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Article

Keywords:

Posted Date: October 27th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-11133621/v1>

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Additional Declarations: No competing interests reported.

Flux Cycling and Voltage Pulse Generation in Nanostructured Gapless Superconductors: A Time-Dependent Ginzburg-Landau Study

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(Dated: September 24, 2026)

Fluxonic devices explore the interaction of quantized magnetic fluxes in superconductors with their surroundings to perform logical operations, magnetic field sensing and photon detection, among other applications. Therefore, knowing in detail the physics governing flux motion is highly important. More specifically, it is important to understand the main set of parameters that affect the thermoelectromagnetic response of the system to dynamic external fields and to be able to predict the feasibility and limitations of certain phenomena for applications. In this work, we applied the time-dependent Ginzburg-Landau formalism to study the trajectories of vortices and the characteristic voltage curves for a particular magnetic field cycle longitudinally applied to a cylindrical superconducting sample. We showed how these cycles vary with different types of impurities and studied whether this approach might produce practically measurable voltage signatures. In fact, we found a characteristic pattern in these signatures that occurs regardless of the number of vortices that penetrate the sample within the cycles.

I. INTRODUCTION

The development of quantum nanoelectronics has increased access to technologies and devices based on purely quantum phenomena. A prominent example is the class of fluxonic devices, which exploit the interaction of quantized magnetic fluxes in superconductors with their environment. These devices have applications in digital logic operations¹, magnetic field sensing², and photon detection, among others^{3,4}. For further research on fluxonic devices, superconducting systems in which voltage is induced by vortex motion are of particular interest. The modelling of vortex dynamics and the associated induced voltage has a long history, starting with the pioneering work of Machida and Kaburaki⁵, who performed the first direct simulation of the time-dependent Ginzburg-Landau (TDGL) equation to obtain the current-voltage characteristics in type-II superconducting thin films. This concept was further developed by Berdiyrov et al.⁶, who demonstrated the formation of kinematic vortex-antivortex lines in strongly driven superconducting stripes. Numerical studies of equilibrium vortex configurations in mesoscopic superconductors with artificial pinning centres have revealed the role of sample topology in vortex stability⁷. Furthermore, time-dependent simulations have demonstrated vortex-based cellular automata for memory applications with switching times below 20 ns⁸. In recent years, interest in controllable vortex motion and voltage generation has increased. Xue et al.⁹ demonstrated open-circuit voltage generation by dragging superconducting vortices with a dynamic pinning potential. A complementary approach was proposed by He et al.¹⁰, who showed switchable reversal of a vortex ratchet using a dynamic pinning landscape. Chen et al.¹¹ demonstrated voltage generation and vortex array manipulation by applying dynamic heat sources to create thermal gradients. Most recently, Duarte et al.¹² performed a comparative analysis of energy dissipation mechanisms and their role in the formation of voltage pulses during vortex expulsion in gapless and gapped nanosized superconductors. Parallel to these efforts, Zotova and Vodolazov¹³ used the TDGL framework to model the detection mechanism of superconducting single-photon detectors, showing that a photon-induced hot spot nucleates a vortex-antivortex pair, whose subsequent motion generates a voltage pulse a scenario directly relevant to both fundamental vortex dynamics and practical sensing applications. In these studies, the voltage induced by vortex motion is commonly treated as an observable of vortex dynamics^{5,6,9}, a control tool for vortex configurations^{10,11}, a measure of energy dissipation¹², or a detection signal in photon counting¹³. However, the conversion of this voltage into a sequence of detectable pulses and, more importantly, the organization of a cyclic detection process based on it remains open conceptual challenges. Our idea is to explore flux cycling in the superconducting channel under a time-varying magnetic field. Moreover, the voltage pulses we obtain are qualitatively similar to those observed in hotspot-based single-photon detectors, although the triggering event is different: here it is a deliberate magnetic-field drive rather than a stochastic photon arrival. The key advantage of our scheme is that the field cycle naturally resets the system to the same operating point after each event, so that the next event can be detected in the same way. In gapless superconductors, as shown in the work of Lee et al.¹⁴, the presence of a finite density of states at zero energy leads to a significant damping mechanism for moving vortices even as $T \rightarrow 0$. This makes the gapless supercon-

ductor an ideal model system for studying the dissipative dynamics of vortices, where the viscous drag coefficient is not exponentially suppressed, fundamentally distinguishing it from conventional BCS superconductors. Furthermore, the works of Lamura et al.¹⁵ and Adhikari et al.¹⁶ demonstrated that the gapless state in real systems is intrinsically linked to granular microstructures and magnetic impurities, which naturally introduce a disordered landscape and additional pinning centres for vortices. Therefore, the choice of a gapless superconductor as a model system is physically justified, as it simultaneously captures both finite dissipation and realistic defect-mediated dissipation mechanisms, providing a more complete description of vortex dynamics than idealized BCS models.

This manuscript is organized as follows: in Sec. II, we outline the theoretical framework, the TDGL formalism, from which we start our study; in Sec. III, we describe the numerical approach for deriving our data; in Sec. IV, we present and discuss the results of our work, ranging from the electromagnetic response of 2D superconductors submitted to a “ramp-up/ramp-down” profile of time-dependent applied magnetic field and a table summarizing the results; and in Sec. V, we discuss broad aspects of our results and present our main conclusions.

II. THEORETICAL FORMALISM

A. Ginzburg-Landau formalism

The Ginzburg-Landau (GL) theory of superconductivity is a phenomenological framework that provides a powerful description of superconducting phenomena near the critical temperature T_c . It draws on an analogy with second-order phase transitions and introduces a complex order parameter Ψ , which characterizes the superconducting state, with $|\Psi|^2$ representing the density of Cooper pairs. A dynamic extension of the GL formalism was developed by Schmid¹⁷, followed by generalizations^{18–20} that include the superconducting gap. These equations describe the time-dependent behaviour of the superconducting condensate. The generalized time-dependent Ginzburg-Landau (GTDGL) equations in dimensionless units are given by:

$$\frac{u}{\sqrt{1 + \gamma^2 |\Psi|^2}} \left(\frac{\partial}{\partial t} + i\varphi + \frac{\gamma^2}{2} \frac{\partial |\Psi|^2}{\partial t} \right) \Psi = (\nabla - i\mathbf{A})^2 \Psi + (1 - T - |\Psi|^2) \Psi \quad (1)$$

$$\left(\frac{\partial \mathbf{A}}{\partial t} + \nabla \varphi \right) = \mathbf{J}_s - \kappa^2 \nabla \times \nabla \times \mathbf{A}, \quad (2)$$

where the superconducting current density is:

$$\mathbf{J}_s = \text{Im} [\Psi^* (\nabla - i\mathbf{A}) \Psi]. \quad (3)$$

Here, the order parameter is expressed in units of its uniform value at zero external magnetic field $|\Psi_u|^2 = \alpha_0 T_c / \beta$, where α_0 and β are the phenomenological Ginzburg-Landau parameters. Distances are expressed in units of the coherence length at zero temperature $\xi(0) = \hbar^2 / 4m\alpha_0 T_c$. The magnetic field is expressed in units of the bulk upper critical field $H_{c2}(0) = \Phi_0 / 2\pi\xi(0)$, where $\Phi_0 = hc/2e$ is the flux quantum. Temperature is measured in units of the critical temperature T_c and time in units of the Ginzburg-Landau time at zero temperature $t_{GL}(0) = \xi^2(0)/Du$, where D denotes the diffusivity. The scalar potential φ is given in units of $\hbar/2et_{GL}(0)$. Finally, $\kappa = \lambda(0)/\xi(0)$ is the Ginzburg-Landau parameter, where $\lambda(0)$ is the London penetration depth at zero temperature. The parameter $\gamma = 2\tau_e \Delta(0)/\hbar$ is related to the gap of the superconductor, where τ_e is the electron-phonon scattering time and $\Delta(0)$ is the superconducting gap at zero temperature. The parameter u is related to the relaxation rate of the superconducting order parameter due to scattering from impurities and was derived under different microscopic conditions. For high concentrations of paramagnetic impurities, Gor'kov and Kopnin²¹ obtained $u = 12$. On the other hand, Kramer and Baratoff²² demonstrated that, for a superconductor with ordinary impurities in the dirty limit $u = 5.79$. This parameter essentially introduces a different rate of time evolution between the order parameter and the vector potential. We compared both of these cases ($u = 12$ and $u = 5.79$) with systems where one ignores this difference of characteristic times. In such cases, this parameter may be taken as $u = 1$.

B. Influence of Heat Dissipation and the Heat Equation

Recent studies have actively employed models accounting for thermal effects in vortex dynamics within the framework of the GTDGL formalism. Duarte et al.¹² reported that energy dissipation caused by vortex motion in gap and gapless superconductors originates from fundamentally different mechanisms. The authors solved the heat conduction equation together with the GTDGL equations and demonstrated that local heating ΔT depends on the sample

85 geometry and vortex velocity, with the key material parameter determining the dissipation magnitude being u . In a
 86 later work of the same group²³, this approach focused on gapless superconductors, where it was shown that heating
 87 leads to asymmetry in vortex cores during dynamical events, similar to the intertype domain.

88 In parallel with these dynamic models, static TDGL studies by de Oliveira et al.²⁴ have revealed that vortices
 89 significantly alter the thermodynamic properties of the system. In particular, the magnetocaloric potential $\Delta S(T, H)$
 90 exhibits a complex pattern of peaks and troughs that correlate with vortex penetration modes. This finding indicates
 91 that even in the static approximation, vortices are thermodynamically active. Their influence on the entropy and
 92 thermal state in this case depends strongly on the temperature and magnetic field.

93 First, the experimental feasibility of detecting such thermal effects has been confirmed by Halbertal et al.²⁵, who
 94 demonstrated thermal dissipation imaging in nanoscale systems via nanoSQUIDS with a sensitivity of $\Delta T \sim 10^{-7}$ K.
 95 In this context, local heating at the scale predicted by the simulations^{12,23} is well within the reach of experimental
 96 detection. Second, these results confirm the conclusions of the classic work of Skocpol, Beasley, and Tinkham²⁶
 97 that thermal effects significantly change the parameters of non-equilibrium superconductivity. Thus, for the correct
 98 modelling of vortex dynamics under realistic conditions within a purely dynamic approach, the dissipation of heat
 99 and thermal gradients, which is accounted for via the heat equation (4) is a necessary element of the model. In
 100 other words, for such small-scale fluxonic devices, such local heating is often significant and should be included in
 101 simulations. To this end, the TDGL equations are coupled with the heat equation²⁰. This heat equation includes a
 102 term that accounts for the dissipative mechanism²⁷. The free energy theorem concerning the dissipative mechanism
 103 was first derived by Schmid¹⁷ and then generalized for gap superconductors²⁸. The heat equation with an additional
 104 term that accounts for energy dissipation in dimensionless units is:

$$C_{\text{eff}} \frac{\partial T}{\partial t} = K_{\text{eff}} \nabla^2 T + \frac{1}{2} W_{\text{Total}}, \quad (4)$$

105 with the dissipation term, $W_{\text{Total}} = W_{\mathbf{A}} + W_{\Psi} + W_{\gamma}$, given by three different contributions:

$$W_{\mathbf{A}} = 2 \left| \frac{\partial \mathbf{A}}{\partial t} \right|^2, \quad (5)$$

$$W_{\Psi} = \frac{2u}{\sqrt{1 + \gamma^2 |\Psi|^2}} \left| \frac{\partial \Psi}{\partial t} \right|^2, \quad (6)$$

$$W_{\gamma} = \frac{2u}{\sqrt{1 + \gamma^2 |\Psi|^2}} \left[\frac{\gamma^2}{4} \left(\frac{\partial |\Psi|^2}{\partial t} \right)^2 \right]. \quad (7)$$

106 In the equations above, $C_{\text{eff}} = C_f T_c \sigma_n / t_{GL}(0) j_0^2$, where C_f is the heat capacity of the material, σ_n is the conductivity
 107 at the normal state and j_0 is the unity of the current density. The effective thermal conductivity coefficient per unit
 108 length is $K_{\text{eff}} = K_f T_c \sigma_n / \xi^2(0) j_0^2$, where K_f is the thermal conductivity of the material. With estimations similar
 109 to those of Vodolazov²⁰, $C_{\text{eff}} = \pi^3/48$ is the effective heat capacity and $K_{\text{eff}} = \pi^4/48u^2$ is the effective thermal
 110 conductivity for the equation in dimensionless units. Dissipation in this model comprises three contributions: $W_{\mathbf{A}}$,
 111 arising from the induced electric field; W_{Ψ} , associated with order parameter relaxation; and W_{γ} , due to the relaxation
 112 of the superelectron density.

113 III. METHODS

114 In this study, we aim to evaluate the thermoelectromagnetic response of the proposed fluxonic systems by quantifying
 115 the characteristic amplitude and time of voltage peaks induced by vortex dynamics in the channel between the edges
 116 of the sample and the inner boundaries of the square central hole. We consider a long superconducting cylinder with
 117 a square cross-section and a concentric square hole. This allows us to assume that the order parameter and the local
 118 magnetic field are invariant along the z direction, simplifying the problem to two dimensions. To describe the currents,
 119 vortex dynamics and thermal dissipation in the superconducting material, we employ the GTDGL approach.

120 According to the microscopic derivation of the TDGL theory, equations are valid for temperatures in the vicinity
 121 of T_c ¹⁹. In our simulations, we set $T_0 = 0.92T_c$ and solve these equations for a gapless superconductor ($\gamma = 0$) in the

Weyl gauge ($\varphi = 0$), which reduces Eqs. (1) and (2) to the simpler form:

$$u \frac{\partial \Psi}{\partial t} = (\nabla - i\mathbf{A})^2 \Psi + (1 - T - |\Psi|^2) \Psi \quad (8)$$

$$\frac{\partial \mathbf{A}}{\partial t} = \text{Im} [\Psi^* (\nabla - i\mathbf{A}) \Psi] - \kappa^2 \nabla \times \nabla \times \mathbf{A} \quad (9)$$

$$C_{\text{eff}} \frac{\partial T}{\partial t} = K_{\text{eff}} \nabla^2 T + \left| \frac{\partial \mathbf{A}}{\partial t} \right|^2 + u \left| \frac{\partial \Psi}{\partial t} \right|^2 \quad (10)$$

In our two-dimensional formulation, the initial conditions are chosen such that the superconductor (defined as a 2D square area, Ω) is in zero magnetic field and in thermal equilibrium with the heat bath at temperature, T_0 :

$$\Psi(t=0)|_{\Omega} = \sqrt{1 - T_0}, \quad (11)$$

$$A(t=0)|_{\Omega} = B(t=0)|_{\Omega} = 0, \quad (12)$$

$$T(t=0)|_{\Omega} = T_0. \quad (13)$$

The boundary conditions are formulated as follows: for the order parameter, we impose zero normal current at both internal and external boundaries. Dirichlet conditions are used for the temperature, which is fixed at a certain temperature, T_0 , on all boundaries, and for the magnetic field, which is set to the applied field H_e on the outer boundary. The magnetic field on the inner boundary H_i is not known a priori and is calculated in runtime using Ampère's law, as described in Ref.²⁹.

$$\vec{n} \cdot (\nabla - i\mathbf{A}) \Psi|_{\partial\Omega} = 0, \quad (14)$$

$$B|_{\partial\Omega_{\text{out}}} = H_e, \quad (15)$$

$$B|_{\partial\Omega_{\text{in}}} = H_i, \quad (16)$$

$$T|_{\partial\Omega} = T_0, \quad (17)$$

The GL equations are gauge invariant, but standard discretization breaks this invariance under the gauge transformations $\Psi' = \Psi \exp(i\chi)$, $A' = A + \nabla\chi$, $\Phi' = \Phi - \frac{1}{c}\partial\chi/\partial t$. Several numerical algorithms have been developed to solve the TDGL equations^{30–32}. We employ the so-called *link-variables* method, which ensures gauge invariance in lattices. This method consists of introducing auxiliary fields and treating the components of the vector potential as angular variables:

$$\mathcal{U}_x(x, y, t) = \exp\left(-i \int_0^x A_x(\xi, y, t) d\xi\right) \quad \leftrightarrow \quad (\partial/\partial x - iA_x)\Psi = \bar{\mathcal{U}}_x \frac{\partial(\Psi \mathcal{U}_x)}{\partial x}, \quad (18)$$

$$\mathcal{U}_y(x, y, t) = \exp\left(-i \int_0^y A_y(x, \xi, t) d\xi\right) \quad \leftrightarrow \quad (\partial/\partial y - iA_y)\Psi = \bar{\mathcal{U}}_y \frac{\partial(\Psi \mathcal{U}_y)}{\partial y}, \quad (19)$$

which was initially proposed by K. Wilson^{33–35}. The voltage was calculated via the line integral of the electric field (obtained with $\mathbf{E} = -\partial\mathbf{A}/\partial t$) along a square integration path with side $2L/3$ (an average between the outer and inner sides), as shown in Fig. (1).

The simulations were executed for the set of parameters $u = \{1, 5.79, 12\}$, $\kappa^2 = \{1, 5, 10\}$, a smaller system size $L = 25.5\xi(0)$ and a larger system with size $L = 38.25\xi(0)$, both cases with a concentric square hole of side $L/3$. Since we focus on mesoscopic samples, the choice of dimensions is deliberate. In Ref. [36], the authors examined the crossover between the macroscopic and mesoscopic vortex regimes in type-II superconductors. For $\kappa^2 = 1$, the critical size that defines the mesoscopic regime follows $L_c^{\kappa^2=1}(T) = 28.7(1 - T_c)^{-0.359}$. For larger κ^2 , this critical size is even greater, confirming that both sizes we examined are well within the mesoscopic regime. We employed a finite differences method for discretizing the system variables where in all cases, the mesh size is $h_x = h_y = 0.3\xi(0)$. Numerical limitations arise in the choice of the time step due to the Euler treatment of the equations and we follow standard numerical criteria for assuring convergence³⁷:

$$\Delta t \leq \min\left(\frac{\delta^2}{4}, \frac{\delta^2 \beta}{4\kappa^2}\right), \quad \text{with} \quad \delta^2 = \frac{2}{\frac{1}{h_x^2} + \frac{1}{h_y^2}}. \quad (20)$$

Note that, specifically in the equation above, β is not one of the Ginzburg–Landau parameters, but rather the relaxation time of the vector potential, which is related to the conductivity from the relation $\beta = \frac{4\pi\sigma D\kappa^2}{c^2}$. The smallest time step obtained from this criterion, for the case of $\kappa^2 = 10$ and $\beta = 1$, was $\Delta t = 2.25 \cdot 10^{-3}$. In our

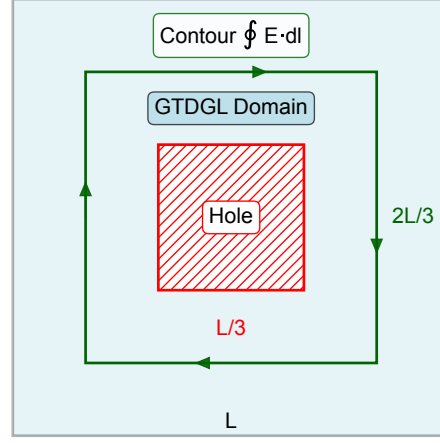


FIG. 1. Example of the integration path (white line) used in our simulations for calculating the voltage around the samples. The voltage was calculated via the line integral of the electric field. The side of the square is $2L/3$ and it is concentric with the inner and outer squares.

calculations we employed an increment, $\Delta t = 5 \cdot 10^{-4}$, which is inside the convergence radius. Furthermore, the system is driven by the time variation of the applied magnetic field. A triangular magnetic field time profile with increments of $\Delta H_e = 10^{-7} H_{c2}(0)$ is applied: the field is ramped up until the minimum vortex number for the given geometry (four vortices) is reached, and then ramped down. This procedure maintains a minimal vortex configuration. Although the field amplitude is chosen to admit four vortices, in some cases, additional vortices may still enter. The complete field cycle proceeds as follows: the field increases linearly until vortices appear, i.e. singular points are observed at the phase, around which there is a gap of 2π ; the field then is decreased linearly. During this cycle, vortices enter the sample and move across the channel to the hole, where they become trapped; upon field reversal, they return through the channel and exit.

IV. RESULTS

We simulated the time-evolution of the measurable parameters, such as the induced voltage around the sample and the dissipated power, for the set of control parameters mentioned in the previous section. The time-evolution results are shown as follows: first, in Fig. (2), we present the results for a smaller system, with side $L = 25.5\xi(0)$, where the first vortex entry is limited to four vortices in total before ramping down the field; second, in Fig. (3), we present a similar case for a 1.5 times larger sample with $L = 38.25\xi(0)$. Further details of this case are expanded in Figs. (4) and (5), where we show several snapshots of the modulus of the order parameter. The idea is to better understand pulse formation in systems with a greater number of vortices. By doing so, we determined that the general picture of the voltage and dissipated power described for the smaller system occurs for other system sizes and slightly larger number of vortices involved in the process. The parameters within the figures are the same for the smaller and larger systems changing the values of $\kappa^2 = \{1, 10\}$ and $u = \{1, 5.79, 12\}$. Additionally, we simulate the case where $\kappa^2 = 5$, but the results are essentially the same as those with $\kappa^2 = 10$. For simplicity, we show only the results for the highest value, which is type-II regime, and compare with the lowest case, $\kappa^2 = 1$, which is closer to the boundary of the intertype domain. Finally, we calculate the amplitude and width of the first and last voltage peaks, which are the most relevant ones, and present them both in the form of Tab. (I) and Fig. (6).

As seen in the plots in Figs. (2) and (3), larger values for u generally reduce the amplitude of the voltage peaks in dimensionless units, yet in dimensional units they actually lead to an increase. The same is true for the dissipated power, generated by the term $\partial \mathbf{A} / \partial t$ in formula (5). The conversion expression between the voltage in CGS units and the dimensionless units for a particular material with critical temperature T_c is as follows:

$$V_{cgs} = V \cdot \frac{4k_B}{\pi e} \cdot T_c u. \quad (21)$$

In short, one could describe the voltage signatures by two noticeable peaks with a strong sign reversal in the middle and a slight change in derivative in the plateau and between these peaks. First, the introduction of the applied field generates a sudden appearance of a current and therefore an induced voltage. The energy dissipation is exhibited in

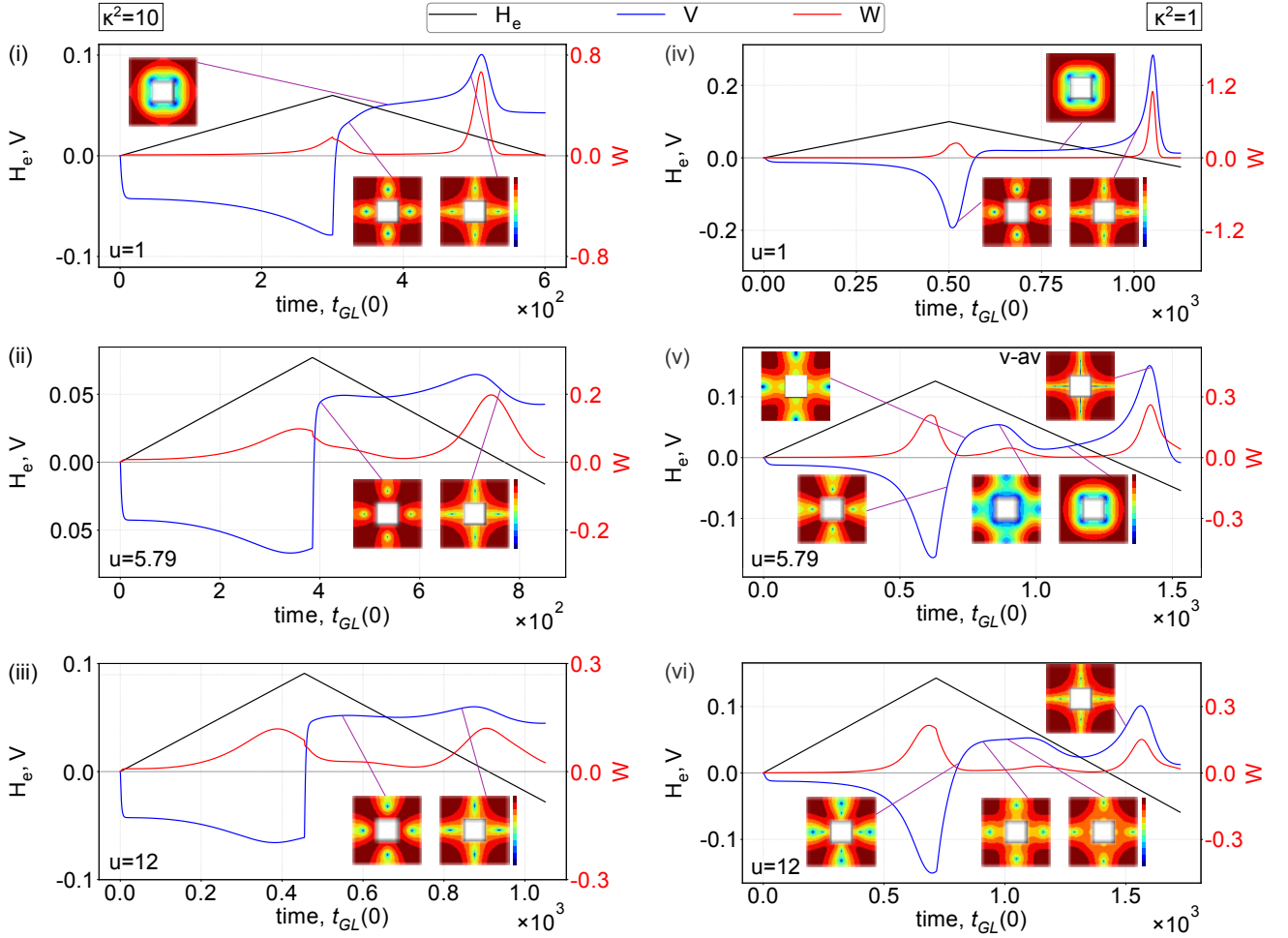


FIG. 2. Voltage signatures (blue lines) for the smaller system, i.e. $L = 25.5\xi(0)$ at $T_0 = 0.92T_c$. On the left column, we show the cases with $\kappa^2 = 10$, where plots (i), (ii) and (iii), represent the results for $u = 1$, $u = 5.79$ and $u = 12$, respectively. The same values for u are applied on the plots on the right, (iv), (v) and (vi), but with $\kappa^2 = 1$. The right scale of the plot (shown in red) represents the dissipated power (red line). The time profiles of the applied magnetic field (black curves) are shown as a guide.

red colour, more specifically, the term W_{total} from the heat equation (4). We show as insets in each panel some of the key events, such as vortex positions and their reconfigurations corresponding to the voltage pulses.

A. Smaller sample — $L = 25.5\xi(0)$

All plots in Fig. (2) show a first peak (or at least an increase in modulus) for the voltages as the field is increased. This is caused by the supercurrent shielding response, prior to vortex penetration and subsequent propagation toward the hole. Upon vortex entry, the applied magnetic field is decreased and the voltage suffers a strong sign reversal and becomes positive. Note that the discontinuity in the time derivative of the applied magnetic field generates a cusp in the voltage and dissipation curves. As the field is reduced (even possibly becoming negative, if the vortices remain trapped in the hole), a second peak arises as vortices leave the hole, move away, and eventually exit the sample. Let us start with the artificial situation where both the order parameter and magnetic field have the same time scale of variation (or more precisely, the same coefficient multiplying the time-derivative terms), i.e. for $u = 1$. This case serves as a guide, the typical voltage signatures become accentuated. With large $\kappa^2 = \{10, 5\}$, a transient regime (where the voltage curve is neither constant nor a peak) was observed. This feature originates from the rearranging of the crowding currents at the corners of the square hole. Although this also occurs in other cases ($u = 5.79$ and $u = 12$), it produces a noticeable signal only in this case, with $u = 1$ and higher values of κ^2 . This result supports

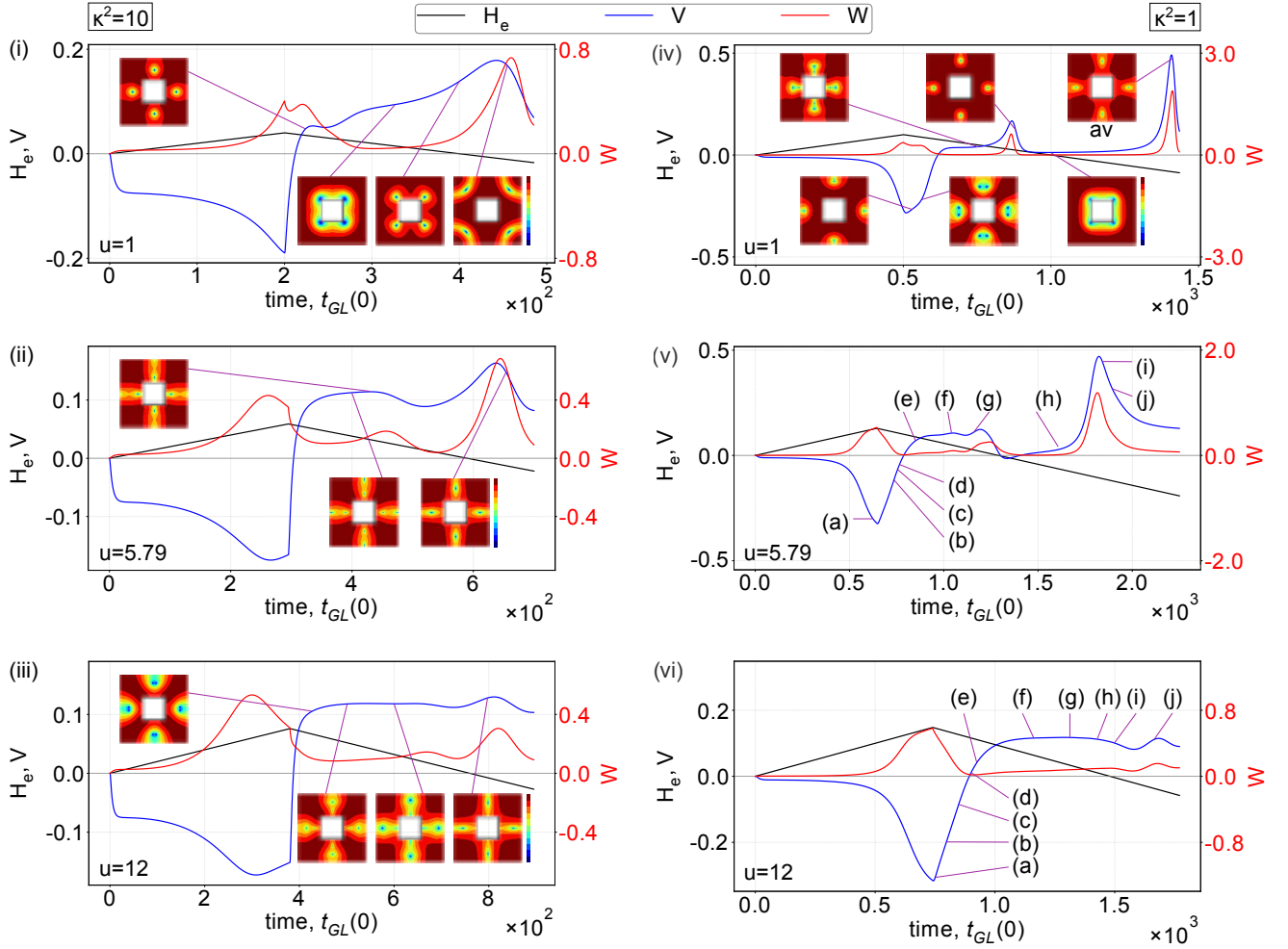


FIG. 3. Voltage signatures (blue lines) for the larger system, i.e. $L = 38.25\xi(0)$ at $T_0 = 0.92T_c$. On the left column, we show the cases with $\kappa^2 = 10$, where plots (i), (ii) and (iii), represent the results for $u = 1$, $u = 5.79$ and $u = 12$, respectively. The same values for u are applied on the plots on the right, (iv), (v) and (vi), but with $\kappa^2 = 1$. The right scale of the plot (shown in red) represents the dissipated power (red line). The time profiles of the applied magnetic field (black curves) are shown as a guide.

the assumption that even slight flux motion can produce a detectable pulse, albeit small, distinguishable from the noise. Additionally, both peaks show a shift in time when vortices are leaving the sample for $u = 5.79$ and $u = 12$, as shown in the left column of Fig. (2). Again, this shift is expected, since u represents the ratio of the characteristic time-scale of variation for the order parameter and the magnetic field. On the opposite extreme of the range of parameters, for the case where $\kappa^2 = 1$, with $u = 5.79$ and $u = 12$, i.e. when one considers essentially different time-scales for the order parameter and magnetic field, two vortices (a first and then a second one into each side) enter the system sequentially, resulting in a total of eight vortices. This causes two large peaks, as mentioned above, but with an additional smaller and wider peak appearing between them. This intermediate peak is associated with a second vortex entry; its amplitude is reduced because this vortex does not traverse the entire channel. More precisely, while the first vortex moves forward and enters the hole, the second vortex moves along the channel but then returns and exits the sample before reaching the hole. Notably, for different values of u , the peaks in the voltage and energy dissipation arising from vortex motion away from a hole and current rearrangement reach their maximum values only after the vortices have already left the sample.

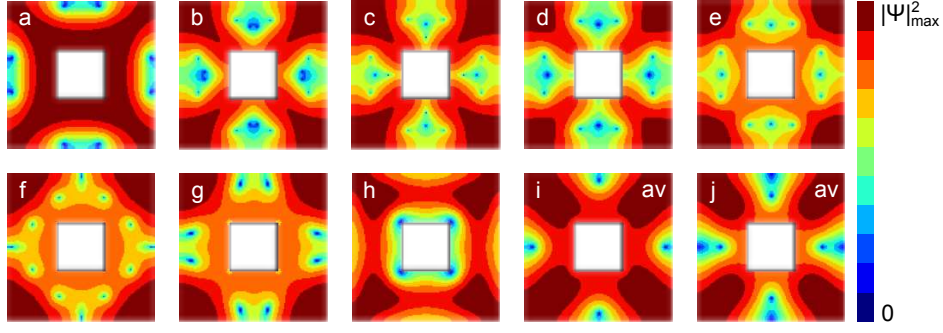


FIG. 4. Time evolution of the modulus of the order parameter marked in plot (v) of Fig. (3). The parameters used here were $u = 5.79$, $\kappa^2 = 1$ and $T_0 = 0.92T_c$.

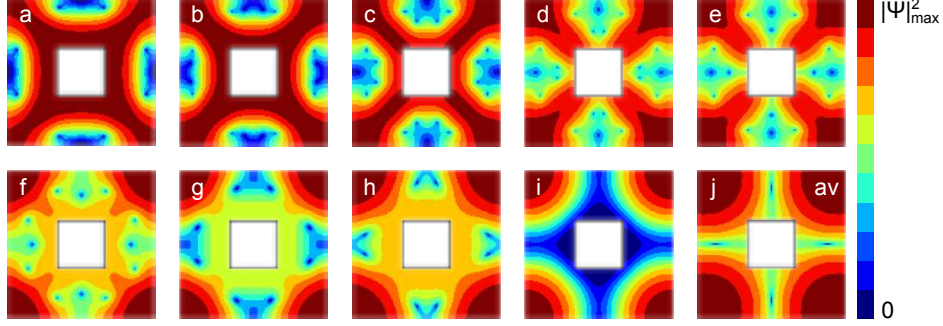


FIG. 5. Time evolution of the modulus of the order parameter marked in plot (vi) of Fig. (3). The parameters used here were $u = 12$, $\kappa^2 = 1$ and $T_0 = 0.92T_c$.

B. Larger sample — $L = 38.25\xi(0)$

In larger systems with $u = 5.79$ and $u = 12$, the topological configurations for the order parameter become considerably more complex. The larger system size allows vortices to enter in pairs, i.e., two vortices from each side simultaneously, as shown in Fig. (3)-(iii). Surprisingly, the voltage curves still look qualitatively similar to those found in the previous case for the smaller system: two pronounced peaks at the first and last flux motion along the sample. As expected, quantitatively, the absolute values increase overall due to the increased (base) area of the sample. As in the smaller system, the larger system with $u = 1$ and higher κ^2 exhibits a noticeable change in derivative or even a negligible peak due to current crowding around the hole, with a greater extent than in the smaller system. Another key distinction is that once the vortices are positioned along the diagonals, the interplay of further field reduction and inter-vortex interactions yields a net force strong enough to expel the vortices. Even more complex cases were found with the two representative systems in Fig. (3), which are shown in the plots (v) and (vi). For the cases where the number of vortices is too large, additional plots are provided in the figures (4) and (5), which show the evolution of the modulus of the order parameter and the shift in temperature in greater detail.

In the smaller system, vortices enter one after another, producing two peaks: a large peak from the first vortex on each side and a smaller peak from the second. In contrast, in the larger system, the second pair enters before the first pair has reached the hole. As a result, the first pair spreads across the sample and stops propagating toward the hole, whereas the second pair rotates, forming a channel through which the vortices move sequentially. This behaviour is illustrated in panels (a) to (e) of Fig. (4) and from (a) to (e) of Fig. (5). The continuous redistribution of vortices results in noise and broadening of the voltage peak, rather than a single change in the derivative as seen in smaller systems for both $u = 5.79$ and $u = 12$. Nevertheless, the moment when vortices leave the sample can still be identified by the voltage decrease. For vortices trapped in the hole, we observed the following behaviour: expelling them requires lowering the applied field to values considerably more negative than those required in the smaller system. Consequently, rather than exiting the hole and leaving the sample, these vortices may instead be annihilated by additional vortices entering from the outer boundary, either in the hole or in the channel between the inner and outer edges. To confirm that the observed phenomenon is indeed vortex/anti-vortex annihilation³⁸, we observed the phase of the order parameter and observed singular points with opposite phase cuts, or a discontinuity of -2π .

Overall, the following behaviour was observed: in all the presented smaller systems, as well as in the larger systems with $u = 12$, $\kappa^2 = \{5, 10\}$ and $u = 5.79$, $\kappa^2 = 5$, the vortices exited the sample in the same manner as they entered. In the larger systems with $u = 1$ and $\kappa^2 = \{10, 5\}$, the vortices tried to exit diagonally. Owing to the relatively strong applied fields in the larger systems, instead of vortices exiting as in the case of the smaller system, in a number of cases, antivortices penetrated into the system and subsequently annihilated either in the superconducting channel or cancelled the flux in the hole. This cancelling of flux was observed for $\kappa^2 = 1$ with $u = \{1, 5.79, 12\}$, whereas for $\kappa^2 = 5$ and $u = 5.79$, annihilation took place in the channel between the inner and outer edges. In addition, for $u = 5.79$ and $\kappa^2 = 1$, the field was high enough for two external vortices to penetrate simultaneously (from the side); the first vortex was annihilated in the hole, whereas the second vortex passed through it. An additional antivortex also entered the system due to the strong field.

Reversing the time derivative of the applied magnetic field not only changes the voltage sign but also sometimes slightly reduces the first peak, depending on the exact moment of reversal. To determine the amplitude of the first peak without clipping, which will later be used to estimate the pulse amplitude, we also performed simulations for all the smaller and larger systems via a simple linear increase in the magnetic field. In this approach, we increase the magnetic field until one vortex is trapped in the hole. In any case, all the systems present qualitatively similar results with similar trends.

The next step, following the analysis of the voltage peak formation, was to estimate the amplitudes and characteristic times of the most pronounced peaks. Using simulation results from both the linear field ramp and the full field cycle (ramp up and down), we obtained estimates of the amplitudes of the first and last voltage peaks, ΔV_f and ΔV_l , respectively. The results in CGS units are summarized in Table (I). As shown in the table, the amplitude of the voltage pulses primarily increases with increasing u . This is generally explained by the fact that vortex penetration into systems with higher values of u requires a higher applied magnetic field, which in turn increases the shielding current and, consequently, the voltage. The same reasoning explains the amplitude increase as κ^2 approaches 1, since departing from the deep type-II regime also requires a higher field. Increasing the system size likewise leads to an amplitude increase. However, as noted earlier, the redistribution of vortices associated with the increased system size can actually reduce the amplitude, as seen in some systems with $u = 12$. If one sets $T = 1$ K, then the largest amplitude is equal to 872 nstatV for the system with $u = 12$ and $\kappa^2 = 1$, and the smallest value is 10.9 nstatV for the system with $u = 1$ and $\kappa^2 = 10$. That is, even the minimum voltage values obtained during simulations are fundamentally measurable via laboratory equipment.

In the same Table (I) we also present the estimated characteristic times (in CGS units) for the first and last peaks, τ^f and τ^l , respectively. The results were obtained from full-cycle simulations by applying the criterion *full width at half maximum* (FWHM). Examining the characteristic times across both parameters reveals two clear trends. The dominant control parameter is u : both τ^f and τ^l increase monotonically as u varies from 1–12 in both smaller and larger systems. This behaviour is physically consistent with the role of u in the TDGL equation, since it multiplies the covariant time derivative $\partial\psi/\partial t$: a larger u corresponds to a more torpid order parameter that relaxes more slowly relative to the electromagnetic dynamics, thereby increasing the time elapsed between the applied perturbation and the appearance of each voltage peak. In contrast, the dependence on κ^2 is comparatively weak and non-monotonic, i.e. this relation is not always consistent between the two system sizes: in the smaller system τ^f tends to increase mildly with κ^2 at fixed u , whereas in the larger system the opposite tendency is more common, with τ^f and τ^l often decreasing or plateauing as κ^2 increases. This suggests that, unlike u , which sets the overall relaxation timescale of the condensate, κ^2 primarily modulates the relative strength of magnetic screening and coupling between ψ and $\mathbf{A}$. This is a feature whose net impact on the dynamics is more sensitive to the sample geometry and size than to u alone. The same characteristic times are also shown in dimensionless units in Fig. (6), which helps visualize trends that are not immediately apparent from the tabulated values. In every case, τ^f increases with u for both system sizes. The behaviour of τ^l is less uniform: it grows monotonically with u for the smaller system, but for the larger system it is non-monotonic and depends on κ^2 , even decreasing with u in some cases (e.g., $\kappa^2 = 10$). This indicates that the sensitivity of the last-peak dynamics to u is modulated by the strength of the magnetic coupling, unlike the first-peak dynamics, which respond to u more uniformly. Nevertheless, when $T_c = 1$ K, the characteristic pulse time in all systems is of the order of 100 ns, suggesting that devices based on this geometry could achieve response times below nanoseconds.

V. CONCLUSION

In this study, we have shown that cycling vortex dynamics in a nanostructured gapless superconductor produces a detectable, reversible sequence of voltage pulses. We directly linked the recombination of vortex configurations in the superconductor volume to changes in voltage and noted that the most prominent events resulting in the largest voltage pulses are the first entry of vortices into the sample and their complete expulsion at the end of the system

TABLE I. Estimation for the amplitudes and characteristic times of the two main voltage peaks: first (ΔV^f , τ^f) and last (ΔV^l , τ^l), respectively, with ΔV measured in $\text{nstatV} = 10^{-9} \text{ statV}$ and τ measured in $\text{ps} = 10^{-12} \text{ s}$.

u	κ^2	Smaller system — $L = 25.5\xi(0)$				Larger system — $L = 38.25\xi(0)$			
		ΔV^f	ΔV^l	τ^f	τ^l	ΔV^f	ΔV^l	τ^f	τ^l
1	1	$64.9 \cdot T_c$	$96.6 \cdot T_c$	$196/T_c$	$97.5/T_c$	$87.4 \cdot T_c$	$137 \cdot T_c$	$3.32/T_c$	$91.5/T_c$
5.79	1	$200 \cdot T_c$	$252 \cdot T_c$	$303/T_c$	$339/T_c$	$391 \cdot T_c$	$625 \cdot T_c$	$478/T_c$	$381/T_c$
12	1	$421 \cdot T_c$	$339 \cdot T_c$	$435/T_c$	$373/T_c$	$872 \cdot T_c$	$108 \cdot T_c$	$572/T_c$	$242/T_c$
1	5	$21.7 \cdot T_c$	$37.0 \cdot T_c$	$149/T_c$	$104/T_c$	$42.9 \cdot T_c$	$48.3 \cdot T_c$	$117/T_c$	$149/T_c$
5.79	5	$47.3 \cdot T_c$	$59.8 \cdot T_c$	$356/T_c$	$317/T_c$	$168 \cdot T_c$	$197 \cdot T_c$	$294/T_c$	$182/T_c$
12	5	$97.7 \cdot T_c$	$85.4 \cdot T_c$	$512/T_c$	$369/T_c$	$384 \cdot T_c$	$152 \cdot T_c$	$402/T_c$	$220/T_c$
1	10	$10.9 \cdot T_c$	$21.2 \cdot T_c$	$209/T_c$	$104/T_c$	$29.0 \cdot T_c$	$39.9 \cdot T_c$	$110/T_c$	$252/T_c$
5.79	10	$23.9 \cdot T_c$	$30.5 \cdot T_c$	$458/T_c$	$342/T_c$	$93.0 \cdot T_c$	$137 \cdot T_c$	$306/T_c$	$203/T_c$
12	10	$48.1 \cdot T_c$	$43.2 \cdot T_c$	$597/T_c$	$374/T_c$	$214 \cdot T_c$	$78.5 \cdot T_c$	$480/T_c$	$203/T_c$

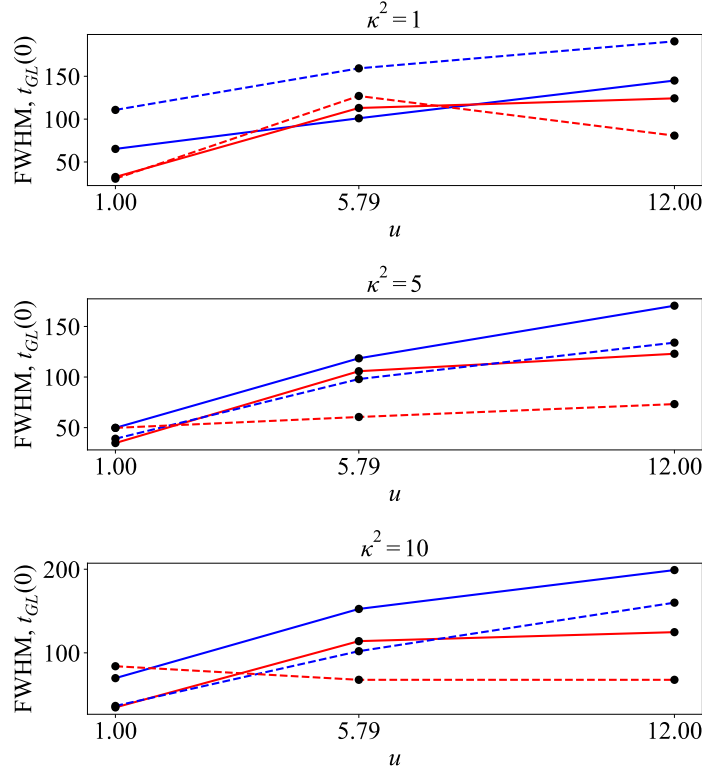


FIG. 6. Characteristic times of the first and last voltage peaks for fixed κ^2 and $u = \{1, 5.79, 12\}$. Solid lines: smaller system (blue – first, red – last). Dashed lines: larger system (blue – first, red – last). Note that, in dimensionless units, we used $t_{GL}(0) = \xi^2(0)/Du$ as unit of time.

evolution. This is natural, as both events fundamentally affect the current profiles within the superconductor. The entry or removal of Abrikosov vortices in a type-II superconductor corresponds to a phase transition, changing the state of the superconducting condensate from a homogeneous Meissner state to a mixed state and back. Other vortex reconfigurations can still influence the voltage response, albeit generally to a much weaker extent. The fact that both vortex entry and exit produce voltage pulses makes the system cycle-reversible. This conforms to the conceptual gap identified in Sec. I: prior studies have largely treated vortex-induced voltage as a passive observable of dynamics, a

control tool, a measure of dissipation, or a one-time detection trigger, whereas our results indicate that a periodically driven vortex system can generate a repeatable, field-controlled cycle of voltage pulses. This raises the possibility that such a mechanism could eventually underlie a cyclic detection scheme, in a spirit similar to the hotspot mechanism exploited in single-photon detectors, although here, the pulse-triggering event is a deliberate magnetic-field drive rather than a stochastic photon arrival.

The simulations involved a long superconducting cylinder with a square cross-section and a concentric square hole. We simulated systems with different lateral sizes L and a hole of fixed size $L/3$. The smaller system size was $L = 25.5\xi(0)$, which provides the minimum vortex configuration for this geometry, whereas a 1.5 times larger system had $L = 38.25\xi(0)$. In the latter, the number of vortices reached 20 at a time. Moreover, such an increase in the vortex number did not qualitatively change the system behaviour in terms of the emerging pulses not associated with vortex redistribution. This demonstrates the resilience of the response with respect to the number of vortices, which may be useful for applications, since ideally tuning the system parameters to obtain four vortices strictly may be difficult. Although the system of equations ruling this system is quite complicated, the underlying physics is governed by a small set of competing factors. Vortex entry and exit produce the dominant voltage peaks, whereas secondary features, such as the transient period associated with current crowding at the corners of the hole for high- κ^2 (particularly 5 and 10 for our simulations), and $u = 1$ systems arise from more subtle reconfigurations and remain distinguishable from noise. Both impurities (higher u) and proximity to the type-I regime ($\kappa^2 \rightarrow 1$) increased the applied field needed to nucleate and expel vortices (critical fields), which in turn increases the shielding current and the resulting voltage amplitude, although at the cost of a shorter pulse duration. Increasing the system size similarly increases the amplitude but can also introduce vortex redistribution effects that partially offset this gain.

In some systems, as the field was reduced, antivortices entered and either annihilated the vortices in the superconductor channel, as shown in Fig. (2)-(v), or cancelled the flux in the hole, as shown in Figs. (3)-(iv), (4)-(i) and (4)-(j), and (5)-(j). Generally, the appearance of antivortices is caused by the fact that, to force vortices out of the hole, a relatively strong magnetic field must be applied. This field, therefore, becomes comparable, by modulus, to that required for vortex penetration at the very beginning of the system's evolution. In our simulations, this effect appears only in systems with $L = 38.25\xi(0)$ and $\kappa^2 = 1$, since both the larger size (which enhances screening currents) and the proximity to the type-I regime increase the required field. First, the elongation of vortices (and antivortices) in similar systems²³ can be caused by the proximity to the Bogomolny point and the intertype domain³⁹⁻⁴¹, in which non-trivial vortex patterns and configurations were observed. Second, the interaction of vortices with the sample boundaries leads to core distortion directly at the edges, and the vortex-antivortex interaction^{38,42,43} further deforms the profiles. As mentioned in the work of Zadorosny⁴⁴, the presence of the hole/antidote deforms the vortex/antivortex profiles into elongated forms, whose low $|\Psi^2|$ extensions overlap within the small effective superconducting area, creating a channel from the antidote to the system's exterior.

Table I summarizes both the estimates for the amplitude and the characteristic times (defined as FWHM - full width at half maximum) for the first and last voltage pulses. The pulse amplitude is qualitatively proportional to the critical temperature and the characteristic time $\propto 1/T_c$. This suggests that superconductors with higher T_c values may be preferable for such system implementations. It has also been demonstrated that increasing the value of u generally leads to an increase in the pulse amplitude, with the caveat that it also increases its characteristic time.

More broadly, these results suggest that nanostructured gapless superconductors could be used as a practical platform for cyclic flux-based sensing. A simple periodic magnetic field is enough to generate a repeatable sequence of voltage pulses with amplitudes well within the reach of standard laboratory equipment and response times reaching the nanosecond scale. Since the pulses come directly from the entry and exit of vortices rather than from a single triggering event, the mechanism could be used for devices that reset themselves each cycle, which is something of clear interest for single-photon detectors, which need to recover quickly between counts, and for flux-based logic or memory elements. Naturally, several steps still separate this proof of principle from an actual device, most notably the effects of thermal and electrical noise and coupling to a real readout circuit, and these steps should be the focus of future work. Additionally, 3D simulations reach the thin-film limit instead of the axially symmetric case might lead to more feasible setups. Nevertheless, the fact that the pulse pattern remains qualitatively robust across different system sizes and vortex numbers is a good sign that this behaviour is not an artefact of a particular geometry but rather a genuine feature of the underlying vortex dynamics.

DATA AVAILABILITY

The datasets generated and analysed during the current study are available from the corresponding author on reasonable request.

ACKNOWLEDGMENTS

The authors thank Alexei Vagov and Arkady Shanenko for useful discussions. T. T. S. thanks HSE University for excellent support as well as all the members of the Centre for Quantum Metamaterials for their collaboration and collegiality. This work was supported by the project “International academic cooperation” of HSE University.

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AUTHOR CONTRIBUTIONS

All authors contributed to the conceptualization and to the definition of the theoretical formalism. R. R. Kozii developed the simulation code, carried out the numerical calculations, and prepared the figures. T. T. Saraiva supervised the work. All authors contributed to the writing and revision of the manuscript.

COMPETING INTERESTS

The authors declare no competing interests.