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Closed-Form Kinematic Modeling and Experimental Dexterity Validation of the Sina_{flex} Hybrid Surgical Manipulator

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Abstract

Hybrid serial–parallel manipulators are increasingly employed in robotic minimally invasive surgery because they combine large workspace reach with high stiffness and dexterity under remote center of motion (RCM) constraints. However, rigorous closed-form kinematic modeling and experimental validation of such systems remain limited.

This study presents a complete analytical kinematic and dexterity framework for the Sina_{flex} patient-side manipulator (PSM), a hybrid surgical robot composed of a proximal RRP positioning chain and a distal non-symmetric spherical parallel mechanism with an independent tool roll. To address the complexity of the coupled serial–parallel structure, a virtual spherical wrist representation is introduced that reformulates the mechanism as an equivalent six degree of freedom serial chain while preserving the RCM constraint and orientation capabilities.

Based on this formulation, closed-form forward and inverse kinematics and a full analytical Jacobian are derived, enabling explicit singularity analysis and quantitative dexterity evaluation using singular value-based performance indices. Monte Carlo workspace reconstruction demonstrates continuous reachable regions compatible with laparoscopic surgical requirements, and forward–inverse kinematic consistency tests confirm numerical accuracy within machine precision.

The proposed model is further validated using 1 kHz joint data recorded during a 90-minute in vivo robotic vasectomy procedure. Analysis of the recorded trajectories shows that the manipulator operates predominantly within well-conditioned regions of its workspace, achieving a global mobility index of 0.8056 across the operational workspace and 0.83246 within the task-specific surgical workspace.

These results provide experimental evidence supporting the analytical model and demonstrate that hybrid serial–parallel architectures can provide reliable dexterity and stable operation for constrained surgical telemanipulation.

Keywords— *Robotic telesurgery; Hybrid manipulators; Remote center of motion; Kinematic modeling; Surgical dexterity; Workspace analysis.*

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1. Introduction

Robot-assisted minimally invasive surgery (MIS) has substantially improved surgical precision, ergonomics, and access to anatomically constrained regions through motion scaling, tremor filtering, and teleportation [1, 2]. In these procedures, the kinematic design of the patient-side manipulator (PSM) plays a critical role in ensuring safety, dexterity, and stable operation. Laparoscopic interventions require surgical instruments to pivot about a fixed remote center of motion (RCM) at the trocar site in order to minimize tissue stress while maintaining maneuverability within the operative field [3, 4]. As a result, workspace reachability, singularity avoidance, and dexterity distribution are fundamental considerations in the design and evaluation of surgical robotic systems [5].

Serial manipulators typically provide large workspaces and relatively simple control but may suffer from reduced structural stiffness and cumulative positioning errors. Parallel mechanisms, on the other hand, offer higher rigidity and precision but often have limited workspace and more complex singularity characteristics. Hybrid serial-parallel architectures therefore provide an attractive compromise, combining the positioning flexibility of serial chains with the stiffness and compact orientation mechanisms of parallel wrists. In particular, spherical parallel mechanisms such as the Agile Eye have been widely considered for distal orientation modules due to their compact geometry and high rotational dexterity[6].

The analysis and synthesis of hybrid manipulators have been extensively studied in recent years. Architectural classifications have demonstrated the versatility of serial-parallel configurations for precision robotic applications [7, 8]. Several investigations have analyzed five degree of freedom hybrid manipulators composed of serial positioning chains and parallel orientation mechanisms, focusing on workspace characteristics and singularity behavior [9, 10]. Analytical studies of spherical parallel wrists have further examined their kinematic properties and singularity free orientation regions [11], while conceptual serial-parallel integrations have shown improvements in stiffness and orientation performance compared with purely serial systems [12]. Dimensional optimization and workspace synthesis approaches have also demonstrated that appropriate design parameters can significantly improve manipulability distribution and reduce singular configurations in hybrid robots [13, 14].

In surgical robotics, maintaining a strict RCM introduces additional kinematic constraints. Various approaches have been proposed to address inverse kinematics and control under these conditions. Real-time inverse kinematics solutions using evolutionary optimization have been reported for RCM-constrained manipulation [15], while manipulability-based strategies have been explored to enhance dexterity during constrained tasks [16]. Constraint consistent control formulations have also been developed to simultaneously enforce task objectives and RCM requirements[17]. Related studies on redundant robotic systems for medical procedures highlight the importance of maintaining dexterity under strict geometric constraints [18]. Broader reviews of surgical robotic systems further emphasize distal dexterity, triangulation capability, and adaptable workspace as key design objectives for operation within confined anatomical environments [19]. In addition, task-specific workspace optimization methods have been proposed to improve surgical accessibility and efficiency [20], and quantitative dexterity metrics have been introduced to relate manipulator kinematics to teleoperation performance [21].

Despite significant progress in the analysis of hybrid manipulators, rigorous closed-form modeling and experimental validation of such systems in surgical applications remain limited. The coupling between serial positioning mechanisms and parallel orientation modules often complicates analytical treatment, particularly for Jacobian derivation, singularity analysis, and dexterity evaluation. Moreover, validation of analytical predictions using real surgical motion data is rarely reported.

In this work, a unified analytical kinematic framework is developed for the *Sinaflex* PSM, a hybrid architecture composed of a serial RRP positioning chain and a non-symmetric spherical parallel orientation mechanism augmented with an independent roll axis. To address the complexity of the coupled system, a virtual spherical wrist abstraction is introduced that reformulates the manipulator as an equivalent six degree of freedom (DOF) serial chain while strictly preserving the RCM constraint. This representation enables closed-form forward and inverse kinematics, analytical Jacobian derivation, and systematic evaluation of workspace and dexterity characteristics.

The remainder of the paper presents the detailed formulation and validation of the proposed model. First, the kinematic structure of the *Sinaflex* PSM and the associated analytical model are derived. Next, the Jacobian formulation and the metrics used for workspace and dexterity analysis are introduced. Simulation and in vivo experimental protocols are then described, followed by results demonstrating kinematic consistency, workspace reconstruction, and dexterity characteristics of the manipulator.

2. Kinematics Modeling

In laparoscopic procedures, the surgical instrument must pivot about a fixed entry point at the trocar site to prevent excessive lateral forces on surrounding tissue while maintaining precise dexterous motion inside the operative cavity. Accurate and stable enforcement of the RCM constraint is therefore a fundamental design requirement. Each surgical arm of the *Sinaflex* is designed to maintain a fixed RCM throughout the task execution. This RCM point is kinematically realized as the geometric intersection of the pitch, yaw, and insertion axes, ensuring that all rotational motions of the instrument shaft occur about a stationary point in space. As illustrated in Fig. 1, this configuration enables safe tool insertion while preserving dexterous orientation control at the distal end of the instrument.

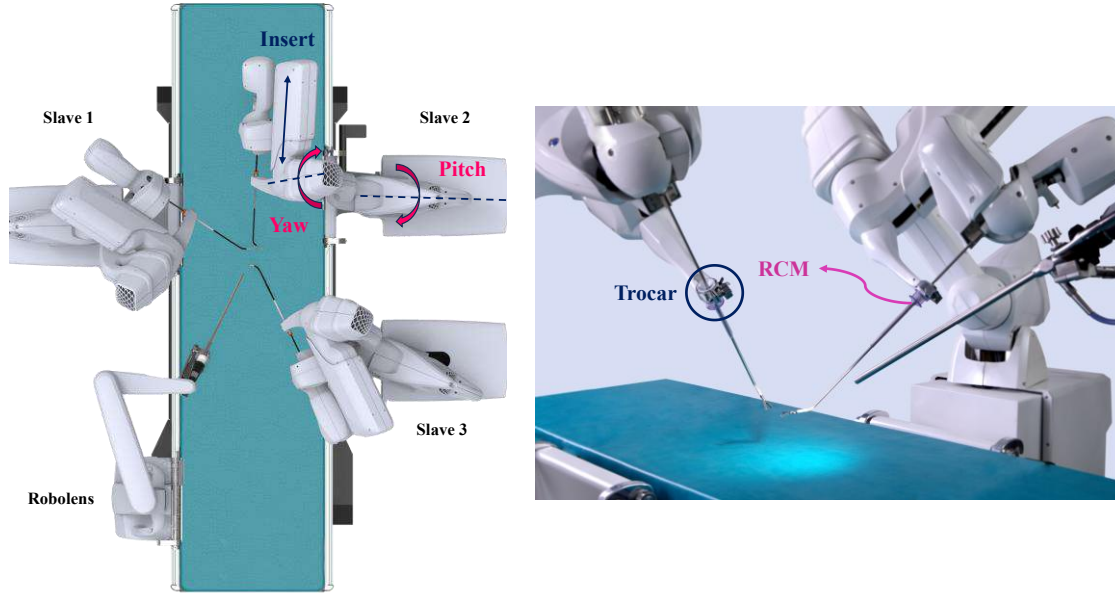


Fig. 1. Schematic representation of the Sina_{flex} PSM.

From a kinematic perspective, the Sina_{flex} manipulator [22] exhibits a hybrid architecture, combining a serial positioning mechanism with a parallel orientation mechanism, as illustrated in Fig. 2. The proximal serial chain primarily governs the positioning and insertion of the instrument, while the distal parallel mechanism—augmented by an independent roll degree of freedom—provides compact, high-stiffness orientation control about the RCM. This architectural choice allows decoupling of translational and rotational motions, which is particularly advantageous for analytical modeling, singularity analysis, and dexterity evaluation in surgical workspaces.

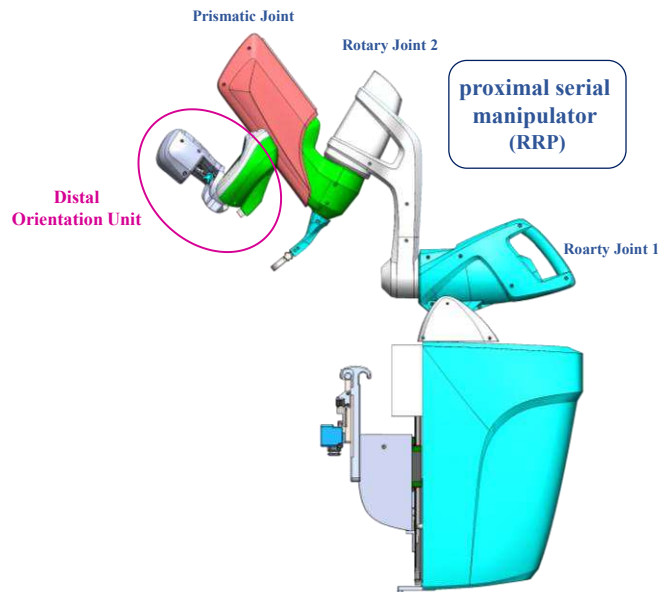


Fig. 2. Hybrid architecture of the Sina_{flex} manipulator.

Direct kinematic modeling of the *Sinaflex* manipulator as a coupled serial–parallel system results in implicit constraint equations that significantly complicate analytical treatment, particularly in the context of Jacobian derivation and dexterity evaluation [23]. To overcome this limitation and obtain a systematic closed-form kinematic formulation using the Denavit–Hartenberg (DH) convention, a virtual spherical wrist abstraction is introduced.

In this framework, the 2 DOF non-symmetric Agile Eye parallel mechanism, together with the independent tool roll joint, is modeled as an equivalent 3 DOF spherical wrist located at the distal end of the manipulator, below the RCM. Consequently, the overall robot can be represented as a 6 DOF serial kinematic chain, consisting of a proximal RRP positioning mechanism followed by a distal 3 DOF virtual wrist.

This abstraction preserves the orientation workspace and rotational capabilities of the physical mechanism while enabling a unified serial representation of the robot. As a result, the forward and inverse kinematics, as well as the associated Jacobian matrix, can be derived analytically.

2.1. Forward Kinematics

The forward kinematics of the manipulator is formulated by decomposing the overall motion into two independent components: (I) the position kinematics of the proximal serial RRP chain, and (II) the orientation kinematics of the distal virtual spherical wrist.

I) Serial RRP Position Kinematics

Let q_1 and q_2 denote the rotational variables of the first and second revolute joints, respectively, and let q_3 represent the linear displacement of the prismatic joint. The two revolute joints constrain the instrument shaft to pivot about the RCM, thereby ensuring that all angular motions occur about the fixed entry point, while the prismatic joint governs insertion and retraction of the tool along its longitudinal axis.

Following the DH convention, coordinate frames are assigned to the first three links as illustrated in Fig. 3, with the base frame $\{0\}$ located at the RCM. The axes are defined such that Z_0 and Z_1 coincide with the axis of the first revolute joint q_1 , while Z_2 is aligned with the axis of the second revolute joint q_2 . The prismatic joint axis Z_3 corresponds to the direction of insertion q_3 . The associated X -axes are assigned to satisfy the DH convention, with X_0 oriented toward the inner side of the mechanism, X_1 oriented upward, and X_2 and X_3 oriented toward the outer side of the robot, as depicted in the figure.

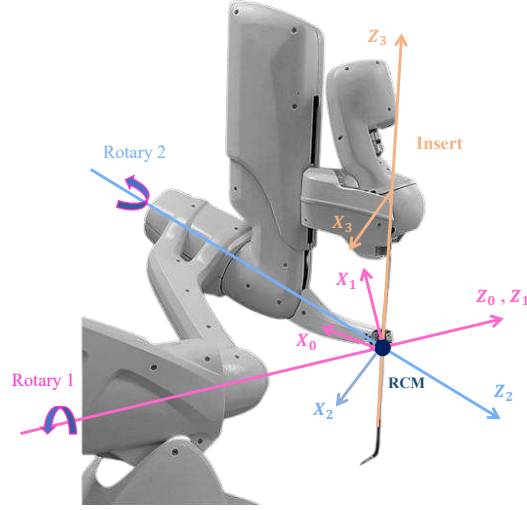


Fig. 3. Denavit–Hartenberg frame assignment for the serial RRP positioning mechanism with respect to the RCM coordinate frame.

The first three rows of the DH parameter table, corresponding to the serial RRP position kinematics, are summarized in Table 1.

Table 1. Denavit–Hartenberg Parameters of the *Sina_{flex}* Manipulator – Serial RRP

<i>joint i</i>	θ_i	d_i	α_{i-1}	a_{i-1}	<i>joint type</i>
1	$\pi - q_1$	0	π	0	<i>Rotary 1</i>
2	q_2	0	$-\mathcal{C}_1$	0	<i>Rotary 2</i>
3	0	q_3	$\pi - \mathcal{C}_2$	0	<i>Insert</i>

Using the DH representation with fixed angular offsets $\mathcal{C}_1$ and $\mathcal{C}_2$, the homogeneous transformation from the RRP serial mechanism end-effector to the RCM center frame is given by:

$$T_{RRP}^{RCM}(q_1, q_2, q_3) = \prod_{i=1}^{i=3} T_i(q_i) \quad 1$$

where $\mathbf{T}_i$ denotes the standard homogeneous transformation associated with the i -th joint.

The position of the wrist center expressed in the RCM frame is then uniquely determined by the joint variables (q_1, q_2, q_3) and inherently satisfies the RCM constraint. This position vector is expressed as:

$$P_{RRP}^{RCM} = P_{Wrist}^{RCM} = [x \quad y \quad z] \quad 2$$

Which represents the Cartesian coordinates of the wrist center resulting from the serial RRP positioning mechanism.

II) Virtual Spherical Wrist Orientation Kinematics

The orientation of the surgical tool is described by the virtual spherical wrist variables $(\theta_4, \theta_5, \theta_6)$, which correspond to yaw, pitch, and axial roll, respectively. The roll angle θ_6 is directly actuated by the independent tool rotation about its longitudinal axis, whereas the yaw and pitch angles (θ_4, θ_5) are obtained through the forward kinematics of the Agile Eye parallel mechanism. Specifically, given the two active rotational joints of the Agile Eye, denoted as AE_1 and AE_2 , the unit direction vector of the tool shaft expressed in the Agile Eye center frame can be computed from the Agile Eye forward kinematics as [6]:

$$\vec{t} = \frac{1}{G^2 + H^2} \left\{ C_\xi \begin{bmatrix} GS_{AE_1} \\ H \\ GC_{AE_1} \end{bmatrix} \pm \sqrt{G^2 + H^2 - C_\xi^2} \right\} \begin{bmatrix} HS_{AE_1} \\ -G \\ HC_{AE_1} \end{bmatrix} \quad 3$$

where the auxiliary variables G and H are defined as:

$$G = S_\lambda S_\gamma S_{AE_2} - C_\lambda C_\gamma, \quad H = S_{AE_1} (C_\lambda S_\gamma S_{AE_2} + S_\lambda C_\gamma) + C_{AE_1} C_{AE_2} S_\gamma \quad 4$$

Here, the shorthand notations $S_x = \sin(x)$ and $C_x = \cos(x)$ are used. The angles λ , γ , and ξ are fixed geometric parameters of the Agile Eye mechanism, while AE_1 and AE_2 denote its actuated joint variables, as illustrated in Fig. 4. It is also worth mentioning, the forward kinematics of the Agile Eye admits two assembly modes corresponding to the sign of the square root term in the analytical solution. In this work, only the physically realizable branch consistent with the robot assembly is selected. Configurations that violate the geometric feasibility condition are explicitly excluded from the workspace analysis to preserve physical validity.

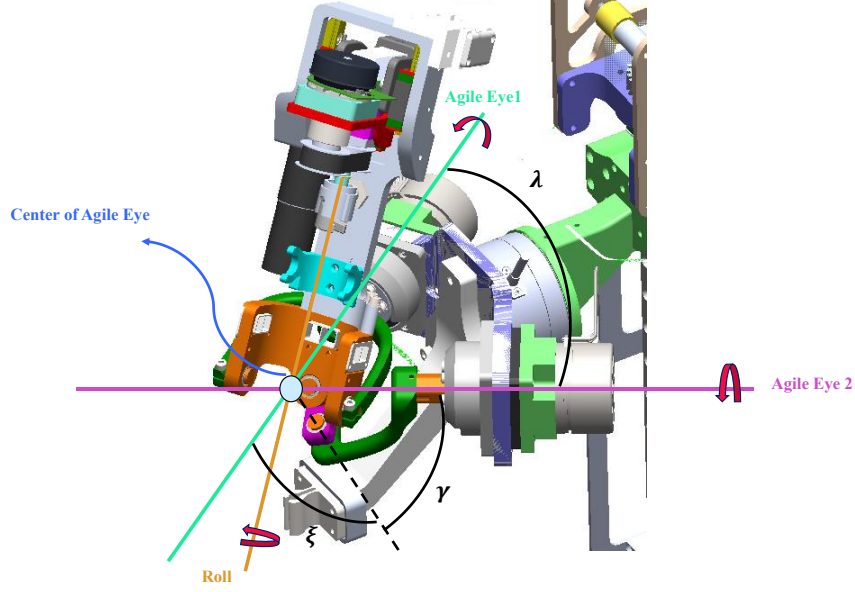


Fig. 4. Forward kinematic mapping of the Agile Eye spherical parallel mechanism.

Since the tool direction vector is a unit vector [6], its components correspond to direction cosines and can equivalently be expressed using spherical coordinates associated with the virtual wrist as:

$$\vec{t} = \begin{bmatrix} x = \sin\left(\frac{\theta_5}{\rho}\right) \cos(\theta_4) \\ y = \sin\left(\frac{\theta_5}{\rho}\right) \sin(\theta_4) \\ z = \cos\left(\frac{\theta_5}{\rho}\right) \end{bmatrix} \quad 5$$

where ρ is a constant inclination ratio that captures the nonlinear transmission between the Agile Eye joint motions and the effective tilt of the tool shaft.

By inverting (5), the yaw and pitch angles of the virtual spherical wrist are obtained as:

$$\theta_4 = \text{atan2}(y, x), \quad \theta_5 = \rho \text{atan2}(\pm\sqrt{1 - z^2}, z) \quad 6$$

with $(x, y, z)^T = \vec{t}$.

Through this mapping, the two active joints of the Agile Eye mechanism are equivalently represented as the yaw and pitch joints of a virtual spherical wrist at the distal end of the manipulator. The complete orientation of the tool frame with respect to the RCM frame is then obtained by appending the axial roll angle θ_6 . The DH frame assignment for the virtual spherical wrist is illustrated in Fig. 5, and the corresponding DH parameters are summarized in the Table 2.

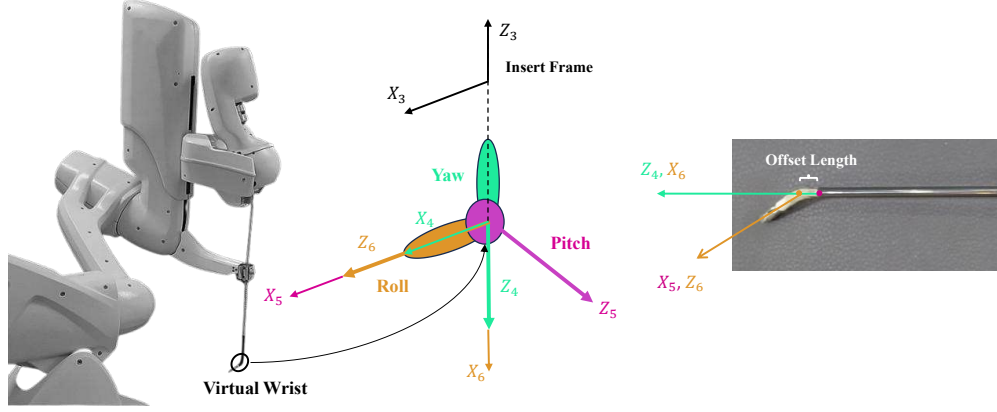


Fig. 5. Denavit–Hartenberg frame assignment of the virtual spherical wrist equivalent to the Agile Eye orientation mechanism.

Table 2. Denavit–Hartenberg Parameters of the Sina_{flex} Manipulator – Virtual Wrist RRR

$joint\ i$	θ_i	d_i	α_{i-1}	a_{i-1}	$joint\ type$
4	$\theta_4 + C_3$	$-d_4$	π	0	<i>Yaw</i>
5	θ_5	0	$-\pi/2$	0	<i>Pitch</i>
6	θ_6	d_6	$-\pi/2$	0	<i>Roll</i>

Using the DH convention for the virtual spherical wrist, the homogeneous transformation from the wrist center to the RRP chain end-effector frame is:

$$T_{Wrist}^{RRP} = T_4(\theta_4)T_5(\theta_5)T_6(\theta_6) \quad 7$$

where each $T_i(\theta_i)$ is the standard DH homogeneous transformation associated with the corresponding virtual wrist joint. Finally, the full forward kinematics of the manipulator from the wrist to the RCM is obtained by concatenating the serial RRP chain and the virtual wrist:

$$T_{Wrist}^{RCM}(q_1, q_2, q_3, \theta_4, \theta_5, \theta_6) = T_{RRP}^{RCM}(q_1, q_2, q_3)T_{Wrist}^{RRP}(\theta_4, \theta_5, \theta_6) \quad 8$$

Where T_{RRP}^{RCM} is the homogeneous transformation of the RRP positioning chain with respect to RCM. This formulation provides a complete analytical mapping from joint space to end-effector pose, fully respecting the RCM constraint while incorporating the orientation capabilities of the distal Agile Eye mechanism and tool roll.

2.2. Inverse Kinematics

The inverse kinematics of the Sina_{flex} PSM aims to determine the joint vector that realizes a desired end-effector pose with respect to the RCM frame. Owing to the decoupled kinematic structure introduced by the virtual spherical wrist abstraction, the inverse kinematics admit a closed-form analytical solution, obtained by sequentially solving the position and orientation subproblems.

2.2.1. Position Inverse Kinematics (RRP Chain)

Let P_{Wrist}^{RCM} denote the position of the wrist center expressed in the RCM frame. The first three joints (q_1, q_2, q_3) form an RRP chain whose geometry enables direct analytical inversion. The forward position relationship of the wrist point is given by:

$$P_{Wrist}^{RCM} = \begin{bmatrix} -q_3\{\sin(C_2)(\sin(q_1)\sin(q_2) + \cos(C_1)\cos(q_2)\sin(q_1)) + \cos(C_2)\sin(C_1)\sin(q_1)\} \\ q_3\{\sin(C_2)(\sin(q_1)\sin(q_2) - \cos(C_1)\cos(q_1)\cos(q_2)) - \cos(C_2)\sin(C_1)\cos(q_1)\} \\ q_3(\cos(C_1)\cos(C_2) - \sin(C_1)\sin(C_2)\cos(q_2)) \end{bmatrix} \quad 9$$

The prismatic joint variable is obtained directly from the Euclidean norm of the wrist position:

$$q_3 = \|P_{Wrist}^{RCM}\| = \sqrt{x^2 + y^2 + z^2} \quad 10$$

Using the z-component of (9), the second revolute joint angle is determined through:

$$\cos(q_2) = \frac{\cos(C_1)\cos(C_2) - z/q_3}{\sin(C_1)\sin(C_2)} \quad 11$$

This relation yields two admissible solutions corresponding to distinct kinematic branches:

$$q_2 = \text{atan2}(\pm\sqrt{1 - \cos^2(q_2)}, \cos(q_2)) \quad 12$$

Once q_2 is selected, the first revolute joint angle q_1 is obtained from the horizontal components (x, y) of the wrist position in (9). Together, these expressions uniquely determine the position-related joint variables, up to the choice of kinematic branch associated with the sign in (12). To ensure numerical robustness, branch selection is performed using a minimum joint space error criterion. This approach guarantees continuity of inverse kinematic solutions and prevents spurious branch switching in the presence of numerical noise.

2.2.2. Orientation Inverse Kinematics

Once the wrist position has been determined, the orientation of the surgical tool end-effector is governed entirely by the virtual spherical wrist positioned below the RCM. This arrangement allows the classical decoupling of inverse kinematics into position and orientation subproblems,

as initially introduced by Pieper [24] and extensively described in standard robotics literature (e.g., Craig[25]).

Accordingly, the orientation inverse kinematics reduces to determining the three wrist joint variables $\theta_4, \theta_5, \theta_6$ that achieve a desired end-effector rotation matrix $R_{Wrist}^{RCM} \in SO(3)$. Let R_{Wrist}^{RCM} denote the rotation of the wrist frame relative to the RCM frame, obtained from the forward kinematics. Due to the spherical structure of the virtual wrist, the orientation mapping can be expressed as:

$$R_{Wrist}^{RCM} = R_{RRP}^{RCM}(q_1, q_2) R_{Wrist}^{RRP}(\theta_4, \theta_5, \theta_6) \quad 13$$

Here R_{RRP}^{RCM} depends solely on the positioning joints and is known once the position inverse kinematics is solved. Explicitly:

$$R_{RRP}^{RCM} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} = \begin{cases} R_{11} = \cos(q_1)\cos(q_2) - \cos(C1)\sin(q_1)\sin(q_2) \\ R_{12} = \cos(C2)(\cos(q_1)\sin(q_2) + \cos(C1)\cos(q_2)\sin(q_1)) - \sin(C1)\sin(C2)\sin(q_1) \\ R_{13} = \sin(C2)(\cos(q_1)\sin(q_2) + \cos(C1)\cos(q_2)\sin(q_1)) + \cos(C2)\sin(C1)\sin(q_1) \\ R_{21} = \cos(q_2)\sin(q_1) + \cos(C1)\cos(q_1)\sin(q_2); \\ R_{22} = \cos(C2)(\sin(q_1)\sin(q_2) - \cos(C1)\cos(q_1)\cos(q_2)) + \sin(C1)\sin(C2)\cos(q_1) \\ R_{23} = \sin(C2)(\sin(q_1)\sin(q_2) - \cos(C1)\cos(q_1)\cos(q_2)) - \cos(C2)\sin(C1)\cos(q_1) \\ R_{31} = \sin(C1)\sin(q_2) \\ R_{32} = \cos(C1)\sin(C2) + \cos(C2)\sin(C1)\cos(q_2); \\ R_{33} = \cos(C1)\cos(C2) - \sin(C1)\sin(C2)\cos(q_2); \end{cases} \quad 14$$

Consequently, the wrist orientation matrix can be isolated as:

$$R_{Wrist}^{RRP} = R_{RRP}^{RCM T} R_{Wrist}^{RCM} = \quad 15$$

$$\begin{bmatrix} -\cos(\theta_4 + C3)\cos(\theta_5)\cos(\theta_6) + \sin(\theta_4 + C3)\sin(\theta_6) & \cos(\theta_4 + C3)\cos(\theta_5)\sin(\theta_6) + \sin(\theta_4 + C3)\cos(\theta_6) & \cos(\theta_4 + C3)\sin(\theta_5) \\ -\sin(\theta_4 + C3)\cos(\theta_5)\cos(\theta_6) - \cos(\theta_4 + C3)\sin(\theta_6) & \sin(\theta_4 + C3)\cos(\theta_5)\sin(\theta_6) - \cos(\theta_4 + C3)\cos(\theta_6) & \sin(\theta_4 + C3)\sin(\theta_5) \\ \sin(\theta_5)\cos(\theta_6) & -\sin(\theta_5)\sin(\theta_6) & \cos(\theta_5) \end{bmatrix}$$

The orientation inverse kinematics is therefore fully formulated in terms of R_{Wrist}^{RRP} . As a result, the third column of R_{Wrist}^{RRP} is sufficient to determine the first two wrist angles:

$$\vec{r}_3 = \begin{bmatrix} \cos(\theta_4 + C3)\sin(\theta_5) \\ \sin(\theta_4 + C3)\sin(\theta_5) \\ \cos(\theta_5) \end{bmatrix} \quad 16$$

From Equation (16), the pitch angle θ_5 is uniquely obtained as:

$$\theta_5 = \text{atan2}(\sqrt{r_{31}^2 + r_{32}^2}, r_{33}) \quad 17$$

Provided that $\sin(\theta_5) \neq 0$, the yaw angle θ_4 is derived from the first two components of $\vec{r}_3$:

$$\theta_4 = \text{atan2}(r_{32}, r_{31}) - C3 \quad 18$$

When $\sin(\theta_5) = 0$, the wrist aligns such that the pitch rotation is fully extended or folded, resulting in a singular configuration (commonly referred to as gimbal lock). In this condition, the yaw angle θ_4 becomes indeterminate because rotations about the yaw and roll axes become coupled. Once θ_4 and θ_5 are determined for a non-singular configuration, the roll angle θ_6 can be uniquely computed from the appropriate elements of R_{Wrist}^{RRP} , completing the orientation solution as follow:

$$\theta_6 = \text{atan2}(R_{Wrist}^{RRP}(3,2), R_{Wrist}^{RRP}(3,1)) = (-\sin(\theta_5)\sin(\theta_6), \sin(\theta_5)\cos(\theta_6)) \quad 19$$

To find the robot actuated joints, the virtual wrist angles (θ_4, θ_5) are mapped to the active joints of the non-symmetric Agile Eye mechanism via a transformation into direction cosines as mentioned in the Equation (5). The resulting direction vector enables the analytical solution of the Agile Eye inverse kinematics [6]:

$$\begin{aligned} AE_1: \theta_1 &= \text{atan2}(x, z) \\ AE_2: \theta_2 &= \text{atan2}(MP \sin(\lambda) - LN, \quad LP \sin(\lambda) + MN) \end{aligned} \quad 20$$

With:

$$\begin{aligned} \sin(\theta_3) &= \frac{-\sin(\theta_1) \sin(\theta_5) \sin(\lambda) - \cos(\theta_5) \cos(\lambda) + \cos(\gamma) \cos(\xi)}{\sin(\gamma) \sin(\xi)} \\ M &= \cos(\theta_3) \sin(\xi) \\ P &= \cos(\theta_1) \sin(\theta_5) \\ L &= \sin(\gamma) \cos(\xi) + \sin(\theta_3) \cos(\gamma) \sin(\xi) \\ N &= \cos(\theta_5) + \sin(\theta_3) \cos(\lambda) \sin(\gamma) \sin(\xi) - \cos(\lambda) \cos(\gamma) \cos(\xi) \end{aligned} \quad 21$$

This mapping ensures precise control of the non-symmetric Agile Eye wrist within the hybrid serial-parallel configuration.

2.3. Jacobian

The differential kinematics of the Sinaflex is described using a geometric Jacobian derived from screw-axis representations of the joints [26]. Each column of the Jacobian corresponds to the instantaneous twist generated by an individual joint. The full Jacobian matrix is partitioned into translational and rotational components. The translational submatrix J_v characterizes how joint velocities produce linear motion of the wrist center, while the rotational submatrix J_ω describes the mapping between joint velocities and angular velocity of the end-effector. Revolute joints contribute to both submatrices, whereas prismatic joints contribute exclusively to the translational component.

For the $Sina_{flex}$ slave manipulator, the Jacobian is constructed using the joint screw axes expressed in the base frame. The complete 6×6 Jacobian is obtained:

$$J = \begin{bmatrix} J_V \\ J_\omega \end{bmatrix} = \begin{bmatrix} z_1 \times (p_e - p_1) & z_2 \times (p_e - p_2) & z_3 & z_4 \times (p_e - p_4) & z_5 \times (p_e - p_5) & z_6 \times (p_e - p_6) \\ z_1 & z_2 & 0 & z_4 & z_5 & z_6 \end{bmatrix} \quad 22$$

Where z_i and p_i denote the axis and position of the i -th joint, respectively, and p_e is the position of the end-effector. This formulation provides a compact geometric representation of the velocity relationship between joint space and task space.

To enable meaningful comparison of translational and rotational performance, the Jacobian is normalized using a characteristic length scaling prior to singular value analysis. This normalization avoids dimensional inconsistency and allows the use of singular value-based dexterity measures. The resulting Jacobian provides a complete description of the local kinematic behavior of the hybrid manipulator and forms the basis for workspace and dexterity evaluation.

3. Workspace and Dexterity

The kinematic performance of the manipulator is evaluated in terms of reachable workspace and dexterity under joint-limit and RCM constraints. Workspace analysis characterizes the spatial region accessible to the wrist center, while dexterity analysis quantifies the robot's ability to generate arbitrary motions within that region.

3.1. Workspace Evaluation

The reachable workspace is computed using Monte Carlo sampling [27] of the admissible joint space. Randomized sampling ensures statistically uniform coverage and avoids the directional bias associated with grid-based discretization. For each sampled configuration, forward kinematics are used to determine the wrist position relative to the RCM. The resulting point cloud represents the set of reachable positions and serves as the basis for subsequent dexterity analysis.

3.2. Dexterity

Dexterity measures a manipulator's ability to generate motion in arbitrary directions within its workspace. To quantify dexterity, several indices commonly used in robotics literature are employed: the inverse of the condition number (I_C) and manipulability (I_M). These indices are defined as:

$$I_C = \frac{1}{k}, \quad k = \frac{\sigma_{max}}{\sigma_{min}}, \quad I_M = \prod_{i=1}^m \sigma_i \quad 23$$

where σ_i denotes the singular values of the Jacobian, and m is the total number of singular values. In this study, all indices are computed at each sampled end-effector position within the workspace

using singular value decomposition (SVD) of the Jacobian. To evaluate the dexterity across the entire workspace, the Global Conditioning Index (GCI) and Global Mobility Index (GMI) are calculated over N discrete workspace points:

$$\widetilde{GCI} = \frac{1}{N} \sum_{i=1}^N I_C, \quad \widetilde{GMI} = \frac{1}{N} \sum_{i=1}^N I_{MN}, \quad I_{MN_n} = \frac{I_{M_n}}{\max(I_{M_1} \dots I_{M_N})} \quad 24$$

Where, I_{MN_n} is the normalized manipulability index at the n -th workspace point, and I_{M_i} is the manipulability index at the i -th point. These global indices provide a concise measure of the overall dexterity of the robot throughout its reachable workspace.

4. Model Validation

To demonstrate the accuracy and practical relevance of the proposed hybrid kinematic model for the Sinaflex PSM, both simulation and in vivo experimental validation were performed. These analyses aim to verify the correctness of the kinematic equations and to quantify the workspace and dexterity of the robot under realistic surgical conditions.

4.1. Simulation

Simulation studies were conducted in MATLAB 2024a (MathWorks, Natick, MA) to evaluate the accuracy and consistency of the proposed kinematic model. Both forward and inverse kinematics equations of the hybrid serial–parallel architecture were implemented. For forward kinematics, the end-effector position and orientation were computed for a large set of joint configurations within the joint limits. These computed poses were then compared to the reference trajectories generated from the inverse kinematics solution to confirm consistency.

The simulation workflow consisted of the following steps:

1. Joint Space Sampling: Serial joints q_1, q_2, q_3 and spherical wrist joints $\theta_4, \theta_5, \theta_6$ were sampled uniformly across their allowable ranges.
2. Forward Kinematics Computation: Using the proposed hybrid kinematic equations, the corresponding end-effector positions P_{EE} and orientations R_{EE} were calculated for each sampled configuration.
3. Inverse Kinematics Verification: For each computed end-effector pose, the inverse kinematics algorithm was applied to retrieve the joint variables. The resulting joint configurations were compared with the original input values to determine the accuracy of the inverse mapping.
4. Error Analysis: The positional error was computed as the Euclidean distance between the original and reconstructed end-effector positions, while the orientation error was measured using the angle of the rotation difference matrix.

Additionally, the Jacobian matrices were evaluated at various points in the workspace, and singularity analysis was performed by examining the conditioning index.

4.2. In vivo experiment

To further validate the proposed kinematic model under realistic surgical conditions, an in vivo experiment was conducted using the Sina_{flex} PSM during a robotic vasectomy procedure. The experiment was performed as illustrated in Fig. 6 using a 90-month-old dog weighing 30 kg with a body length of 110 cm. The animal was provided by the laboratory animal facility of Tehran University of Medical Sciences (TUMS) and was not recruited from a private owner. The animal was maintained under institutional care, with dietary management initiated three days before the surgical procedure.



Fig. 6. The vasectomy surgery on a dog using Sina_{flex} robotic telesurgery system.

The study was conducted in accordance with the applicable ethical principles for animal research and was approved by the Biomedical Research Ethics Committee under the Research Ethics Committees of the Vice-Chancellor in Research Affairs, Tehran University of Medical Sciences, Iran, under approval ID IR.TUMS.VCR.REC.1399.485. The procedure was carried out in a sterilized environment at the Iran Advanced Clinical Training (IACT) Center, Imam Khomeini Hospital Complex. The main surgeon operated the Sina_{flex} telesurgery system remotely from Sina Hospital, located approximately 7 km from the surgical site.

General anesthesia was administered throughout the procedure. Anesthesia was induced intravenously using ketamine at 5 mg/kg and diazepam at 0.2 mg/kg, followed by endotracheal intubation and maintenance with isoflurane. Cefazolin (22 mg/kg) was administered as prophylactic antibiotic therapy, while tramadol (3 mg/kg) was administered for analgesia. Intravenous saline was administered at a rate of 10 mL/kg/h. Physiological parameters, including oxygen saturation (SpO₂), heart rate, and respiratory rate, were monitored throughout the anesthetic period by veterinary personnel.

The robotic vasectomy procedure lasted approximately 90 min. During the experiment, the duration of the surgical procedure and its individual steps, together with the kinematic variables of the PSM, were recorded. The acquired kinematic data included the joint positions and velocities of the manipulator and were sampled at 1 kHz throughout the procedure. These experimentally recorded trajectories were subsequently used to reconstruct the actual region of the robot workspace utilized during surgery and to evaluate the dexterity characteristics predicted by the proposed analytical model.

Following completion of the procedure, the animal was recovered from anesthesia and remained under institutional postoperative care for approximately three weeks. No euthanasia was performed as part of this study. The postoperative course was successful, and the animal remained under institutional observation and care throughout the follow-up period.

5. Results and Discussion

5.1. Kinematic Consistency and Model Validation

The correctness of the proposed analytical model was first assessed through forward–inverse kinematic consistency tests. A large number of joint configurations were sampled within the admissible joint limits of the manipulator, and the corresponding end-effector poses were computed using the forward kinematics. These poses were subsequently used as inputs to the closed-form inverse kinematics in order to reconstruct the joint variables.

To illustrate the consistency of the solution, a representative subset of 50 randomly selected configurations is shown in Fig. 7. The reconstructed joint variables coincide with the original inputs with negligible numerical deviation. Across the entire sampled dataset, the residual errors remained within machine precision, confirming the mathematical equivalence between the forward and inverse formulations.

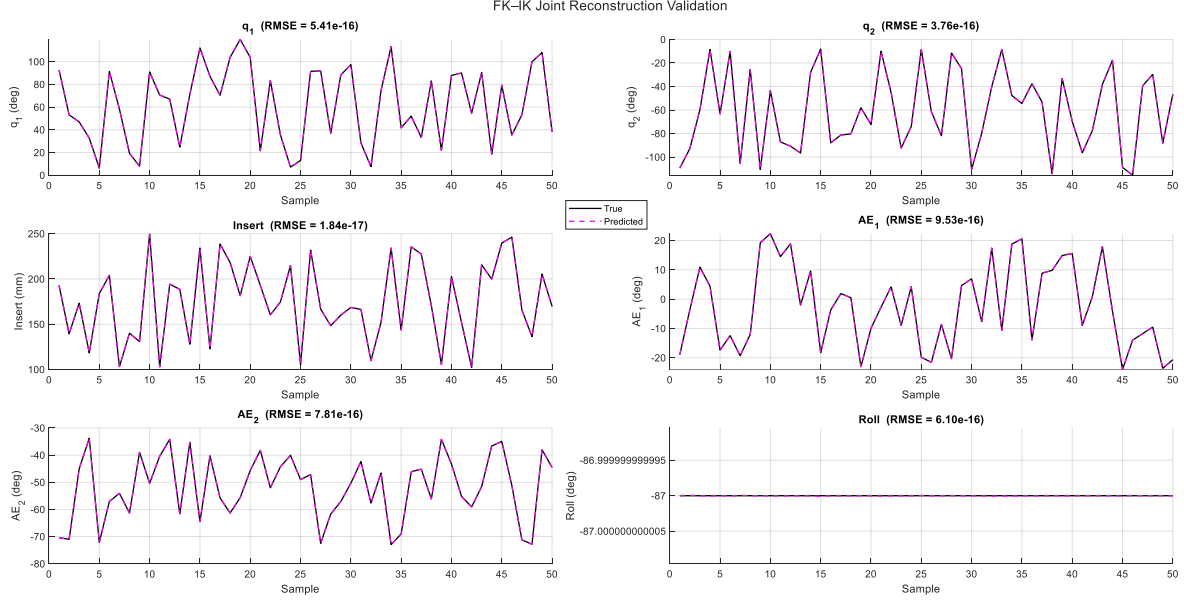


Fig. 7. Forward-inverse kinematic consistency test of the Sina_{flex} manipulator.

These results demonstrate that the proposed virtual spherical wrist abstraction accurately preserves the kinematic behavior of the underlying hybrid serial-parallel mechanism. In particular, the mapping between the Agile Eye orientation subsystem and the equivalent serial wrist representation does not introduce approximation or loss of information.

From a practical perspective, this property is critical for surgical teleoperation, where reliable inverse kinematics must operate continuously at high control frequencies. The absence of discontinuities, instability, or unintended branch switching across the sampled configurations indicates that the analytical formulation is well suited for real-time implementation.

Quantitative reconstruction errors were evaluated by comparing the original and reconstructed end-effector poses. The positional error was defined as the Euclidean distance between the two wrist positions, while the orientation error was computed from the angle of the rotation difference matrix. The resulting mean errors are summarized in Table 3. The extremely small values confirm the numerical stability and accuracy of the proposed kinematic model.

Table 3. Forward-inverse kinematic reconstruction error of the Sina_{flex} manipulator.

Metric	Mean error	Unit
Position	$4.132e - 17$	<i>m</i>
Orientation	$4.768e - 09$	<i>rad</i>

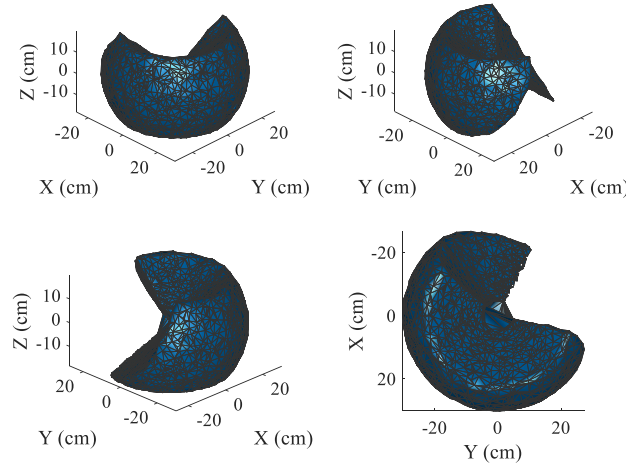
5.2.Workspace Reconstruction

The reachable workspace of the Sina_{flex} PSM was reconstructed using a Monte Carlo sampling strategy over the admissible joint space. A total of 50,000 random configurations were generated within the physical joint limits, and the wrist center position relative to the RCM was computed using the forward kinematics.

The resulting point cloud, illustrated in Fig. 8, defines the geometric boundary of the manipulator workspace. The envelope forms a continuous region centered around the RCM, reflecting the pivoting constraint imposed by the RRP positioning mechanism. This structure is consistent with the expected kinematic behavior of laparoscopic manipulators, where the instrument shaft rotates about a fixed entry point while maintaining insertion capability.

Importantly, the reconstructed workspace does not exhibit disconnected regions or internal voids within the feasible joint range. This indicates smooth kinematic accessibility and confirms that the hybrid serial–parallel architecture provides continuous coverage of the operational region.

Operational Workspace Envelope ($568.1 \times 574.7 \times 379.7 \text{ mm}^3$)



(a)

Reachable Workspace Envelope ($606.5 \times 612.1 \times 434.0 \text{ mm}^3$)

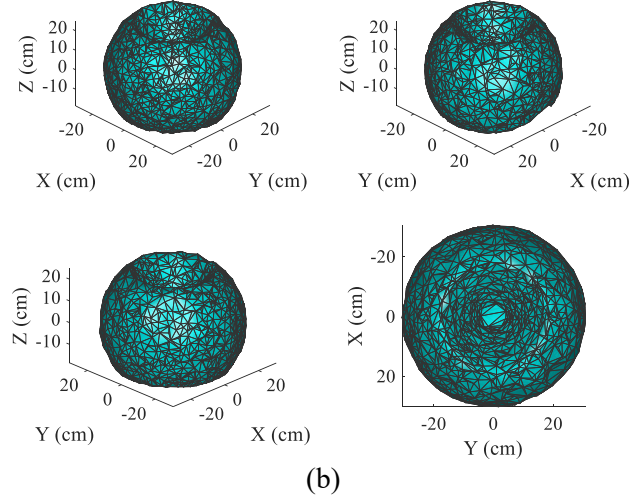


Fig. 8. Monte Carlo reconstruction of the Sina_{flex} PSM workspace. (a) Reachable workspace of the wrist center relative to the RCM. (b) Workspace boundary envelope obtained from 50,000 sampled configurations.

The computed reachable workspace volume is approximately $[606.5 \times 612.1 \times 434.0 \text{ mm}^3]$, while the operational workspace under surgical constraints corresponds to $[568.1 \times 574.7 \times 379.7 \text{ mm}^3]$. These dimensions are compatible with the spatial requirements of minimally invasive procedures.

To further evaluate practical relevance, the end-effector trajectories recorded during the in vivo vasectomy procedure were projected onto the reconstructed workspace (Fig. 9). The surgical motion lies well within the central region of the reachable space, indicating that the manipulator operates away from kinematic boundaries during typical clinical use. This observation supports the suitability of the Sina_{flex} architecture for confined surgical environments.

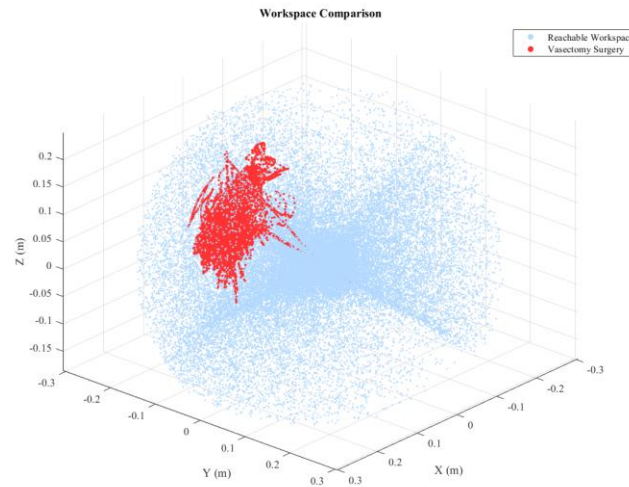


Fig. 9. Surgical trajectories within the robot workspace.

5.3. Dexterity Distribution in the Workspace

The dexterity of the manipulator was evaluated by computing the analytical Jacobian at each sampled configuration and extracting singular value–based performance indices. In particular, the conditioning index and normalized manipulability were used to characterize the quality of motion transmission throughout the workspace.

Fig. 10 shows the spatial distribution of the conditioning index computed for 50,000 sampled configurations as well as for the experimentally recorded surgical trajectories. High-performance regions, corresponding to conditioning values close to 1, are primarily located in the central portion of the workspace and along the lateral operating regions. These areas correspond to configurations where the manipulator operates away from joint limits and singular postures.

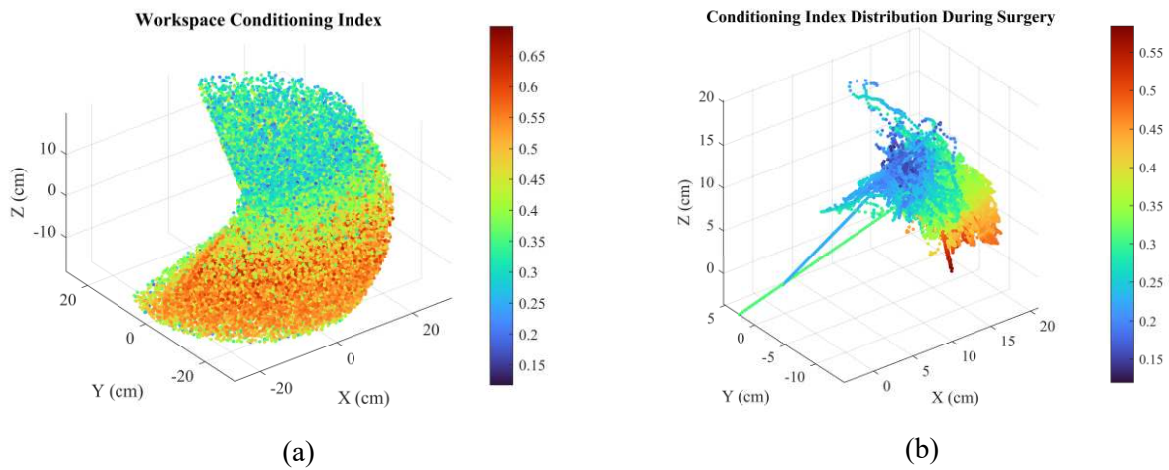


Fig. 10. Conditioning index distribution in the workspace. (a) Operational workspace. (b) Vasectomy surgical workspace.

To further visualize the kinematic performance, the normalized manipulability index was mapped across both the operational workspace and the task-specific surgical workspace (Fig. 11).

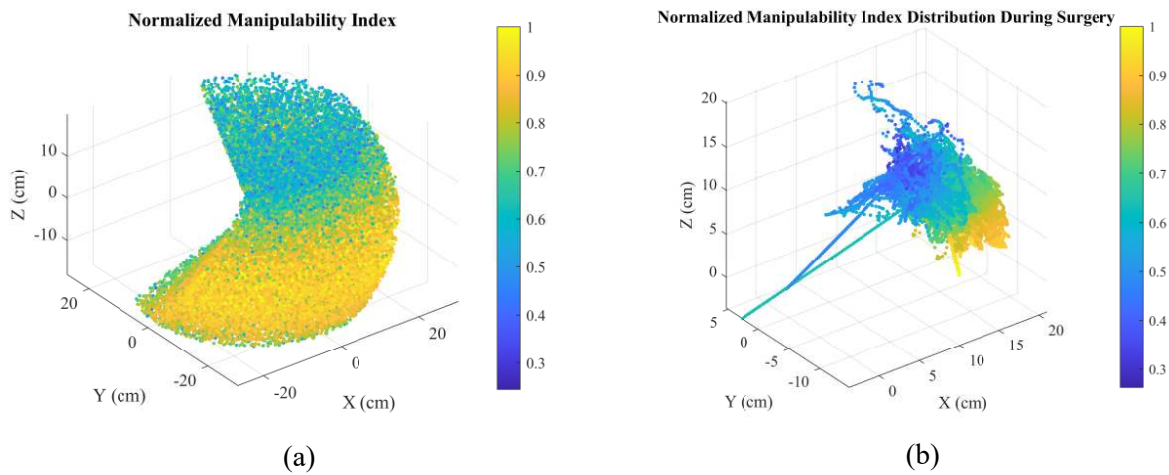


Fig. 11. Normalized manipulability across the workspace. (a) Operational workspace. (b) Surgical task workspace.

The results reveal a gradual decrease in manipulability near the workspace boundaries, particularly in configurations approaching extreme pitch angles of the RRP chain. However, the degradation occurs smoothly rather than abruptly, indicating that the hybrid architecture mitigates severe singular behavior. Global dexterity metrics were computed to summarize overall performance. The Global Conditioning Index (GCI) and Global Mobility Index (GMI) were evaluated for both the complete operational workspace and the subset corresponding to the vasectomy procedure. The results are reported in Table 4.

Table. 4. The dexterity of the robot for the operational and surgical workspaces derived from the simulation and animal study.

Dexterity Index	Operational workspace	Vasectomy surgery workspace
$\mathbf{Max}(I_C)$	0.6970	0.58408
$\mathbf{Min}(I_C)$	0.1175	0.11945
$\mathbf{GCI}$	0.4737	0.44176
$\mathbf{Max}(I_{MN})$	1	1
$\mathbf{Min}(I_{MN})$	0.2447	0.26293
$\mathbf{GMI}$	0.8056	0.83246

The manipulator achieves a GMI of 0.8056 across the operational workspace and 0.83246 within the task-specific surgical workspace. These values indicate that the robot maintains favorable kinematic conditioning across a substantial portion of its reachable region. Notably, the slightly higher value observed in the surgical workspace suggests that the procedure naturally occurs within regions of improved dexterity.

From a surgical perspective, this alignment between task execution and high-performance regions is desirable, as it reduces the likelihood of encountering near-singular configurations during teleoperation and contributes to stable and precise instrument manipulation.

6. Conclusion

This study presented a comprehensive analytical framework for the kinematic modeling and performance evaluation of the Sina_{flex} PSM, a hybrid surgical robot combining a serial RRP positioning mechanism with a non-symmetric spherical parallel wrist and an independent roll axis. By introducing a virtual spherical wrist abstraction, the coupled serial–parallel architecture was reformulated as an equivalent six-degree-of-freedom serial system while strictly preserving the RCM constraint. This representation enabled closed-form derivation of forward and inverse kinematics, explicit Jacobian formulation, and systematic singularity and dexterity analysis.

Simulation results confirmed the internal consistency and numerical stability of the analytical model. Forward–inverse kinematic reconstruction errors remained within machine precision,

demonstrating that the virtual wrist representation accurately captures the kinematic behavior of the physical mechanism. Monte Carlo workspace reconstruction further revealed a continuous reachable region centered on the RCM with no disconnected domains, indicating smooth accessibility across the admissible joint range.

Dexterity analysis based on singular value decomposition of the Jacobian showed favorable conditioning throughout the central operational region of the workspace. High manipulability values were concentrated within the regions most frequently used during surgical tasks, while gradual performance degradation was observed only near the workspace boundaries. Global dexterity metrics confirmed this behavior, yielding a Global Mobility Index of 0.8056 across the operational workspace and 0.83246 within the task-specific surgical workspace.

Importantly, the analytical predictions were supported by in vivo experimental data obtained during a robotic vasectomy procedure. The recorded joint trajectories indicated that the manipulator operates predominantly within well-conditioned regions of its workspace, providing empirical validation of the theoretical analysis. This agreement between model predictions and real surgical motion demonstrates the practical relevance of the proposed framework for evaluating hybrid surgical manipulators.

Beyond the specific *Sinaflex* platform, the modeling strategy introduced in this work provides a general methodology for analyzing hybrid serial–parallel robots operating under RCM constraints. The virtual wrist formulation simplifies analytical treatment while preserving the essential kinematic characteristics of the mechanism, enabling rigorous workspace and dexterity assessment. Such analytical tools are valuable for guiding mechanism design, optimizing surgical robot performance, and supporting the development of robust control strategies for teleoperated minimally invasive surgery.

Future work will focus on incorporating dynamic modeling, task-specific optimization of manipulator configuration, and real-time control methods that leverage the analytical formulation to further enhance surgical precision and safety.

Ethics declarations

Competing interests: The authors declare no competing interests.

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Ethics Statement

The study was conducted in accordance with the applicable ethical principles for animal research and was approved by the Biomedical Research Ethics Committee under the Research Ethics Committees of the Vice-Chancellor in Research Affairs, Tehran University of Medical Sciences, Iran, under approval ID IR.TUMS.VCR.REC.1399.485.

Data availability

The datasets used and/or analysed during the current study available from the corresponding author on reasonable request.

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