

Offshore Wind Farm-Induced Ocean Circulation Over Variable Bathymetry: A Multiscale LES--Shallow-Water Approach

Massimo Della Monica

Sapienza University of Rome

Matteo Bernardini

`matteo.bernardini@uniroma1.it`

Sapienza University of Rome

Stefano Leonardi

The University of Texas at Dallas

Research Article

Keywords: wind farm, ocean circulation, Ekman pumping, large-eddy simulation, shallow water equations, response surface methodology

Posted Date: October 14th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-11128847/v1>

License:   This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Additional Declarations: No competing interests reported.

Offshore Wind Farm-Induced Ocean Circulation Over Variable Bathymetry: A Multiscale LES–Shallow-Water Approach

Massimo Della Monica^{1,2}, Matteo Bernardini^{2*}, Stefano Leonardi¹

^{1*}Department of Mechanical Engineering, The University of Texas at Dallas, Richardson, Texas, 75080-3021, USA.

^{2*}Department of Mechanical and Aerospace Engineering, Sapienza University of Rome, Via Eudossiana 18, Rome, 00184, Italy.

*Corresponding author(s). E-mail(s): matteo.bernardini@uniroma1.it;

Abstract

The objective of this work is to investigate the effects of bathymetric variations on the mesoscale ocean response to offshore wind farms, with particular focus on wind-driven upwelling and downwelling, using a novel multiscale, multi-fidelity modeling framework. The framework couples high-fidelity large-eddy simulations (LES), which resolve turbine wakes and turbulent mixing, with a two-dimensional shallow water model of the ocean featuring a diagnostically reconstructed vertical velocity, so that the circulation response can be represented over extended spatial and temporal scales while preserving the key features of the flow. A preliminary statistical analysis, based on the shallow water model forced by an idealized Gaussian wind stress and on a full factorial design of experiments, identifies the wind speed as the dominant controlling parameter, with basin depth and wind farm size playing comparable secondary roles and latitude a minor one. LES of a turbine row representative of the Horns Rev 1 wind farm are then performed with an in-house code and validated against manufacturer, experimental and literature data. The LES velocity field is upscaled to the mesoscale by means of physics-based response surfaces, and the resulting wind stress is used to force the shallow water model over flat and ramped bathymetries. The results show that even mild bathymetric gradients dominate the horizontal circulation, whereas the surface elevation dipole remains discernible and the wind-stress-driven Ekman contribution to the vertical velocity remains clearly identifiable within the present diagnostic reconstruction.

Keywords: wind farm, ocean circulation, Ekman pumping, large-eddy simulation, shallow water equations, response surface methodology

Article highlights

- A multiscale LES–shallow-water framework couples turbine-resolved wakes to the mesoscale ocean response.
- Wind speed dominates wind-farm-induced upwelling; farm size and basin depth follow, latitude matters least.

- Even mild bathymetric slopes dominate the horizontal circulation induced by an offshore wind farm.

1 Introduction

Offshore wind capacity has been growing rapidly worldwide [20, 49], and large wind farms are

increasingly deployed in shallow shelf seas, where their interaction with the underlying ocean cannot be neglected. Over the past two decades, several studies have analyzed the impact of offshore wind farms on surface ocean variability and circulation [36]. The results obtained strongly depend on the simplifications and assumptions made in the construction of the models. Thus, there exists some discrepancy in the results based on the parametrization used by the different authors.

Broström [6] first investigated the potential environmental impacts of offshore wind farms on local ocean circulation. Using a simplified analytical model, he showed how a large wind farm could alter local wind patterns and, consequently, the surface ocean response. He primarily employed a reduced gravity model, simulating two layers of water with different densities [46]. Broström showed how large wind farms can generate a coherent structure of upwelling and downwelling, which the model represents as thinning and thickening of the surface layer, i.e., as vertical water motions toward the surface or the bottom. Broström suggested that these processes could affect temperature, primary productivity, and local ecosystems.

Paskyabi and Fer [33] used simplified numerical models to analyze the response of the surface ocean to the presence of wind farms accounting for the influence of surface waves. They represented a wind farm as a fixed rectangular region and described the associated wind deficit using a U-shaped profile. The results showed strong agreement with those of Broström and highlighted a pronounced sensitivity of the oceanic response to wave forcing at the surface.

In a subsequent study by Paskyabi [32], multiple wind farms in shallow waters were analyzed. By combining empirical wake models with 2D and 2DV (namely, 2D models with prognostic vertical velocity) oceanic models, the study highlighted how ocean dynamics can be modified in terms of turbulent mixing, wave propagation, currents, stratification, and upwelling processes. The results showed that the presence of the wind farm modifies the stratification of the water column compared to reference conditions, generally amplifying the effects on local circulation.

Ludewig [28] showed that sustained winds blowing from a constant direction at moderate

speeds over several hours are required to generate a surface elevation dipole in the ocean. Under these conditions, the resulting circulation cells can extend over tens of square kilometers, leading to thermocline displacements and enhanced vertical mixing.

Segtnan and Christakos [39] investigated the impact of wind farm configuration (turbine layout, rotor size, hub height) on vertical circulation, using highly simplified models for the wind turbine wakes. The simulations, spanning two days, compared two wind farm layouts: one with straight rows and one with curved rows. The results showed that the wind farm layout influences the wind stress and, consequently, the ocean vertical velocity as well as the local upwelling and downwelling patterns. In shallow waters, these variations are primarily due to changes in the horizontal flow introduced by the wind farm, rather than by Ekman pumping.

Finally, Raghukumar et al. [35] studied the effects of wind farms on the Californian coast. Their analyses indicated that the wind farms alter the wind stress gradient, reducing upwelling near the coast and increasing it further offshore. Overall, the average intensity of upwelling shows little variation, but the local pattern is altered, with potential implications for marine ecosystems.

The early studies reviewed here provided a qualitative description of the upwelling and downwelling mechanisms arising near wind farms, whereas more recent works have focused on real-world case studies. Nevertheless, there remains significant scope for deeper investigations using low-fidelity models, particularly because statistical analyses of these phenomena are still limited. In particular, a systematic characterization of upwelling and downwelling dynamics in relation to wind farm configurations and environmental variables is still lacking.

The present work addresses this gap with a twofold contribution. First, a systematic statistical characterization of wind-farm-induced upwelling and downwelling is carried out with a two-dimensional shallow water model forced by an analytical (Gaussian) wind stress, quantifying the influence of latitude, basin depth, wind forcing and wind farm size through a full factorial design of experiments. Second, a multiscale, multi-fidelity framework is proposed in which the wind field

over a real wind farm (Horns Rev 1) is obtained from large-eddy simulations (LES), extended to the mesoscale by physics-based response surfaces and used to force the shallow water model; to our knowledge, this is the first direct coupling between an ocean model and an LES-informed wind field reconstructed at the scale of an entire wind farm. The framework is then used to investigate the effects of bathymetry variations, which have so far received limited attention. In all cases the vertical velocity is diagnostically reconstructed from the depth-averaged solution, trading the completeness of fully 3D approaches for transparency and low computational cost. The paper is organized as follows. Section 2 presents the shallow water model and the vertical velocity reconstruction; Section 3 reports the statistical analysis; Section 4 describes the LES solver, the Horns Rev 1 setup and the mesoscale reconstruction, together with their validation; Section 5 presents the coupled results over flat and variable bathymetry, and Section 6 draws the conclusions. The validation of the in-house shallow water solver is reported in Appendix A.

2 Shallow Water Model

The shallow water equations (SWE) adopted in this work are solved for the depth-averaged horizontal velocity of a rotating ocean basin, $\bar{\mathbf{u}} = (\bar{u}, \bar{v})^T$, and for the water column height $h(x, y, t)$, under the forcing of a surface wind stress $\boldsymbol{\tau} = (\tau_x, \tau_y)^T$ and of a non-flat bottom described by the bathymetry function $b(x, y)$, as illustrated in Fig. 1. Throughout the paper the overbar denotes depth averaging, $\eta = b + h$ is the free-surface elevation measured from the reference datum $z = 0$, and the sea surface height anomaly (SSHA) denotes the departure of η from its value at rest.

The following nondimensionalization of the variables is used

$$\begin{aligned} \bar{u}^* &= \frac{\bar{u}}{U_0}, & \bar{v}^* &= \frac{\bar{v}}{U_0}, & h^* &= \frac{h}{H_0}, \\ b^* &= \frac{b}{H_0}, & t^* &= ft, & x^* &= \frac{x}{L}, & y^* &= \frac{y}{L}, \end{aligned}$$

where L is the characteristic dimension of the wind farm, H_0 is the mean water column height at rest, U_0 is the characteristic velocity, and f is the Coriolis parameter. The nondimensional components of the imposed wind stress can be written instead

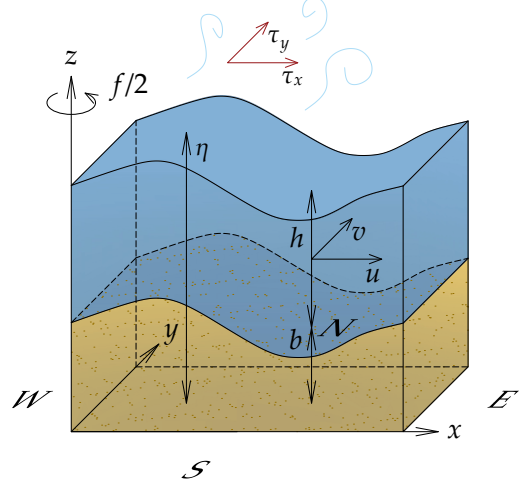


Fig. 1 Schematic of the 2D rotating shallow ocean problem with external wind forcing and non-flat bathymetry

as

$$\tau_x^* = \frac{\tau_x}{\Delta\tau}, \quad \tau_y^* = \frac{\tau_y}{\Delta\tau},$$

where $\Delta\tau$ represents the characteristic wind stress variation induced by the wind downstream of the wind farm, as introduced by Broström [6].

Now, in the absence of wind stress forcing, the problem is defined by only two nondimensional groups. The nondimensional equations can be expressed, for example, in terms of the Rossby and Burger numbers [46], defined as

$$Ro = \frac{U_0}{fL} \quad \text{and} \quad Bu = \frac{gH_0}{(fL)^2},$$

where g is the gravitational acceleration. Both of the considered numbers describe the dynamics of oceanic basins where the Coriolis force cannot be neglected. That said, the SWE employed herein are rewritten in vector form as

$$\frac{\partial h^*}{\partial t^*} + Ro \nabla^* \cdot (h^* \bar{\mathbf{u}}^*) = 0$$

and

$$\begin{aligned} \frac{\partial (h^* \bar{\mathbf{u}}^*)}{\partial t^*} + Ro \nabla^* \cdot (h^* \bar{\mathbf{u}}^* \otimes \bar{\mathbf{u}}^*) \\ + \frac{Bu}{Ro} \frac{1}{2} \nabla^* h^{*2} = -h^* \bar{\mathbf{u}}^{*\perp} - \frac{Bu}{Ro} h^* \nabla^* b^* \\ + Ro \Delta \tau^* \boldsymbol{\tau}^* \end{aligned} \quad (1)$$

where

$$\nabla^* = \left(\frac{\partial}{\partial x^*}, \frac{\partial}{\partial y^*} \right), \quad \bar{\mathbf{u}}^* = \begin{pmatrix} \bar{u}^* \\ \bar{v}^* \end{pmatrix},$$

$$\bar{\mathbf{u}}^{*\perp} = \begin{pmatrix} -\bar{v}^* \\ \bar{u}^* \end{pmatrix}, \quad \boldsymbol{\tau}^* = \begin{pmatrix} \tau_x^* \\ \tau_y^* \end{pmatrix},$$

and

$$\Delta\tau^* = \frac{\Delta\tau}{\rho_w U_0^2} \frac{L}{H_0}.$$

One immediate observation from Eq. (1) is that, with the characteristic velocity

$$U_0 \sim \frac{\Delta\tau}{f\rho_w H_0}$$

suggested by the Ekman balance, the wind stress and Coriolis terms are of comparable magnitude. The pressure-gradient and bathymetric terms both carry the factor Bu/Ro , which is much greater than one in the cases considered (typically $Ro \sim 10^{-2}$ and $Bu \sim 10^3$ in the configurations of Sections 3 and 5); they must, however, be considered jointly, since they combine into $(Bu/Ro) h^* \nabla^* \eta^*$, with $\eta^* = h^* + b^*$, and exactly balance for a lake at rest ($\eta^* = \text{const}$). The dynamical relevance of bathymetry can therefore not be inferred from the prefactor alone: it depends on how the bottom topography modifies the evolving free-surface and depth-averaged velocity fields, an aspect that is examined numerically in Section 5.2.

2.1 Numerical Code

A 2D shallow water model, when incorporating a bathymetric forcing, can be numerically prone to instability [53]. Therefore, the method proposed by Ziveri [54] is chosen to include the bathymetric forcing and ensure a stable, constant solution (namely, to achieve a still-water equilibrium-preserving scheme). Part of the numerical setup of the schemes is also taken from Ziveri [54].

Specifically, a numerical model based on the MUSCL scheme with flux limiter, FORCE-type flux reconstruction with Minmod limiter, and time integration using the Strang splitting algorithm [45] is employed. The finite-volume transport step is explicit in time, with Courant numbers between 0.9 and 1 in the cases reported in Appendix A, and it preserves the still-water

equilibrium in the presence of bathymetry (well-balanced property) [54]. The only boundary conditions implemented are of the open boundary type, as employed, for example, by Ludewig [28]. Specifically, extrapolation-type boundary conditions are used, which correspond to homogeneous Neumann conditions (zero gradient), which provide a simple, approximately transmissive treatment for the long waves considered here and were found to produce limited reflection in the validation cases of Appendix A.

The scheme is formally second-order accurate in space and time, with the two half-steps of the Strang splitting devoted to the source terms and the intermediate full step to the hyperbolic (transport) problem, whose finite-volume update also incorporates the bathymetric source term. The Coriolis and wind-stress sources are advanced with a Crank–Nicolson ($\theta = 1/2$) scheme: since the Coriolis term is linear in the conserved variables and the wind stress does not depend on them, the implicit update reduces to the inversion of a constant 3×3 matrix involving the Jacobian of the Coriolis source. The computational cost of a typical mesoscale run of Sections 3 and 5 is of the order of minutes.

A number of validation cases for the in-house SWE solver Q3DFLOOD (Q3D Finite voLume Ocean mODEl), implemented in MATLAB, are reported in Appendix A.

2.2 Diagnostic Vertical Velocity

Although the 2D shallow water model does not explicitly resolve the vertical velocity, it is possible to reconstruct a simplified representation based on a set of idealized physical assumptions, thereby obtaining reasonable qualitative insights into the Ekman vertical velocity as well. The reconstructed quantity should therefore not be interpreted as a prognostic prediction of the three-dimensional vertical circulation, but as a diagnostic estimate consistent with the depth-averaged solution and with the imposed wind stress curl, under the assumptions introduced below.

One possible approach to model the vertical velocity is to prescribe its profile, as illustrated, for example, by Lu et al. [27], where an Ekman vertical velocity profile is imposed, characterized by a sinusoidal decay that goes to zero both at the surface and at the depth of the pycnocline, with the

maximum velocity located at the bottom of the top Ekman layer. Other studies, in contrast, solve prognostically for the vertical velocity profile using 2DV or 3D models [2]. The current work proposes instead a first-order reconstruction of the vertical velocity. The following model is based on Ekman's theory for a barotropic ocean, as described, for example, by Cushman-Roisin and Beckers [12].

In a well-mixed basin of constant and moderate depth H_0 , three zones are distinguished: the upper Ekman layer, the lower Ekman layer, and the interior layer between them [12]. The velocity at the base of the upper Ekman layer, driven by wind stress, is given by [46]

$$w_E^T = \frac{1}{\rho_w f} (\nabla \times \boldsymbol{\tau}),$$

while the velocity at the top of the lower Ekman layer is [46]

$$w_E^B = \frac{1}{\rho_w f} (\nabla \times \boldsymbol{\tau}_B),$$

where $\boldsymbol{\tau}_B$ is the bottom stress. When working with depth-averaged velocities, $\boldsymbol{\tau}_B$ should contain contributions from the surface stress because the two Ekman layers are not fully independent [13]. Their coupling depends on the ratio H_0/d , where the Ekman layer thickness d is [46]

$$d = \sqrt{\frac{2\nu}{f}},$$

with ν a characteristic vertical eddy viscosity. For $H_0/d \gtrsim 2$, however, the wind influence on the bottom layer becomes negligible [13].

At this point, it is important to recall that, as discussed in Section 2, the bottom friction was neglected in the construction of the model. Although bottom pumping directly depends on bottom friction, it is not immediately obvious that neglecting bottom pumping in the reconstruction of the Ekman vertical velocity is equally justified. However, this is physically motivated: it is well known in the literature that the velocity w_E^B depends linearly on the height of the bottom Ekman layer [46], which in turn becomes small if the bottom shear is small.

It is therefore proposed to linearly interpolate the only three vertical velocity values available in

the water column. The first is $w(\eta)$ at the free surface, which is set to zero: this amounts to a rigid-lid approximation, justified by the fact that surface elevation gradients are small for the phenomena considered here, and it keeps the model simple. The second is w_E^T , imposed at a distance d_E below the surface and referred to simply as w_E hereafter. The third is the bottom velocity w_B , which has been assumed to vanish so far but does not in general: with non-flat (and non-deforming) bathymetry it can be determined as [18]

$$w_B \simeq \bar{\mathbf{u}} \cdot \nabla b,$$

which, strictly speaking, is a term that acts within the bottom Ekman layer and does not depend on the layer's height [52]. Note that w_B should be of the same order of magnitude as w_E ; otherwise, a linear interpolation between the two may not be physically meaningful. A graphical representation is provided in Fig. 2.

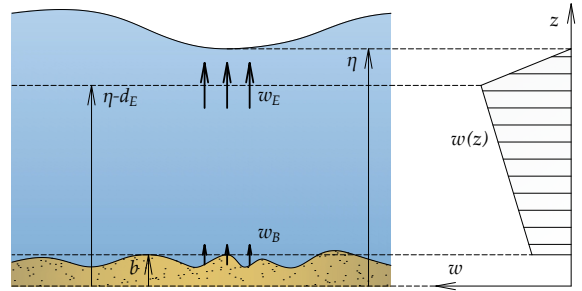


Fig. 2 Proposed first-order reconstruction of the vertical velocity profile

Before proceeding further, it is useful to clarify the physical motivation and the limitations of the proposed linear reconstruction. A physical argument for a low-order profile of w between the base of the top Ekman layer and the bottom (after neglecting the velocity at the top of the bottom Ekman layer, as discussed above) can be drawn from the β -plane vorticity balance [46]: stepping momentarily out of a purely 2D context, one can write [12]

$$\frac{\partial w}{\partial z} = \frac{\beta}{f_0} v,$$

and since in the interior layer the horizontal velocities practically do not vary with depth [12], $\partial w / \partial z$ is approximately uniform within this layer. The shallow water model is solved here on an f -plane,

so that this argument should be regarded as a motivation rather than as a derivation: the linear interpolation is a first-order closure that satisfies the available boundary and interface constraints, not a complete representation of the vertical structure. The profile between $w(\eta)$ and w_E is likewise interpolated linearly for modeling simplicity; it is the only part of the profile that is, strictly speaking, prescribed rather than physically motivated, although it still satisfies the boundary condition imposed at the water surface.

Finally, the thickness of the top Ekman layer, d_E , must be estimated. This can be done empirically as [27]

$$d_E = \frac{0.4}{f} \sqrt{\frac{\|\boldsymbol{\tau}\|}{\rho_w}} \simeq \frac{0.4}{f} \sqrt{\frac{\tau_0}{\rho_w}},$$

where, for simplicity, it is assumed constant along the entire basin, with f being constant and where τ_0 is a characteristic wind stress, such as the undisturbed wind stress (not $\Delta\tau$). Note that d_E is the empirical counterpart of the Ekman depth d introduced above; since ν is not known a priori, d_E is used operationally in the criterion $H_0 \gtrless 2d_E$.

It is proposed herein to linearly interpolate the velocities

$$\begin{aligned} w(\eta) &= 0, \\ w(\eta - d_E) &= w_E = \frac{1}{\rho_w f} \left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y} \right), \\ w(b) &= w_B = \bar{u} \frac{\partial b}{\partial x} + \bar{v} \frac{\partial b}{\partial y}. \end{aligned}$$

Explicitly, for $H_0 > 2d_E$, the vertical velocity is given by

$$w = \begin{cases} w_B + (w_E - w_B) \frac{\sigma}{\sigma_E} & 0 \leq \sigma < \sigma_E \\ w_E \frac{1 - \sigma}{1 - \sigma_E} & \sigma_E \leq \sigma \leq 1 \end{cases}, \quad (2)$$

where the σ -coordinate [41] is defined as

$$\sigma = \frac{z - b}{h},$$

and the parameter σ_E as

$$\sigma_E = \frac{\eta - d_E - b}{h}.$$

For $H_0 \leq 2d_E$, and in the absence of additional physical information, it is proposed to assume $\sigma_E = 0.5$, and consequently write

$$\tilde{w} = \begin{cases} w_B + 2(w_E - w_B)\sigma & 0 \leq \sigma < 0.5 \\ 2w_E(1 - \sigma) & 0.5 \leq \sigma \leq 1 \end{cases}.$$

Now, according to Cushman-Roisin and Deleersnijder [13], it is found that the Ekman vertical velocity, for a very shallow basin ($H_0 \ll d$) and working with vertically averaged velocities, scales linearly with the height of the basin itself, reaching zero as $H_0 \rightarrow 0$. Therefore, it is proposed to reconstruct w , for $H_0 \leq 2d_E$, as

$$w = \tilde{w}\tilde{F}, \quad (3)$$

where $\tilde{F} = H_0/(2d_E)$ is a linear function that goes from 0, for $H_0/d_E = 0$, to 1, for $H_0/d_E = 2$. Although the actual scaling of w with depth is not perfectly linear within the considered range of H_0/d_E (again, according to Cushman-Roisin and Deleersnijder [13]), the proposed linear approximation does not introduce significant error compared to the results of Cushman-Roisin and Deleersnijder [13]. In the variable-bathymetry cases of Section 5.2, $\tilde{F}$ is evaluated with the local water-column depth $h(x, y, t)$ in place of H_0 .

Given the simplicity of the presented model, its validation is based on two main checks: the correct computation of the Ekman vertical velocity, and that of the velocity at the bottom. In the first case, the reconstructed Ekman vertical velocity was checked against the values reported by Eliassen [16] for a Gaussian wind-stress distribution with $L = 5$ km, $\tau_0 = 0.1$ N/m² and $\Delta\tau = 0.05$ N/m² (the stress model of Section 3); since w_E follows analytically from the imposed stress curl, the agreement is essentially exact, with residual differences attributable only to the different discretizations. The second check can be carried out directly on the results reported in Section 5.2, without the need for a dedicated section. Accordingly, in the remainder of the paper the reconstructed vertical velocity is used to identify the location and the indicative magnitude of wind-farm-induced upwelling and downwelling, rather than as a fully resolved three-dimensional velocity field.

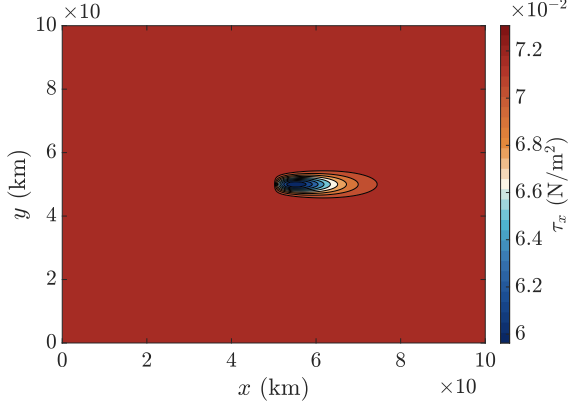


Fig. 3 Imposed x -direction wind stress, constant in time

3 Statistical Analysis Based on a Gaussian Wind Stress Model

For the statistical analysis, an analytical (Gaussian) wind stress model is employed, specifically the one proposed by Broström [6].

The effective wind stress acting on the ocean basin is assumed to have components

$$\tau_x(x, y) = \tau_0 - \Delta\tau \exp\left(-\left(\frac{2y}{\alpha_\tau L + \beta_\tau x}\right)^2\right) \max\left(\frac{x}{L} e^{-x/L}, 0\right) \quad (4)$$

and

$$\tau_y(x, y) = 0,$$

with $\alpha_\tau = 0.8$, $\beta_\tau = 0.2$, and $\Delta\tau = 0.5\tau_0$ (Fig. 3).

A constant wind stress upstream of the wind farm, unaffected by disturbances, is assumed to be

$$\tau_0 = \rho_a C_D \|\bar{\mathbf{u}}_{10}\|^2, \quad (5)$$

with $\rho_a = 1.2 \text{ kg/m}^3$ being the air-side density [33] (the LES of Section 4 adopts the standard sea-level value 1.225 kg/m^3 ; the 2% difference is immaterial for the present purposes), C_D the wind drag coefficient calculated by Smith [43] with the correlation

$$C_D = (0.61 + 0.063\|\bar{\mathbf{u}}_{10}\|) 10^{-3},$$

and $\|\bar{\mathbf{u}}_{10}\|$ the modulus of the undisturbed wind speed at 10 m above sea level, here assumed uniform and directed along the positive x -axis.

When a Gaussian wind stress is used to force a shallow water model, several parameters can be identified that influence upwelling and downwelling motions. In particular, the following analysis focuses on the geographical location of the wind farm and the wind characterization, the latter being in turn dependent on the design of the wind farm itself.

The geographical location of the wind farm is undoubtedly characterized by the latitude φ , but also by the average depth H_0 . The wind model described by Eq. (4) is primarily characterized by $\|\bar{\mathbf{u}}_{10}\|$, and by the characteristic length of the wind farm, L .

Building on this, two output variables are defined to quantify the upwelling response, namely

$$UI = \max\{-SSHA\},$$

$$WEK = \max\{w(\sigma = \sigma_E)\}, \quad (6)$$

where UI represents the maximum surface anomaly in the upwelling zone (hence the negative sign), and WEK represents the maximum vertical velocity in the top Ekman layer (also in upwelling). The maxima in Eq. (6) are evaluated over the entire ocean basin, and the two objective functions are therefore expressed as functions of the form

$$UI = UI(H_0, L, \|\bar{\mathbf{u}}_{10}\|, \varphi),$$

$$WEK = WEK(H_0, L, \|\bar{\mathbf{u}}_{10}\|, \varphi).$$

Table 1 Design variables and corresponding levels

Design variable	Levels
Depth at rest (H_0)	30, 90, 150 m
Characteristic length (L)	3, 5, 7 km
Wind speed at 10 m ($\ \bar{\mathbf{u}}_{10}\ $)	2.5, 5, 7.5 m/s
Latitude (φ)	30°, 45°, 60°

Table 1 shows the levels of the chosen design variables. The design of experiments (DoE) is a full factorial 3^4 , i.e., 81 runs. All runs share the same numerical setup: a $100 \times 100 \text{ km}^2$ domain discretized by a 101×101 grid, a Courant number of 0.9, and an initial condition at rest ($h = H_0$, $\bar{u} = \bar{v} = 0$). Each case is advanced for 5 days, and

both indicators are evaluated at the end of the run.

The range of H_0 covers shallow to moderately deep shelf basins and is chosen so that the shallowest level falls in the regime $H_0 \leq 2d_E$, where the reconstructed vertical velocity depends on the depth (Section 2). The characteristic length L and the wind speed at 10 m are selected from the literature [33]. The latitudes span a wide range, from about 30°N (the latitude of Houston, Texas) to 60°N (close to that of Anchorage, Alaska).

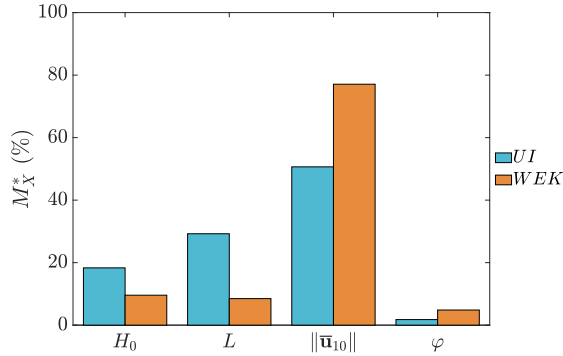


Fig. 4 Normalized main effects M_X^* of the design variables on the two upwelling indicators

Figure 4 shows the normalized main effects M_X^* of each design variable X on the two indicators, expressed as percentages. For each design variable X with $L_X = 3$ levels, the main effect is computed as $M_X = \sum_{i=1}^{L_X} (\bar{Y}_{X=i} - \bar{Y})^2$, where $Y \in \{UI, WEK\}$, $\bar{Y}_{X=i}$ is the response averaged over all runs with X held at its i -th level, and $\bar{Y}$ is the overall mean over the 81 runs; the normalized effects $M_X^* = 100 M_X / \sum_{X'} M_{X'}$, thus sum to 100 by construction. As expected, the wind speed is by far the most influential parameter (about 50% for UI and 77% for WEK). The basin depth and the characteristic length of the wind farm have effects of comparable magnitude, both larger on UI than on WEK . The latitude, i.e., the Coriolis parameter, has a marginal influence on the magnitude of both indicators (about 5% or less), although it plays an important role in shaping the spatial structure of the solution: varying the latitude rotates the dipolar $SSHA$ pattern, while leaving its overall excursion nearly unchanged (not shown; see also Section 5).

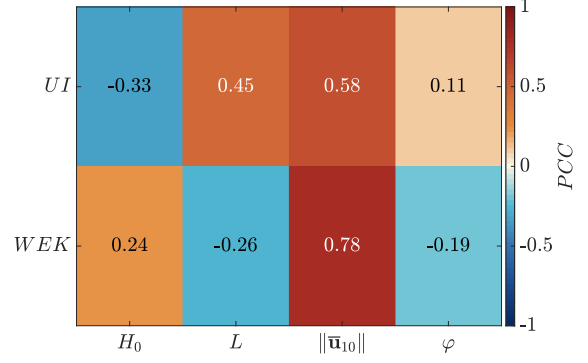


Fig. 5 Matrix of Pearson correlation coefficients (PCC) between design variables and upwelling indicators

Finally, Fig. 5 shows the matrix of Pearson correlation coefficients (PCC) between design variables and upwelling indicators, computed following the same procedure as in Della Monica and Bernardini [14]; each entry quantifies the sign and strength of the linear association between the corresponding pair of variables. As expected, the wind speed at 10 m is strongly and positively correlated with both indicators (PCC = 0.58 for UI and 0.78 for WEK). Less obvious is the opposite response of the two indicators to the depth: as H_0 increases, UI decreases (PCC = -0.33) while WEK increases (PCC = 0.24). The dependence on the characteristic length L shows the reverse pattern (UI increases, PCC = 0.45; WEK decreases, PCC = -0.26), a trend recently highlighted also by Eliassen [16]. The correlations with latitude are weak ($|\text{PCC}| \leq 0.19$) and of opposite sign for the two indicators: higher latitudes slightly reduce the maximum Ekman velocity, consistently with the $1/f$ scaling of w_E , and slightly increase the surface anomaly.

4 Wind Farm LES Modeling

The statistical analysis of Section 3 relies on an idealized, analytical description of the wind stress. To investigate the role of bathymetry under a realistic forcing, an alternative approach is now pursued: the wind field over a real wind farm is obtained from LES performed with the in-house computational fluid dynamics (CFD) code UTD-WF [37], developed at the University of Texas at Dallas, and is then upscaled to the

mesoscale before being used to force the shallow water model. This section describes the LES solver (Section 4.1), the multi-domain strategy adopted to extend the LES field to the scale of an entire wind farm (Section 4.2.1), the Horns Rev 1 setup (Section 4.2.2), and the validation of both the LES (Section 4.2.3) and the reconstruction (Sections 4.2.4 and 4.2.5).

4.1 UTD-WF Code

In this section, following the usual LES notation, a tilde denotes filtered quantities, whereas the overbar denotes time averaging (as in the figure labels of Sections 4.2.4 and 4.2.5), in contrast with Section 2, where it denotes depth averaging. The UTD-WF code solves the filtered Navier–Stokes equations for incompressible flow in nondimensional form. The governing equations are

$$\begin{aligned}\frac{\partial \tilde{U}_i}{\partial t} + \frac{\partial \tilde{U}_i \tilde{U}_j}{\partial x_j} &= -\frac{\partial \tilde{P}}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 \tilde{U}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} + f_i, \\ \frac{\partial \tilde{U}_j}{\partial x_j} &= 0.\end{aligned}$$

Here, $\tilde{U}_i$ and $\tilde{P}$ denote filtered velocity and pressure, while f_i represents body forces accounting for turbine blades. The Reynolds number is defined as $Re = U_\infty D / \nu$, where U_∞ is the inflow velocity, D the rotor diameter, and ν the kinematic viscosity; in the present simulations $Re = 5.44 \times 10^7$, corresponding to full-scale atmospheric conditions.

The subgrid-scale (SGS) stress tensor is

$$\tau_{ij} = \widetilde{U_i U_j} - \tilde{U}_i \tilde{U}_j.$$

The deviatoric SGS stresses are modeled using the Boussinesq hypothesis,

$$\tau_{ij}^d = -\nu_{\text{sgs}} \left(\frac{\partial \tilde{U}_i}{\partial x_j} + \frac{\partial \tilde{U}_j}{\partial x_i} \right) = -2\nu_{\text{sgs}} \tilde{S}_{ij},$$

where $\tilde{S}_{ij}$ is the filtered strain-rate tensor and ν_{sgs} is the SGS eddy viscosity. The classical Smagorinsky model [42] is adopted, as in [9], that is

$$\nu_{\text{sgs}} = (C_s \tilde{\Delta})^2 \sqrt{2 \tilde{S}_{ij} \tilde{S}_{ij}},$$

with filter width

$$\tilde{\Delta} = (\Delta x_1 \Delta x_2 \Delta x_3)^{1/3}.$$

The tower and nacelle are modeled using the Immersed Boundary Method (IBM) [31], which enables representation of complex geometries on a Cartesian grid. Grid points located inside the solid body are forced to zero velocity, while spatial derivatives near the boundary are computed using the actual distance between the grid nodes and the immersed surface.

The turbine blades are represented using a Rotating Actuator Disk Model (RADM) [10]. Each blade is discretized into sections where lift and drag forces per unit length are computed using 2D airfoil data,

$$F_L = \frac{1}{2} \rho_a U_{\text{rel}}^2 c_l(\alpha) c, \quad F_D = \frac{1}{2} \rho_a U_{\text{rel}}^2 c_d(\alpha) c,$$

where c is the chord length, c_l and c_d are the sectional lift and drag coefficients, ρ_a is the air density, and U_{rel} is the relative velocity at the blade section. The relative velocity and angle of attack are

$$U_{\text{rel}} = \sqrt{U_x^2 + (U_\theta - \Omega r)^2},$$

$$\alpha = \arctan \left(\frac{U_x}{U_\theta - \Omega r} \right) - \phi,$$

with ϕ the local twist angle of the blade section. The resulting aerodynamic forces are distributed over the rotor disk using a Gaussian kernel, that is

$$\mathbf{f} = \frac{\mathbf{F} \mathcal{G}}{\iint_A \mathcal{G} \, dA},$$

with spreading function

$$\mathcal{G} = \exp \left(- \left(\frac{x - x_c}{\varepsilon} \right)^2 \right) \exp \left(- \left(\frac{\theta - \theta_{\text{blade}}}{\pi/B} \right)^2 \right),$$

where B is the number of blades. The parameter ε controls the disk thickness, and, following standard practice, is set to $\varepsilon \simeq 2\tilde{\Delta}$ [9].

The rotor angular velocity Ω evolves according to the angular momentum balance [5]

$$I\dot{\Omega} = M_{\text{aero}} - M_{\text{gen}},$$

where I is the rotor inertia. The generator torque follows a quadratic control law [26],

$$M_{\text{gen}} = k_{\text{gen}}\Omega_{\text{gen}}^2,$$

with $\Omega_{\text{gen}} = N_{\text{gear}}\Omega$. The torque gain determining the equilibrium operating condition is [5]

$$k_{\text{gen}} = \frac{1}{N_{\text{gear}}^2} \frac{1}{2} \rho_a \pi R^5 \frac{C_P(\lambda_{\text{opt}})}{\lambda_{\text{opt}}^3},$$

where $R = D/2$ is the rotor radius, C_P the power coefficient and λ_{opt} the optimal tip-speed ratio.

4.2 Horns Rev 1 Case Study

Simulating a complete offshore wind farm, consisting of tens to hundreds of turbines, with an LES solver remains computationally expensive even on high-performance computing (HPC) systems. To overcome this limitation, the present work extends the LES domain with regions in which a regression-based model of the wake development within the turbine array is employed. The framework allows the velocity field resolved for a single turbine row, aligned with the inflow direction, to be extrapolated to the scale of a complete wind farm with a regular (non-staggered) layout, capturing both the streamwise and the spanwise decay of the velocity deficit. The principal advantage of informing the ocean model with an LES-derived wind field is that the reconstruction retains, before the projection onto the mesoscale grid, the streamwise and spanwise wake-deficit structure obtained from the high-fidelity simulation, which is difficult to reproduce with purely analytical wake formulations.

4.2.1 Multi-Domain Approach

The effects of wind farms on ocean circulation are typically represented by prescribing wake-induced forcing through analytical wake models, which are then used to drive ocean solvers of varying complexity, ranging from shallow water formulations to fully 3D Navier–Stokes models such as FVCOM [8] or ROMS [32, 40]. To the authors’

knowledge, no previous study directly couples an ocean model with a farm-scale wind field derived from high-fidelity LES; the closest work is that of Guzmán Hernández et al. [21], which considers a line of wind turbines on a domain of a few kilometers and does not address mesoscale dynamics.

The UTD-WF code allows for the simulation of wind turbines deployed over complex domains. In the present study, since the turbines are assumed to be installed offshore, the computational setup is idealized as a rectangular domain with the turbines placed over a flat surface.

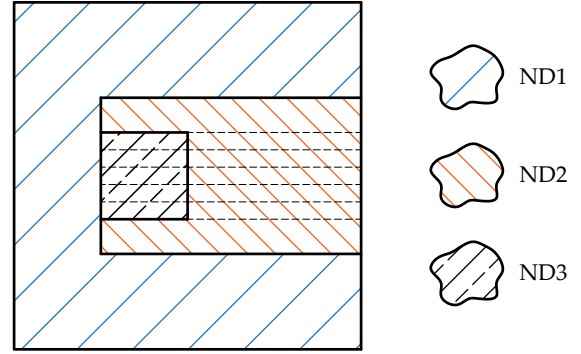


Fig. 6 Schematic of the nested-domain layout, composed of three embedded domains (ND1, ND2, and ND3). Horizontal dashed lines highlight the arrangement of the wind farm into rows of wind turbines; a westerly wind (from left to right) is assumed

The challenge here is thus to develop a strategy to consistently extend LES results obtained at relatively small spatial scales (high fidelity) to an ocean basin described at the mesoscale (low fidelity). Following an approach adopted in the literature [38], the computational domain is subdivided into nested domains (ND), as illustrated in Fig. 6. In the innermost domain (ND3), the velocity field around a wind turbine array is resolved by LES.

By simulating a single row of turbines with periodic boundary conditions applied at the sides (north and south), the row can be repeated multiple times in the spanwise direction, effectively generating a wind farm of any desired width. The idea behind ND3 is shown in Fig. 7.

The intermediate domain (ND2) employs a response surface model (RSM) to reconstruct the velocity field based on the LES results obtained

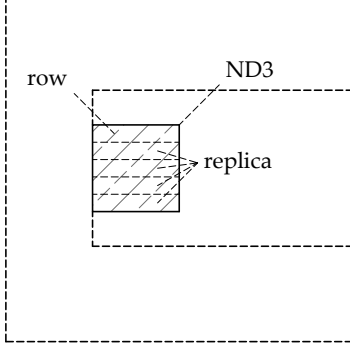


Fig. 7 Detail of the LES-resolved domain ND3. A row of wind turbines is replicated in the spanwise direction to represent a full wind farm, taking advantage of the periodic boundary conditions

in ND3. In the outermost domain (ND1), a freestream velocity is prescribed.

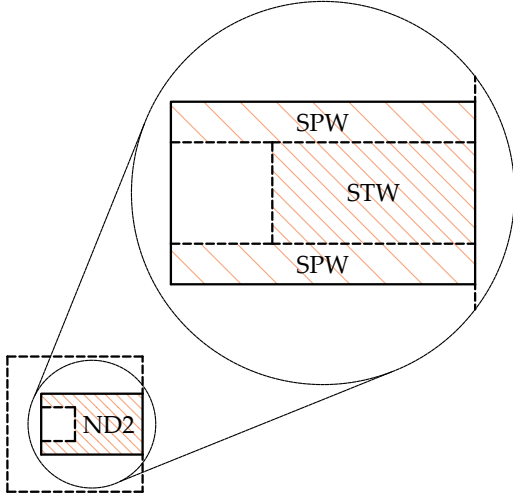


Fig. 8 Detail of the RSM-based domain ND2, showing the lateral (SPW) and far-wake (STW) regions

Figure 8 shows the subdomains into which ND2 is divided, depending on the chosen approximation of Eq. (8), which differs between the streamwise far-wake region (STW) and the spanwise lateral region (SPW) with respect to the LES domain ND3. In the STW region, the velocity deficit of a row of wind turbines is modeled by an RSM of the form

$$f_{STW}(x^*, y^*) = \frac{p_1}{(x^* + p_2)^2} - \frac{p_3 y^{*2}}{(x^* + p_2)^4}, \quad (7)$$

where $x^* = x/D$, $y^* = y/D$, D is the rotor diameter, and the vector of unknown parameters is defined as

$$\boldsymbol{\vartheta}_{STW} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix},$$

and is identified by training the RSM using suitable LES data. The RSM in Eq. (7) can be recovered by modeling the wind velocity deficit using the well-known model proposed by Bastankhah and Porté-Agel [3], which reads

$$\frac{\Delta U}{U_\infty} = \left(1 - \sqrt{1 - \frac{D^2 C_T}{8\sigma^2}}\right) \exp\left(-\frac{r^2}{2\sigma^2}\right), \quad (8)$$

where C_T is the thrust coefficient, r is the radial distance from the wake axis, and σ is the standard deviation of the Gaussian profile, which can be a function of the downstream distance x . Specifically, one can write

$$1 - \sqrt{1 - \frac{D^2 C_T}{8\sigma^2}} \simeq \frac{D^2 C_T}{16\sigma^2}$$

and, assuming a linear wake expansion $\sigma = kx + \sigma_0$, with k the wake expansion coefficient,

$$\exp\left(-\frac{r^2}{2(kx + \sigma_0)^2}\right) \simeq \left(1 - \frac{y^2}{2(kx + \sigma_0)^2}\right),$$

having assumed $x \gg z$ and $x \gg y$, i.e., having evaluated the deficit close to hub height, so that $r^2 \simeq y^2$, and sufficiently far downstream that $y \ll \sigma(x)$, so that the exponential can be expanded to leading order in $y^2/(2\sigma^2)$; the approximation is therefore appropriate in the far-wake region.

Assuming further that the individual deficits can be linearly superposed, i.e.,

$$\frac{\Delta U_{TOT}}{U_\infty} = \frac{\Delta U_{TOT}(x, r)}{U_\infty} = \sum_{j=1}^{N_T} C(x - x_j) G(r, x - x_j),$$

where x_j is the position of the j -th turbine along the row, $j = 1, \dots, N_T$, and C and G denote the amplitude and the Gaussian shape function of the individual wake deficit in Eq. (8), a reasonable approximation in the far-wake (FW) region of the

total cumulative deficit of N_T turbines in a row is, in nondimensional terms,

$$\frac{\Delta U_{FW}}{U_\infty} \simeq \frac{N_T C_T}{16(kx^* + \sigma_0^*)^2} \left(1 - \frac{y^{*2}}{2(kx^* + \sigma_0^*)^2} \right),$$

where $\sigma_0^* = \sigma_0/D$. The formula just obtained can be rewritten in the form of the RSM in Eq. (7).

In the SPW region, the following model for the velocity deficit of a row of wind turbines is proposed instead

$$f_{SPW}(x^*, y^*) = \chi(x^*) \exp\left(-\frac{y^{*2}}{(q_1 x^* + q_2)^2 + q_3}\right), \quad (9)$$

where q_1 , q_2 , and q_3 are free parameters, and $\chi(x^*)$ can be recovered from LES and STW data by setting $y^* = 0$ and performing the training at the appropriate $z^* = z/D$. If $\chi(x^*)$ is assumed to be known (for example, from a previous training), the actual vector of unknown parameters reduces to

$$\boldsymbol{\vartheta}_{SPW} = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}.$$

The RSM in Eq. (9) can be recovered by again assuming linear superposition of the individual deficits and by approximating the resulting Gaussian mixture with a single equivalent Gaussian, which gives, in the near-wake (NW) region,

$$\frac{\Delta U_{NW}}{U_\infty} \simeq \overline{C}_{eq}(x^*) \exp\left(-\frac{y^{*2}}{2\sigma_{eq}^2}\right).$$

The zeroth-, first-, and second-order moments of a Gaussian mixture of N_T components can be computed as [11]

$$m_0 = \sum_{i=1}^{N_T} \omega_i, \quad m_1 = \sum_{i=1}^{N_T} \omega_i \xi_i, \\ m_2 = \sum_{i=1}^{N_T} \omega_i (\xi_i^2 + \sigma_i^2),$$

where ω_i , ξ_i and σ_i denote the weight, the mean and the standard deviation of the i -th component. Assuming

$$\sigma_{eq}^2 \simeq \frac{1}{N_T} \sum_{i=1}^{N_T} \sigma_i^2,$$

a cumulative velocity deficit of the form

$$\frac{\Delta U_{NW}}{U_\infty} \simeq \overline{C}_{eq}(x^*) \exp\left(-\frac{y^{*2}}{2N_T^{-1} \sum_{i=1}^{N_T} \sigma_i^2}\right), \quad (10)$$

can be recovered. The assumption on σ_{eq}^2 is introduced as an analytical ansatz rather than as an exact representation of the mixture weights: despite its simplicity, it closely approximates the cumulative wake obtained from the linear superposition of the individual deficits, and provides useful reference values for the RSM parameters (Section 4.2.4). Assuming again a linear wake expansion, $\sigma = kx + \sigma_0$, the deficit in Eq. (10) can be recast in the form of the RSM in Eq. (9).

4.2.2 CFD Case Modeling

Validating the approach presented in the previous section required the use of a real-world benchmark. Accordingly, the proposed case study is based on the Horns Rev 1 wind farm, located off the west coast of Denmark [22]. It consists of 80 bottom-fixed turbines arranged in 8 rows of 10 turbines each.

Horns Rev 1 is frequently used as a reference for wind farm simulations due to the extensive availability of experimental data in the literature [22, 48]. These datasets include measurements of wind speed and direction at multiple heights, turbine power output, turbulence intensity, and wake interactions, making the site well-suited for the validation of numerical models.

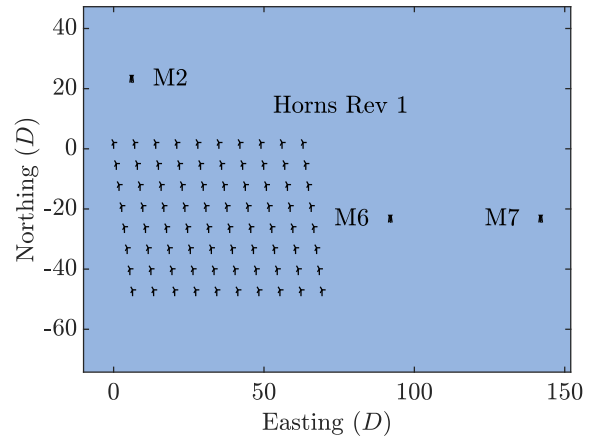


Fig. 9 Positions of the M2, M6, and M7 masts relative to the Horns Rev 1 wind farm

In particular, the wind measurements used in this study, taken from Eriksson et al. [17] and Volker et al. [48], were collected from masts located near the wind farm (see Fig. 9) and correspond to an average wind direction of 270° (westerly wind). The M2 mast was used to assess the wind conditions prior to the installation of the wind farm [34] (it should be noted that M2 is slightly lower than the turbine hub height, 62 m versus 70 m [34]). On the other hand, M6 and M7 (70 m high) typically record wind data within the wind farm wake.

The actual case study analyzed in this work represents a slightly simplified version of the original wind farm: specifically, the layout is assumed to be perfectly rectangular rather than the rhomboidal configuration of Horns Rev 1 (see again Fig. 9). This simplification has been previously employed in the literature [48], and provides a realistic but manageable benchmark.

The computational domain is a rectangular parallelepiped. Referring to the quantities defined in Fig. 10, and expressing all lengths in units of the rotor diameter D , the domain dimensions are $L_X = 126D$, $L_Y = 7D$, and $L_Z = 14D$. The streamwise spacing between the inlet and the first turbine, as well as between consecutive turbines, is $L_S = 7D$.

The mesh is a Cartesian grid, uniform in the horizontal directions, consisting of 5760 nodes in the streamwise direction, 240 in the spanwise direction, and 320 in the vertical direction; along the vertical direction, grid refinement is applied from $z = 0$ up to $z = 4D$ to better resolve the boundary layer over the ocean surface.

The turbines are Vestas V80 with rotor diameter $D = 80$ m and hub height of 70 m above sea level. In the literature, the V80 is modeled in slightly different ways from one study to another [29, 51], since a complete tabulated characterization of its blade over the entire span is not openly available [29]. Here, the rotor diameter and hub height are exactly those of the Vestas V80, whereas the blade characterization (twist-angle distribution, sectional lift and drag polars $c_l(\alpha)$ and $c_d(\alpha)$, and C_P – C_T tables) is taken from the Gamesa G80 lookup tables already implemented in UTD-WF, the G80 being a turbine of similar rated power and dimensions. Regarding the control strategy, the optimal tip-speed ratio $\lambda_{\text{opt}} =$

8.3 and the gearbox ratio $N_{\text{gear}} = 101$ are consistent with publicly available V80 data, although different sources report slightly different values: they are treated here as fixed modeling assumptions, whereas in real turbines they may vary with the operating conditions [29]. The resulting power output is cross-validated against the manufacturer power curve in Section 4.2.3 [15, 44].

As mentioned above, for the wind farm under consideration, westerly winds with direction 270° are commonly studied in the literature [48]. Following this, the inflow wind in the present simulations is aligned with the turbine row. To mimic the atmospheric boundary layer at the inlet, a mean velocity profile is prescribed by the following law

$$\frac{U(z)}{U_{\text{hub}}} = \left(\frac{z}{z_{\text{hub}}} \right)^\alpha,$$

where z denotes the vertical coordinate measured from the mean surface height, z_{hub} represents the hub height above the mean sea surface, and α is the shear exponent, taken as $\alpha = 0.1$ based on reference values from the literature [47, 50]. The freestream velocity $U_\infty = 10$ m/s is the reference velocity used in the definition of Re and in the nondimensionalization, and does not correspond to the wind speed at a specific height: the profile is anchored so that the hub-height velocity equals $0.8 U_\infty = 8$ m/s, the operating condition targeted in the validation of Section 4.2.3. The reference air density is assumed equal to 1.225 kg/m³. No turbulence intensity is prescribed at the inlet.

As for the remaining numerical settings, the filtered equations are advanced in time with a third-order Runge–Kutta scheme, with the time step adjusted so that the maximum CFL number equals 1, and the Smagorinsky constant is set to $C_s = 0.09$. The mean velocity profile described above is prescribed at the inlet, while a radiative (convective) outflow condition, with convection velocity equal to U_∞ , is applied at the outlet; the sea surface (the bottom of the computational domain) is treated as a no-slip wall, and a free-slip condition is imposed at the top of the domain. The flow statistics discussed below are collected after discarding an initial transient of 1 h ($450 D/U_\infty$). Table 2 summarizes the main LES parameters.

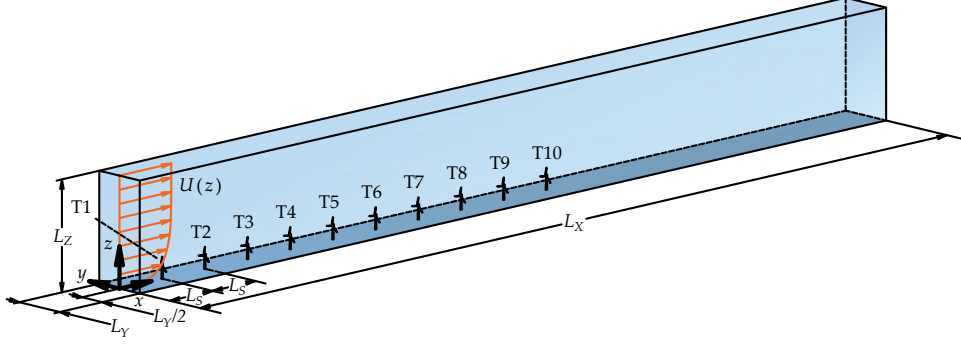


Fig. 10 CFD domain for the LES case study and corresponding turbine numbering

Table 2 Characteristic parameters of the LES setup

Variable	Value
Rotor diameter (D)	80 m
Hub height (z_{hub})	70 m
Domain size (L_X)	$126D$
Domain size (L_Y)	$7D$
Domain size (L_Z)	$14D$
Turbine spacing (L_S)	$7D$
Grid resolution	$5760 \times 240 \times 320$
Time integration	third-order Runge-Kutta
Maximum CFL number	1
Smagorinsky constant (C_s)	0.09
Freestream velocity (U_∞)	10 m/s
Hub-height velocity	$0.8 U_\infty$
Inlet profile exponent (α)	0.1
Air density	1.225 kg/m^3
Inlet turbulence intensity	none (laminar inflow)

4.2.3 LES Validation

The reliability of the CFD turbine model is first assessed by evaluating the power output of each turbine.

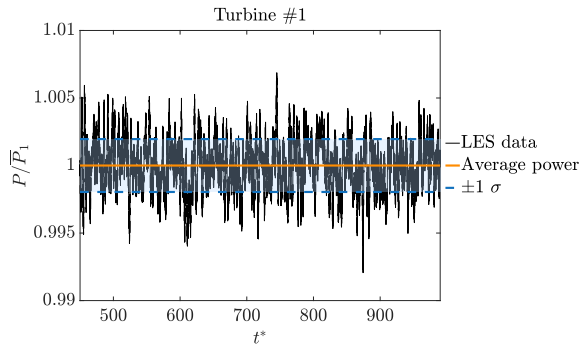


Fig. 11 Power output of the first turbine of the row and its standard deviation, normalized by the mean power of the same turbine

Figure 11 shows the power produced by the first turbine, averaged over a time window of 1.2 h (540 stored flow fields). The sample standard deviation over the same window is reported as a measure of the dispersion of the instantaneous power about its mean, i.e., of the power fluctuations.

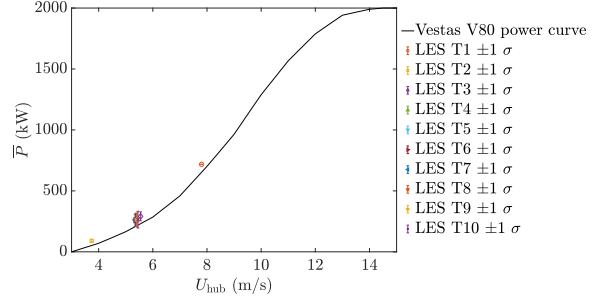


Fig. 12 Vestas V80 manufacturer power curve [15, 44] compared with the LES-predicted mean power of each turbine of the row (T1–T10), plotted against the mean hub-height inflow velocity $1D$ upstream of the rotor; error bars denote $\pm 1\sigma$

For validation, the manufacturer power curve of the Vestas V80 [44], also reported by Draper et al. [15], is used as a reference (Fig. 12). The mean power of each turbine in the row, computed from the LES torque, is plotted against the mean inflow velocity at hub height, obtained by averaging the velocity over a disk of area equal to the rotor area located $1D$ upstream of each turbine. The LES predictions follow the manufacturer curve well over the range of inflow velocities encountered along the row (the lower part of the curve), with the first turbine lying slightly below it.

Figure 13 compares the power output along a turbine row with experimental measurements and

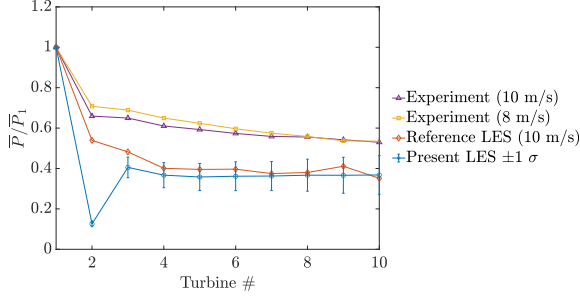


Fig. 13 Normalized power output along the turbine row: experiments (hub-height wind speed 10 and 8 m/s) and reference LES (10 m/s), both reported by Ivanell [25], compared with the present LES (error bars: $\pm 1\sigma$)

reference LES results from Ivanell [25]. Experimental data are available for hub-height inflow velocities of 10 m/s and 8 m/s, while the present LES corresponds to a hub-height velocity of 8 m/s.

A moderate discrepancy between experiments and LES is observed, mainly due to different averaging procedures. Experimental data were averaged over a directional sector of $\pm 2.5^\circ$, while the present LES assume a fixed wind direction of 270° . Even small variations in wind direction can significantly affect wake interactions and the power production of downstream turbines.

A larger difference appears for turbine T2, which shows lower power in the present LES compared to the reference results. This behavior is likely associated, at least in part, with the turbulence-free inflow used in the simulations, which limits wake recovery downstream of T1. Consequently, T2 operates in a strongly decelerated but relatively steady flow, consistent with the low power fluctuations observed. Farther downstream, wake recovery increases as turbulence develops, leading to larger power variability along the turbine row.

Overall, the LES results show physically consistent trends and reasonable agreement with experimental observations.

4.2.4 Multi-Domain Approach Validation

By construction, the wake reconstruction model cannot capture transient effects or interactions between turbine rows (see also Section 4.2.5). Therefore, instantaneous LES data (an example is shown in Fig. 14) must first be time-averaged to obtain a representative velocity field.

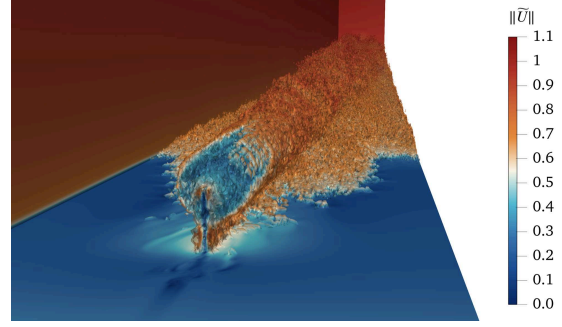


Fig. 14 Example of nondimensional filtered velocity magnitude at $t = 2.2$ h, with Q-criterion isosurfaces highlighting vortical structures. Perspective view of the row from the first turbine (T1)

Following Section 4.2.3, the mean field is obtained by averaging the same 540 flow fields ($\Delta t_{\text{avg}} = 1.2$ h). The reference frame in Fig. 10 is shifted $7D$ downstream so that the origin corresponds to turbine T1. Hereafter $\bar{U}$ denotes the time-averaged streamwise velocity, $\Delta\bar{U}$ the corresponding velocity deficit, normalized in the figures by the hub-height inflow velocity U_{hub} , and $\langle \bar{U} \rangle$ the assembled mesoscale field of Section 4.2.5.

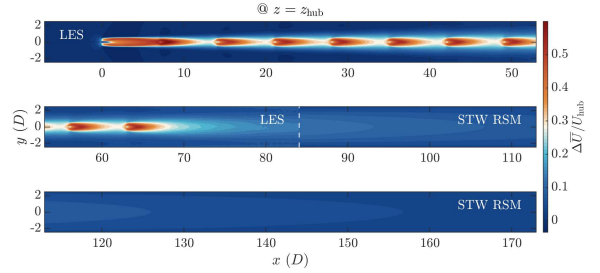


Fig. 15 Averaged streamwise velocity deficit field at hub height, showing the LES solution merged with the STW RSM reconstruction

Figure 15 shows the far-wake extension of the LES velocity deficit reconstructed using the STW response surface introduced in ND2. The predicted wake evolution is consistent with the LES continuation.

Further validation is provided in Fig. 16, which compares LES, RSM reconstruction, and experimental data from Eriksson et al. [17]. The RSM accurately reproduces the LES profile in the overlapping far-wake region. Experimental measurements show limited wake recovery downstream of Horns Rev 1 due to coastal effects [23]. In the present model, however, gradual recovery toward

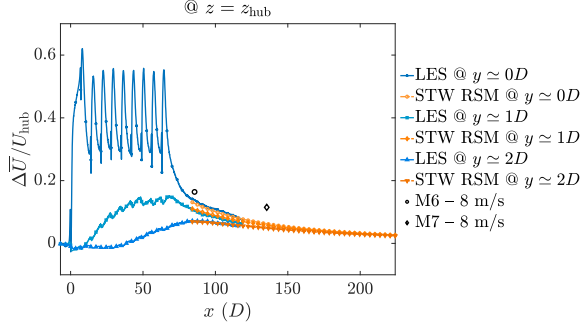


Fig. 16 Averaged velocity deficit at hub height. Comparison of LES and STW RSM results with experimental measurements from masts M6 and M7 (Eriksson et al. [17])

freestream conditions is assumed. Similar behavior has been reported in other LES studies [17]. Since the RSM does not capture turbine-row interactions, the recovery follows that of a single row, yet remains consistent with LES in the far wake.

Experimental data are unavailable for near-wake validation; therefore only LES results are considered to validate the SPW RSM reconstruction.

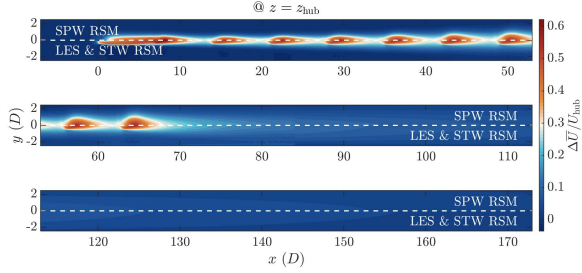


Fig. 17 Averaged velocity deficit field at hub height: LES ($y \leq 0$) versus SPW RSM lateral reconstruction ($y > 0$)

Figure 17 compares the LES ($y \leq 0$) with the RSM reconstruction ($y > 0$). Good agreement is observed for the turbines downstream of T1, with minor discrepancies near the rotors. The accuracy gradually decreases toward the last turbines (notably T9 and T10) because of the analytical assumptions of the model: the approximation $\sigma^2 \sim x^2$ limits the representation of the wake expansion, and the radial velocity deficit becomes harder to capture as turbine interactions accumulate. Since the SPW domain of ND2 is used only for the far lateral wake reconstruction, the approximation nevertheless remains acceptable.

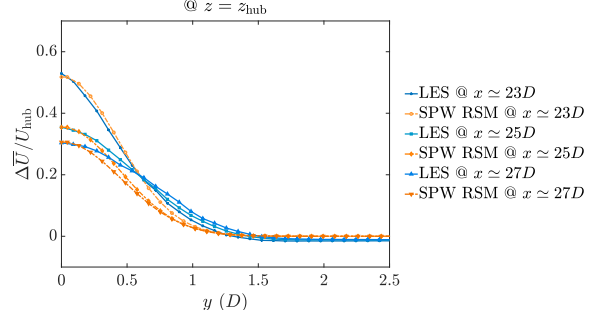


Fig. 18 Averaged velocity deficit at hub height. Comparison of LES and SPW RSM in the wake of turbine T4

For completeness, Fig. 18 shows spanwise sections at different downstream positions in the wake of turbine T4. LES and RSM predictions are generally consistent. Far from the turbine axis, the RSM predicts full recovery to the freestream velocity, whereas the LES indicates a slight acceleration above the freestream, as expected near the edges of a wind farm; this effect cannot be reproduced by the RSM by construction.

Finally, some technical details on the training of the RSMs are provided. To improve numerical conditioning, the free parameters are rescaled (e.g., $p_1 = p_{1,0} \tilde{p}_1$, with $p_{1,0}$ a fixed scaling factor and $\tilde{p}_1$ the corresponding normalized parameter) and the optimization is performed on the normalized variables. The scaling factors and the fitted normalized parameters of both RSMs are listed in Table 3. The STW RSM is trained on the region $84 \leq x^* \leq 119$, over a spanwise extent of about $4D$ centered on the row axis, and yields a coefficient of determination $R^2 = 0.87$ on the training data; the SPW RSM is trained on $56 \leq x^* \leq 91$, over a spanwise extent of about $1D$, with $R^2 = 0.88$.

Table 3 Scaling factors and fitted normalized parameters of the STW and SPW RSMs ($p_i = p_{i,0} \tilde{p}_i$ and $q_i = q_{i,0} \tilde{q}_i$, $i = 1, 2, 3$)

i	$p_{i,0}$	$\tilde{p}_i$	$q_{i,0}$	$\tilde{q}_i$
1	1250.0	1.37	0.0283	0.9385
2	15.9	1.90	-0.4412	1.0050
3	1562.5×10^3	1.86	0.3234	1.0008

For the STW RSM, the parameters are estimated by ordinary least squares (OLS). The

condition number of the design matrix $\mathbf{A}$,

$$CN(\mathbf{A}) = \frac{\sigma_{\max}(\mathbf{A})}{\sigma_{\min}(\mathbf{A})},$$

is of order 10^1 , indicating a well-conditioned problem that does not require regularization, and the solution is robust with respect to the choice of the training region.

For the SPW RSM, correlations between parameters (see Eq. (9)) increase the condition number to about 10^2 . Although not critical, this motivates the use of centered ridge regression [4, 24],

$$\min_{\boldsymbol{\vartheta}} \|\mathbf{A}\boldsymbol{\vartheta} - \mathbf{b}\|_2^2 + \lambda_R \|\boldsymbol{\vartheta} - \boldsymbol{\vartheta}_0\|_2^2,$$

where $\boldsymbol{\vartheta}$ is the parameter vector, $\boldsymbol{\vartheta}_0$ the penalty center and λ_R the regularization coefficient. The analytical solution is used as penalty center, and λ_R is selected by k -fold cross-validation [24] with $k = 5$, yielding $\lambda_R \sim 10^{-2}$, sufficient to stabilize the solution without introducing significant bias.

4.2.5 Wind Farm Assembly and Projection onto the Ocean Model Grid

Before proceeding with the coupling to the shallow water model, the whole mesoscale velocity field is reconstructed (from LES data, RSMs, and the imposed freestream).

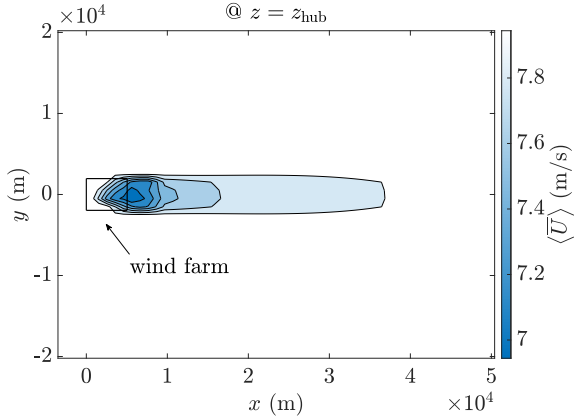


Fig. 19 Reconstructed streamwise velocity field $\langle \bar{U} \rangle$ at hub height

Figures 19 and 20 show the assembled velocity field projected onto a grid with spatial resolution $\Delta x = \Delta y = 1008$ m, evaluated at $z =$

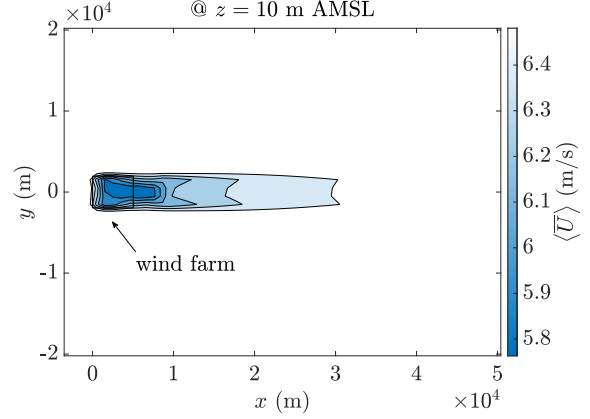


Fig. 20 Reconstructed streamwise velocity field $\langle \bar{U} \rangle$ at 10 m above mean sea level

z_{hub} and at $z = 10$ m above mean sea level, to enable comparison with the results of Volker et al. [48]. The projection is first performed onto a finer intermediate grid and subsequently onto a 100×100 grid in order to mitigate potential aliasing effects. The computational domain covers an area of $100.8 \times 100.8 \text{ km}^2$. It is worth emphasizing that, although using LES data interpolated onto a coarse grid may seem counterintuitive, it enables the mesoscale footprint of the wind farm (the magnitude and extent of the wake deficit, in the region where Ekman pumping develops most strongly) to be derived from high-fidelity simulations, even though turbine-scale features are necessarily not resolved on the ocean model grid, whose spacing exceeds the turbine spacing. Lower-fidelity models, such as those implemented in the Weather Research and Forecasting (WRF) model [48], may not capture all the flow details.

Focusing on the region surrounding the wind farm, the projection onto the ocean model grid highlights some differences between the two horizontal slices. In particular, at the center of the wind farm, the velocity deficit region appears moderately more extended at $z = 10$ m. This behavior is mainly due to the increased flow disturbance induced by the turbine towers and the stronger interaction among turbine rows at lower elevations compared to hub height [21].

Another aspect is the wake recovery process, which at $z = 10$ m is slightly more gradual than at hub height in the vicinity of the wind farm, although the far-wake region appears to develop in the same way. A difference in magnitude is

also evident, since a power-law velocity profile was prescribed at the inlet.

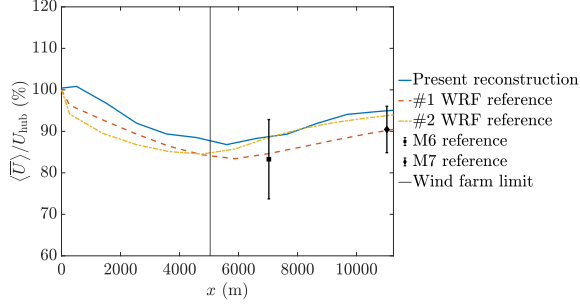


Fig. 21 Streamwise section of the velocity field interpolated onto the ocean model grid, along with reference data from Volker et al. [48], including both experimental measurements and WRF-based simulations at hub height. The vertical line marks the downstream edge of the wind farm; $x = 0$ corresponds to its upstream edge

To further assess the influence of the projection onto the ocean model mesh on the obtained results, Fig. 21 presents a comparison with the results of Volker et al. [48], which consider the same wind farm as in the present study, simulated using two different schemes within the WRF model.

The reconstructed field is in good overall agreement with the reference data. At the mast locations, the predicted wake deficit falls within the measurement uncertainty of M6 and M7. Moreover, the 10% velocity deficit threshold is reached at $x \simeq 8$ km, in agreement with the WRF scheme denoted as “#2 WRF reference” in Fig. 21, whose overall decay pattern is also closely matched; within the wind farm, the reconstruction is instead closer to the “#1 WRF reference” curve.

A qualitative consideration can also be made regarding the wake shape (see Fig. 19). The reconstructed wake in the present study appears much more homogeneous than that reported by Volker et al. [48], as it is based on an extension that does not account for row-to-row interactions. Furthermore, the reconstructed wake does not exhibit the tail deviation induced by the Coriolis effect.

5 LES–SWE Coupling

The reconstructed mesoscale wind field of Section 4.2.5 is now used to force the shallow water model through the wind stress term in Eq. (1).

Only the streamwise component of the wind is retained: the effective, wake-affected streamwise velocity, evaluated at 10 m above mean sea level (Fig. 20), rather than the undisturbed wind speed $\|\bar{\mathbf{u}}_{10}\|$, is converted into a stress field $\tau_x(x, y)$ through Eq. (5) and the drag correlation of Smith [43], while $\tau_y = 0$. The ocean domain coincides with the 100.8×100.8 km² projection grid of Section 4.2.5, with the wind farm located at the center of the basin. The Coriolis parameter and the water density are the same as in Appendix A, $f = 1.2 \times 10^{-4} \text{ s}^{-1}$ and $\rho_w = 1025 \text{ kg/m}^3$. Starting from a basin at rest ($h = H_0$, $\bar{u} = \bar{v} = 0$), the wind stress is applied impulsively at $t = 0$ and the simulation is advanced for 6 hours, as discussed below.

5.1 Order-of-Magnitude Considerations on Varying Bathymetry

In the first part of this study, a constant-depth ocean was assumed, allowing Ekman pumping to be characterized clearly, without interference from other phenomena. Subsequent analysis considers interactions with bathymetry variations, as such variations can introduce contributions that may obscure the Ekman pumping phenomena.

For example, consider a seabed with a slope from west to east (or vice versa—for now, only the magnitude of the resulting vertical velocity is of interest). In this case, the following can be written

$$w_B \simeq \bar{u} \frac{\partial b}{\partial x}.$$

Assuming, for instance, a 100 m difference in elevation over 5 km and a maximum current velocity of 0.025 m/s, values similar to those modeled by Segtnan and Christakos [39], the result is $w_B \simeq 43 \text{ m/day}$. This value is more than ten times the estimate made by Ludewig [28] for the maximum magnitude of the Ekman pumping.

Apart from the study by Segtnan and Christakos [39], in which vertical velocity variations near a wind farm are attributed to bathymetry changes but discussed only qualitatively, and the work of Paskyabi [32], which considers the effects of a constant slope using a 2DV shallow water model, to the authors’ knowledge no systematic investigation of the interaction between wind

farms and varying bathymetry is available in the literature.

5.2 Combining LES-Based Wind Stress and Bathymetry

Four test cases are now proposed, labeled A, B, C, and D (Fig. 22).

Case A corresponds to a flat bathymetry with a 40 m basin depth, and case B to a flat bathymetry with a 20 m depth. Case C has the same average depth of 20 m, but the bottom shoals linearly from 30 m at the western boundary to 10 m at the eastern boundary along a ramp located between $x = 20$ km and $x = 80$ km (Fig. 22); case D is similar to C, except that a flat platform, 20 km wide and 20 m deep, is introduced at the location of the wind farm. These bathymetries are idealized. To avoid numerical issues, the bathymetry was smoothed with a moving-average filter. In addition, the streamwise grid resolution was refined by a factor of ten (1000×100 cells), since bathymetry-induced forcing requires finer grids [54]; this refinement did not alter the wind stress forcing, and the diagnosed vertical velocity remained practically identical to that obtained with the coarser streamwise grid.

The motivation behind cases C and D is to investigate how circulation behaves near the wind farm when the flow is disturbed by bathymetry variations along its entire length by a ramp (case C) versus when the disturbance is limited to the upstream and downstream regions of the wind farm (case D).

Regarding the simulation time, a period of 6 hours was adopted to characterize the early mesoscale response of a shallow, well-mixed basin under persistent wind forcing. Although this duration may seem short compared with the time typically required for full flow development [32], it has a physical basis. The Ekman layer develops in response to a constant wind, and its vertical pumping approaches a steady state with a characteristic timescale of order $1/f$, a well-known result from the equations describing the evolution of the Ekman boundary layer [46]. At latitudes where $f \sim 10^{-4} \text{ s}^{-1}$, this corresponds to roughly 3 hours, so that the adopted interval spans a few Ekman timescales, although it is shorter than the inertial period $2\pi/f$. It should be noted that, in the present model, w_E is diagnosed from the curl of

the steady wind stress (Section 2) and is therefore available from the onset of the forcing; the 6-hour horizon concerns the development of the surface elevation, of the depth-averaged velocities and, through them, of the bottom velocity w_B . Since bottom friction is neglected, the depth-averaged solution retains inertial oscillations (cf. Appendix A) and is not interpreted as a fully equilibrated ocean state: the comparison among cases A–D is performed at the same, physically representative forcing time, which is sufficiently long for a clear wind-farm-induced surface and circulation pattern to develop. This choice is also consistent with Ludewig [28], who simulated deeper basins using a baroclinic ocean model and found that effective pumping occurs over roughly 8 hours, and with the observation that, in real atmospheric conditions, maintaining a steady wind forcing for more than about ten hours is unlikely [19].

In general, it was found that even a modest bathymetry variation tends to strongly influence the wind-driven dynamics, at times even dominating them. Focusing specifically on the *SSHA*, for example, a change from 40 m to 20 m in depth results in an increase in the water surface excursion; the magnitude of the *SSHA* dipole after 6 hours is of order 10^{-5} m (Fig. 23). The $\simeq 1$ km grid spacing, chosen to match the resolution of the projected LES-based stress field (Section 4.2.5), does not constrain this result: refining the ocean-model grid did not lead to appreciable differences. More interesting, however, is the case with a ramped bathymetry, where the dipolar structure progressively diffuses, with the dipole extending further in the longitudinal direction (Fig. 23). The results are similar for both a single ramp and a double ramp. It is important to stress that the bathymetric gradient considered here is relatively mild; stronger gradients would, as already observed qualitatively by Segtnan and Christakos [39], lead to vertical velocities potentially exceeding the Ekman pumping, in which case the reconstruction model implemented in Q3DFLOOD would not perform reliably.

The effects of bathymetry are even more pronounced on the horizontal velocities. Figure 24 shows that the streamwise velocity pattern is similar for the two flat bathymetries, but its magnitude roughly doubles from 40 m to 20 m depth, as observed for the *SSHA*; this is the expected

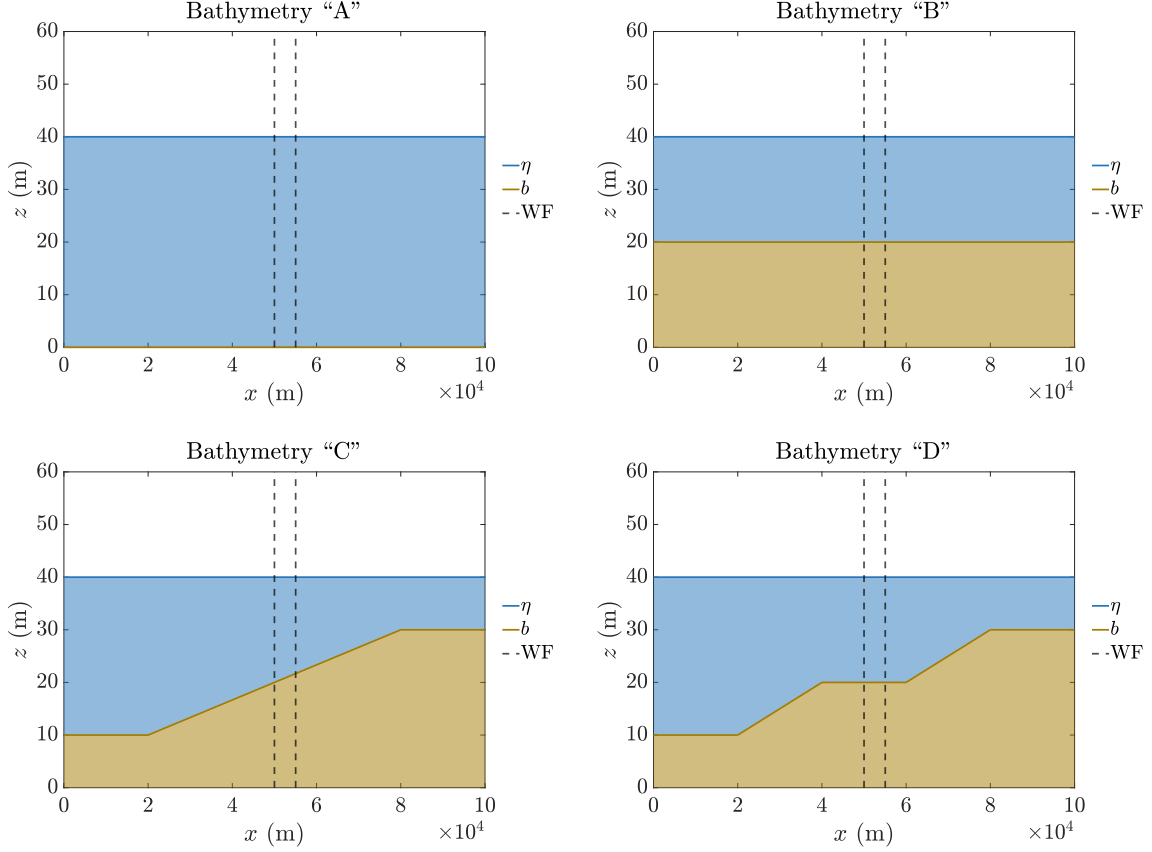


Fig. 22 Bathymetry of the four test cases (W–E depth variations only), used to assess the combined influence of topography and LES-based wind stress on the ocean circulation; the dashed lines mark the wind farm (WF) extent

$1/H_0$ scaling of the depth-averaged wind-driven transport (cf. Appendix A). In the ramped cases, steep velocity gradients appear at the ramps, particularly in case D, whereas over the flat platform beneath the wind farm of case D the velocity is nearly uniform; the footprint of the wind farm nevertheless remains clearly visible.

The spanwise velocity (Fig. 25) behaves similarly: its magnitude nearly doubles from 40 m to 20 m for flat bathymetry, while in the ramped cases the structure of $\bar{v}$ closely resembles that of $\bar{u}$, indicating that the bathymetry-induced reorganization of the circulation dominates the local wind-farm signature. Distinctive features can nevertheless be identified: around the center of the wind farm, $\bar{u}$ is symmetric whereas $\bar{v}$ is anti-symmetric. Note also that the nearly uniform background value of $\bar{v}$ visible in case A ($\simeq -0.02$ m/s) corresponds to the basin-wide inertial oscillation of the depth-averaged flow discussed in

Appendix A, $\bar{v}(t) = \tau_0 / (f \rho_w H_0) (\cos ft - 1)$, evaluated at $t = 6$ h with the undisturbed upstream stress: the wind-farm-induced pattern is superimposed on this background, which should be kept in mind when interpreting the color scales.

The strength of the bathymetric response is consistent with the fact that bottom-topography variations enter the hydrostatic balance (Section 2) and can substantially reorganize the depth-averaged circulation: since the wind-driven transport $h \bar{u}$ is nearly uniform along the ramp, the depth-averaged velocity varies roughly in inverse proportion to the local depth (Fig. 24), and even the mild gradients considered here produce horizontal-velocity variations larger than those directly associated with the wind farm forcing. For reference, $Ro \simeq 1.2 \times 10^{-2}$ and $Bu \simeq 1.8 \times 10^3$ in case A, and $Ro \simeq 2.4 \times 10^{-2}$ and $Bu \simeq 8.9 \times 10^2$ in cases B–D, so that $Bu/Ro \gg 1$ in all cases. The resemblance between $\bar{v}$ and $\bar{u}$ is

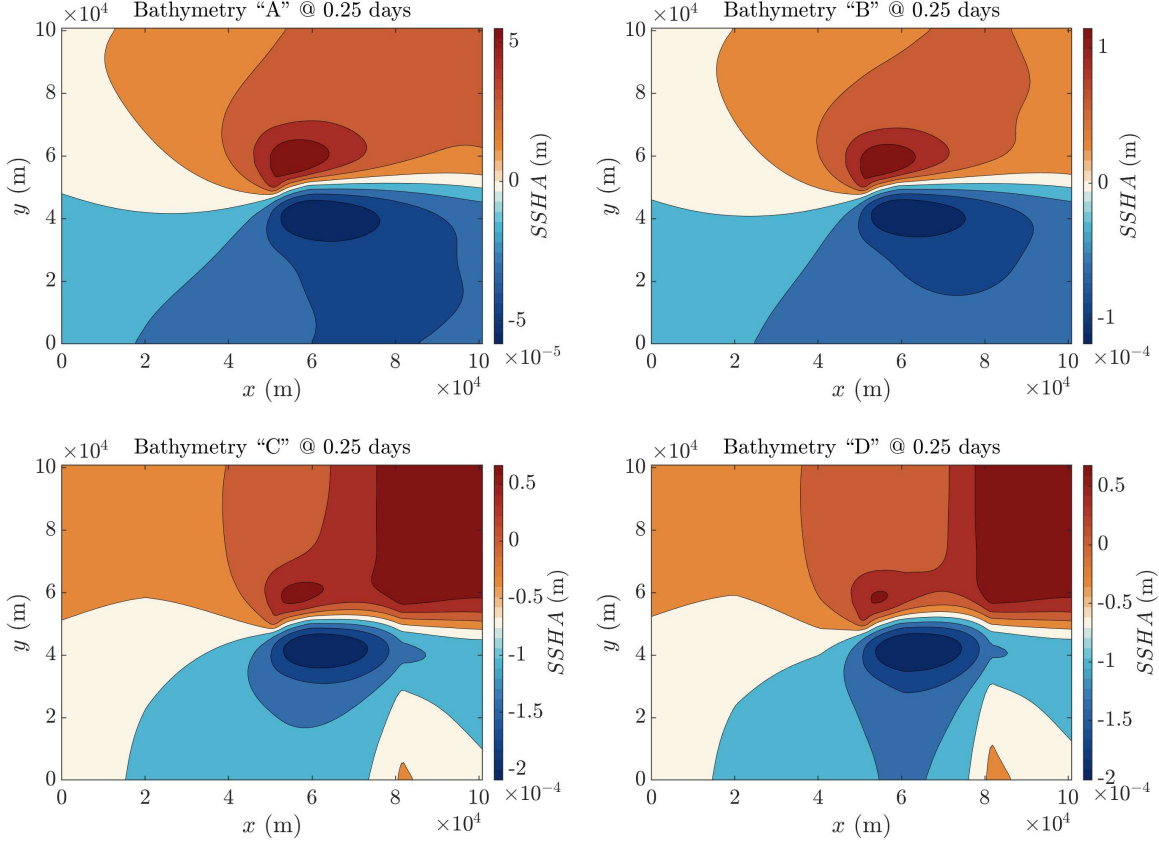


Fig. 23 SSHA for test cases A, B, C, and D at a simulation time of 6 hours (0.25 days)

due to the Coriolis coupling, since a term proportional to $\bar{u}$ appears in the spanwise momentum equation; without rotation, $\bar{v}$ would be expected to recover the classic quadrupole pattern of cases A and B.

It is also interesting to examine the Ekman vertical velocity. In both cases A and B the basin is shallower than $2d_E$, so that the reconstructed vertical velocity at $\sigma = \sigma_E = 0.5$ is $w = \bar{F}w_E$, with $\bar{F} = H_0/(2d_E)$ (Eq. (3)): by construction of the diagnostic model, which follows the top-to-bottom Ekman layer scaling of Cushman-Roisin and Deleersnijder [13], halving the depth halves the vertical velocity. Accordingly, Fig. 26 shows maxima of approximately 1.5 m/day for the 20 m deep basin and roughly 3 m/day for the 40 m deep one. The latter value is of the same order as the estimate reported by Ludewig [28] for a 60 m deep basin under similar atmospheric conditions.

Finally, Fig. 27 shows a vertical section at $x = 65$ km, i.e., 10 km downstream (east) of the

wind farm, to illustrate how the bottom velocity affects the vertical velocity profile across the basin in cases C and D. The maximum bathymetric gradient considered here is of order 10^{-4} (20 m over 60 km in case C, 10 m over 20 km for each ramp of case D), i.e., between one and two orders of magnitude smaller than the representative value of 2×10^{-2} discussed in Section 5.1 with reference to Segtnan and Christakos [39]. Accordingly, the bottom-induced contribution w_B appears in Fig. 27 as a nearly uniform background across the basin, of magnitude comparable to but smaller than the wind-farm-induced dipole, which remains clearly identifiable. For larger gradients, as discussed in Section 5.1, the bathymetric contribution can even overshadow the Ekman pumping [39].

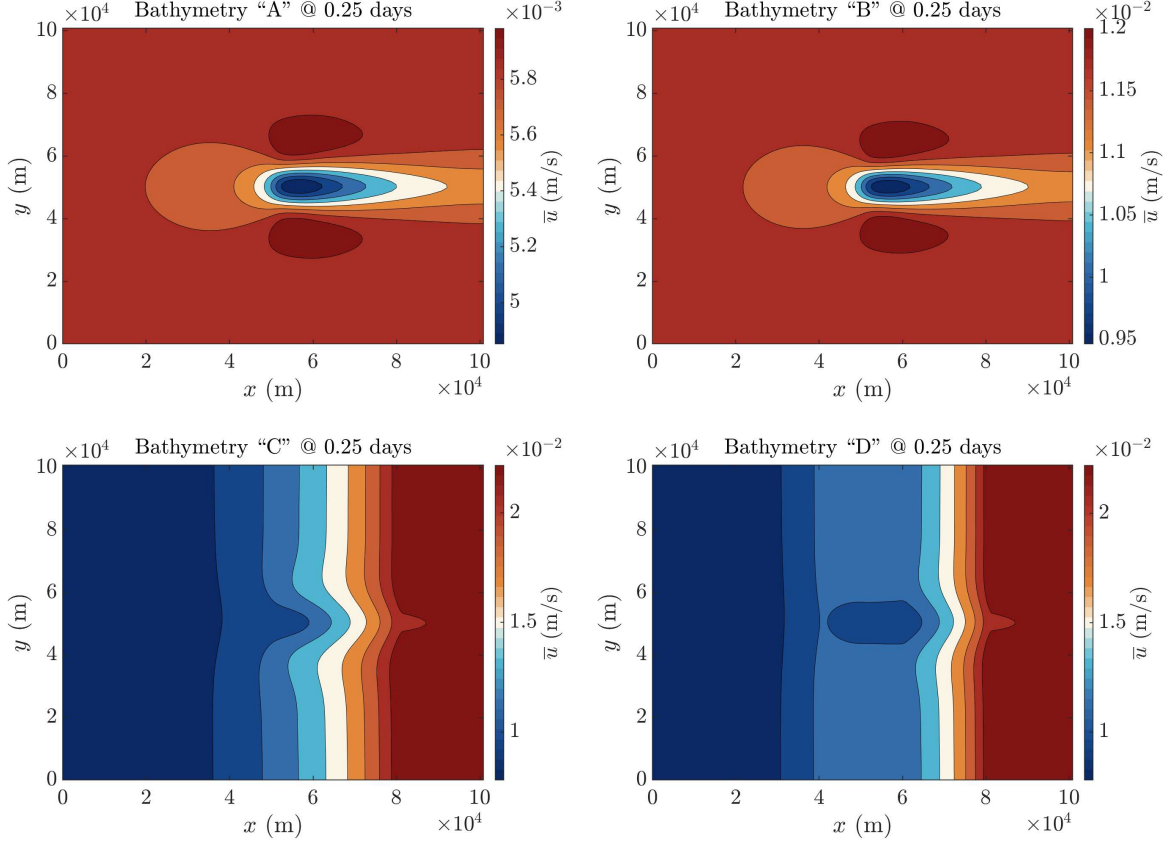


Fig. 24 $\bar{u}$ for test cases A, B, C, and D at a simulation time of 6 hours (0.25 days)

6 Conclusions

In this work, the impact of offshore wind farms on the local ocean circulation has been characterized through a novel multiscale framework that couples high-fidelity LES of wind turbines with a mesoscale shallow water model of the ocean featuring a diagnostically reconstructed vertical velocity.

Starting from a simplified Gaussian wind stress model, this study has investigated how a shallow water model, despite relying on depth-averaged horizontal velocities, can capture pumping phenomena induced by the wind deficit downstream of a wind farm. In addition, a diagnostic model for the oceanic vertical velocity has been proposed based on physical considerations. It should be emphasized that the reconstructed vertical velocity is physically consistent only at the mesoscale

under investigation and under specific assumptions regarding the characteristic velocities and length scales involved.

The role of the Coriolis force in the wind farm–ocean interaction has been quantified. The statistical analysis indicates that the wind intensity is the primary parameter controlling Ekman pumping, whereas the Coriolis force has a limited impact on its magnitude but is crucial in determining the spatial structure of the solution, in particular the position of the upwelling/downwelling cells and of the associated horizontal velocity patterns (Figs. 23 and 25). If the analysis is restricted to a small basin and to local rather than mesoscale dynamics, neglecting the Coriolis force may be justifiable. The size of the wind farm and the (flat) basin depth exert comparable influences on the circulation. It should be noted that the statistical analysis explores depths between 30 and 150 m under an idealized forcing, whereas the LES-based cases consider shallower basins (20–40 m); the two

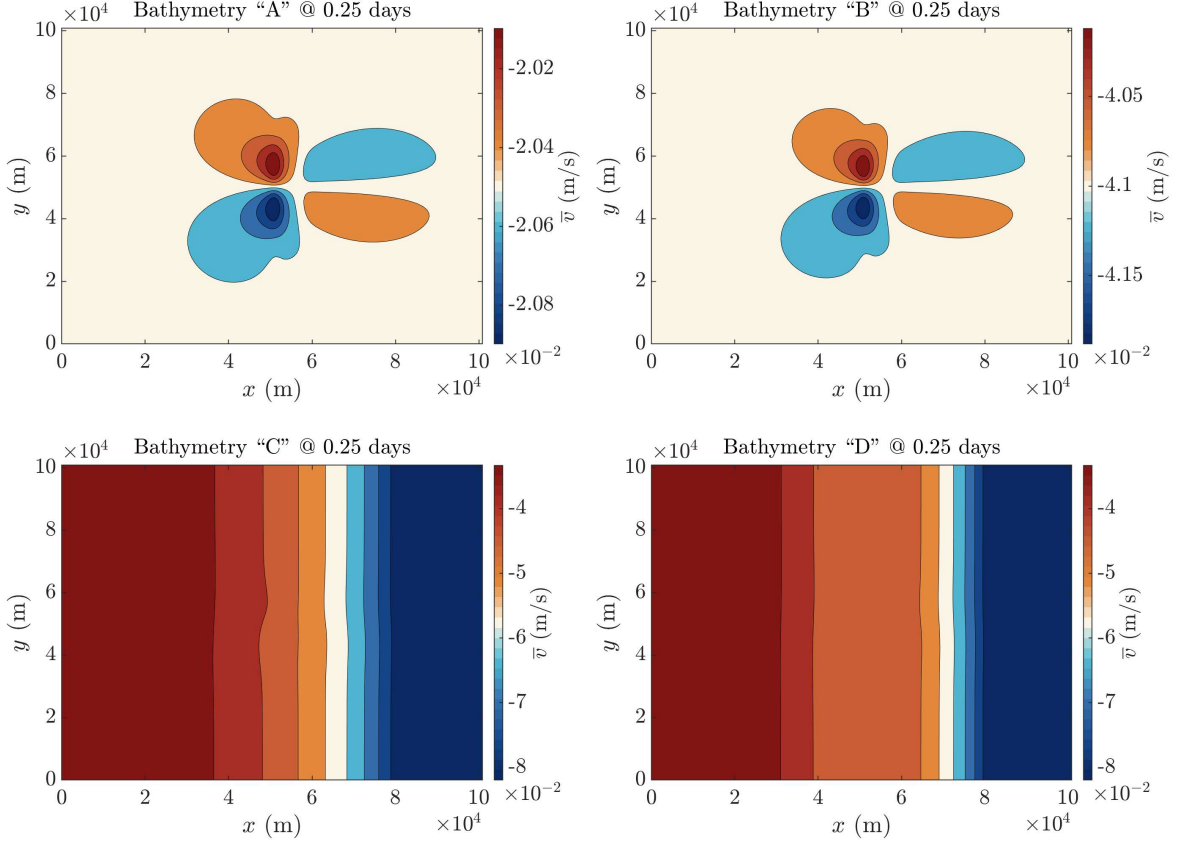


Fig. 25 $\bar{v}$ for test cases A, B, C, and D at a simulation time of 6 hours (0.25 days)

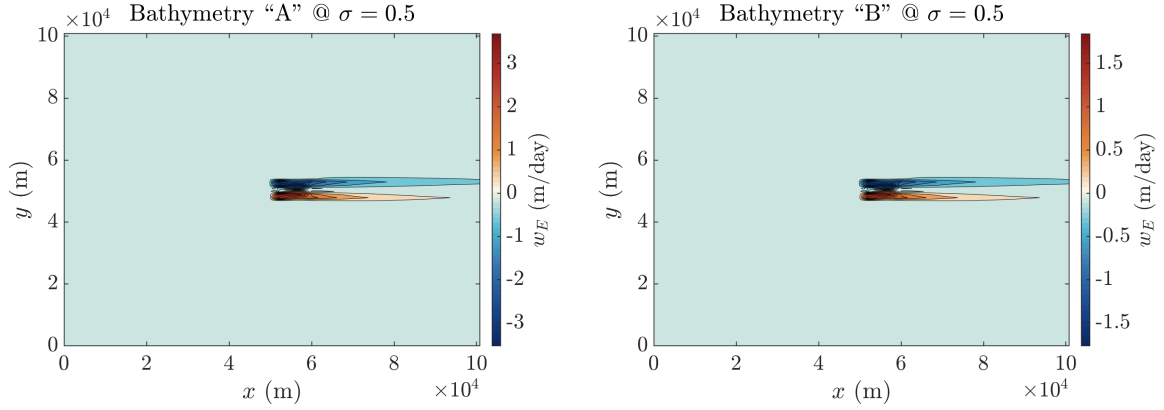


Fig. 26 Reconstructed vertical velocity $w(\sigma_E) = \tilde{F} w_E$ at $\sigma = \sigma_E = 0.5$ for test cases A and B (the colorbar label w_E in the panels refers to this quantity)

sets of results are therefore complementary rather than directly comparable.

LES of a turbine row representative of the Horns Rev 1 wind farm were performed, and the

resulting velocity field was extended to the farm

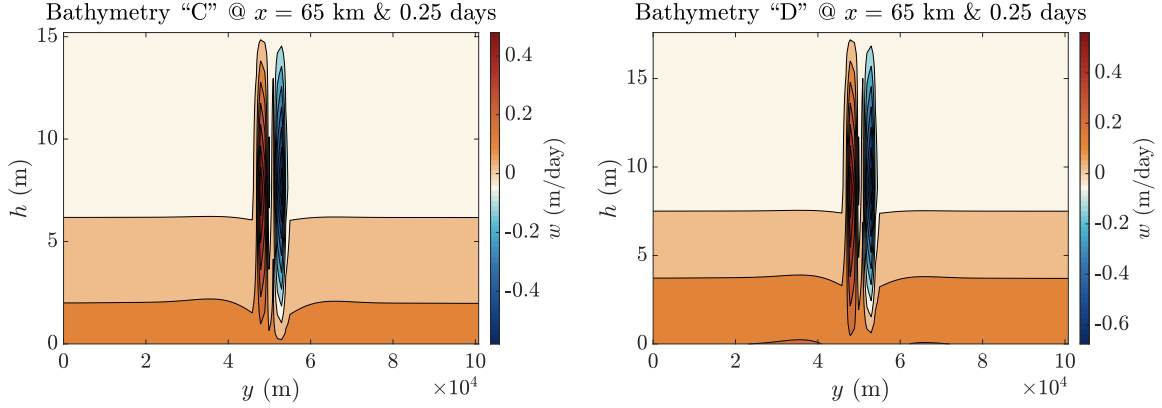


Fig. 27 Vertical cross-section at $x = 65$ km (10 km downstream of the wind farm) of the vertical velocity field for test cases C and D

scale and to the mesoscale using an RSM technique. Although this approach currently has limitations stemming from the simplifying assumptions underlying the RSM (neglect of yaw misalignment, transient phenomena and row-to-row interactions in the far wake), the reconstructed velocity field compares well with the results reported in the literature for the same configuration.

Using the LES-based wind stress field, both the influence of the wind farm on ocean circulation and the role of bathymetry were quantified for basins with flat bottoms of varying depth, as well as for ramped bathymetries (with depth variations assumed only along the W–E direction). The results show how small bathymetric variations can influence certain aspects of the ocean circulation induced by the wind farm. In particular, the horizontal velocity field is more strongly affected by bathymetry, while the classical dipolar structure that can be seen in terms of surface elevation is altered but remains discernible. Within the present diagnostic framework, the weak bathymetric gradients considered here strongly reorganize the depth-averaged horizontal circulation while leaving the wind-stress-driven Ekman signature of the wind farm clearly identifiable in the reconstructed vertical velocity.

Overall, within the limits of the present diagnostic reconstruction, wind-farm-induced Ekman pumping emerges as a relevant factor for the local circulation primarily when wind farms are deployed in regions with very weak bathymetric gradients, at least under strong, unidirectional

wind forcing. Its dependence on latitude is limited, whereas the size of the wind farm and the basin depth play a major role in determining the overall response.

The present framework rests on a number of simplifying assumptions that bound the validity of the results: the ocean is barotropic and well mixed, bottom friction is neglected, the wind forcing is steady and unidirectional, surface gravity waves, whose role was highlighted by Paskyabi and Fer [33], are not included, the reconstructed wind field neglects row-to-row interactions, transient effects and yaw misalignment, and the bathymetries considered are idealized. Relaxing these assumptions, in particular by including stratification and bottom friction and by extending the LES to multiple rows, is the natural continuation of this work.

Statements and Declarations

Competing interests. The authors declare that they have no competing interests.

Funding. No funding was received for conducting this study.

Data availability. The data presented in this study are partially available from the corresponding author, upon request.

Ethics approval and consent to participate. Not applicable: this study does not involve human participants, human data or animals.

Consent for publication. Not applicable.

Materials availability. Not applicable.

Code availability. The in-house solvers used in this work (Q3DFLOOD and UTD-WF) are not publicly available.

Author contribution. All authors contributed to the conception and design of the study, and all authors read and approved the final manuscript.

Appendix A In-House Shallow Water Code Validation

To ensure the accuracy of many of the results obtained in this work, it was necessary to validate the in-house shallow water solver Q3DFLOOD (Section 2.1), by comparing its outputs with benchmark and analytical solutions.

Three validation cases were selected. The first, an analytical case, verifies the correct implementation of the wind stress forcing and of the Strang splitting algorithm. The second, a well-known benchmark, verifies the Coriolis forcing, the flux computation and the open-boundary treatment. The third assesses the discretization of the bathymetric source term.

A.1 Wind-Stress-Forced Velocity Oscillation Validation Case

For an initial validation, a relatively simple test case is considered, specifically the problem defined by the initial conditions

$$\begin{aligned}\bar{u}(x, y, 0) &= 0 \text{ m/s}, & \bar{v}(x, y, 0) &= 0 \text{ m/s}, \\ h(x, y, 0) &= H_0,\end{aligned}$$

with $H_0 = 20 \text{ m}$, and a constant wind stress forcing characterized by $\tau_x(x, y) = \tau_0$ and $\tau_y(x, y) = 0$, with $\tau_0 = 0.009 \text{ N/m}^2$. Additionally, $f = 1.2 \times 10^{-4} \text{ s}^{-1}$ and $\rho_w = 1025 \text{ kg/m}^3$ are assumed. Finally, homogeneous Neumann boundary conditions for each variable enforce zero gradients in both directions.

The resulting system of ordinary differential equations (ODEs) can be written simply as

$$\begin{cases} \frac{dh}{dt} = 0 \\ \frac{d\bar{u}}{dt} = f\bar{v} + \frac{\tau_0}{\rho_w H_0} \\ \frac{d\bar{v}}{dt} = -f\bar{u} \end{cases},$$

whose analytical solution is

$$\begin{pmatrix} h \\ \bar{u} \\ \bar{v} \end{pmatrix} = \begin{pmatrix} H_0 \\ \frac{\tau_0}{f\rho_w H_0} \sin(ft) \\ \frac{\tau_0}{f\rho_w H_0} (\cos(ft) - 1) \end{pmatrix},$$

where $\bar{u} = \bar{u}(t)$ and $\bar{v} = \bar{v}(t)$, since the solution is constant in space for both velocities.

The numerical case is run on a 401×401 grid with a Courant number $\text{Co} = 0.9$.

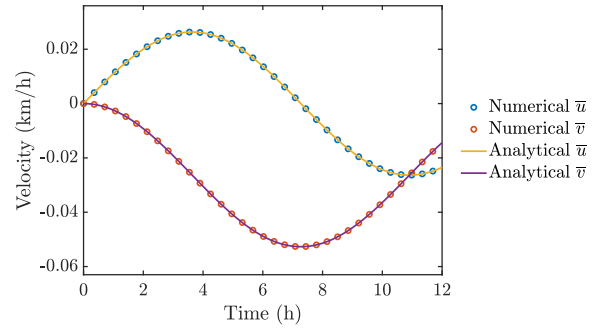


Fig. A1 Numerical and analytical solutions for the constant-wind-stress problem (numerical solutions under-sampled)

Figure A1 shows a comparison between numerical and analytical results (the numerical results are undersampled for readability); here, the numerical solution is obtained using the two-step Lax–Wendroff scheme [55] for flux reconstruction combined with the van Leer limiter. Identical results are obtained when using the FORCE scheme together with the Minmod limiter.

In addition to the excellent agreement between the numerical and analytical solutions, it can be observed that an integral average yields mean velocity values such that $\bar{u}_{\text{avg}} = 0$ and $\bar{v}_{\text{avg}} = -\tau_0/(f\rho_w H_0)$. Essentially, in this test case, a

net mass transport (southward) orthogonal to the wind direction (assumed to blow from west to east) is captured.

The case with a non-zero wind stress in the y -direction also gave equally satisfactory results (not shown), as did those with non-zero initial conditions.

A.2 2D Geostrophic Adjustment Validation Case

In the following second validation case, the behavior of the solver in the presence of discontinuities is studied, and the correct integration of the Coriolis forcing is ensured in the case of a spatially non-constant solution. The test, referred to in the literature as “2D geostrophic adjustment” [30], is considered here.

The case consists of an initial asymmetric water column subjected to strong rotation, which induces a 2D geostrophic adjustment that the numerical scheme must be able to reproduce. Since the solution is expected to develop discontinuities, the FORCE scheme with the Minmod limiter is selected. The computational domain is $V = [-10, 10] \text{ m} \times [-10, 10] \text{ m}$, the bathymetry is flat, and the initial conditions are defined by

$$h(x, y, 0) = 1 + \frac{A_\eta}{2} \left[1 - \tanh \left(\frac{r_\lambda - R_i}{R_E} \right) \right],$$

$$r_\lambda = \sqrt{\lambda x^2 + \frac{y^2}{\lambda}},$$

where

$$A_\eta = 0.5, \quad \lambda = 2.5,$$

$$R_E = 0.1, \quad R_i = 1,$$

in consistent units of measurement, and $\bar{u}(x, y, 0) = \bar{v}(x, y, 0) = 0 \text{ m/s}$. The acceleration due to gravity and the Coriolis frequency are $g = 1 \text{ m/s}^2$ and $f = 1 \text{ s}^{-1}$, respectively. Homogeneous Neumann boundary conditions are applied at the boundaries. In some studies, wave-transmissive boundary conditions are employed instead [7]; however, their behavior is similar to that of the Neumann boundary conditions adopted here.

The numerical solution is first computed on a 101×101 grid with $\text{Co} = 1$, and then on a 401×401 grid, also with a unit Courant number.

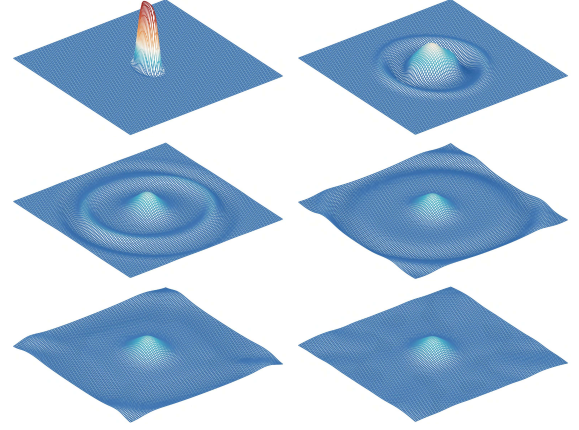


Fig. A2 Surface disturbance at various times on the 101×101 grid. From left to right and top to bottom: 0 s, 4 s, 8 s, 12 s, 16 s, 20 s

Initially, the elliptical water column is not in equilibrium and evolves in a non-axial manner due to the effect of rotation. Two successive shock waves (gravity waves) are then generated, leaving behind a small smooth bulge that slowly rotates clockwise (consistent with $f > 0$). First, in Fig. A2, the water surface disturbance is shown for various times, computed on the 101×101 grid. An excellent qualitative agreement with the results of Navas-Montilla and Murillo [30] can be observed, with long waves leaving the domain with little to no reflection.

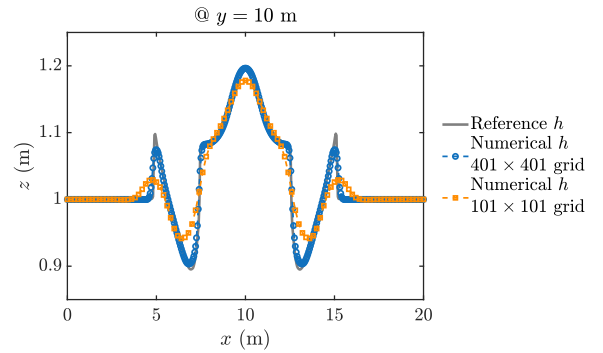


Fig. A3 Reference and present numerical solutions, on the 101×101 and 401×401 grids, for the 2D geostrophic adjustment problem at $t = 4 \text{ s}$ (horizontal axes shifted by 10 m)

For a quantitative comparison, numerical results for the water column elevation at time $t = 4 \text{ s}$ and at the plane $y = 10 \text{ m}$ are also shown, along with the reference solution (again

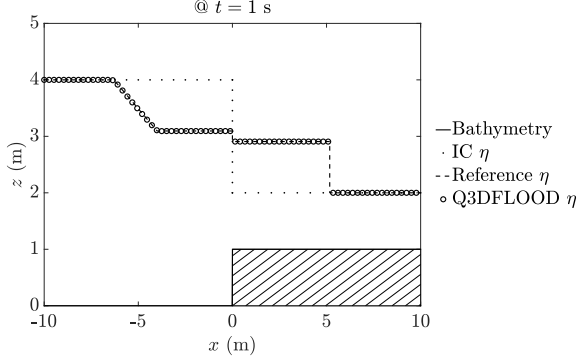


Fig. A4 Surface elevation at an arbitrary constant value of y , at $t = 1$ s

from Navas-Montilla and Murillo [30]), in Fig. A3. It can be observed that the expected behavior of the solution's evolution is reproduced by the numerical scheme, and the two solutions are in excellent agreement.

A.3 Shock Propagation Over a Step Bottom Validation Case

In the third and final test case, a well-known benchmark problem is considered, whose analytical solution was derived by Alcrudo and Benkhaldoun [1].

The test consists of a 1D bottom step problem. The initial conditions for the water depth are defined as

$$h(x, y, 0) = \begin{cases} h_L & x < 0 \\ h_R & x > 0 \end{cases},$$

with $h_L = 4$ m and $h_R = 2$ m. The initial velocity is zero everywhere, $\bar{u}(x, y, 0) = \bar{v}(x, y, 0) = 0$, and the bathymetry is prescribed as a step function,

$$b(x, y) = \begin{cases} 0 & x < 0 \\ b_0 & x > 0 \end{cases},$$

with $b_0 = 1$ m. It is also assumed $g = 9.81$ m/s².

The computational domain is defined as $V = [-L_x/2, L_x/2] \times [-L_y/2, L_y/2]$, with $L_x = 20$ m and $L_y = 0.2$ m. The grid resolution consists of 2000 elements in the x -direction and 20 elements in the y -direction. A Courant number $Co = 0.9$ is imposed. As the solution is expected to be discontinuous, a FORCE scheme is selected for flux reconstruction, with Minmod limiter.

Figure A4 shows, at a fixed value of y (the solution being identical along any transverse section), the bathymetry, the initial condition, the reference surface elevation from Alcrudo and Benkhaldoun [1], and the solution computed with Q3DFLOOD. The latter is undersampled for visualization purposes.

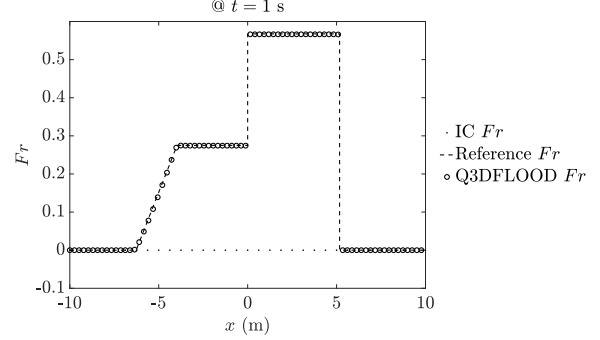


Fig. A5 Froude number at an arbitrary constant value of y , at $t = 1$ s

The initial discontinuity generates a rarefaction wave propagating to the left and a shock wave propagating to the right. The flow remains subcritical throughout the domain, as confirmed by the Froude number distribution shown in Fig. A5. Again, the Q3DFLOOD solution in the latter plot is undersampled for visualization purposes.

Finally, it is worth emphasizing that in the presence of variable bathymetry a sufficiently fine grid resolution is essential, as for second-order accurate schemes the error in the elevation η scales as Δx^2 [54].

References

- [1] Alcrudo F, Benkhaldoun F (2001) Exact solutions to the Riemann problem of the shallow water equations with a bottom step. *Computers & Fluids* 30(6):643–671
- [2] Asdar S, Ciani D, Buongiorno Nardelli B (2024) 3D reconstruction of horizontal and vertical quasi-geostrophic currents in the North Atlantic Ocean. *Earth System Science Data* 16(2):1029–1046
- [3] Bastankhah M, Porté-Agel F (2014) A new analytical model for wind-turbine wakes. *Renewable Energy* 70:116–123

- [4] Belli E (2022) Smoothly adaptively centered ridge estimator. *Journal of Multivariate Analysis* 189:104882
- [5] Bernardoni F, et al (2021) Identification of wind turbine clusters for effective real time yaw control optimization. *Journal of Renewable and Sustainable Energy* 13(4):043301
- [6] Broström G (2008) On the influence of large wind farms on the upper ocean circulation. *Journal of Marine Systems* 74:585–591
- [7] Castro M, López J, Madroñal C (2008) Finite volume simulation of the geostrophic adjustment in a rotating shallow-water system. *SIAM Journal on Scientific Computing* 31:444–477
- [8] Cazenave P, Torres R, Icarus Allen J (2016) Unstructured grid modelling of offshore wind farm impacts on seasonally stratified shelf seas. *Progress in Oceanography* 145:25–41
- [9] Ciri U (2015) Large-Eddy Simulations of Wind Turbines. Master’s thesis, Università degli Studi di Pisa
- [10] Ciri U, et al (2015) Effects of the subgrid-scale modeling in the large-eddy simulations of wind turbines and wind farms. In: *Proceedings of the 10th Workshop of Direct and Large-Eddy Simulation*, Limassol, Cyprus
- [11] Colgate Kottler H, Lindberg J, Rodriguez J (2025) Method of moments for Gaussian mixtures: Implementation and benchmarks. URL <https://arxiv.org/abs/2502.07648>, [arXiv:2502.07648](https://arxiv.org/abs/2502.07648)
- [12] Cushman-Roisin B, Beckers J (2011) *Introduction to Geophysical Fluid Dynamics: Physical and Numerical Aspects*. International Geophysics, Academic Press, Waltham, MA
- [13] Cushman-Roisin B, Deleersnijder E (2019) Top-to-bottom Ekman layer and its implications for shallow rotating flows. *Environmental Fluid Mechanics* 19(5):1105–1119
- [14] Della Monica M, Bernardini M (2025) Strategies for CFD Investigation and Optimization of the External Aerodynamics of a Sounding Rocket. *Aerotecnica Missili & Spazio* <https://doi.org/10.1007/s42496-025-00299-9>
- [15] Draper M, Guggeri A, Usera G (2016) Modelling one row of Horns Rev wind farm with the Actuator Line Model with coarse resolution. *Journal of Physics: Conference Series* 753:082028. <https://doi.org/10.1088/1742-6596/753/8/082028>
- [16] Eliassen F (2025) The Effect of Offshore Wind Farms on Upper Ocean Variability and Marine Ecosystems. Master’s thesis, The University of Bergen, available at <https://hdl.handle.net/11250/5271598>
- [17] Eriksson O, et al (2017) The Long distance wake behind Horns Rev I studied using large eddy simulations and a wind turbine parameterization in WRF. *Journal of Physics: Conference Series* 854:012012
- [18] Ferrari S, Saleri F (2004) A new two-dimensional Shallow Water model including pressure effects and slow varying bottom topography. *ESAIM: Modélisation mathématique et analyse numérique* 38(2):211–234
- [19] Floeter J, et al (2022) Chasing the offshore wind farm wind-wake-induced upwelling/downwelling dipole. *Frontiers in Marine Science* 9
- [20] Global Wind Energy Council (GWEC) (2025) Global offshore wind report 2025. Tech. rep., GWEC, URL <https://www.gwec.net/reports/globaloffshorewindreport>
- [21] Guzmán Hernández M, et al (2025) Numerical Simulations of an Array of Offshore Turbines and its Effect on the Local Circulation. In: *APS Division of Fluid Dynamics Annual Meeting 2025*, Houston, Texas
- [22] Hansen K, et al (2012) The impact of turbulence intensity and atmospheric stability on power deficits due to wind turbine wakes at Horns Rev wind farm. *Wind Energy*

- 15(1):183–196
- [23] Hasager C, et al (2013) Wind Farm Wake: The Horns Rev Photo Case. *Energies* 6(2):696–716
 - [24] Hastie T (2020) Ridge Regularization: An Essential Concept in Data Science. *Technometrics* 62(4):426–433
 - [25] Ivanell S (2009) Numerical computations of wind turbine wakes. PhD thesis, Royal Institute of Technology, Stockholm, Sweden
 - [26] Johnson K, et al (2006) Control of variable-speed wind turbines: standard and adaptive techniques for maximizing energy capture. *IEEE Control Systems Magazine* 26(3):70–81
 - [27] Lu W, et al (2017) One-dimensional ocean model with three types of vertical velocities: a case study in the South China Sea. *Ocean Dynamics* 67(2):253–262
 - [28] Ludewig E (2015) On the Effect of Offshore Wind Farms on the Atmosphere and Ocean Dynamics, vol 31. Springer, Cham
 - [29] Moens M, Chatelain P (2022) Correlations Between Wake Phenomena and Fatigue Loads Within Large Wind Farms: A Large-Eddy Simulation Study. *Frontiers in Energy Research* 10:881532. <https://doi.org/10.3389/fenrg.2022.881532>
 - [30] Navas-Montilla A, Murillo J (2018) 2D well-balanced augmented ADER schemes for the Shallow Water Equations with bed elevation and extension to the rotating frame. *Journal of Computational Physics* 372:316–348
 - [31] Orlandi P, Leonardi S (2006) DNS of turbulent channel flows with two- and three-dimensional roughness. *Journal of Turbulence* 7:N73
 - [32] Paskyabi M (2015) Offshore Wind Farm Wake Effect on Stratification and Coastal Upwelling. *Energy Procedia* 80:131–140. 12th Deep Sea Offshore Wind R&D Conference, EERA DeepWind 2015
 - [33] Paskyabi M, Fer I (2012) Upper Ocean Response to Large Wind Farm Effect in the Presence of Surface Gravity Waves. *Energy Procedia* 24:245–254
 - [34] Pryor S, et al (2013) Benchmarking of Wind Farm Scale Wake Models in the EERA - DTOC Project. In: ICOWES2013 Conference, Lyngby, Denmark
 - [35] Raghukumar K, et al (2023) Projected cross-shore changes in upwelling induced by offshore wind farm development along the California coast. *Communications Earth & Environment* 4(1):116
 - [36] Renner A (2025) Current knowledge and key gaps in understanding of offshore wind farm impacts on the physical marine environment. *ICES Journal of Marine Science* 82(12):fsaf235
 - [37] Santoni C, et al (2015) Development of a high fidelity CFD code for wind farm control. In: 2015 American Control Conference (ACC), pp 1715–1720
 - [38] Santoni C, et al (2020) One-way mesoscale-microscale coupling for simulating a wind farm in North Texas: Assessment against SCADA and LiDAR data. *Wind Energy* 23(3):691–710
 - [39] Segtnan O, Christakos K (2015) Effect of Offshore Wind farm Design on the Vertical Motion of the Ocean. *Energy Procedia* 80:213–222. 12th Deep Sea Offshore Wind R&D Conference, EERA DeepWind 2015
 - [40] Shchepetkin A, McWilliams J (2005) The regional oceanic modeling system (ROMS): a split-explicit, free-surface, topography-following-coordinate oceanic model. *Ocean Modelling* 9(4):347–404
 - [41] Sikirić M, Janeković I, Kuzmić M (2009) A new approach to bathymetry smoothing in sigma-coordinate ocean models. *Ocean Modelling* 29(2):128–136
 - [42] Smagorinsky J (1963) General circulation experiments with the primitive equations.

- Monthly Weather Review 91(3):99–164
- [43] Smith S (1980) Wind Stress and Heat Flux over the Ocean in Gale Force Winds. *Journal of Physical Oceanography* 10(5):709–726
 - [44] The Wind Power (2025) Vestas V80/2000. https://www.thewindpower.net/turbine_en_30_vestas_v80-2000.php, accessed: 31 August 2026
 - [45] Toro E (2001) *Shock-Capturing Methods for Free-Surface Shallow Flows*. Wiley, Chichester
 - [46] Vallis G (2006) *Atmospheric and Oceanic Fluid Dynamics*. Cambridge University Press, Cambridge, U.K.
 - [47] Visbeck J, et al (2024) Aerodynamic effects of leading-edge erosion in wind farm flow modeling. *Wind Energy Science* 9(9):1811–1826
 - [48] Volker P, et al (2015) The Explicit Wake Parametrisation V1.0: a wind farm parametrisation in the mesoscale model WRF. *Geoscientific Model Development* 8(11):3715–3731
 - [49] WindEurope (2025) Latest wind energy data for Europe - Autumn 2025. Tech. rep., WindEurope, URL <https://windeurope.org/data/products/latest-wind-energy-data-for-europe-autumn-2025/>
 - [50] Witha B, et al (2019) WRF model sensitivity studies and specifications for the NEWA mesoscale wind atlas production runs. <https://doi.org/10.5281/zenodo.2682604>
 - [51] Wu Y, Porté-Agel F (2015) Modeling turbine wakes and power losses within a wind farm using LES: An application to the Horns Rev offshore wind farm. *Renewable Energy* 75:945–955
 - [52] Zavala Sansón L, Van Heijst G (2002) Ekman effects in a rotating flow over bottom topography. *Journal of Fluid Mechanics* 471:239–255
 - [53] Zhou J, et al (2001) The Surface Gradient Method for the Treatment of Source Terms in the Shallow-Water Equations. *Journal of Computational Physics* 168(1):1–25
 - [54] Ziveri C (2008) 2D finite volume numerical modeling of free surface flows with topography. PhD thesis, Università degli Studi di Parma
 - [55] Zwas G (1972) On Two Step Lax-Wendroff Methods in Several Dimensions. *Numerische Mathematik* 20:350–355