

Risk-at-Risk: A Quantile-Based Indicator of Diversification Vulnerability

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Abstract

This paper introduces the *Risk-at-Risk*, a dynamic measure of the relationship between the internal downside-risk structures of risky and hedge assets. We first use the Cross-Quantilogram to estimate lower-tail dependence within each group and aggregate these relationships into separate risk concentration measures. We then estimate the local relationship between the two concentration states through a rolling regression, with the slope coefficient defining the *Risk-at-Risk*. Using daily data from 1997 to July 2026, we find substantial time variation in this relationship, including pronounced changes around the global financial crisis, the early 2010s, and 2017-2018. The lower-tail specification differs materially from median and Pearson-based dependence and is only weakly associated with direct lower-tail dependence between risky and hedge assets. Emerging markets exhibit higher diversification vulnerability and faster post-crisis recovery than advanced economies. Overall, the *Risk-at-Risk* provides a complementary indicator of diversification vulnerability.

Keywords: systemic risk ; risk-at-risk ; cross-quantilogram ; tail risk ; risk concentration ; diversification

1 Introduction

Portfolio diversification is most valuable precisely when it is hardest to obtain: during periods of market stress, when assets that ordinarily offer distinct sources of return and protection can become more closely dependent (Torrente and Uberti, 2021; Ergen, 2014; Zhou and Qin, 2021). Although the literature measures direct cross-asset dependence, connectedness, and the safe-haven properties of individual assets, it remains unclear whether downside risk concentration within equity markets becomes systematically aligned with downside risk concentration within assets commonly used to hedge equity risk.

To examine this issue, we distinguish between risky assets and hedge assets. The risk-asset group consists of national equity-market indices spanning advanced and emerging economies, examined both jointly and within the two market groups. The hedge asset group comprises major currencies, gold, the US dollar index, and long-term US Treasury-yield indicators, markets commonly used by investors when hedging against risky assets (Baur and Lucey, 2010; Baur and McDermott, 2010; Ciner *et al*, 2013; Cheema *et al*, 2022). Under normal conditions, these groups play distinct portfolio roles: risky assets provide exposure to equity-market returns, whereas hedge assets may provide diversification when their behavior remains sufficiently distinct (Ciner *et al*, 2013; Flavin *et al*, 2014). However, the risk structures of these groups are not constant over time. Changes in the dependence among assets within each group can alter the extent to which they provide distinct sources of protection, as well as risk diversification (Ergen, 2014; Zhou and Qin, 2021). Monitoring how these internal risk structures evolve over time is therefore important for understanding how diversification conditions and potential diversification vulnerability may change across market environments.

The literature offers several approaches to characterizing risk: VaR measures individual

tail risk, CoVaR assesses system risk conditional on institutional stress, and connectedness measures aggregate pairwise relationships into system-wide indicators (Adrian and Brunnermeier, 2016; Kupiec, 1998; Billio *et al*, 2012; Diebold and Yilmaz, 2014). More recent approaches extend these ideas to different regions of the return distribution, including quantile based and Cross-Quantilogram measures (Ando *et al*, 2022; Baumöhl *et al*, 2022; Qian *et al*, 2022). These approaches provide important information about dependence, interconnectedness, and the propagation of shocks within financial systems.

A related literature examines the role of hedge and safe haven assets in portfolio diversification. This literature shows that their protective properties can vary substantially across market conditions, while stronger dependence during adverse states can reduce diversification benefits (Baur and Lucey, 2010; Baur and McDermott, 2010; Flavin *et al*, 2014; Klein, 2017; Cheema *et al*, 2022). Thus, diversification conditions are state dependent, particularly when downside dependence changes.

Taken together, existing studies offer measures of bilateral return co-movement, system-wide connectedness, direct associations between risk and hedge assets, and the protective properties of individual hedge assets. They do not, however, examine whether downside risk becomes concentrated within both the risk-asset and hedge asset groups on the same temporal window. This question differs from one of direct transmission: it concerns the temporal alignment of two internal risk-concentration processes. We therefore examine the co-movement between the internal downside-risk structures of risk and hedge assets.

To address this gap, we first estimate risk concentration separately for risky assets and hedge assets by aggregating pairwise lower-tail co-movement among the assets within each group. Each risk-concentration measure therefore characterizes the internal downside-risk structure of one asset group. We then estimate rolling regressions of hedge asset risk concentration on risk-asset risk concentration and define the resulting slope coefficient as the *Risk-at-Risk*. A positive *Risk-at-Risk* value indicates that, within a given

rolling estimation period, higher risk concentration among risky assets is associated with higher risk concentration among hedge assets. *Risk-at-Risk* therefore measures the relationship between two internal downside-risk structures rather than direct transmission between the two groups. As a dynamic monitoring indicator of diversification vulnerability, high positive *Risk-at-Risk* values identify periods in which the two structures are more closely aligned; when this alignment is accompanied by elevated risk concentration in both groups, the assets may offer fewer independent sources of downside exposure.

Using this framework, we estimate the proposed indicator for global, advanced, and emerging equity markets. The results show substantial time variation in the relationship between the risk concentrations of risk and hedge assets, with stronger positive relationships during several periods of financial stress. The lower-tail specification also follows a materially different trajectory from the median specification, indicating that the relationship observed during adverse-return states cannot be inferred from dependence around the center of the return distribution. The estimates for advanced and emerging economies further reveal differences in the evolution and variability of this relationship across markets.

This study makes three contributions. First, it shifts attention from individual asset pairs and direct cross-asset associations to the time-varying alignment of within-group lower-tail risk concentration across risk and hedge assets. Second, it offers a monitoring perspective on diversification vulnerability by identifying periods in which the two groups may provide less distinct sources of downside exposure. Third, by applying the framework to the full sample and separately to advanced and emerging equity markets, the study provides international evidence on how this alignment varies over time and differs between lower-tail and median return conditions.

The remainder of the paper describes the economic motivation and statistical methodology, presents the empirical results, examines the behavior of the proposed measure across

different market groups, and discusses its implications and limitations.

2 Measuring Dependence and Risk Concentration

Understanding dependence among financial assets is central to portfolio diversification, because the extent to which assets provide distinct sources of risk depends on how strongly their returns move together. To characterize this dependence, researchers commonly rely on correlation-based measures such as the Pearson correlation coefficient. These measures are widely used because of their simplicity and ease of interpretation, providing a useful summary of the average linear co-movement between financial assets. Correlation-based approaches have consequently played an important role in the study of interdependence across financial markets, including international equity markets (Eun and Shim, 1989).

However, while correlation-based measures provide valuable information about asset co-movement, they may overlook important aspects of the dependence structure in financial markets. In particular, these measures summarize relationships around the mean but do not capture variation throughout the entire distribution, potentially discarding valuable information. This is especially relevant because financial dependence can change substantially during periods of market stress, when relationships between assets may differ from those observed under more typical conditions and may not be homogeneous throughout the return distribution. Evaluating the entire distribution is especially relevant when the economic interest lies in large downside movements, since dependence associated with extreme losses can differ materially from that implied by conventional correlation measures (Ergen, 2014). Thus, average dependence remains informative but may provide only partial information when relationships vary across different regions of the return distribution (Ando *et al*, 2022; Han *et al*, 2016).

One way to address this limitation is to examine dependence at different points of the

return distribution rather than summarizing it through a single average relationship. Quantile-based measures provide this flexibility by allowing researchers to investigate how dependence changes across different regions of the distribution and under different market conditions. Existing evidence shows that financial connectedness can vary substantially across quantiles, with extreme and more typical market states exhibiting different dependence structures (Qian *et al*, 2022). In practice, the quantile of interest can therefore be selected according to the economic question being examined. In our analysis, we are particularly interested in adverse market outcomes. The lower-tail of the return distribution captures infrequent but severe negative returns and therefore provides a natural setting for examining downside risk. Rather than being imposed by the methodology, this focus reflects our interest in the dependence associated with large losses. We therefore concentrate primarily on the lower-tail while using other regions of the distribution for comparison.

Our analysis can be viewed in levels as described in figure 1. For the first level, which is the asset level, we follow Han *et al* (2016), using the Cross-Quantilogram to measure dependence between financial assets at the selected quantile. By focusing on the lower-tail, we examine whether adverse-return events in one asset tend to occur together with adverse-return events in another. This distinction is important because dependence observed during large negative returns may differ substantially when measured at the mean or at the tails. The Cross-Quantilogram therefore provides a way to characterize downside dependence directly, while retaining the flexibility to examine other regions of the return distribution when appropriate (Han *et al*, 2016). In our setting, this dependence additionally captures the extent to which the lower-tail states of two assets are associated contemporaneously. Another important distinction is that because the measure characterizes association rather than direction, it does not require the assumption that shocks are transmitted from one asset to another.

The second-level analysis evaluates dependence among assets within the same group.

Building on Baumöhl *et al* (2022) and Qian *et al* (2022), we measure lower-tail dependence between pairs of financial assets and aggregate these pairwise relationships within economically defined asset groups. We refer to the resulting measure as *risk concentration*. A higher level of risk concentration indicates that adverse-return states are more strongly associated across the assets in the group, reflecting a more concentrated internal downside-risk structure. Because it aggregates contemporaneous associations rather than directional effects, risk concentration describes the extent to which adverse states occur jointly within a group; it does not establish whether one asset transmits risk to another.

The third level of analysis examines how the internal risk-concentration structures of the two asset groups evolve in relation to one another. The relevant question is not simply whether either group exhibits a high level of risk concentration at a particular point in time, but whether higher concentration in risky assets is systematically associated with higher concentration in hedge assets. This relationship is economically relevant from a diversification perspective because it indicates whether the groups' internal downside-risk structures are becoming more closely aligned. As this relationship strengthens, the diversification roles of the two groups may become less distinct, even when the risk concentration of either group alone does not fully reveal this vulnerability (Torrente and Uberti, 2021; Ergen, 2014; Flavin *et al*, 2014). We refer to this dynamic relationship between the two risk-concentration measures as *Risk-at-Risk*.

A positive relationship between the risk-concentration measures of different asset groups may reflect common global conditions, distinct shocks occurring in the same period, or related changes in dependence without direct risk transmission between groups. Such co-movement indicates interdependence, but not necessarily contagion or direct shock propagation. This distinction is consistent with research separating common-factor co-movement from contagion through changing transmission mechanisms (Forbes and Rigobon, 2002; Bekaert, Campbell R Harvey, *et al*, 2005), and with the safe-haven literature's distinction among common shocks, idiosyncratic shocks, and contagion (Flavin *et al*, 2014).

Accordingly, *Risk-at-Risk* measures the association between the internal downside-risk structures of different asset groups without having to assume an underlying spillover mechanism.

An important distinction is that our perspective differs from connectedness approaches whose principal objective is to characterize the propagation of shocks or uncertainty across a financial system. Measures based on forecast error variance decompositions, for example, quantify the extent to which shocks associated with one variable contribute to the forecast uncertainty of others (Diebold and Yilmaz, 2014), while quantile connectedness approaches extend related questions to different regions of the return distribution (Ando *et al*, 2022). Our framework instead first characterizes the internal downside risk structure of risky assets and hedge assets separately and then examines the dynamic relationship between the resulting group level concentration states. The next section develops the statistical methodology used to measure lower-tail dependence, construct the two risk concentration measures, and estimate their relationship through the *Risk-at-Risk*.

3 Statistical Methodology

This section describes the methodology used to construct the *Risk-at-Risk* indicator. We first measure lower-tail dependence between individual financial assets using the Cross-Quantilogram. We then aggregate these measures within the risky asset and hedge asset groups to construct group-level risk-concentration measures. Finally, we estimate the local relationship between the two risk concentration series through a rolling regression, with the resulting slope coefficient defining the *Risk-at-Risk*.

3.1 Risk Dependence

Following the conceptual motivation developed in Section 2, we measure pairwise lower-tail dependence using the Cross-Quantilogram of Han *et al* (2016). Unlike Pearson correlation, which summarizes average linear co-movement and relies on finite second moments, the Cross-Quantilogram is constructed from quantile-hit processes and allows dependence to be examined at selected regions of the conditional return distribution. It is therefore well suited to measuring lower-tail dependence without imposing a linear relationship between asset returns.

Specifically, we define lower-tail risk dependence between two assets as the contemporaneous association between the events that their returns fall below their respective conditional quantiles. These events are represented using quantile-hit indicators.

Formally, define the adverse-return event for asset i at time t as

$$\mathbf{1}\{y_{it} \leq q_{i,t}(\tau)\} = 1, \quad (1)$$

where y_{it} denotes the return of asset i at time t , and $q_{i,t}(\tau)$ denotes its τ -quantile. Following the value-at-risk convention, we focus on the lower-tail by setting $\tau = 0.05$, so that the indicator identifies observations in the lower 5% of the return distribution.

For two financial assets with return series y_{1t} and y_{2t} , the Cross-Quantilogram of Han *et al* (2016) measures dependence between their quantile-hit processes. Specifically, for quantile levels $\tau_1, \tau_2 \in (0, 1)$, it measures the dependence between the events

$$\{y_{1t} \leq q_{1,t}(\tau_1)\} \quad \text{and} \quad \{y_{2t} \leq q_{2,t}(\tau_2)\}. \quad (2)$$

In our application, we set $\tau_1 = \tau_2 = 0.05$ and evaluate the relationship at lag zero. The

resulting statistic therefore captures the contemporaneous dependence between adverse-return events of the two assets within each rolling estimation window. These pairwise measures are subsequently aggregated across assets within each group to construct the group-level risk-concentration measure.

3.2 Risk Concentration

Using the risk dependence measure, we construct a risk-concentration measure that summarizes the overall dependence among assets within a given group. Risk concentration aggregates the pairwise dependence between the assets in the group into a single measure, thereby characterizing the strength of the group’s internal dependence structure.

The risk concentration, denoted as $RC(\tau)$, is determined by a re-scaled summation of risk dependence between each pair of assets, where τ signifies the quantile level. A higher value of risk concentration indicates a greater degree of cross-country interdependence within the equity markets. Note that the summation of risk dependence is scaled by a ratio of total time periods and the number of assets considered to standardize the risk concentration, ensuring that $RC(\tau)$ asymptotically converges to a standard normal distribution in the absence of risk dependence.¹ This normalization accounts for the size of the dataset T and the number of assets N . Under the alternative hypothesis where risk dependence is present, the risk concentration escalates to infinity as the level of risk dependence intensifies.

Formally, let $\hat{\rho}_{ij,t}(\tau)$ denote the sample lag zero Cross-Quantilogram between assets i and j , estimated using a window of T observations ending at estimation point t . For a group containing N assets, risk concentration is defined as

$$RC_t(\tau) = \sqrt{\frac{2T}{N(N-1)}} \sum_{i < j} \hat{\rho}_{ij,t}(\tau). \quad (3)$$

The summation includes all $N(N-1)/2$ distinct asset pairs. Because the Cross-Quantilogram is signed, $RC_t(\tau)$ measures net rather than absolute dependence. Higher values indicate stronger net lower-tail dependence within the group, whereas negative pairwise relationships reduce the aggregate measure.

The scaling factor adjusts the aggregate measure for the estimation-window length and the number of pairwise relationships entering the calculation (Wang *et al*, 2021). We use this scaling as a normalization and interpret $RC_t(\tau)$ primarily through its evolution within each asset group rather than through comparisons of its absolute magnitude across groups of different sizes.

We construct $RC_t(\tau)$ separately for risky assets, denoted $RC_t^E(\tau)$, and hedge assets, denoted $RC_t^H(\tau)$. Each measure therefore characterizes the internal downside structure of one economically defined asset group. As discussed in Section 2, we categorize all financial assets considered into two groups: risky assets and hedge assets. Therefore, we measure the risk concentration for each type of asset separately. Risky assets are constructed using equity returns from various countries, while hedge assets comprise commonly used assets employed by portfolio managers to diversify the risk of equity markets. $RC_t(\tau)$ should be interpreted as an indicator of internal downside concentration rather than as a direct measure of realized portfolio diversification.

To allow the internal dependence structure to evolve over time, we estimate risk concentration using rolling windows of 500 daily observations, advanced in 20-trading-day steps, following the rolling specification used in related Cross-Quantilogram applications (Qian *et al*, 2022). The selection of the estimation window reflects a trade-off between finite-sample precision and responsiveness to changing market conditions. At $\tau = 0.05$,

a 500-observation window contains approximately 25 lower-tail observations per asset on average. A substantially shorter window could therefore produce noisy and unstable pairwise Cross-Quantilogram estimates, whereas a considerably longer window could smooth over recent changes in the underlying dependence structure. The 500-observation window provides a practical compromise, using approximately two years of daily market information while retaining sensitivity to changes over time.

Advancing the estimation window by 20 trading observations produces approximately monthly updates of the risk-concentration measures while retaining daily information within each window. Successive values of $RC_t(\tau)$ should therefore be interpreted as overlapping estimates of the group’s internal dependence structure rather than as estimates based on independent samples.

Standardized aggregations of pairwise dependence are not unique to our framework. Related measures have been constructed using Pearson correlations (Wang *et al.*, 2021), while system-level co-movement has also been examined through variance-based connectedness measures (Diebold and Yilmaz, 2014). The distinguishing feature of $RC_t(\tau)$ is that it aggregates quantile-specific dependence and therefore characterizes connectedness during lower-tail states rather than average co-movement.

3.3 The Risk-at-Risk Indicator

The construction of the *Risk-at-Risk* indicator aims to assess the relationship between risk concentration in risky assets and hedge assets. This is achieved through a regression framework, where the risk concentration of hedge assets $RC_{hedge}(\tau)$ is regressed on the risk concentration of risky assets $RC_{risky}(\tau)$.

Here the estimated slope coefficient β captures the relationship between the risk con-

centrations of the two groups of assets. A positive and sizable β indicates a strong positive relationship between the risk concentrations of hedge assets and equity markets. Consequently, higher risk concentration in equity markets is associated with higher risk concentration in hedge assets. From a risk diversification standpoint, a positive and substantial β suggests that risk diversification using hedge assets may be less effective, highlighting the challenges in effectively diversifying risk from equity markets to hedge assets.

The *Risk-at-Risk* measures the local relationship between the internal risk concentration of risky assets and that of hedge assets. Having constructed $RC_t^E(\tau)$ and $RC_t^H(\tau)$ at common estimation points, we estimate

$$RC_s^H(\tau) = \alpha_t(\tau) + \beta_t(\tau)RC_s^E(\tau) + \varepsilon_s. \quad (4)$$

over a rolling window containing the most recent 60 risk concentration observations. The slope coefficient $\beta_t(\tau)$ defines the *Risk-at-Risk* at estimation point t .

Therefore, β provides an indicator of diversification vulnerability. A higher value of β indicates a stronger relationship between the risk concentration of risky and hedge assets, suggesting that the internal downside-risk structures of the two groups are becoming more closely aligned. During periods of market stress, when risk concentration within equity markets increases, a positive and elevated β indicates that risk concentration within hedge assets tends to increase as well, pointing to greater diversification vulnerability.

The indicator constructed in this section provides, to the best of our knowledge, a distinct way of measuring the co-movement between the tail risk concentrations of two asset groups. It is a scalar and continuous measure that summarizes this relationship through a rolling regression and permits statistical inference around the estimated coefficient. Tail

co-movement has been examined at the system level using parametric tools. For example, Ando *et al* (2022) estimate a quantile vector autoregression with common factors to trace how the topology of a credit network varies with the size of the shock. Their approach, however, characterizes a model-based network structure rather than providing a single indicator of the relationship between the tail-risk concentration of two asset groups. Because β is estimated over rolling windows and accompanied by confidence intervals at each date, it also provides a direct framework for monitoring changes in this relationship over time. As Betz *et al* (2016) note, tail dependence is particularly relevant for financial surveillance.

4 Empirical Analysis

This section provides an overview of the data used in our empirical analysis, encompassing equity market returns and hedge asset returns. We also provide details on the methodology for measuring risk dependence, risk concentration, and the dynamic relationship between them. Furthermore, estimates of *Risk-at-Risk* are presented to offer general insights into our analysis.

4.1 Data

The equity sample consists of a broad set of national stock market indices divided into advanced and emerging market groups, covering the period from January 1997 through July 2026. The data were collected from publicly accessible third-party sources, including Yahoo Finance, The Wall Street Journal, and other online and API-based financial data sources, using historical observations for the variables included in the analysis. The full candidate universe contains 39 equity indices. The advanced market group includes indices from the United States, United Kingdom, France, Germany, Japan, Canada, South

Korea, Netherlands, Switzerland, Ireland, Sweden, Greece, Australia, Singapore, New Zealand, Israel, Italy, Spain, Norway, and Poland. The emerging market group contains indices from Taiwan, Thailand, Malaysia, Brazil, Turkey, Indonesia, the Philippines, Chile, Saudi Arabia, China, India, Russia, Mexico, South Africa, Argentina, Vietnam, the United Arab Emirates, and Colombia.

The hedge group contains assets commonly associated with international portfolio diversification, including major currencies, gold, the US dollar index, and US Treasury yield indicators. The candidate set includes the US dollar index (DXY), Chinese yuan (CNY), Japanese yen (JPY), Swiss franc (CHF), pound sterling (GBP), Canadian dollar (CAD), Australian dollar (AUD), 10-year and 30-year US Treasury yields, and gold (XAU).

The selection of emerging market assets and hedge assets is motivated by their importance in international portfolio management. Emerging market assets can provide opportunities for portfolio diversification, although their diversification properties depend on the characteristics of the markets and the sources of cross-market dependence. In particular, evidence from emerging equity markets documents substantial cross-sectional variation in expected returns associated with firm- and market-level characteristics (Hanauer and Lauterbach, 2019), underscoring the heterogeneity within this asset class. Moreover, the diversification benefits of emerging-market equities can be particularly sensitive to dependence in adverse market states (Ergen, 2014; Zhou and Qin, 2021). These considerations motivate the inclusion of emerging-market equities in our analysis alongside advanced-market equities.

On the other hand, hedge assets play a crucial role in managing risk and preserving capital during periods of market uncertainty. Gold, in particular, has been extensively studied as a potential hedge and safe haven for equity and financial-market risk. Evidence also suggests that gold can provide long-run protection against inflation, although its hedging effectiveness varies across markets and time horizons (Bampinas and Panagiotidis, 2015;

Hoang *et al*, 2016). More generally, the safe-haven properties of gold and other non-equity assets can vary substantially across market conditions, with their effectiveness depending on the nature and severity of market stress (Baur and Lucey, 2010; Baur and McDermott, 2010; Flavin *et al*, 2014).

4.2 *AR filter*

When assessing risk dependence, we adopt a cautious approach by avoiding direct use of raw data for asset returns. Instead, we opt for a methodological refinement wherein each return series undergoes fitting via an autoregressive (AR) model, resulting in residuals known as AR-filtered asset returns. This strategic choice is motivated by the prevalent observation of long-term dependence in financial returns, a phenomenon that can introduce distortions when constructing dynamic measures for risk dependence and risk concentration of assets.

Long-term dependence poses a particular challenge in capturing market signals during significant events, as such signals may become obscured by other observations within the data window or conflated with signals from previous periods. This phenomenon is particularly evident in traditional frameworks where market correlation is assessed using the Pearson correlation coefficient and raw data are employed without AR-filtering, often leading to signal smoothing at individual time points.

However, our methodological framework addresses this issue effectively through two key mechanisms. Firstly, the utilization of AR-filtered return series helps mitigate long-run dependence, ensuring that the residuals used for risk concentration predominantly reflect current market shocks and changes. Consequently, substantial changes in residual values of returns are attributed to present market dynamics rather than historical factors.

Secondly, we use the Cross-Quantilogram methodology instead of correlation, selecting quantiles such as $\tau = 0.05$ or 0.1 to represent the tails of the return distribution. This strategic choice allows us to retain the influence of significant changes in the tails, particularly during market crises, where extreme observations in the distribution are prioritized. Importantly, these extreme observations are less influenced by other data points compared to measures of central tendency like the mean.

This approach underscores the value of quantile-based measures in capturing extreme events or risks in financial markets, where rare but impactful occurrences can significantly shape overall market dynamics. Consequently, the adoption of the Cross-Quantilogram over correlation proves advantageous, as it adeptly captures the heterogeneous relationship between variables across different quantiles, particularly how the tails of one variable correlate with the tails of another.

In contrast, correlation primarily reflects the average change in one variable if the other changes by a certain amount, a metric that may not fully encapsulate the dynamics of extreme events or tail risks. By embracing these methodological refinements, our research methodology offers a robust and insightful framework for assessing risk dependence and concentration in financial markets.

4.3 Risk Concentration

As discussed in Section 2, the risk concentration metric is constructed based on the total amount of risk dependence within a set of financial assets. This metric measures the overall level of risk connectedness among these assets. We separately compute the risk concentration measures for global equity markets and a basket of hedge assets. This differentiation allows us to capture the distinct dynamics within each category, which can further assist our understanding of *Risk-at-Risk* in the financial system in later sections.

Figure 1 presents the lower-tail specification at $\tau = 0.05$. Equity risk concentration varies substantially over time and increases markedly around periods of severe market stress. In particular, concentration rises around the global financial crisis and reaches another pronounced high during the COVID-19 period. This pattern is consistent with prior evidence that financial connectedness varies substantially during periods of market stress (Billio *et al*, 2012). The subsequent decline indicates that the exceptional internal downside concentration observed during the pandemic was not permanent.

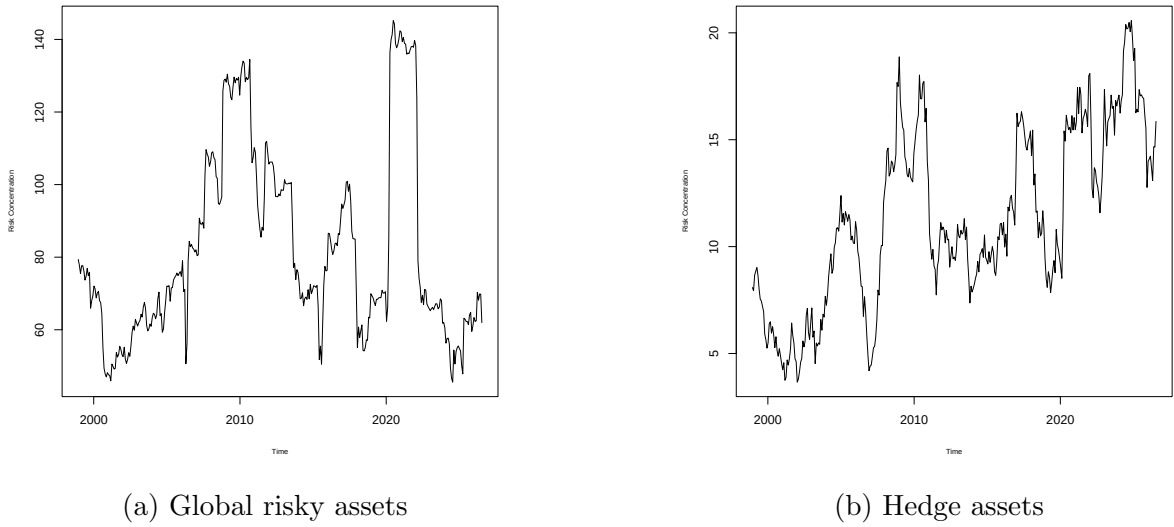


Figure 1: Lower-tail risk concentration at $\tau = 0.05$. Panel (a) reports the internal lower-tail risk concentration of the global risky-asset group, and Panel (b) reports the corresponding concentration of the hedge asset group.

The hedge asset group likewise displays substantial time variation in lower-tail risk concentration, although its evolution differs from that of equity markets. Concentration increases around the global financial crisis, during parts of the late 2010s, and throughout the COVID-19 and post-COVID-19 periods. This variation during major stress episodes is consistent with evidence that the behavior and protective properties of traditional safe-haven assets changed materially between the global financial crisis and the COVID-19 pandemic (Cheema *et al*, 2022). However, the two groups do not exhibit a one-to-one correspondence: at several points, equity risk concentration changes markedly while hedge asset concentration follows a different pattern.

This comparison alone does not determine whether changes in the two concentration measures are systematically related. We therefore use *Risk-at-Risk* to estimate their dynamic relationship directly. Figure 2 presents the corresponding estimates at the median, $\tau = 0.50$. The median and lower-tail specifications exhibit materially different dynamics, indicating that dependence during adverse-return states cannot be inferred from dependence around the center of the return distribution.

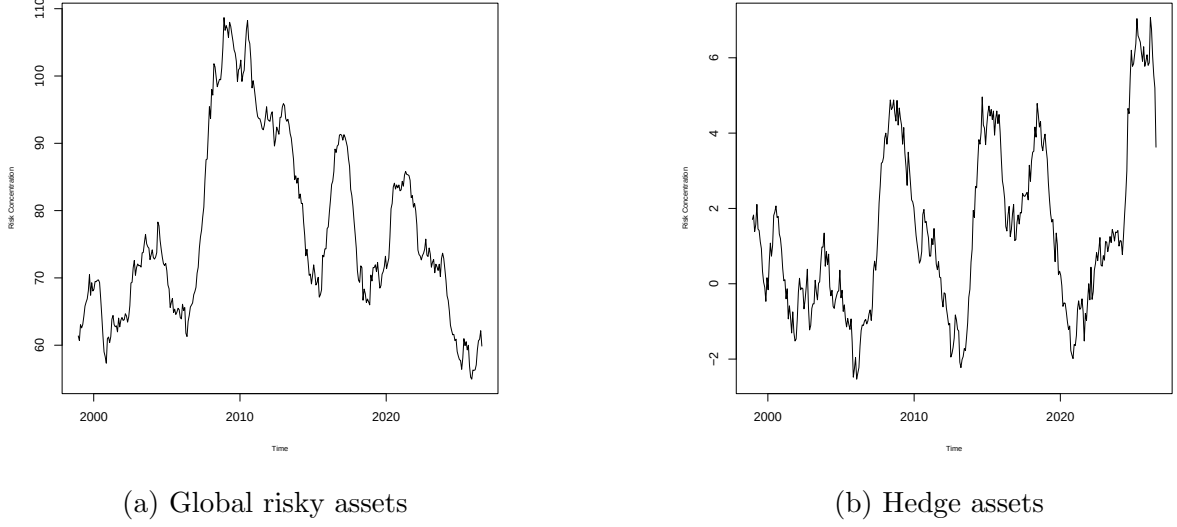


Figure 2: Median risk concentration at $\tau = 0.50$. Panel (a) reports the internal risk concentration of the global risky-asset group around the center of the conditional return distribution, and Panel (b) reports the corresponding concentration of the hedge asset group.

4.4 Dynamic Relationship between Risk Concentrations

The *Risk-at-Risk* indicator acts as a dynamic measure, offering insights into market changes and responses to shocks in the financial landscape. To capture the evolving nature of *Risk-at-Risk*, we implement a rolling window scheme for estimating risk dependence and risk concentration, thereby tracking risk dynamics through $\beta(t)$. While the AR-filter and quantile-based approach enhance our ability to capture real-time market shifts, the choice of window size in the rolling window scheme also plays a crucial role in capturing recent developments or events in the data.

Besides the given differences between average behavior and tail described, we examine the rolling relationship between equity and hedge risk concentration. Figure 3 presents the lower-tail *Risk-at-Risk*, $\beta_t(\tau = 0.05)$.

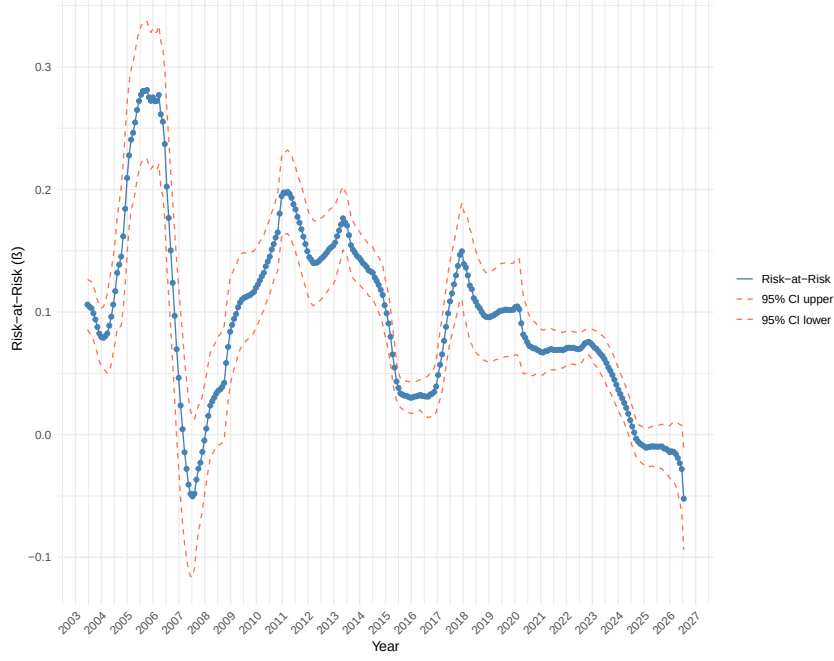


Figure 3: Lower-tail *Risk-at-Risk* at $\tau = 0.05$.

Figure 3 illustrates the evolution of the *Risk-at-Risk* coefficient from mid-2003 to mid-2026. The blue line shows the estimated coefficient, $\beta_t(\tau = 0.05)$, while the red dashed lines indicate its 95% confidence interval. The coefficient is positive from mid-2003 and rises before the subprime crisis, reaching a peak shortly before the onset of the global financial crisis. This pattern indicates a strengthening alignment between the internal downside-risk structures of equity and hedge assets, implying greater diversification vulnerability.

A second period of relatively strong positive association occurs during the early 2010s. The coefficient also increases markedly around 2017-2018, indicating a renewed strengthening in the relationship between the two risk-concentration measures. These episodes show that substantial variation in *Risk-at-Risk* is not confined to a single crisis period.

The COVID-19 episode provides an important contrast. Although equity risk concentra-

tion increases sharply, $\beta_t(\tau = 0.05)$ does not exhibit a comparable increase. This occurs despite broader evidence of a pronounced change in cross-asset connectedness around the COVID-19 outbreak (Bouri *et al*, 2021). This result shows that a rise in equity risk concentration, RC^E does not necessarily imply a stronger relationship between the concentration states of equity and hedge assets. Following the pandemic, the coefficient declines towards zero and becomes negative near the end of the sample.

An advantage of *Risk-at-Risk* is that it enables monitoring without requiring each change in the relationship to be attributed to a specific event. For example, US-China trade tensions, Federal Reserve rate hikes, Brexit, and emerging-market turbulence may all have contributed to changing market conditions in 2018. Related evidence documents substantial changes in stock-market connectedness during the US-China trade-war period, including stronger effects in downside market conditions (Bissoondoyal-Bheenick *et al*, 2022). Rather than identifying the separate effect of each event, the indicator summarizes their potential association with the alignment of the two groups' internal downside-risk structures. Positive and rising values of $\beta(t)$ signal increasing diversification vulnerability, as the risk concentration of hedge assets becomes more closely aligned with that of risky assets.

Figure 4 presents the median specification, $\beta_t(\tau = 0.50)$. The coefficient is generally positive during the first part of the sample and reaches its highest levels in the early 2010s, but becomes predominantly negative from approximately the middle of the 2010s onward. This divergence from the lower-tail specification is economically important because it shows that the relationship between the internal risk structures of the two asset groups depends on the region of the return distribution. This is consistent with previous evidence that financial dependence and connectedness can differ substantially between tail and central market states (Ergen, 2014; Zhou and Qin, 2021; Qian *et al*, 2022). During much of the later sample, a positive lower-tail association coexists with a negative relationship at the median.

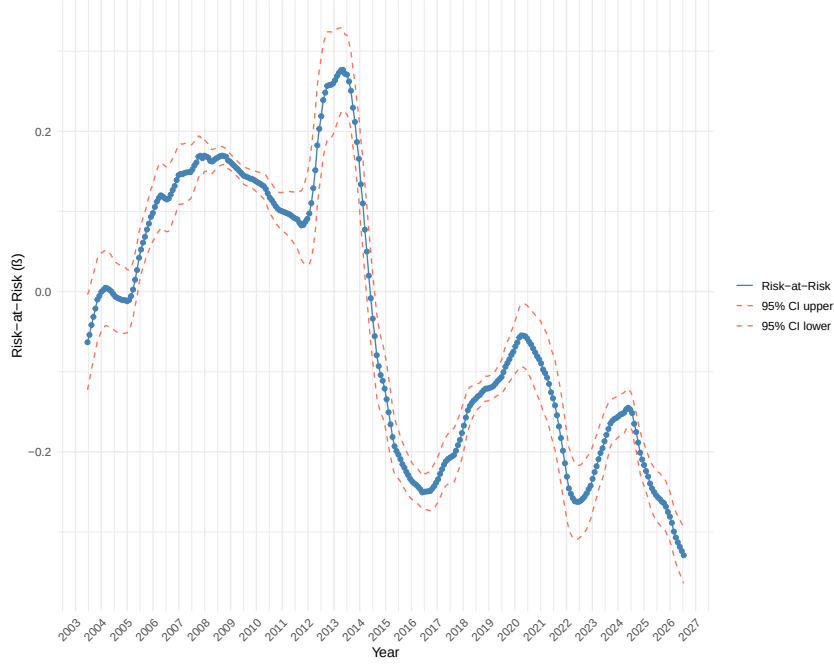


Figure 4: Median *Risk-at-Risk* at $\tau = 0.50$.

5 Comparison between Risk-at-Risk and other measures

Having documented the evolution of the *Risk-at-Risk* across the global market specifications, we next examine how the indicator relates to alternative measures of financial dependence. The purpose of these comparisons is to determine whether the dynamics captured by the *Risk-at-Risk* resemble those obtained from more conventional dependence measures or reflect a different empirical object. We consider two benchmarks that provide complementary comparisons. First, we compare the quantile-based results with a Pearson correlation-based specification to assess how the indicator changes when dependence is characterized through a conventional average relationship rather than through quantiles. Second, we construct direct cross-group dependence using the Cross-Quantilogram, preserving the same quantile methodology and rolling estimation design while changing the relationships being measured. Together, these comparisons help clarify whether the *Risk-at-Risk* reflects features of the underlying dependence estimator itself or the distinct structure through which dependence is aggregated and related across the two asset groups.

5.1 Comparison with Pearson-Based Coefficient

We first compare the quantile-based *Risk-at-Risk* with an equivalent construction based on Pearson correlation. For each rolling window, we calculate pairwise Pearson correlations among the residuals within the risky asset group and within the hedge asset group, and aggregate them using the same normalization employed for risk concentration. We then apply the same 60-observation rolling regression used in the *Risk-at-Risk* construction. The resulting Pearson-based slope coefficient therefore preserves the aggregation and second-stage regression structure of the proposed indicator while replacing the Cross-Quantilogram with conventional linear correlation.

This comparison provides a benchmark for assessing whether the dynamics captured by the indicator depend on how dependence is characterized. Pearson correlation summarizes average linear co-movement across the return distribution, whereas the Cross-Quantilogram at $\tau = 0.50$ measures dependence around the center of the conditional return distribution. Comparing the resulting slope coefficients helps determine whether the relationship between the internal dependence structures of risky and hedge assets changes when dependence is measured through a quantile-based rather than a conventional correlation-based approach.

Figure 5 shows that the two specifications share several broad movements during the earlier part of the sample but diverge substantially over time. Both coefficients rise around the Global Financial Crisis and again during the early 2010s, indicating periods in which the relationship between the internal dependence structures is visible under both approaches. From the mid 2010s onward, however, their trajectories separate more clearly. The Pearson-based coefficient gradually returns toward positive values and remains positive for much of the period from 2020 through 2024, while the $\tau = 0.50$ *Risk-at-Risk* remains predominantly negative and declines further toward the end of the sample. The comparison therefore indicates that the dependence measure itself materially affects the

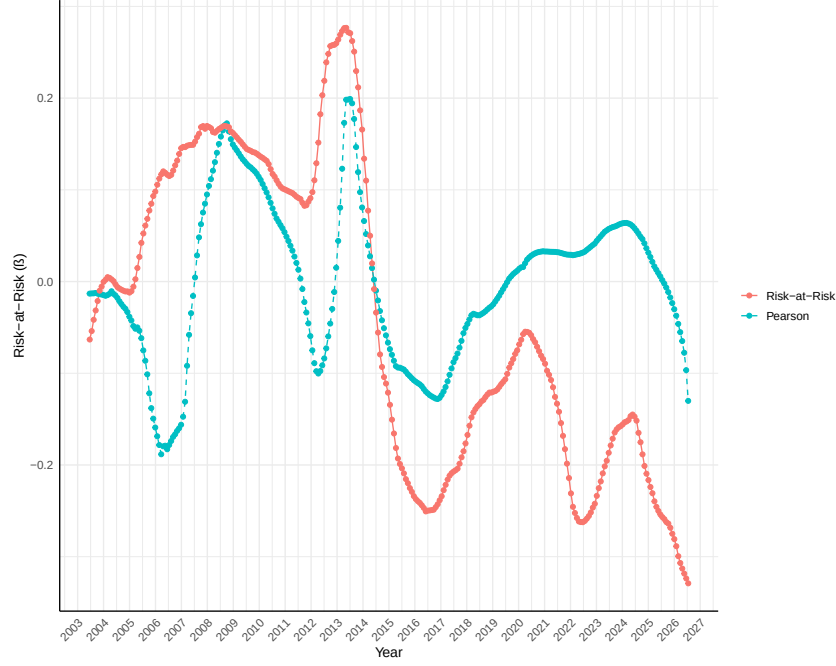


Figure 5: Comparison of the Pearson-based coefficient with the median *Risk-at-Risk* at $\tau = 0.50$.

dynamics captured by the indicator.

5.2 Comparison with Direct cross-group CQ

We next compare the lower-tail *Risk-at-Risk* with a direct cross-group Cross-Quantilogram benchmark. For each 500-observation rolling window, we estimate the lag-zero Cross-Quantilogram at $\tau = 0.05$ for every pair formed by one risky asset and one hedge asset, and average the resulting pairwise coefficients. The benchmark therefore uses the same residual series, Cross-Quantilogram estimator, quantile level, rolling window length, and 20-observation step employed in the construction of the lower-tail risk concentration measures. To make the temporal information set more comparable with the second stage *Risk-at-Risk* regression, we then calculate a 60 state rolling average of the direct cross-group CQ before comparing the two series.

This comparison is particularly informative because it holds the underlying lower-tail de-

pendence methodology largely constant while changing the object being measured. The direct cross-group CQ summarizes contemporaneous lower-tail dependence between risky and hedge assets across groups. The *Risk-at-Risk*, in contrast, first measures lower-tail dependence among assets within each group separately and then estimates the local relationship between the resulting internal concentration states. The comparison therefore helps determine whether the dynamics of the *Risk-at-Risk* simply reflect direct downside dependence between the two groups or whether they capture a different aspect of how the internal dependence structures of risky and hedge assets evolve in relation to one another.

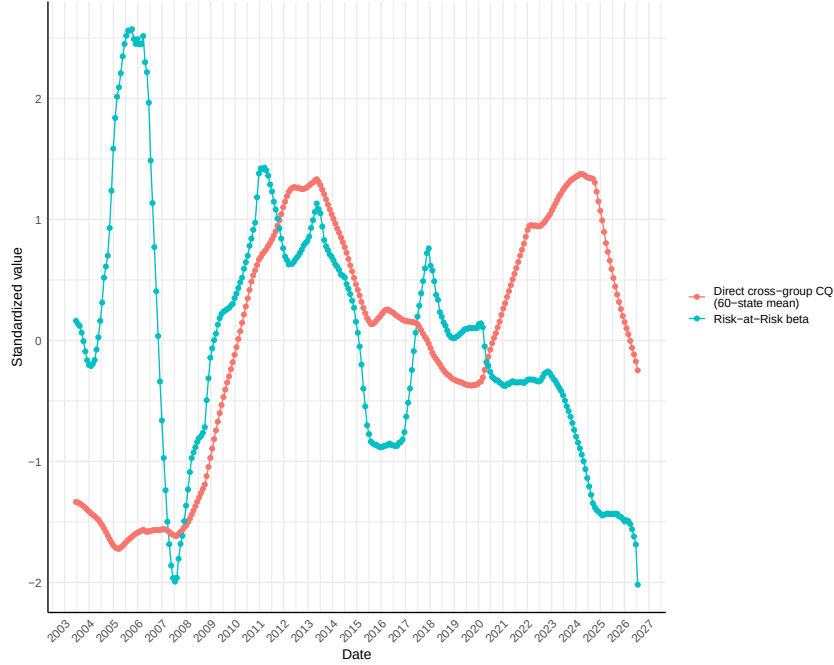


Figure 6: Comparison of direct cross-group lower-tail dependence with the lower-tail *Risk-at-Risk*.

Figure 6 shows substantial differences between the standardized direct cross-group CQ benchmark and the lower-tail *Risk-at-Risk*. Despite using the same Cross-Quantilogram framework and closely aligned rolling specifications, the same-date Pearson correlation between the two series is only -0.138 . Their trajectories also diverge markedly in several periods. In particular, direct cross-group dependence rises strongly during the early 2020s, while the *Risk-at-Risk* remains negative and subsequently declines. The weak association between the two measures is therefore consistent with the interpretation that

direct downside dependence across groups and the relationship between their internal downside concentration structures represent empirically distinct dimensions of financial dependence.

Taken together, the two comparisons clarify the empirical object captured by the *Risk-at-Risk*. The Pearson comparison shows that the dynamics of the indicator depend on how financial dependence is characterized, while the direct Cross-Quantilogram comparison shows that even when the underlying quantile estimator and rolling design are held largely constant, direct cross-group dependence can evolve differently from the relationship between the groups' internal concentration structures. The *Risk-at-Risk* should therefore be interpreted as capturing the evolving relationship between internal dependence states rather than as another measure of direct dependence between risky and hedge assets.

To understand why the tail-based measure does not exhibit a similar increase as the median-based measure in 2009, we delve into two aspects. First, from a technical standpoint, as discussed previously, tail-based measures estimate dependence across assets mainly using data around the tails, thereby limiting accuracy due to the number of observations available for estimation. In contrast, the median-based approach benefits from a larger number of observations, resulting in narrower confidence intervals. Second, because $\beta(t)$ is estimated as the slope from a regression of hedge asset risk concentration on equity risk concentration, its value depends on both the covariance between the two concentration measures and the variance of equity risk concentration. Therefore, when (b) exhibits a notable increase in $\beta(t)$ while (a) does not, this may indicate that the internal dependence structures of the two groups became more closely related around the median, while their lower-tail concentration structures remained less aligned. Third, these heterogeneous responses may have changed the internal lower-tail dependence of the hedge group without producing a similar change in the lower-tail dependence of global equity markets. In contrast, dependence around the median may have become more closely aligned across the two groups during the recovery period.

6 Risk-at-Risk across Advanced and Emerging Markets

We next examine whether the relationship between equity and hedge risk concentration differs across advanced and emerging markets. The hedge group remains unchanged, while equity risk concentration is reconstructed separately using the two regional groups. This analysis allows us to examine heterogeneity in the evolution of the relationship without interpreting differences in the raw coefficient as direct rankings of systemic risk.

6.1 *Advanced Economies*

The lower-tail risk concentration of advanced equity markets varies substantially over time. Pronounced increases occur around the global financial crisis and, most notably, during the COVID-19 period. The median specification follows a different and generally smoother trajectory, reinforcing the distinction between dependence during adverse states and dependence around the center of the distribution.

The lower-tail *Risk-at-Risk*, shown in Figure 7, rises during the period preceding the global financial crisis and again during the early 2010s. Another visible increase occurs around 2017-2018. The coefficient remains positive through much of the subsequent period before declining after 2023 and becoming negative near the end of the sample.

The median-based coefficient, shown in Figure 8, follows a markedly different trajectory. It rises strongly through the first part of the sample and reaches a high during the early 2010s, but subsequently falls and becomes predominantly negative from approximately the middle of the decade onward. The contrast is particularly clear after 2015, when the lower-tail relationship remains positive for much of the period while the median relationship is negative.

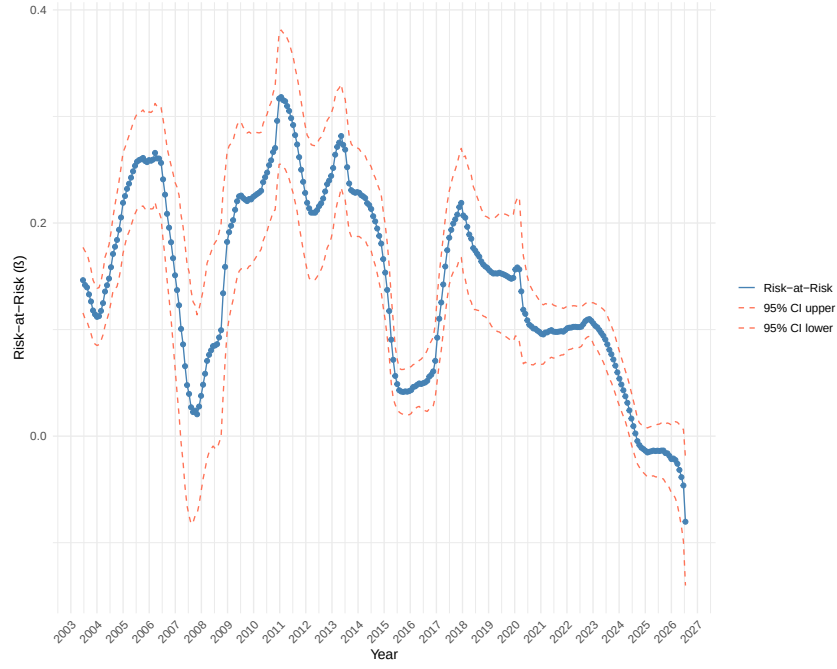


Figure 7: Lower-tail *Risk-at-Risk* at $\tau = 0.05$ for advanced economies

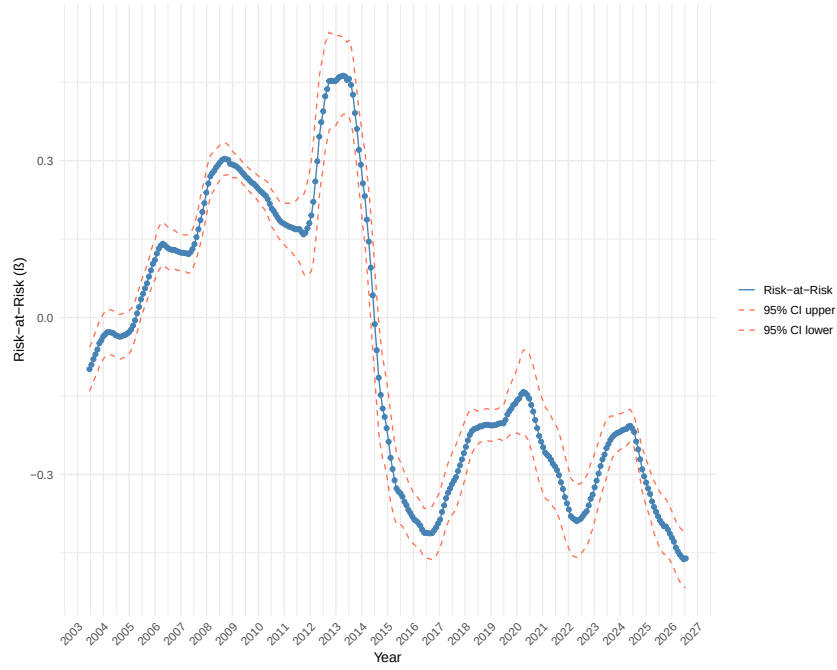


Figure 8: Median *Risk-at-Risk* at $\tau = 0.50$ for advanced economies

6.2 Emerging Economies

The emerging-market estimates also reveal strong time variation. Lower-tail equity risk concentration increases substantially around the global financial crisis and again during the COVID-19 period, before declining sharply afterward. The median specification follows a different pattern, particularly during the later part of the sample.

The lower-tail *Risk-at-Risk*, presented in Figure 9, exhibits larger fluctuations than the corresponding advanced-market series. The coefficient changes sharply around the global financial crisis, becomes strongly positive during the early 2010s, and rises again around 2017-2018. It remains positive through much of the COVID-19 and post-COVID-19 period before declining and eventually becoming negative toward the end of the sample.



Figure 9: Lower-tail *Risk-at-Risk* at $\tau = 0.05$ for emerging economies

The median specification, shown in Figure 10, again differs substantially. It reaches a pronounced positive peak during the early 2010s and then falls sharply, becoming predominantly negative from the middle of the 2010s onward. The persistence of this divergence between the lower-tail and median specifications indicates that the distinction

between adverse-state and central dependence is not driven solely by the advanced-market sample.

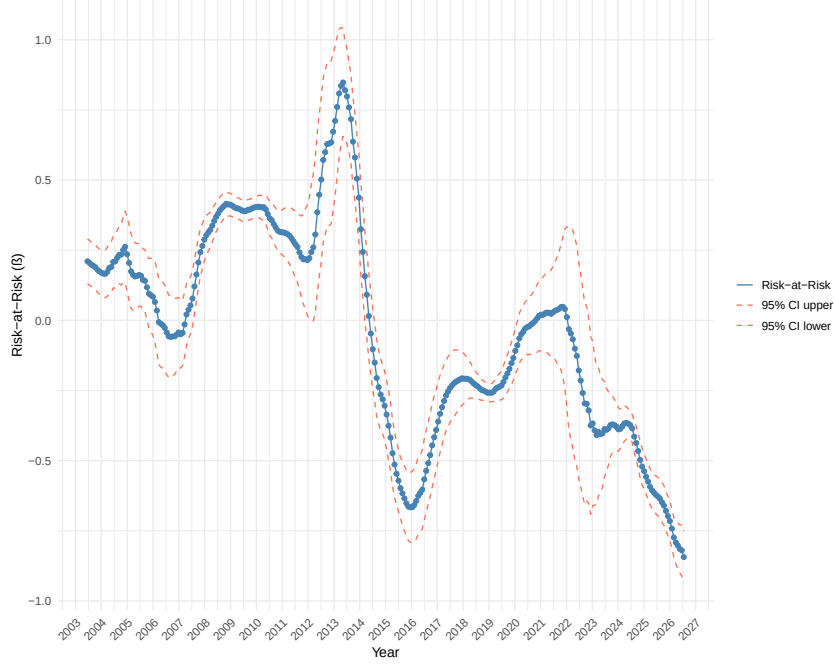


Figure 10: Median *Risk-at-Risk* at $\tau = 0.50$ for emerging economies

6.3 Comparison across Market Groups

Taken together, the advanced and emerging estimates reveal both common dynamics and meaningful heterogeneity. Both groups show important changes in lower-tail *Risk-at-Risk* around the global financial crisis, the early 2010s, 2017-2018, and the post-COVID-19 period. The emerging-market coefficient, however, displays substantially larger positive and negative fluctuations. This greater variability is consistent with the broader literature documenting distinctive and time-varying volatility dynamics in emerging equity markets (Bekaert and Campbell R. Harvey, 1997).

These differences should not be interpreted solely by ranking the absolute values of β_t . Because the regression slope depends on both the covariance between the two concentration series and the variance of equity risk concentration, its magnitude is affected by the scale and variability of the underlying series. The regional comparison is therefore

most informative in terms of the timing, direction, persistence, and variability of the relationship.

The regional results also illustrate that higher internal risk concentration and higher *Risk-at-Risk* are distinct phenomena. Advanced equity markets can exhibit high levels of internal downside concentration while the emerging-market *Risk-at-Risk* displays larger fluctuations. Thus, the level of RC^E alone does not determine the strength of its relationship with hedge asset concentration.

Most importantly, the lower-tail and median coefficients display similar qualitative divergence across both market groups. From approximately the middle of the 2010s onward, median-based relationships are predominantly negative, whereas the lower-tail relationship remains positive through much of the same period. This cross-group consistency provides additional evidence that the *Risk-at-Risk* captures a relationship that is specific to adverse-return states rather than a simple reflection of dependence around the center of the distribution.

Emerging markets also tend to experience faster declines in *Risk-at-Risk* after reaching local peaks, which may reflect their higher growth potential, stronger capital inflows, and faster recovery of investor confidence following periods of financial stress. In addition, market integration and risk concentration do not always move together. During some periods, market integration increases while risk concentration declines, suggesting that broader return co-movement does not necessarily translate into stronger lower-tail dependence within the equity group. After approximately 2015, however, these measures appear to become more closely aligned in emerging markets, possibly reflecting global developments such as the slowdown in China and changes in US monetary policy.

7 Conclusion

This paper introduces the *Risk-at-Risk*, a dynamic indicator of the relationship between the internal downside-risk structures of risky and hedge assets. The framework first aggregates lower-tail Cross-Quantilogram dependence within each group into separate risk concentration measures and then estimates their local relationship through a rolling regression. The resulting coefficient captures whether higher internal downside concentration among risky assets is systematically associated with higher concentration among hedge assets, without requiring direct transmission between the groups.

The empirical results show substantial time variation in this relationship. Lower-tail *Risk-at-Risk* strengthens around several important periods, including the years preceding the global financial crisis and around 2017-2018. The COVID-19 episode also shows that high equity risk concentration does not necessarily imply a high *Risk-at-Risk*, reinforcing the distinction between the level of internal concentration and the relationship between concentration states. The lower-tail and median specifications follow materially different trajectories, particularly after the mid-2010s.

Comparisons with alternative measures further support the distinctiveness of the indicator. The *Risk-at-Risk* differs from an analogous Pearson-based measure and exhibits only a weak relationship with direct lower-tail dependence between risky and hedge assets. The latter captures adverse dependence between the groups, whereas the *Risk-at-Risk* captures the relationship between their separately estimated internal downside structures. Advanced and emerging markets also display common turning points but substantial heterogeneity in the variability of this relationship.

While this paper focuses on monitoring *risk at risk* in global financial markets, the methodology can be adapted flexibly. For instance, if one is interested in assessing diver-

sification vulnerability within a particular country, the risk concentration of risky assets can incorporate common risky assets within that country’s financial market or equity returns from different industries.

The *Risk-at-Risk* therefore relates the internal downside concentration states of economically distinct asset groups through a dynamic scalar indicator. We interpret this relationship as a measure of diversification vulnerability rather than realized diversification failure or causal risk transmission. The framework remains subject to limitations associated with overlapping rolling windows and generated-variable uncertainty, which should be addressed in future work.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Declaration of Generative AI and AI-Assisted Technologies in the Manuscript Preparation Process

During the preparation of this work, the authors used OpenAI Codex solely for grammar, language, and minor clarity improvements. The tool was not used to generate or

modify data, statistical estimates, or figures. The authors reviewed and approved the final manuscript and take full responsibility for its content.

Data Availability

The data used in this study were obtained from publicly accessible third-party financial data sources, including Yahoo Finance, The Wall Street Journal, and other online and API-based sources. The underlying observations are not redistributed where the applicable terms of those sources do not permit redistribution.

Notes

1. While this asymptotic result is empirically grounded, further theoretical validation of the asymptotic distribution is warranted. As discussed in Wang *et al* (2021), the standardized summation of $\hat{\rho}_{ij}$ conforms to a normal distribution as $N, T \rightarrow \infty$ when $\hat{\rho}_{ij}$ is computed using the Pearson correlation coefficient. Considering that the time series analyzed in this paper are AR-filtered and stationary, we anticipate that $RC(\tau)$ will exhibit a similar convergence towards a normal distribution. However, further verification of the asymptotic distribution in a Cross-Quantilogram framework is necessary.

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