unstable contour of the excavation face in the inclined fault does not extend at the interface between the inclined fault zone and the upper rock layer, but instead extends upward from the fault interface to the region above the excavation face (Fig. 4).

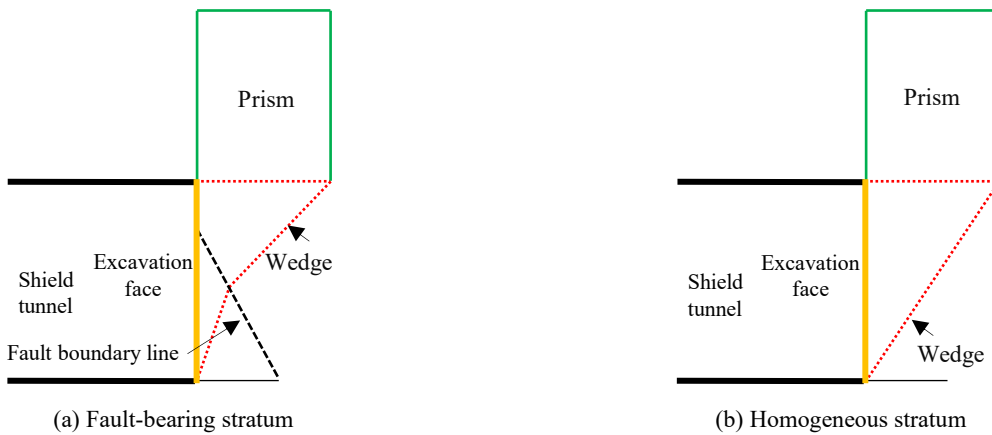


Fig.4 Instability modes of shield tunnel excavation face.

Therefore, comparison of the unstable regions and their contours displays that the instability contour of the excavation face in the inclined fault (Fig. 4(a)) was significantly different from the failure pattern predicted by the traditional wedge model (Fig. 4(b)) and and therefore could not be directly represented by the conventional wedge failure mechanism.. Consequently, this study proposes an improved wedge model based on the conventional wedge model and derives a formula to calculate the ultimate support pressure of the excavation face in inclined fault strata.

### 3. Calculation Method for Ultimate Support Pressure of Tunnel Excavation Face

The ultimate support pressure at the tunnel excavation face is essentially determined by the critical stress equilibrium state of the sliding wedge, which is closely related to the rupture angle of the surrounding stratum. The conventional wedge model assumes a fixed empirical rupture angle, which may not adequately represent layered fault strata with pronounced mechanical heterogeneity. For tunnel

sections crossing an inclined fault, the stratum interface divides the excavation-face instability wedge into an upper fault fracture zone and a lower intact-rock zone, each characterized by a different rupture angle. Establishing a rational compatibility relationship between the two rupture angles is central to improving the model. In this study, an associated plastic flow rule was introduced to improve the conventional wedge model, thereby enabling the rupture angle to be determined adaptively in inclined fault strata.

In plastic mechanics, to describe the magnitude and direction of plastic strain increment during plastic deformation, the plastic strain increment  $d\varepsilon^p$  is defined as follows:

$$d\varepsilon^p = d\lambda \frac{\partial g}{\partial \sigma} \quad (1)$$

where  $\lambda$  is a non-negative plastic multiplier, and  $\partial g / \partial \sigma$  denotes the gradient of the plastic potential with respect to stress..  $d\lambda$  represents the magnitude of  $d\varepsilon^p$ .  $\partial g / \partial \sigma$  defines the direction of the plastic strain increment, normal to the plastic potential surface in stress space. and represents the direction of  $d\varepsilon^p$ .

If the plastic potential surface  $g$  coincides with the yield function  $f$ , the flow rule is termed an associated plastic flow rule as follows:

$$d\varepsilon^p = d\lambda \frac{\partial f}{\partial \sigma} \quad (2)$$

To simplify the derivation and better represent the observed failure characteristics of the excavation face in faulted strata, the following assumptions are adopted for the modified wedge model:

(1) The ultimate support pressure acting on the upper fault stratum and lower intact-rock stratum of the excavation face is equal ( $P_1 = P_2$ ).

(2) The surrounding geomaterial obeys the Mohr–Coulomb yield criterion.

(3) The rock and soil mass obeys the associated plastic flow rule. When the support pressure at the excavation face decreases to the critical ultimate value  $P_f$ , the entire wedge-shaped failure body slides downward along the rupture surface with a uniform velocity. The rupture angles of the upper fault stratum and lower intact stratum measured relative to the horizontal plane are defined as  $\varphi_1$  and  $\varphi_2$ , respectively (Fig. 5). Combined with the Mohr–Coulomb yield criterion and multi-layer stratum deformation coordination principle, a quantitative coordination relationship between the two rupture angles was derived (Eq. 3).

$$\beta_1 = \beta_2 + \varphi_1 - \varphi_2 \quad (3)$$

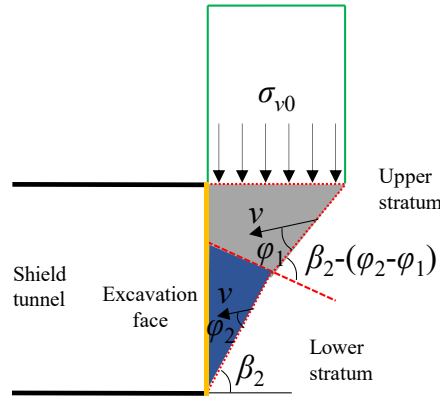


Fig.5 Schematic diagram of sliding direction of the upper and lower wedges.

The excavation face stratum was divided into two parts by the inclined fault interface. The upper fault fracture zone is characterized by unit weight  $\gamma_1$ , cohesion  $c_1$ , and internal friction angle  $\varphi_1$ , while the lower intact rock stratum is characterized by unit weight  $\gamma_2$ , cohesion  $c_2$ , and internal friction angle  $\varphi_2$ . The force analysis of the overall wedge-shaped failure body is presented in Fig. 6.

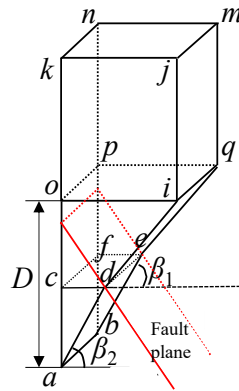


Fig.6 Stress analysis diagram of excavation face.

To determine the geometry and stress state of the layered wedge body, the coordinates of the intersection between the key structural lines in the model were determined geometrically. The coordinates of key point  $d$  at the lower stratum rupture surface are defined as  $(d_x, d_y)$ , and the coordinates of key point  $s$  at the upper stratum rupture surface are defined as  $(s_x, s_y)$ . The coordinate parameters were determined from the linear intersection relationships of the wedge model (Fig. 7).

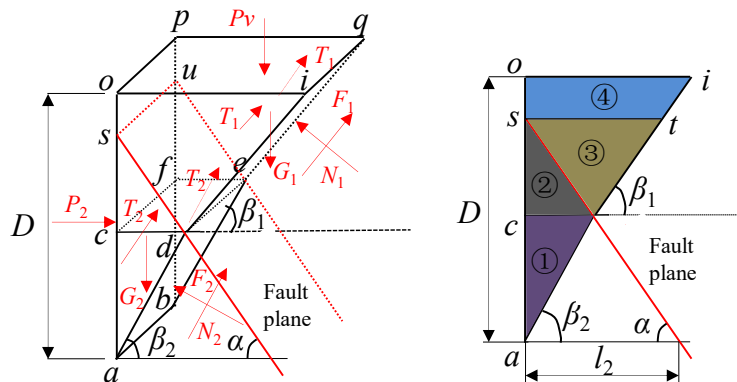


Fig.7 Force analysis diagram of excavation face wedge body.

Point  $d$  is the intersection of lines  $ad$  and  $sd$ , and its coordinates are determined by solving the following linear equations:

$$\tan \beta_2 d_x = -\tan \alpha (d_x - l_2)$$

$$d_x = \frac{l_2 \tan \alpha}{\tan \alpha + \tan \beta_2}$$

$$d_y = \frac{l_2 \tan \alpha \tan \beta_2}{\tan \alpha + \tan \beta_2}$$

where

$$D_1 = D - \frac{l_1 \tan \alpha \tan \beta_2}{\tan \alpha + \tan \beta_2}$$

Point  $s$  is the intersection of lines  $oa$  and  $sd$ , and its coordinates are determined by solving the corresponding linear equations:

$$s_y = l_2 \tan \alpha$$

where  $\alpha$  denotes the dip angle of the fault interface,  $l_2$  is the vertical distance between the bottom of the tunnel excavation face and the fault interface, and  $D$  is the tunnel diameter.

### 3.1 Gravity of Wedge Body

#### (1) Gravity of upper wedge of block ①

$$G_{21} = \frac{\gamma_2 B l_2^2}{2} \frac{\tan^2 \alpha \tan \beta_2}{\tan \alpha + \tan \beta_2} \quad (4)$$

where  $\gamma_2$  is the unit weight of the lower intact rock stratum, and  $B$  is the width of the prism body.

#### (2) Gravity of upper wedge of block ②

$$G_{22} = \frac{\gamma_2 B l_2^2}{2} \frac{\tan^3 \alpha}{(\tan \alpha + \tan \beta_2)^2} \quad (5)$$

$$G_2 = G_{21} + G_{22} \quad (6)$$

#### (3) Gravity of upper wedge of block ③

$$G_{13} = \frac{\gamma_1 B l_2^2}{2} \frac{\tan^3 \alpha}{\tan \alpha + \tan \beta_2} \left[ \left( \frac{\tan \alpha}{\tan \alpha + \tan \beta_2} \right) \frac{1}{\tan \beta_1} + \frac{\tan \beta_1}{\tan \alpha + \tan \beta_1} \right] \quad (7)$$

#### (4) Gravity of upper wedge of block ④

$$G_{14} = \frac{\gamma_1 B l_2 \tan \alpha}{2} \left[ \left( \frac{\tan \alpha}{\tan \alpha + \tan \beta_2} \right) \frac{1}{\tan \beta_1} + \frac{\tan \beta_1}{\tan \alpha + \tan \beta_1} \right] (D - l_2 \tan \alpha) \\ + \left[ \left( D - \frac{l_2 \tan \alpha \tan \beta_2}{\tan \alpha + \tan \beta_2} \right) \frac{1}{\tan \beta_1} + \frac{l_2 \tan \alpha \tan \beta_1}{\tan \alpha + \tan \beta_1} \right] (D - l_2 \tan \alpha) \quad (8)$$

$$G_1 = G_{13} + G_{14} \quad (9)$$

### 3.2 Shear Force on Vertical Sliding Surface

(1) Shear force  $T_{21}$  acting on block ①

$$T_{21} = \int_0^{\frac{l_2 \tan \alpha \tan \beta_2}{\tan \alpha + \tan \beta_2}} \left[ c_2 + K_2 \sigma_{z_2}(z_2) \tan \varphi_2 \right] \frac{z}{\tan \beta_2} dz \quad (10)$$

where

$$\sigma_{z_1}(z_1) = \sigma_{v0} + \gamma_1 z_1 \quad (11)$$

$$0 \leq z_1 \leq D_1$$

$$\sigma_{z_2}(z_2) = \sigma_{v0} + \gamma_1 D_1 + \gamma_2 z_2 \quad (12)$$

$$0 \leq z_2 \leq D_2$$

$$P_v = \frac{\sigma_{v0} B^2}{\tan((\beta_1 + \beta_2)/2)}$$

$$\sigma_{v0} = \frac{\gamma BL - 2c(B+L)}{2K_0 \tan \varphi(B+L)} \left( 1 - e^{-\frac{2K_0 \tan \varphi(B+L)}{BL} z} \right) + q_0 e^{-\frac{2K_0 \tan \varphi(B+L)}{BL} z}$$

Following the active earth pressure theory, the lateral earth pressure coefficient is taken as  $K_0 = 1 - \sin \varphi$ .  $K_1$  and  $K_2$  denote the lateral earth pressure coefficients of the upper fault stratum and lower intact stratum, respectively, and  $L$  is the axial length of the prism in the wedge model. The corresponding expression for  $L$  is derived as follows:

$$L = \frac{D}{\tan((\beta_1 + \beta_2)/2)}$$

By substituting the rupture-angle parameters in Eq. (12) into the geometric expression for  $L$ , the resulting expression for the prism length is obtained as follows:

$$T_{21} = \frac{1}{2} \left( \frac{l_2 \tan \alpha \tan \beta_2}{\tan \alpha + \tan \beta_2} \right)^2 \frac{c_2 + K_2 (\sigma_{v0} + \gamma_1 D_1) \tan \varphi_2}{\tan \beta_2} + \frac{1}{3} \left( \frac{l_2 \tan \alpha \tan \beta_2}{\tan \alpha + \tan \beta_2} \right)^3 \frac{K_2 \gamma_2 \tan \varphi_2}{\tan \beta_2} \quad (13)$$

where  $K_2 = 1 - \sin \varphi_2$  is the lateral earth pressure coefficient of the lower intact rock stratum.

(2) Shear force  $T_{22}$  acting on block ②

$$T_{22} = \int_0^{\frac{l_2 \tan \alpha}{\tan \alpha + \tan \beta_2}} \left[ c_2 + K_2 \sigma_{z_2}(z_2) \tan \varphi_2 \right] \left( l_2 - \frac{z}{\tan \alpha} \right) dz \quad (14)$$

Substituting Eq. (12) into the above equation gives:

$$T_{22} = \left[ c_2 + K_2 \tan \varphi_2 (\sigma_{v0} + \gamma_1 D_1) \right] \frac{l_2^2 \tan^2 \alpha}{\tan \alpha + \tan \beta_2} + \frac{1}{2} \left( \gamma_2 l_2 K_2 \tan \varphi_2 - \frac{c_2 + K_2 \tan \varphi_2 (\sigma_{v0} + \gamma_1 D_1)}{\tan \alpha} \right) \left( (l_2 \tan \alpha)^2 - \left( \frac{l_2 \tan \alpha \tan \beta_2}{\tan \alpha + \tan \beta_2} \right)^2 \right) - \frac{1}{3} \frac{\gamma_2 K_2 \tan \varphi_2}{\tan \alpha} \left( (l_2 \tan \alpha)^3 - \left( \frac{l_2 \tan \alpha \tan \beta_2}{\tan \alpha + \tan \beta_2} \right)^3 \right) \quad (15)$$

(3) Shear force  $T_{13}$  acting on Block ③

$$T_{13} = \int_{\frac{l_2 \tan \alpha \tan \beta_2}{\tan \alpha + \tan \beta_2}}^{l_2 \tan \alpha} \left[ c_1 + K_1 \sigma_{z1} (z_1) \tan \varphi_1 \right] \left( \frac{z}{\tan \beta_1} - \frac{l_2 \tan \alpha \tan \beta_2}{\tan \beta_1 (\tan \alpha + \tan \beta_2)} + \frac{l_2 \tan \alpha \tan \beta_1}{\tan \alpha + \tan \beta_1} - l_2 + \frac{z}{\tan \alpha} \right) dz \quad (16)$$

Substituting Eq. (11) into the above equation gives:

$$T_{13} = (c_1 + K_1 \sigma_{v0} \tan \varphi_1) \left( \frac{l_2 \tan \alpha \tan \beta_1}{\tan \alpha + \tan \beta_1} - \frac{l_2 \tan \alpha \tan \beta_2}{\tan \beta_1 (\tan \alpha + \tan \beta_2)} - l_2 \right) \left( l_2 \tan \alpha - \frac{l_2 \tan \alpha \tan \beta_2}{\tan \alpha + \tan \beta_2} \right) + \frac{1}{2} \left( (l_2 \tan \alpha)^2 - \left( \frac{l_2 \tan \alpha \tan \beta_2}{\tan \alpha + \tan \beta_2} \right)^2 \right) \left( K_1 \gamma_1 \tan \varphi_1 \left( \frac{l_2 \tan \alpha \tan \beta_1}{\tan \alpha + \tan \beta_1} - \frac{l_2 \tan \alpha \tan \beta_2}{\tan \beta_1 (\tan \alpha + \tan \beta_2)} - l_2 \right) + (c_1 + K_1 \sigma_{v0} \tan \varphi_1) \left( \frac{1}{\tan \beta_1} + \frac{1}{\tan \alpha} \right) \right) + \frac{1}{3} K_1 \gamma_1 \tan \varphi_1 \left( \frac{1}{\tan \beta_1} + \frac{1}{\tan \alpha} \right) \left( (l_2 \tan \alpha)^3 - \left( \frac{l_2 \tan \alpha \tan \beta_2}{\tan \alpha + \tan \beta_2} \right)^3 \right) \quad (17)$$

where  $K_1 = 1 - \sin \varphi_1$  is the lateral earth pressure coefficient of the upper fault fracture zone.

(4) Shear force  $T_{14}$  acting on block ④

$$T_{14} = \int_{l_2 \tan \alpha}^D \left[ c_1 + K_1 \sigma_{z1} (z_1) \tan \varphi_1 \right] \left( \frac{z}{\tan \beta_1} - \frac{l_2 \tan \alpha \tan \beta_2}{\tan \beta_1 (\tan \alpha + \tan \beta_2)} + \frac{l_2 \tan \alpha \tan \beta_1}{\tan \alpha + \tan \beta_1} \right) dz \quad (18)$$

Substituting Eq. (11) into the above equation gives:

$$T_{14} = (c_1 + K_1 \sigma_{v0} \tan \varphi_1) \left( \frac{l_2 \tan \alpha \tan \beta_1}{\tan \alpha + \tan \beta_1} - \frac{l_2 \tan \alpha \tan \beta_2}{\tan \beta_1 (\tan \alpha + \tan \beta_2)} \right) (D - l_2 \tan \alpha) + \frac{1}{2} \left( K_1 \gamma_1 \tan \varphi_1 + \frac{c_1}{\tan \beta_1} \right) (D^2 - l_2^2 \tan^2 \alpha) + \frac{1}{3} \frac{K_1 \gamma_1 \tan \varphi_1}{\tan \beta_1} (D^3 - l_2^3 \tan^3 \alpha) \quad (19)$$

### 3.3 Shear Force on Inclined Sliding Surface

The shear forces  $F_1$  and  $F_2$  acting along the inclined sliding surface are given by:

$$\begin{cases} F_1 = \frac{c_1 B D_1}{\sin \beta_1} + N_1 \tan \varphi_1 \\ F_2 = \frac{c_2 B D_2}{\sin \beta_2} + N_2 \tan \varphi_2 \end{cases} \quad (20)$$

### 3.4 Excavation Face Support Pressure

Based on static equilibrium, separate force decomposition and balance analysis were performed on the upper fault wedge body and lower intact rock wedge body in the direction tangential and normal to the rupture surface:

Force equilibrium equation for the lower intact-rock wedge at the critical sliding state:

$$\begin{cases} N_1 = G_1 \cos \beta_1 + P_v \cos \beta_1 - Q_1 \cos \beta_1 + P_1 \sin \beta_1 \\ F_1 = G_1 \sin \beta_1 + P_v \sin \beta_1 - Q_1 \sin \beta_1 - 2T_1 - P_1 \cos \beta_1 \end{cases} \quad (21)$$

Force equilibrium equation for the upper fault-wedge at the critical sliding state:

$$\begin{cases} N_2 = P_2 \sin \beta_2 + G_2 \cos \beta_2 + Q_2 \cos \beta_2 \\ F_2 = Q_2 \sin \beta_2 + G_2 \sin \beta_2 - P_2 \sin \beta_2 - 2T_2 \end{cases} \quad (22)$$

A unified analytical expression for the ultimate support pressure of the excavation face in inclined fault strata was derived by combining the two equilibrium balance equations and eliminating intermediate variables.

$$\begin{aligned} P = G_2 & \frac{\sin \beta_2 - \cos \beta_2 \tan \varphi_2}{\sin \beta_2 \tan \varphi_2 + \cos \beta_2} \\ & + \frac{\left[ 2T_1 \sin \beta_1 + c_1 BD_1 - P_v \sin \beta_1 (\sin \beta_1 - \cos \beta_1 \tan \varphi_1) \right] (\sin \beta_2 - \cos \beta_2 \tan \varphi_2)}{\sin \beta_1 (\cos \beta_1 \tan \varphi_1 - \sin \beta_1) (\sin \beta_2 \tan \varphi_2 + \cos \beta_2)} \\ & + \frac{\left[ G_1 (\sin \beta_1 - \cos \beta_1 \tan \varphi_1) + P_v (\sin \beta_1 - \cos \beta_1 \tan \varphi_1) \right] (\sin \beta_2 - \cos \beta_2 \tan \varphi_2)}{(\cos \beta_1 \tan \varphi_1 - \sin \beta_1) (\sin \beta_2 \tan \varphi_2 + \cos \beta_2)} \\ & - \frac{(2T_2 \sin \beta_2 - c_2 BD_2)}{\sin \beta_2 (\cos \beta_1 \tan \varphi_1 - \sin \beta_1)} \end{aligned} \quad (23)$$

where

$$T_1 = T_{13} + T_{14}$$

$$T_2 = T_{21} + T_{22}$$

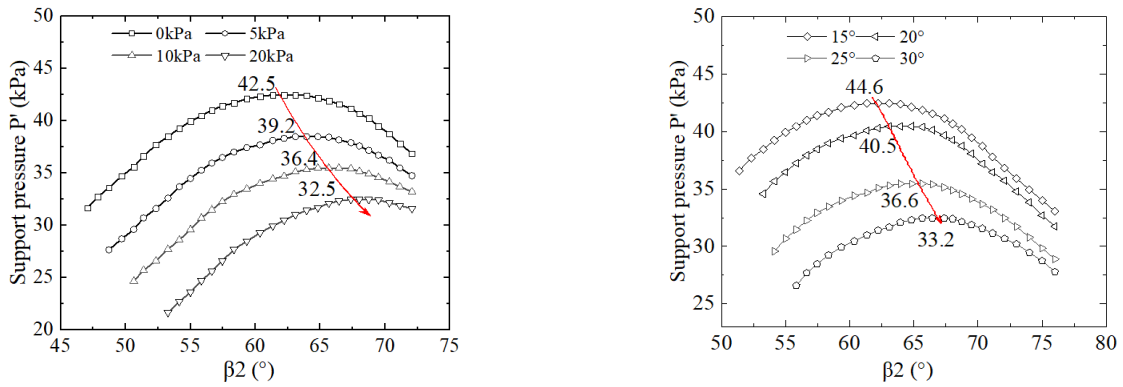
$$G_1 = G_{13} + G_{14}$$

$$G_2 = G_{21} + G_{22}$$

This resulting expression exhibits that when the strength parameters of faults and intact rock are specified, the ultimate support pressure  $P$  depends only on the assumed instability fracture angle  $\beta_2$ , which can be obtained by deriving the support pressure.

#### 4. Parameter Analysis of Ultimate Support Pressure at the Tunnel Excavation Face

In the proposed theoretical model, the rupture angle  $\beta_2$  of the intact rock is an unknown critical parameter, and the ultimate support pressure is a continuous single-variable function of  $\beta_2$ , with  $\beta_2$  varying from  $0^\circ$  to  $90^\circ$ . In this study, an iterative traversal calculation was performed to obtain the relationship curve between  $\beta_2$  and excavation face support pressure. The critical support pressure corresponding to excavation-face instability was determined at the extremum of the support-pressure curve, where its derivative with respect to  $\beta_2$  is zero. The relationship between rupture angle  $\beta_2$  of the intact rock and ultimate support pressure is presented in Fig. 8.



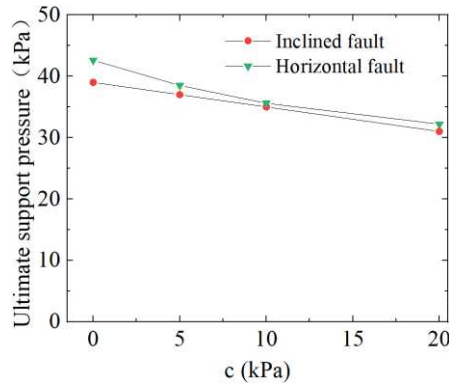
(a) Constant internal friction angle of the intact rock with variable cohesion

(b) Constant cohesion of the intact rock with variable internal friction angle

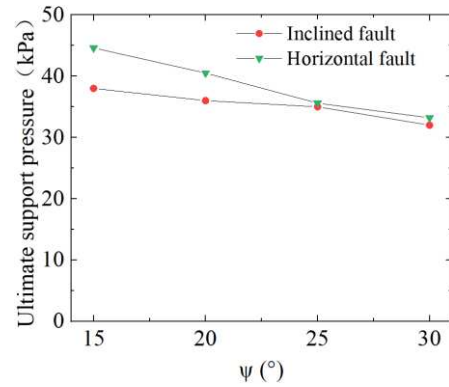
Fig.8 Relationship between rupture angle  $\beta_2$  of intact rock and ultimate support pressure.

The parameter analysis results depict that the ultimate support pressure at the excavation face decreased as the stratum strength parameters increased. For a fixed internal friction angle of the intact rock, the ultimate support pressure decreased continuously, and the critical rupture angle gradually increased as cohesion increased. Similarly, for fixed cohesion, support pressure decreased, while the rupture angle increased as the internal friction angle increased, indicating that higher stratum shear strength enhances excavation-face stability and reduces the required support pressure.

To further evaluate the accuracy and applicability of the modified theoretical model, the theoretical predictions were compared with FLAC3D simulation results under multiple parameter conditions. The comparison curves of the theoretical and numerical predictions are presented in Fig. 9.



(a) Constant internal friction angle of intact rock with variable cohesion



(b) Constant cohesion of intact rock with variable internal friction angle

Fig.9 Comparison between numerical and theoretical predictions of ultimate support pressure.

The results indicate that under inclined fault conditions, the ultimate support pressure at the tunnel excavation face decreased from 42 to 31 kPa as soil cohesion increased from 0 to 20 kPa and the internal friction angle increased from 15° to 30°. Higher shear strength of the surrounding rock mass enhances its self-stabilization capacity, thereby reducing the minimum support pressure required to maintain excavation face stability. The theoretical predictions are consistent with the numerical simulation results in terms of variation trends. Under the initial parameter condition ( $c = 0$  kPa,  $\varphi = 15^\circ$ ), the theoretical prediction was 43 kPa, and the numerical simulation was 38 kPa, with an absolute error of 5 kPa and a maximum relative error of 13.2% across all working conditions. When the cohesion increased to 10 kPa and the internal friction angle reached 25°, both the theoretical and numerical predictions were approximately 35 kPa with a relative error of approximately 0%. When the cohesion increased to 20 kPa and the internal friction angle reached 30°, the theoretical solution was 32 kPa and the numerical solution was 31 kPa, with a minimum relative error of 3.2%.

The overall relative error of the proposed model ranged from 3.2% to 15%, and the error gradually decreased as the stratum shear strength parameters increased, indicating good agreement between the theoretical and numerical results. The slightly conservative theoretical predictions may provide sufficient safety redundancy for practical tunnel support design. The comparison results indicate that the proposed theoretical model can accurately reflect the mechanical evolution mechanism of excavation face instability in inclined fault strata under the assumptions adopted in this study. The model exhibits potential for engineering application and may be used for stability evaluation and support parameter optimization for shield tunneling through fault fracture zones.

## 5. Conclusions

In this study, the influence of the fault dip angle on the excavation-face stability was considered. Starting from the conventional wedge instability model for homogeneous strata, an associated flow rule was introduced to improve the wedge model. The performance of the improved model was evaluated against three-dimensional numerical simulations for inclined fault strata. The conclusions are as follows:

(1) The instability characteristics of the excavation face in inclined fault strata are different from those in a homogeneous stratum; specifically, the fault interface restricts the outward extension of the unstable region, and the unstable region extends upward from the fault interface toward the region above the excavation face. Therefore, an improved wedge calculation model that considers the influence of the fault dip angle was proposed.

(2) The associated flow rule was introduced to establish the coordination relationship between the rupture angles of the upper and lower strata. Based on the limit equilibrium approach, the analytical expression for the limit support pressure at the excavation face in an inclined fault strata was derived, and the corresponding calculation process was established.

(3) Through the comparison with three-dimensional numerical simulations, the numerical simulation results were consistent with the variation trends predicted by the theoretical model, and the relative error across all parameter conditions ranged from 3.2% to 15%, indicating good agreement between the theoretical and numerical results, which can satisfy the requirements of tunnel engineering design.

(4) The stratum strength parameter and fault dip angle are the key factors controlling the limit support pressure at the excavation face in the fault section. The proposed analytical method in this study can quantitatively estimate the limit support pressure of the excavation face in fault strata, which may be useful for preliminary assessment of construction support parameters and evaluating excavation-face stability during slurry-shield tunneling through the fault fracture zones.

## Acknowledgments

The authors gratefully acknowledge the financial support of the Shandong Provincial Natural Science Foundation (Grant No. ZR2024QE046, ZR2024ME019), the Scientific Research Program Project of China Railway Construction Corporation Limited (CRCC) in the Field of Underground Space Utilization (Grant No. 2024-W09), and the Major Science and Technology Special Project of China Railway 14th Bureau Group Co., Ltd. (Grant No. 14J2024-Z01).

**Conflicts of interest**

The authors declare that the work described has not been published before; that it is not under consideration for publication anywhere else; that its publication has been approved by all co-authors; that there is no conflict of interest regarding the publication of this article.

**Conflicts of interest**

The datasets generated and/or analysed during the current study are not publicly available due to geotechnical site confidentiality restrictions but are available from the corresponding author on reasonable request.

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