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A Study on the Mechanical Properties of Spherical Contacts in Low-Temperature Environments

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Abstract

With the rapid development of polar engineering and cryogenic equipment technology, the contact mechanical behavior of mechanical interfaces in cryogenic environments has gradually become a key factor determining system reliability. Traditional Kogut-Etsion (KE) contact models are largely limited to assumptions at room temperature, and their empirical formulas exhibit non-physical sudden drops in stiffness at the boundaries between segments, making it difficult to accurately describe the actual contact characteristics in cryogenic environments. To address this issue, this paper conducts a fully parameterized study of the contact mechanics of 316L austenitic stainless steel spheres in a low-temperature environment ranging from 200 K to 300 K. First, by introducing functions for the elastic modulus and yield strength that dynamically evolve with temperature, an improved KE contact mechanics theoretical framework incorporating temperature gradients was established; to address the defect of a sudden drop in contact stiffness at the transition point between the elastic and plastic evolution stages ($\omega^* = 6$) in the original model, a local boundary correction method based on cubic Hermite interpolation polynomials was proposed. The results indicate that low-temperature effects significantly increase the material's elastic modulus and yield strength, driving the overall evolution of the critical yield parameter and contact stiffness of the micro-convex body. In particular, when investigating the coupled effects of size and temperature, it was found that increasing the radius of curvature of the contact surface can effectively amplify the strengthening effect of low-temperature environments on the material's yield strength; as the temperature decreases, the critical contact load curve exhibits a more pronounced nonlinear upward inflection in the low-temperature region, indicating that structures with large radii of curvature can achieve a more significant increase in yield load-bearing capacity under cryogenic conditions. Furthermore, the model modified using the cubic Hermite correction not only achieves strict first-order continuity at the interface between mechanics and rheology, thereby completely eliminating sudden changes in stiffness, but also preserves the classical theoretical values in the high-strain region without loss of accuracy. This study provides a more rigorous theoretical foundation for the analysis of microscopic contact mechanisms, the optimization of structural load-bearing performance, and stiffness prediction in cryogenic service environments.

Keywords: Contact mechanics; Low-temperature environment; Size effect; Cubic Hermite interpolation; Contact stiffness

1. Introduction

With the rapid development of engineering in extremely cold regions, deep-space exploration, and cryogenic equipment technology [1], the contact mechanical behavior of mechanical interfaces in cryogenic environments has gradually become a key factor determining system operational accuracy and service reliability. In the field of contact mechanics, contact between rough surfaces is typically simplified to the contact problem between an equivalent single micro-protrusion (sphere) and a rigid plane. Early contact models, such as the Hertz [2] elastic model and the Abbott-Firestone [3] fully plastic model, could only describe a single deformation state. To cover a broader range of deformation states, Kogut and Etsion [4] proposed the well-known KE contact model based on the finite element method (FEM). This model divides the contact process into three stages: purely elastic, elastoplastic, and fully plastic. However, because the KE model employs empirical formulas with abrupt discontinuities during the elastoplastic evolution stage, it suffers from a non-physical abrupt drop in contact force and contact stiffness at the boundaries between regions, which can easily lead to computational divergence during coupled simulations or stiffness matrix iteration.

To overcome the discontinuity issues in piecewise models, several researchers have conducted in-depth investigations. Zhao [5] et al.'s ZMC model employs logarithmic and quartic polynomial functions to smooth the elastoplastic transition region. The research by Jackson and Green (JG model) [6] further pointed out that a material's hardness or the average contact pressure in the fully plastic region is not constant but highly dependent on the evolution of the contact geometry and material properties (such as the ratio of yield strength to elastic modulus). Subsequently, Shankar and Mayuram (SM model) [7] elaborated in detail on the evolution of the elastic core and plastic region within a single micro-protrusion as a function of the interference amount. Furthermore, nonlinear factors in rough-surface contact—such as residual stresses after unloading, interactions between micro-protrusions, and spherical inclusions within the material—have all been shown to significantly influence contact mechanical behavior [8,9,10].

Although the aforementioned models have yielded fruitful results regarding the mechanisms of contact mechanics, existing classical contact models (including the KE model and the JG model, among others) are largely limited to assumptions at room temperature. Studies have shown that ambient temperature has a significant impact on the mechanical properties of materials [11,12]; for example, in the study of ice tribology, the indentation hardness of materials and the evolution of contact behavior are highly dependent on dynamic changes in temperature [1,13]. For 316L austenitic stainless steel, which is commonly used in engineering, low-temperature environments significantly alter its elastic modulus and yield strength; however, there remains a significant research gap regarding the evolution of the KE model under low-temperature gradients, changes in contact stiffness, and the coupling effects of geometric dimensions.

To address these issues, this paper conducts a fully parameterized study of the contact mechanical properties of 316L austenitic stainless steel spheres in a low-temperature environment ranging from 200 K to 300 K. First, by introducing functions for the elastic modulus and yield strength that dynamically evolve with temperature, an improved KE contact mechanics theoretical framework incorporating temperature gradients was established. Second, to address the shortcoming of the original KE model—a sudden drop in contact stiffness at the boundary between the elastic and plastic evolution stages ($\omega^* = 6$)—this paper proposes a local boundary correction method based on cubic Hermite interpolation polynomials. This method ensures strict continuity of the first-order derivative at the mechanorheological boundary, completely eliminates theoretical discontinuities in the model, and preserves the classical theoretical values in the high-deformation region without loss of accuracy. Finally, this study thoroughly investigates the coupled evolution laws of different spherical contact radii R and temperature gradients. The research reveals that increasing the radius of curvature at the interface can effectively amplify the strengthening effect of low-temperature environments on material yield strength, resulting in a more significant increase in the critical contact yield load under cryogenic conditions. This study provides a more rigorous theoretical foundation for the analysis of microscopic contact mechanisms and the precise prediction of stiffness in low-temperature service environments.

2. Development of a Mechanical Model for Spherical Contact in Low-Temperature Environments

Spherical joint surfaces, as a typical form of point contact in mechanical systems, are widely found in low-temperature valve sealing pairs, low-temperature ball bearings, liquefied natural gas (LNG) transmission equipment, superconducting mechanical bearings, and components operating in extreme high-altitude environments [14]. Their contact deformation characteristics directly determine the contact stiffness, sealing performance, and fatigue life of the entire system under low-temperature operating conditions. Under the coupled effects of variable loads and cryogenic environments, the evolution of elastic-plastic deformation at the contact interface serves as the core foundation for evaluating low-temperature load-bearing capacity, predicting cold-brittle failure, and optimizing contour design. Existing research has largely focused on analytical solutions for ideal smooth surfaces under Hertzian elastic point contact at room temperature; however, the temperature dependence of material elastic modulus and yield strength in cryogenic environments, the evolution of surface topography, and the thermal - force interactions in cryogenic environments still require further refinement.

To this end, this section takes the contact surface between two spheres as the subject of study. Based on Hertz's theory and the fundamental assumptions of elastoplastic contact mechanics, it introduces temperature-corrected relationships for material mechanical property parameters, derives the relationships between the contact load P , contact stiffness K , contact

area A , and the normal interference angle ω at different deformation stages under temperature effects, and establishes an analytical model for contact deformation at the sphere-sphere interface that accounts for low-temperature effects, thereby providing a theoretical basis for the analysis of elastoplastic deformation at the sphere-sphere contact interface in low-temperature environments and for verification through finite element simulation.

2.1 Full Parametric Study Parameters

To comprehensively investigate the coupled effects of low-temperature environments and geometric dimensions on the contact interface between spheres, this paper conducted a fully parameterized study. The core geometric dimensions, environmental temperature gradients, material property parameters, and finite element mesh characteristics involved in the model were systematically varied across a broad range of engineering design conditions; the specific parameter settings are shown in Table 1.

Table 1: Summary of Key Parameters for the Fully Parametrized Study

Parameter Categories	Parameter Name	Values and Ranges
Environment	Temperature Gradient Range T	200 K to 300 K
Geometry	Equivalent Radius of Curvature R	5, 10, 15, 20 mm
Materials	Material Grade	316L Austenitic Stainless Steel
Finite Element Mesh	Minimum grid size	4.47×10^{-3} mm

2.2 A Semi-Analytical Mechanism for Contact Correction Accounting for the Low-Temperature Strengthening Effect

When extending room-temperature contact models to extremely cold environments, simply substituting constants into the final mechanical equations will not accurately reflect the elastoplastic behavior of materials under complex operating conditions. The classical Kogut-Etsion (KE) contact model is typically derived based on material parameters under room-temperature conditions. However, in low-temperature environments or those with large temperature gradients, the mechanical properties of materials undergo significant dynamic evolution. To address this, the “semi-analytical contact correction method” proposed in this abstract is not merely a simple numerical scaling of existing macroscopic equations. Instead, it starts from the constitutive relations of continuum mechanics, maps thermodynamic mechanisms to microscopic dislocation resistance, and thereby physically modifies the initial yield boundary of the contact domain.

2.2.1 Extensions and Applicability of the Temperature-Dependent von Mises Yield Criterion

For the 316L austenitic stainless steel studied in this paper, due to its highly stable face-centered cubic (FCC) crystal structure, the material does not exceed the lower critical toughness-brittleness transition temperature [15] (DBTT; typically, the DBTT for austenitic stainless steels is well below 77 K or even lower) within the temperature range of 200 K to 300 K, and neither martensitic transformation nor significant cleavage fracture occurs in this temperature range [16]. Under these physical conditions, the dominant mechanism of plastic deformation remains dislocation slip within the crystal lattice. Therefore, the von Mises equivalent stress assumption within the macroscopic continuum mechanics framework remains strictly valid. However, the original room-temperature model assumes that the yield surface is static in stress space. In extremely cold environments, the sharp decrease in thermal activation energy leads to a significant increase in the resistance of the crystal lattice to dislocation motion [17], necessitating the introduction of a thermomechanically coupled constitutive relationship

This paper retains the stress-displacement framework of the von Mises yield criterion but introduces a temperature scalar field T to derive a dynamic, extended yield surface equation that accounts for low-temperature effects:

$$f(\sigma_{ij}, T) = \sqrt{\frac{3}{2} S_{ij} S_{ij}} - \sigma_s(T) \leq 0 \quad (1)$$

In this equation, S_{ij} represents the deviatoric stress tensor; $\sigma_s(T)$ represents the yield stress limit that evolves dynamically with temperature. This extended equation explicitly addresses the core mechanism of low-temperature contact: temperature does not alter the shear-dominated yield behavior under a multiaxial stress state (the left-hand side of the equation contains no temperature term), but rather drives an isotropic, nonlinear expansion of the entire yield surface in stress space by dynamically amplifying the flow resistance ($\sigma_s(T)$ on the right-hand side of the equation).

2.2.2 Critical Displacement Evolution Driven by “Asynchronous Reinforcement”

To solve for the yield boundary of the aforementioned expansion, the evolution functions of the constitutive parameters within this temperature range were derived. The elastic modulus, as a macroscopic parameter characterizing the interatomic bonding forces in a material, increases as the temperature decreases; the physical mechanism behind this phenomenon stems from the reduction in the vibrational energy of the microscopic crystal lattice. This study first adopts the semi-empirical formula proposed by Varshni based on the Einstein oscillator model [18]:

$$E(T) = E_0 - \frac{S}{e^{\frac{T}{\theta}} - 1} \quad (2)$$

Given that this study focuses on the temperature range of 200 K to 300 K, changes in the material's physical properties have not yet exhibited significant nonlinear discontinuities within

this relatively narrow temperature range. To balance the complexity of analytical derivation with the efficiency of engineering calculations, a first-order Taylor expansion of the Varshni exponential model was performed near 293 K, and regression analysis was conducted using relevant experimental data points to extract the temperature sensitivity coefficient of the elastic modulus in this temperature range, which is approximately $4.0 \times 10^{-4} \text{ K}^{-1}$. The fitting results show that within this range, the correlation coefficient R^2 between the linear approximation model and the complex exponential model is 0.981, with a minimal maximum relative error, fully meeting engineering accuracy requirements. Therefore, this study adopts the following simplified linear approximation formula:

$$E(T) = E_{293}[1 + 4.0 \times 10^{-4} \text{ K}^{-1}(293\text{K} - T)] \quad (3)$$

In this equation, E_{293} is the elastic modulus of the material at room temperature (293 K).

Compared to the modulus of elasticity, the yield strength exhibits a more pronounced temperature sensitivity. Based on an analysis of tensile test data for 316L austenitic stainless steel from the NIST [19] database, the yield point of metallic materials increases significantly at low temperatures due to a sharp rise in the resistance to dislocation motion; within the 200 K to 300 K range defined in this paper, its evolution follows an approximately linear trend. By performing a linear fit on the NIST experimental data within this temperature range (correlation coefficient $R^2 = 0.962$), the temperature coefficient was determined to be $2.0 \times 10^{-3} \text{ K}^{-1}$. Based on this, the following corrected model was established:

$$\sigma_s(T) = \sigma_{s293}[1 + 2.0 \times 10^{-3} \text{ K}^{-1}(293\text{K} - T)] \quad (4)$$

In this equation, σ_{s293} represents the yield strength of the material at room temperature (293 K).

The literature indicates that the Poisson's ratio varies relatively little with temperature. Laplanche [20] et al. reported that the Poisson's ratio of CoCrFeMnNi alloys remained at $\nu = 0.25 \pm 0.01$ over the temperature range of 200–1000 K, with a variation of less than 4%. Materials such as 316L austenitic stainless steel also exhibit similar characteristics; therefore, this study treats the Poisson's ratio as a constant.

2.3 Derivation of Elasto-Plastic Correction Formulas for the Low-Temperature Spherical Contact Model

2.3.1 Equivalent Geometry and Temperature-Dependent Material Properties

Let the radii of two 316L austenitic stainless steel spheres be R_1 and R_2 , respectively. The equivalent radius of curvature at the point of contact between the two spheres is determined jointly by their geometric dimensions:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R = \frac{R_1 R_2}{R_1 + R_2} \quad (5)$$

For the sake of analysis, this paper examines the contact between two identical spheres. Since the two spheres have equal radii, i.e., $R_1 = R_2$, the equivalent radius is $R = R_1/2$;

Since both contacting bodies are made of the same type of 316L austenitic stainless steel, at any given low temperature T , their elastic moduli are $E_1(T) = E_2(T) = E(T)$, and their

Poisson's ratios are $\nu_1 = \nu_2 = \nu$. Substituting these values into the formula for the equivalent elastic modulus yields an equivalent modulus that depends solely on a single material:

$$E^*(T) = \frac{E(T)}{2(1 - \nu^2)} \quad (6)$$

2.3.2 Initial Yield Threshold

When the normal deformation between two spheres in contact reaches a critical value, the maximum equivalent stress at the contact sub-surface reaches the yield strength $\sigma_s(T)$ of 316L austenitic stainless steel at the current temperature. At this point, the contact state transitions from purely elastic to elastoplastic. Substituting the simplified $E^*(T)$ into the classical KE critical parameter formula [4], the critical interference amount (normal deformation) $\omega_c(T)$ can be expressed as:

$$\omega_c(T) = R \left(\frac{\pi C (1 - \nu^2) \sigma_s(T)}{E(T)} \right)^2 \quad (7)$$

Where C is a function of the Poisson's ratio; for 316L austenitic stainless steel ($\nu = 0.3$), based on the von Mises yield criterion, $C = 1.295 e^{(0.736\nu)} \approx 1.614$.

The critical contact load $P_c(T)$ can be expressed as:

$$P_c(T) = \frac{2\pi^3 C^3 (1 - \nu^2)^2}{3} R^2 \frac{[\sigma_s(T)]^3}{[E(T)]^2} \quad (8)$$

The critical contact area $A_c(T)$ can be expressed as:

$$A_c(T) = \pi R \omega_c(T) = \pi R^2 \left(\frac{\pi C (1 - \nu^2) \sigma_s(T)}{E(T)} \right)^2 \quad (9)$$

As can be seen from Equation 8, the critical load P_c is directly proportional to the temperature-dependent material property ratio. As the ambient temperature decreases, the increase in the yield strength of 316L austenitic stainless steel is typically greater than the increase in its modulus of elasticity, which means that at low temperatures, the material has a harder time entering the plastic yield stage.

2.3.3 Segmental Contact Mechanics Equations

A dimensionless deformation parameter ω^* that varies with temperature is introduced as the core indicator for determining the contact state: $\omega^* = \frac{\omega}{\omega_c(T)}$. Based on the classical KE elastoplastic contact model and the dimensionless deformation parameter ω^* that accounts for temperature effects, the mechanical evolution of the contact between two spheres at low temperatures can be strictly divided into four stages. The evolution of the normal contact load P and the true contact area A as a function of the deformation ω , as well as the contact stiffness $K = \frac{dP}{d\omega}$, for each stage are as follows:

1) Purely Elastic Stage ($0 < \omega^* \leq 1$):

At this stage, the maximum von Mises stress inside the micro-protrusion has not yet reached the material's yield strength, and fully reversible elastic deformation occurs in the

contact region. The contact load and area follow the classical Hertz contact theory and are governed by the temperature-modulated elastic modulus:

$$P_e(T, \omega) = \frac{2}{3(1-\nu^2)} E(T) R^{\frac{1}{2}} \omega^{\frac{3}{2}} \quad (10)$$

$$A_e(\omega) = \pi R \omega \quad (11)$$

$$K_e(T, \omega) = \frac{E(T)}{1-\nu^2} R^{\frac{1}{2}} \omega^{\frac{1}{2}} \quad (12)$$

2) First Stage of Elasto-Plastic Contact ($1 < \omega^* \leq 6$):

When the dimensionless interference quantity exceeds the critical point ($\omega^* > 1$), the micro-convex body begins to yield, and a plastic nucleus forms beneath the contact surface, surrounded by an elastic region. Based on the finite element fitting rules of the KE model, the load and area formulas, taking temperature effects into account, are derived as follows:

$$P_{ep1}(T, \omega) = P_c(T) \times 1.03 \left(\frac{\omega}{\omega_c(T)} \right)^{1.425} \quad (13)$$

$$A_{ep1}(T, \omega) = A_c(T) \times 0.93 \left(\frac{\omega}{\omega_c(T)} \right)^{1.136} \quad (14)$$

$$K_{ep1}(T, \omega) = 1.03 \times 1.425 \frac{P_c(T)}{\omega_c(T)} \left(\frac{\omega}{\omega_c(T)} \right)^{0.425} \quad (15)$$

3) Second Stage of Elasto-Plastic Contact ($6 < \omega^* \leq 110$):

As the extrusion intensifies further, the internal plastic core expands to the surface of the micro-bulge, and the contact zone gradually becomes dominated by plastic flow, although some elastic recovery force still remains. At this point, the semi-empirical evolution equation is truncated at the boundary point $\omega^* = 110$:

$$P_{ep2}(T, \omega) = P_c(T) \times 1.40 \left(\frac{\omega}{\omega_c(T)} \right)^{1.263} \quad (16)$$

$$A_{ep2}(T, \omega) = A_c(T) \times 0.94 \left(\frac{\omega}{\omega_c(T)} \right)^{1.146} \quad (17)$$

$$K_{ep2}(T, \omega) = 1.40 \times 1.263 \frac{P_c(T)}{\omega_c(T)} \left(\frac{\omega}{\omega_c(T)} \right)^{0.263} \quad (18)$$

4) Fully Plastic Contact Stage ($\omega^* > 110$)

When the deformation exceeds the critical value of 110 times, the contact region enters a fully plastic yield state. According to the Abbott-Firestone truncation model, the average contact pressure at this point is always equal to the material hardness (typically approximated as $H \approx 2.8\sigma_s$). The contact area and the load exhibit a significant linear relationship:

$$P_p(T, \omega) = 5.6\pi R \sigma_s(T) \omega \quad (19)$$

$$A_p(\omega) = 2\pi R \omega \quad (20)$$

$$K_p(T, \omega) = 5.6\pi R \sigma_s(T) \quad (21)$$

3. Analysis of the Mechanical Properties of Spherical Contact in Low-Temperature Environments and a Corrected Model

3.1 The Effect of Low-Temperature Effects on the Mechanical Properties of Spherical Contacts

3.1.1 Evolution of Contact Load as a Function of Interference at Different Low Temperatures

Figure 1 shows the contact load versus normal interference ($P-\omega$) curves from the elastic stage to the maximum elastoplastic stage for five low-temperature conditions ranging from 213 K to 293 K. As the normal interference ω increases, the contact load P rises nonlinearly and monotonically; in the range of small interference amounts, the curves at different temperatures almost coincide, indicating that contact behavior in the elastic stage is primarily governed by the elastic modulus E ; since E varies little with temperature, the temperature effect is not yet significant; as the interference amount increases and the system enters the elastoplastic stage, the curves gradually diverge; the lower the temperature, the greater the contact load corresponding to the same interference amount, and the spacing between the curves continues to widen as ω increases. The underlying physical mechanism is that, in the elastoplastic stage, yield has already occurred in the central contact region, and the load-deformation relationship is now dominated by the yield strength σ_s . At low temperatures, the increase in σ_s leads to greater resistance to plastic deformation; consequently, a larger external load must be applied for the same interference amount. Furthermore, the continuous expansion of the plastic zone with increasing interference amount further amplifies the temperature effect. Furthermore, it can be observed that the lower the temperature, the longer the elastoplastic contact segment becomes, and the greater the critical load P_c required to enter the plastic region.

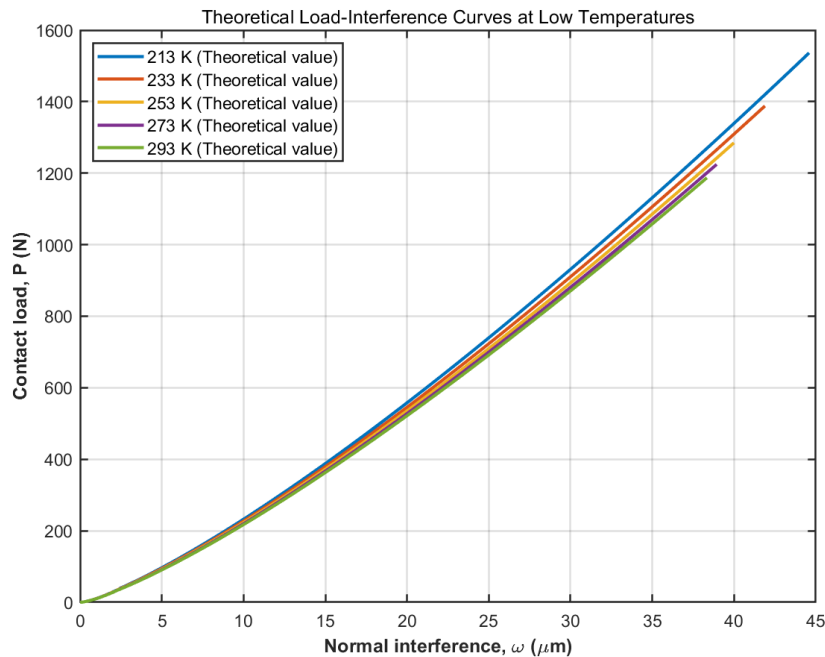


Figure 1: Curves showing the relationship between theoretical load and normal interference at different low temperatures

3.1.2 Temperature Response of the Critical Contact Area A_c

Figure 2 shows the evolution of the critical contact area A_c over the temperature range of 200 K to 300 K. The critical contact area is the contact area at which the contacting bodies first yield; it directly characterizes the maximum contact scale that can be maintained during the elastic contact phase. As shown in the figure, A_c exhibits a monotonically increasing trend as the temperature decreases, and the curve displays a downward-convex profile—that is, the rate of decrease is relatively rapid at the low-temperature end (200–233 K), while it tends to flatten at the room-temperature end (273–293 K). Specifically, A_c is approximately $5.53 \times 10^{-3} \text{ mm}^2$ at 293 K and rises to $6.36 \times 10^{-3} \text{ mm}^2$ at 213 K, representing an increase of about 15.01% from 293 K to 213 K. The physical mechanism behind this trend lies in the fact that at low temperatures, the material's yield strength σ_s increases, causing the critical load P_c to rise; the contacting bodies must withstand a greater external load before initial yielding occurs. At this point, the contact spot has already expanded to a larger size, so A_c increases accordingly; the increased slope of the curve at the low-temperature end reflects the amplifying effect on A_c caused by the nonlinear increase in σ_s as temperature decreases. The increase in A_c implies that the elastic contact phase at low temperatures can be maintained over a larger contact area—that is, the elastoplastic segment is longer and the transition to plasticity occurs later—which is consistent with the delayed inflection point at which the low-temperature curve deviates from the elastic solution in the aforementioned P – ω curve.

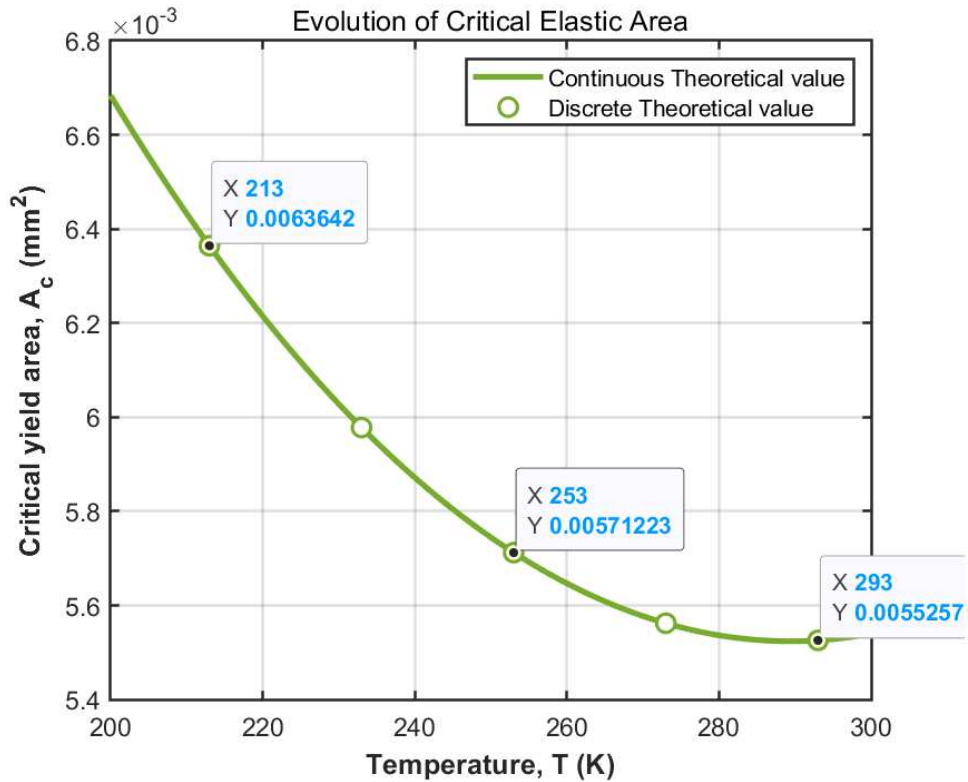


Figure 2: Critical contact area (A_c) curve under a temperature gradient

3.1.3 Temperature Response of the Maximum Contact Stiffness ($K_{\max}$) in the Elasto-Plastic Stage

Figure 3 shows the evolution of the maximum elastoplastic stiffness $K_{\max}$ for spherical contacts in the temperature range of 200 K to 300 K prior to entering the fully plastic stage. The maximum elastoplastic stiffness is defined as the slope $dP/d\omega$ of the P - ω curve at the point of maximum elastoplasticity, which characterizes the ability of the contacting bodies to resist deformation at the end of the elastoplastic stage. As shown in the figure, $K_{\max}$ exhibits a monotonically increasing trend as temperature decreases, with a steeper rate of decline at lower temperatures and a flatter trend near room temperature. Specifically, it is approximately 39.28 MN/m at 293 K and rises to 43.50 MN/m at 213 K, representing an increase of about 10.74% from 293 K to 213 K. The physical mechanism behind this trend lies in the fact that at low temperatures, the material's yield strength σ_s increases. On the one hand, this increases the critical load P_c and extends the elastic region, causing the contact body to enter the plastic phase only after undergoing greater deformation; on the other hand, the increased resistance to plastic deformation results in a steeper load-deformation slope during the elastoplastic phase, thereby correspondingly increasing $K_{\max}$. The increased slope of the curve at the low-temperature end reflects the amplification effect on stiffness caused by the nonlinear increase in σ_s as temperature decreases. The increase in $K_{\max}$ indicates that, at low temperatures, the contact body can maintain a higher resistance to deformation during the elastoplastic stage, which is consistent with the previously mentioned patterns of increased critical load P_c , increased critical elastic area A_c , and the upward shift of the P - ω curve at low temperatures.

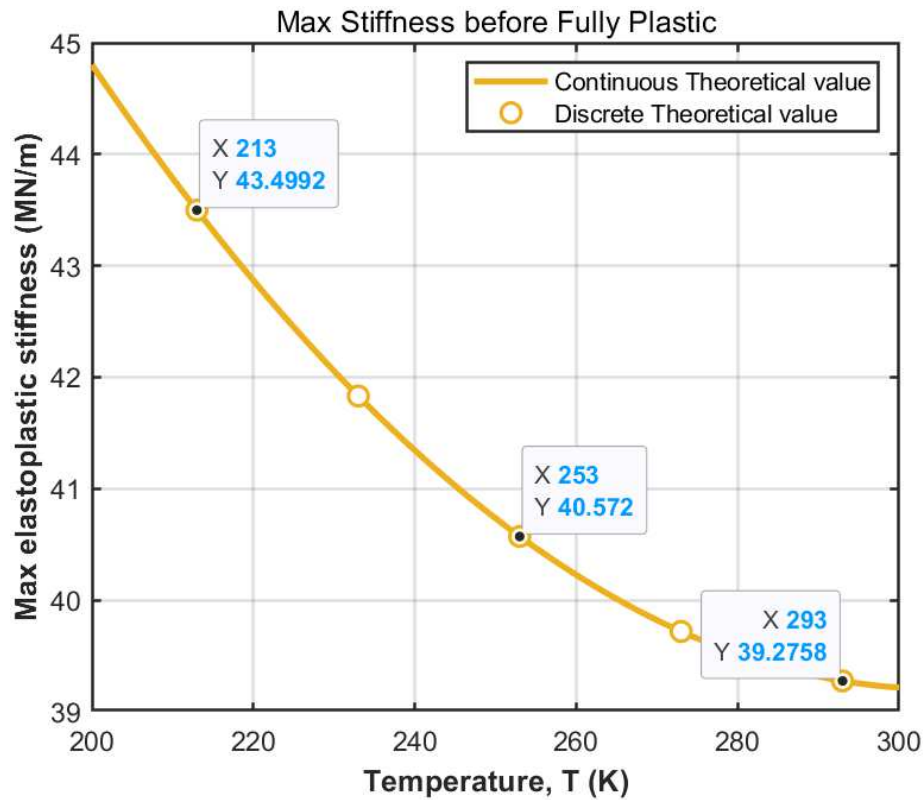


Figure 3: Evolution of the maximum contact stiffness ($K_{\max}$) prior to the fully plastic stage

under a temperature gradient

3.2 The Effects of Geometric Dimensions and Temperature Gradients on Spherical Contact and Finite Element Validation

3.2.1 The Coupled Effect of Temperature and Radius on the Critical Contact Load

The critical yield load P_c is a key indicator characterizing the elastic load-bearing limit of micro-convex bodies. As shown in Equation 8, the critical load P_c at which initial yield occurs during spherical contact is governed by the coupling of the geometric dimension R and temperature-dependent material parameters. In terms of geometry, P_c is proportional to R^2 , indicating that an increase in the sphere's radius will significantly raise the external load required for initial yielding, suggesting a pronounced size effect in the contact behavior; in terms of temperature, P_c exhibits a cubic dependence on the yield strength $\sigma_s(T)$ and a negative quadratic dependence on the elastic modulus $E(T)$, with the temperature sensitivity of $\sigma_s(T)$ being dominant; an increase in the material's yield strength at low temperatures leads to a nonlinear increase in P_c . Four sets of spheres with different radii ($R = 5, 10, 15$, and 20 mm) were selected for theoretical curve evolution analysis within a discrete temperature range of 200 K to 300 K.

Figure 4 shows the theoretical curves and discrete calculation values of the critical load P_c for the four sphere radii $R = 5, 10, 15$, and 20 mm in the temperature range of 200 K to 300 K. It can be observed that an increase in the contact radius of curvature not only significantly raises the absolute value of the load but also causes the slope of the P_c – T curve to become markedly steeper in the cryogenic region; all four curves exhibit a monotonically increasing trend as the temperature decreases, reflecting the fundamental principle that material yield strength increases at low temperatures, requiring a greater external load to achieve initial yield. At the low-temperature end ($T = 213$ K), the differences in P_c among the different radii are extremely pronounced: for $R = 5$ mm, it is only 4.99 N, rising to 19.95 N at $R = 10$ mm, reaching 44.90 N at $R = 15$ mm, and further increasing to 79.82 N at $R = 20$ mm. Taking $R = 5$ mm as the reference, the P_c values for radii that are 2, 3, and 4 times this value are approximately 4, 9, and 16 times, respectively, strictly adhering to the quadratic relationship $P_c \propto R^2$, thereby directly verifying the scaling law of the geometric terms in the analytical formula; More crucially, low temperatures amplify the absolute disparity of the size effect. When the temperature drops from 300 K to 200 K, the absolute increase in P_c for a sphere with $R = 20$ mm can reach more than ten times that of a sphere with $R = 5$ mm. This stems from the fact that R^2 in the P_c expression is related to the temperature-dependent material parameter term $[\sigma_s(T)]^3 / [E(T)]^2$ in the P_c expression. At low temperatures, changes in material parameters are amplified by the geometric term R^2 , making the critical load of larger spheres more sensitive to temperature variations. The good agreement between the discrete theoretical values and the continuous theoretical curve indicates consistency in the computational results. This phenomenon suggests that structures with large radii of curvature can more sensitively capture and amplify the strengthening effect

of low temperatures on material yield strength.

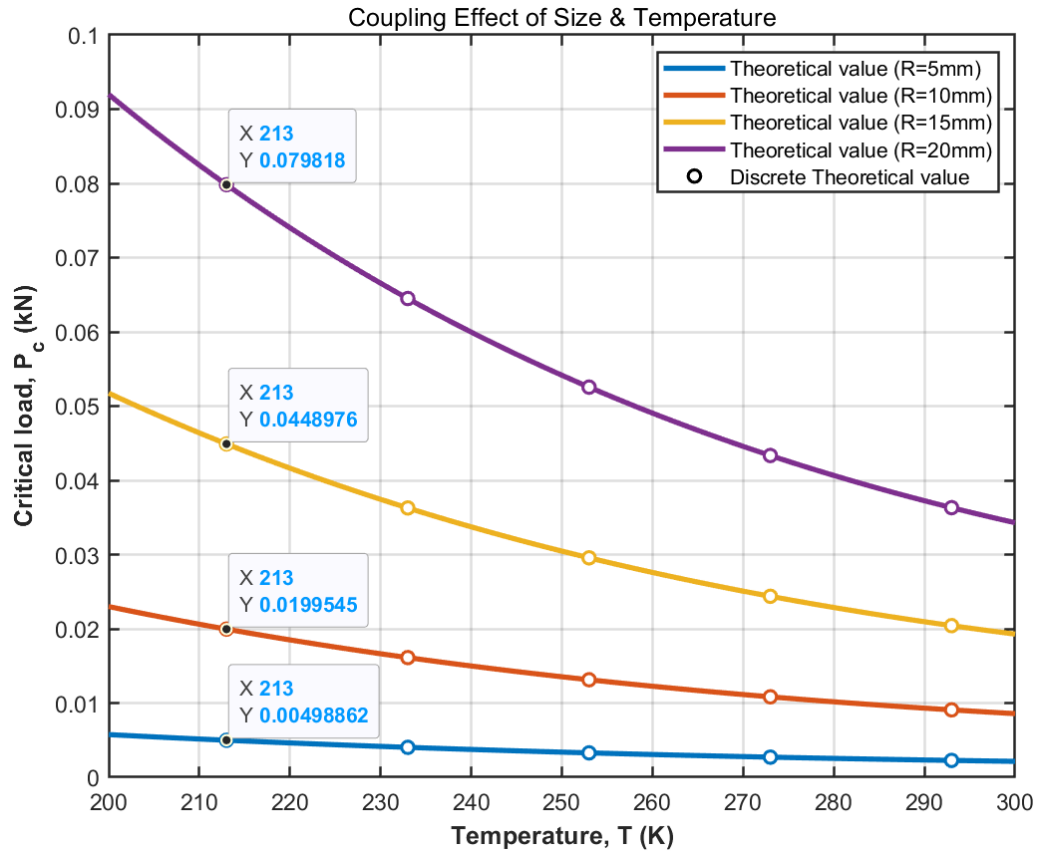


Figure 4: Curves showing the variation of the critical contact load P_c with temperature T for different equivalent sphere radii

3.2.2 ABAQUS Finite Element Validation

To investigate the effects of temperature and geometry on the mechanical properties of deformable spheres under contact, this paper simplifies the calculations and utilizes contact symmetry to equate the “sphere-sphere” contact model to a “sphere + rigid plane” contact model in finite element simulations. Subsequently, a contact model between a deformable sphere and a rigid plane is established using the ABAQUS finite element method [21]; the contact configuration is shown in Figure 5.

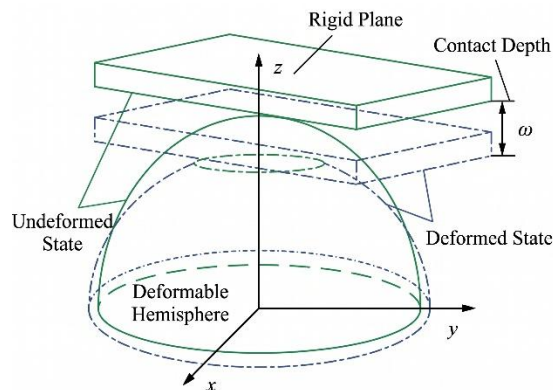


Figure 5: Diagram of the contact between a deformable sphere and a plane

Given that the geometric configuration of the spherical contact, the load conditions, and the material properties are all rotationally symmetric about the contact center axis, and that the stress and strain fields within the contact zone do not vary in the circumferential direction, the three-dimensional spherical contact problem can be reduced to a two-dimensional axisymmetric model for analysis by utilizing this axisymmetry. In this study, the spherical model is made of 316L austenitic stainless steel with the following material properties: density $\rho = 7850 \text{ kg/m}^3$, Young's modulus $E = 2.1 \times 10^5 \text{ MPa}$, Poisson's ratio $\nu = 0.3$, and yield strength $\sigma_s = 380 \text{ MPa}$.

As shown in Figure 6, the model employs a face-to-face contact configuration, with the mesh around the contact interface refined to a minimum mesh size of $4.47 \times 10^{-3} \text{ mm}$. As shown in Figure 7, the boundary conditions are set such that the nodes on the hemisphere's axis of symmetry cannot move radially, and the nodes at the bottom of the sphere are fixed in all directions. Elasto-plastic contact simulation analyses were performed under various temperature and geometric conditions.

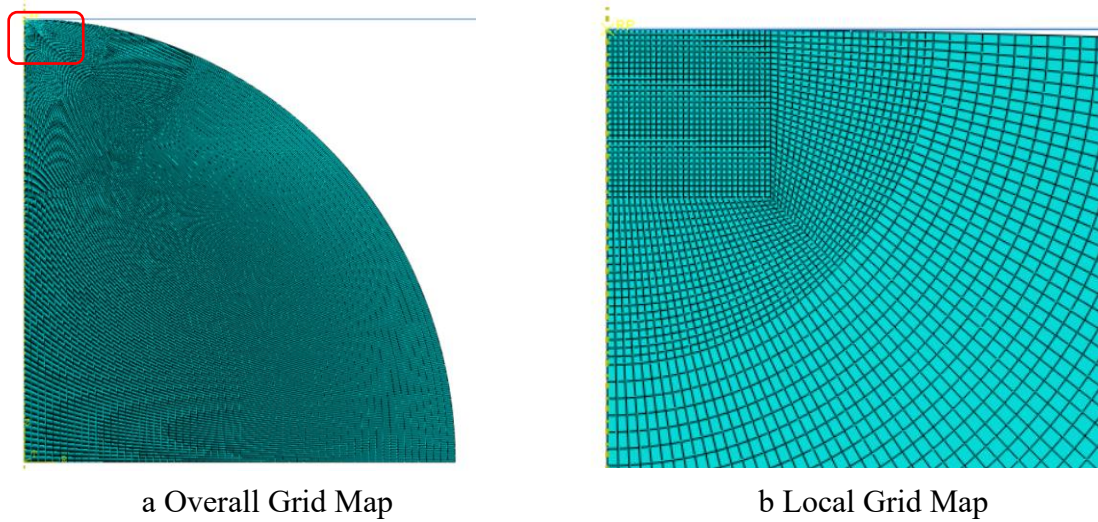


Figure 6: Finite Element Mesh Diagram

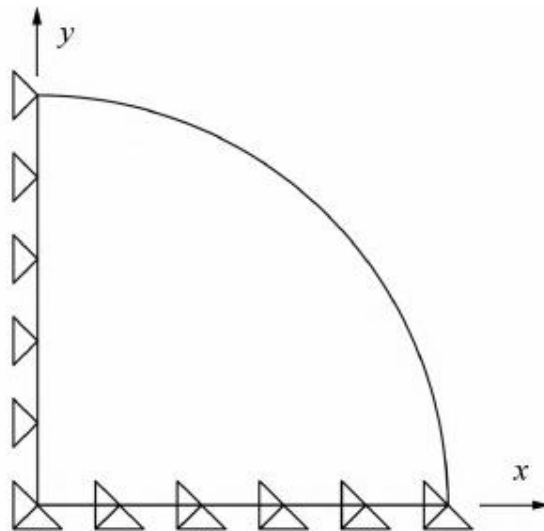


Figure 7: Schematic of the model and boundary conditions used in the FEM

To verify and compare results, different sphere radii R were set. Figure 8 shows the finite element simulation results for sphere-to-sphere contact at a low temperature of 213 K using four equivalent radii of curvature ($R = 5, 10, 15$, and 20 mm). The equivalent plastic strain PEEQ was used as the initial yield criterion to determine the critical load corresponding to the first occurrence of plastic deformation at the contact center. All four subfigures are PEEQ contour plots of a two-dimensional axisymmetric model. It can be seen that under the critical load, a plastic region with $PEEQ > 0$ appears directly beneath the contact area, expanding inward from the contact surface in an approximately hemispherical shape, indicating that initial yielding has occurred at the contact center. As the sphere radius increases, both the load and displacement corresponding to yield increase significantly: for $R = 5$ mm, the critical load is approximately 4.68 N and the normal interference is approximately $0.59 \mu\text{m}$; for $R = 20$ mm, the critical load increases to approximately 79.63 N and the interference to approximately $2.43 \mu\text{m}$. Furthermore, when $R = 20$ mm enters the elastoplastic stage, the PEEQ (4.49×10^{-4}) is significantly greater than that of the other three radii (approximately 2.8×10^{-4}), reflecting more thorough plastic deformation of the larger contact body at the moment of yield. The data windows for the load-displacement curves in each subfigure further verify the location of the yield point—the curves exhibit nonlinear growth in the elastic region and a decrease in slope upon entering the plastic region, which corresponds one-to-one with the behavior shown in the PEEQ contour plot.

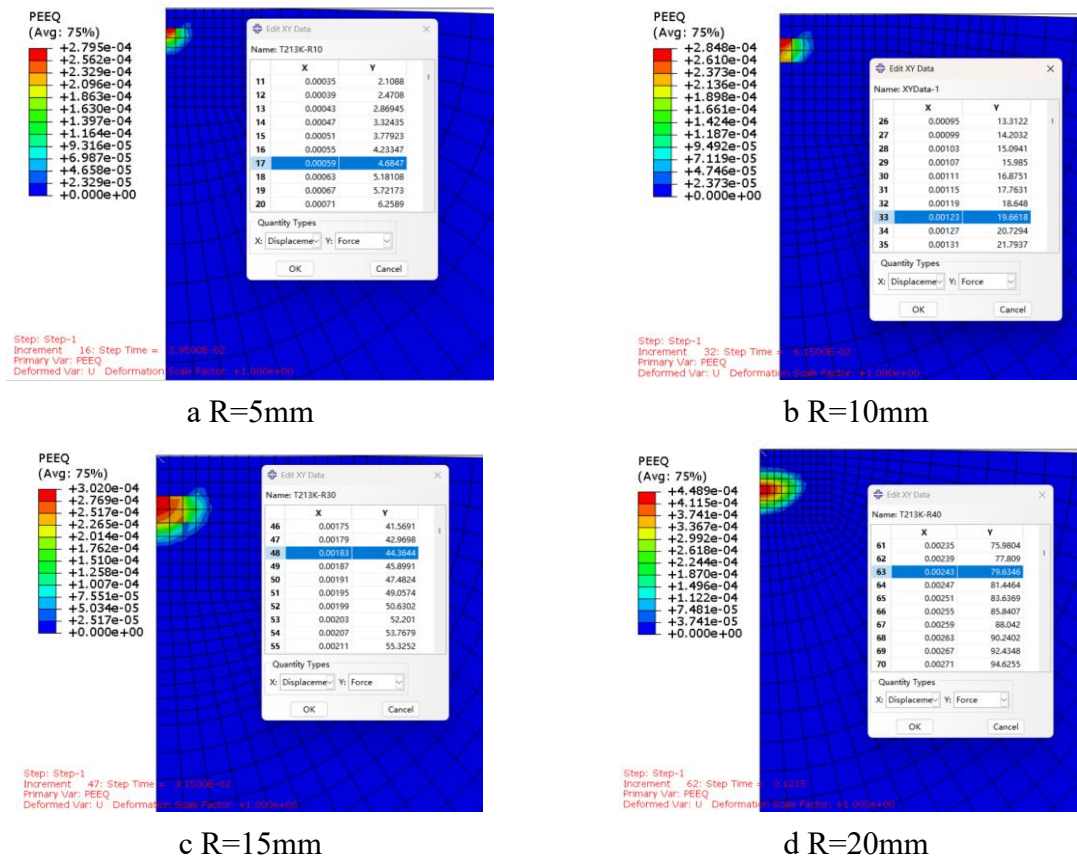


Figure 8: Finite Element Verification of the Critical Contact Load P_c for Different Equivalent Radii at 213K

Table 2 presents a comparison of finite element simulation results and theoretical analytical values for critical loads under four equivalent radii of curvature ($R = 5, 10, 15$, and 20 mm). Overall, the simulation results agree well with the theoretical values, with relative errors ranging from 0.24% to 6.21%, verifying the correctness and reliability of the established theoretical analytical model and the finite element modeling method. In terms of error distribution, the simulated values are slightly lower than the theoretical values under all operating conditions, indicating that the finite element results are conservative. This is related to factors such as the penalty function stiffness of the contact algorithm and the mesh discretization accuracy in the numerical simulation. More importantly, the error shows a monotonically decreasing trend as the sphere radius increases: the error is greatest at $R = 5$ mm (6.21%), while at $R = 20$ mm, it is only 0.24%, indicating that the finite element calculations for larger-sized contact models are more accurate; the relatively large errors under small-radius conditions are primarily due to the contact patch size shrinking as R decreases, resulting in an insufficient number of elements within the contact zone for a given mesh size, which reduces the computational accuracy in stress concentration regions. Furthermore, the simulated values also approximately follow the scaling law $P_c \propto R^2$, further confirming the quadratic effect of geometric dimensions on the critical load. In summary, the finite element results successfully reproduce the theoretical predictions across the entire size range, verifying the reliability of both the theoretical analytical model and the numerical method.

Table 2: Comparison of Simulated and Theoretical Contact Loads for Different Equivalent Radii of Curvature

Equivalent Curvature Radius R (mm)	Simulation Values (N)	Theoretical Value (N)	Error
5	4.68	4.99	6.21%
10	19.66	19.95	1.45%
15	44.36	44.90	1.20%
20	79.63	79.82	0.24%

3.3 A Modified Contact Mechanics Model Based on Cubic Hermite Interpolation

3.3.1 Limitations of the KE Contact Stiffness Model

As can be seen from Equations (12), (15), and (18), the KE model uses different stiffness expressions for the elastic and elastoplastic stages, respectively, resulting in a discontinuity in the K – ω relationship during the transition phase of contact deformation [22]. Figure 9 shows the evolution curves of the contact stiffness K of the original KE model as a function of the normal interference ω under three low-temperature conditions: 293 K, 253 K, and 213 K. As shown in the figure, all three curves exhibit a similar three-segment pattern: in the small-

deformation stage, K increases sharply with ω , reaching a local peak (approximately 22 – 23 MN/m) at $\omega \approx 2.5 \mu\text{m}$; Subsequently, the stiffness drops sharply, falling to approximately 18 – 20 MN/m within an extremely short deformation range, forming a distinct discontinuous inflection point. The magnitude of this sudden drop is about 3 MN/m, accounting for 13%–15% of the stiffness value at that point; after passing the transition point, K increases approximately linearly with ω until the maximum elastoplastic deformation is reached. The root cause of this sudden drop lies in the discontinuity of the first-order derivative at the transition point between the two-stage analytical formulas, which causes a step change in the stiffness value. In terms of the effect of temperature, the three curves generally follow the pattern of “lower temperature, higher stiffness”—for the same interference amount, the 213 K curve is always at the top, the 253 K curve is in the middle, and the 293 K curve is the lowest; Taking $\omega = 35 \mu\text{m}$ as an example, K is approximately 41.5 MN/m at 213 K and approximately 37.1 MN/m at 293 K; lower temperatures increase the contact stiffness by about 12%. It is worth noting that regardless of temperature variations, the phenomenon of a sudden drop in stiffness persists, indicating that this defect is an inherent property of the model itself and is independent of temperature conditions.

In implicit finite element solutions, contact stiffness serves as a core component of the Jacobian matrix; its discontinuity leads to singularity in the tangent stiffness matrix and oscillating iterative residuals, which in severe cases can even cause computational divergence. Particularly in parametric analyses involving multiple operating conditions in the low-temperature range, this flaw significantly increases computational cost and reduces the reliability of results. Therefore, to ensure the nonsingularity of the Jacobian matrix and computational convergence in low-temperature finite element co-simulation, this section conducts an in-depth analysis of the localized sudden drop in stiffness in the original model. Based on a comparison of various numerical interpolation algorithms, a stiffness smoothing correction model based on cubic Hermite interpolation is proposed.

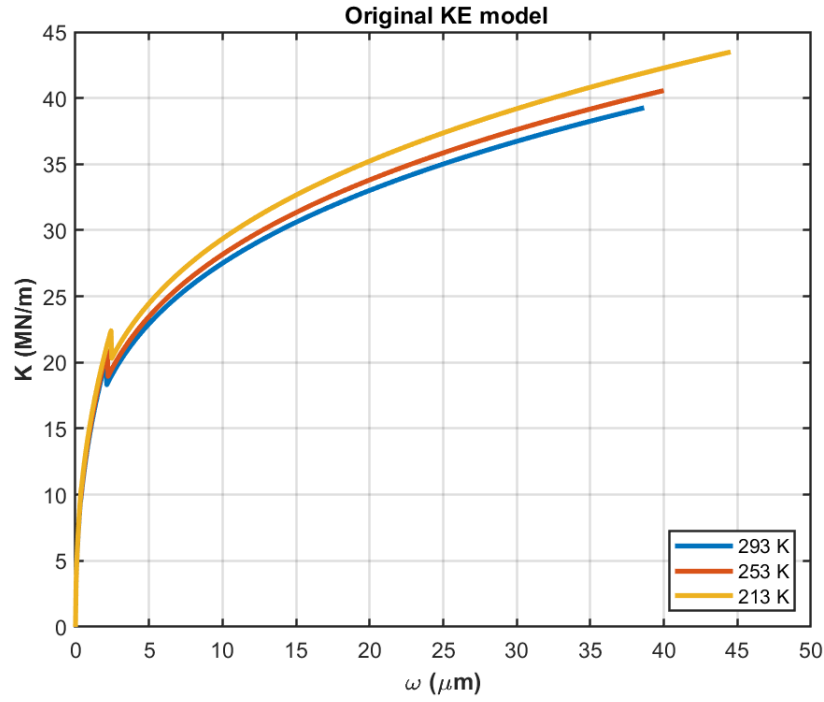


Figure 9: Variation of contact stiffness K based on the KE model as a function of the normal interference angle ω at different temperatures

3.3.2 Third-Order Hermite Interpolation Correction

To eliminate this sudden change in stiffness, one common numerical method is to use cubic spline interpolation. This method constructs a cubic polynomial $S_i(\omega^*) = a_i \omega^* + b_i \omega^{*2} + c_i \omega^{*3} + d_i \omega^{*4}$ within each subinterval and solves it using the continuity conditions for second-order derivatives at adjacent nodes. The advantage of cubic spline interpolation lies in its ability to achieve C^2 -level global smoothness, thereby completely eliminating numerical discontinuities. However, its limitations are equally significant: spline interpolation is essentially a global interpolation algorithm with weak local adjustment capabilities. To forcibly satisfy the global smoothness constraint, the interpolated curve is highly prone to Runge oscillations between nodes, and cannot naturally guarantee that the contact stiffness strictly monotonically increases with the deformation. Furthermore, the slopes estimated by this method at adjacent nodes cannot precisely match the analytical derivatives of the original contact constitutive model, resulting in a disconnect from the inherent phased physical mechanisms of the KE model.

Another correction method commonly used in engineering is progressive exponential decay, or the single power-law method. Assuming the contact load follows the relationship $P = a \omega^b$, the contact stiffness is given by $K = a b \omega^{b-1}$. The advantages of this method are that the expression is extremely concise, the parameters are easy to obtain, and it inherently possesses high-order continuity, facilitating rapid engineering calculations. However, its core limitation lies in the fact that a globally uniform power-law relationship cannot reflect the actual staged evolution of contact. The internal dislocation motion and rheomechanical mechanisms of a sphere differ markedly across the purely elastic, initial yielding, elastoplastic expansion, and

fully plastic stages. Using a single power-law exponent or forcibly adding an exponential decay compensation term makes it difficult to achieve an accurate fit for all deformation stages simultaneously.

To overcome the shortcomings of the two methods mentioned above, the cubic Hermite interpolation method is introduced to construct local transition windows. Five key physical cut-off points from the original KE model are selected as nodes. Through a cubic Hermite monotonic interpolation algorithm, the stiffness curve $K(\omega)$ is ensured to be C^1 continuous and strictly monotonically increasing, thereby eliminating the abrupt changes at the nodes present in the original model. First, the coordinates of the five key nodes (ω^*_i, K_i) are calculated:

1. Node 0 (initial point $\omega^* = 0$):

$$K_0 = 0$$

2. Node 1 (initial yield point $\omega^* = 1$):

$$K_1 = \frac{E(T)}{1 - 0.3^2} \sqrt{5 \times 10^{-3}} \cdot \omega_c = 1.0989 E(T) \sqrt{5 \times 10^{-3}} \omega_c$$

3. Node 2 (Phase 1/Phase 2 transition point $\omega^* = 6$):

Since the original KE model is discontinuous at this point, this paper uses the arithmetic mean of the node values from the preceding and following stages as the transition node value, i.e., $K_6 = (K_6^- + K_6^+)/2$.

$$K_6 = 0.5 \times \frac{p_c}{\omega_c} [1.46775 \times 6^{0.425} + 1.7682 \times 6^{0.263}] \approx 3.0215 \frac{p_c}{\omega_c}$$

This value is not uniquely derived from the original KE theory formula, but rather represents an engineering approximation used to construct a continuous transition. The primary purpose of using the average value is to position the modified node at the center of the minimum envelope interval between the left and right theoretical values—without favoring the constitutive behavior of either side—thereby reducing the impact of unilateral displacement on the Hermite transition shape. To evaluate the sensitivity of this approximation, a weighting parameter λ can be introduced, expressing the node value as $K_6(\lambda) = (1-\lambda) K_6^- + \lambda K_6^+$, where $0 \leq \lambda \leq 1$; in this paper, $\lambda = 0.5$ is taken as the baseline. Since the sudden drop in stiffness at this node in the original KE model is approximately 13%–15% of the node value, the adjustment of the node value relative to the theoretical values on the left and right sides when $\lambda = 0.5$ is approximately half of the sudden change, i.e., about 6.5%–7.5% of the left-side node stiffness scale. Therefore, this treatment does not introduce a node correction magnitude greater than the original jump. Further examination of the three weight values $\lambda = 0.25, 0.50$, and 0.75 shows that the node values are consistently confined within the interval defined by K_6^- and K_6^+ , and the correction effect propagates only through the transition window $[\omega^* = 1, 10]$, while the original KE theoretical baseline for $\omega^* > 10$ remains unchanged. Based on the boundedness requirement of monotonic Hermite interpolation, no additional overshoot will result from the node averaging strategy as long as the node values do not exceed the physical range of adjacent nodes. Therefore, the average value at Node 2 has a certain influence on the local shape of the transition zone but does not alter the load–stiffness baseline outside the transition zone or the

main conclusions throughout the paper regarding temperature, dimensional, and stiffness continuity. This paper explicitly defines this influence as a local engineering approximation to avoid presenting it as a strictly unique theoretical solution. The criteria for the sensitivity analysis of node values are as follows: $\lambda = 0.5$ is used as the baseline, with $\lambda = 0.25$ and $\lambda = 0.75$ serving as left and right offset conditions, respectively; the maximum stiffness deviation, transition zone area, and monotonicity of the curve are compared across these three conditions. The core of the correction strategy in this paper is not to force Node 2 to “equal the analytical value at a certain stage,” but rather to constrain Node 2 between the left and right theoretical endpoints and to use a Hermite local construction for the transition. Therefore, changes in the node weights primarily affect the local curvature near $\omega^* = 6$, without altering the original theoretical baseline in the intervals $\omega^* < 1$ and $\omega^* > 10$. Consequently, the average treatment of Node 2 can be regarded as a low-sensitivity local parameter rather than a key parameter controlling the global results.

4. Node 3 (End of the second transition zone, $\omega^* = 10$):

$$K_{10} = 1.7682 \times \frac{P_c}{\omega_c} \times 10^{0.263} \approx 3.2393 \frac{P_c}{\omega_c}$$

5. Node 4 (elasto-plastic stability cutoff point $\omega^* = 110$):

$$K_{110} = 1.7682 \times \frac{P_c}{\omega_c} \times 110^{0.263} \approx 6.0683 \frac{P_c}{\omega_c}$$

The final mathematical expression is as follows: For any interference quantity falling within the i -th interval $\omega \in [\omega_i, \omega_{i+1}]$, its corrected stiffness $K_{Hermite}(\omega)$ is expressed in terms of cubic Hermite basis functions as:

$$K_{Hermite}(\omega) = K_i \cdot H_{00}(s) + K_{i+1} \cdot H_{10}(s) + h_i \cdot d_i \cdot H_{01}(s) + h_i \cdot d_{i+1} \cdot H_{11}(s) \quad (22)$$

In the formula: step size $h_i = \omega_{i+1} - \omega_i$;

Normalized Local Coordinates $s = \frac{\omega - \omega_i}{h_i} \in [0, 1]$;

d_i and d_{i+1} are the derivatives calculated at the nodes using the third-order Hermite monotonic interpolation algorithm;

$H(s)$ is the standard cubic Hermite basis function:

$$H_{00}(s) = 2s^3 - 3s^2 + 1 \quad (23)$$

$$H_{10}(s) = -2s^3 + 3s^2 \quad (24)$$

$$H_{01}(s) = s^3 - 2s^2 + s \quad (25)$$

$$H_{11}(s) = s^3 - s^2 \quad (26)$$

Table 3 compares the differences in characteristics among cubic spline interpolation, the power-index function method, and cubic Hermite interpolation in the fitting of contact curves. In terms of continuity, cubic spline interpolation achieves global C^2 continuity, the power-index function method maintains only C^0 continuity, and cubic Hermite interpolation satisfies local C^1 continuity; In terms of monotonicity guarantees, cubic spline interpolation is prone to numerical oscillations, the exponential function method exhibits good monotonicity, and the cubic Hermite interpolation method ensures strictly monotonically increasing behavior;

regarding physical node matching, the first two methods perform poorly, whereas the cubic Hermite interpolation method perfectly matches the data at each physical node; In terms of local correction capability, cubic spline interpolation performs averagely; any correction using the power-index function method alters the curve’s global characteristics, whereas the cubic Hermite interpolation method possesses extremely strong local correction capability; regarding phasic representation, the first two methods perform poorly, while the cubic Hermite interpolation method performs strongly. Overall, the cubic Hermite interpolation method outperforms the other two methods in terms of monotonicity assurance, physical node agreement, local correction capability, and step representation, making it more suitable for characterizing the curve characteristics of physical quantities—such as contact stiffness—that vary continuously with temperature and dimensions and require strict monotonicity.

Table 3: Comparison of the Characteristics of Three Interpolation Methods

	Cubic Spline Interpolation	Power-Function Method	Cubic Hermite interpolation
Continuity Level	Global C^2 Continuous	C^0 Continuous	Locally C^1 Continuous
Monotonicity Guarantee	Prone to volatility	Better	Strictly Monotonically Increasing
Physical Node Matching	Weak	Weak	Perfect
Local correction capability	Moderate	Change Global Properties	Extremely Strong
Phased Expression	Weak	Weak	Strong

As shown in Figure 10, the comparison curves of the four contact stiffness models clearly demonstrate the superior performance of the cubic Hermite interpolation method (purple solid line) in terms of numerical correction: The original KE model (black dashed line) exhibits a steep, non-physical cliff-like drop at the rheological boundary marked by the vertical dashed line, whereas the cubic spline interpolation (orange-red solid line), in an attempt to forcibly close the discontinuity, produces a noticeable downward oscillation in the transition region, thereby disrupting the monotonic increase in stiffness with deformation; Meanwhile, although the power-law fitting curve (yellow solid line) remains smooth, it shifts upward overall in the subsequent large-deformation region ($\omega > 5 \mu\text{m}$), completely distorting the theoretical load-carrying characteristics at the far end. In contrast, cubic Hermite interpolation not only achieves a smooth, oscillation-free, and strictly monotonically increasing transition within the characteristic slit but also overlaps seamlessly with the original KE theoretical baseline beyond the transition zone, successfully suppressing local non-physical discontinuities while maintaining strict, lossless fidelity to the global contact constitutive behavior.

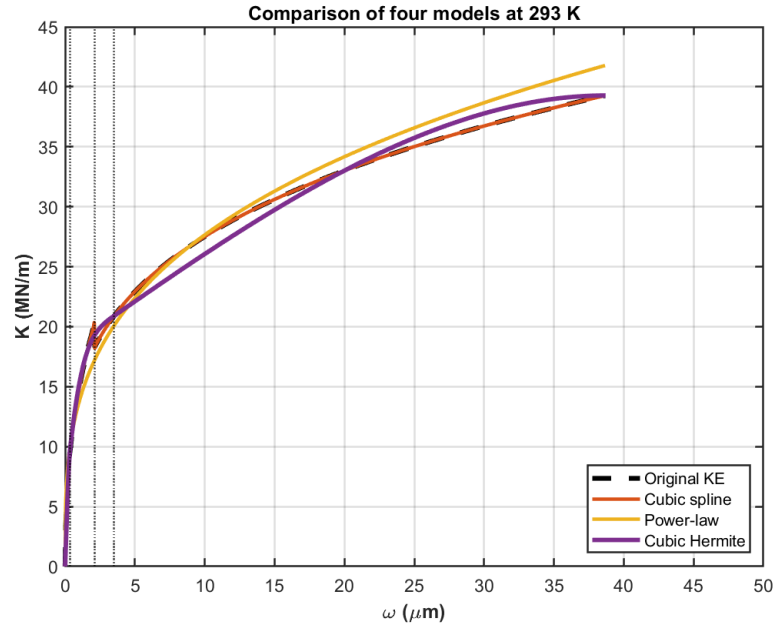


Figure 10: Comparison of Three Different Correction Models with the KE Model

4. Conclusion

This paper presents a fully parameterized study of the elastoplastic contact behavior of 316L austenitic stainless steel in a low-temperature environment ranging from 200 K to 300 K. An improved KE contact mechanics theoretical framework that accounts for temperature gradients was established, and a method for correcting local contact stiffness based on cubic Hermite interpolation polynomials was proposed. The main conclusions of the study are as follows:

1. The low-temperature strengthening effect significantly enhances the deformation resistance and load-bearing capacity of the contact interface: As the temperature decreases, the material's elastic modulus and yield strength increase, causing the critical yield load P_c , critical contact area A_c , and maximum elastoplastic stiffness $K_{\max}$ —prior to entering the fully plastic stage—to all exhibit a nonlinear, monotonically increasing trend. Low temperatures effectively delay the onset of initial yielding, enabling the material to maintain elastic load-bearing capacity over a broader range of contact scales.

2. There is a strong coupling and amplification effect between geometric dimensions and the low-temperature environment: The critical contact load strictly follows a scaling law that is proportional to the contact curvature radius R^2 . Contact bodies with large curvature radii are able to more sensitively capture and amplify the strengthening effect of low temperatures on the material's yield strength, thereby achieving a multi-fold increase in ultimate yield load-bearing capacity under cryogenic conditions;

3. The cubic Hermite interpolation method successfully corrects the numerical stiffness defects in the classical model: The original KE model exhibits a non-physical sudden drop in stiffness accounting for approximately 13%–15% at the transition from pure elasticity to

elastoplasticity ($\omega^* = 6$). Compared to the concave oscillations of cubic spline interpolation and the theoretical distortions at the far end of the power-law function method, the cubic Hermite-corrected model achieves local C^1 -class continuity and strictly monotonically increasing behavior while seamlessly overlapping with the original KE theoretical baseline outside the transition zone. This method perfectly achieves the smooth suppression of local non-physical discontinuities and the lossless preservation of global contact constitutive characteristics.

In summary, the findings of this study not only refine the elastic-plastic contact mechanics mechanism for spherical joint surfaces in the low-temperature range but also provide a more rigorous theoretical model to support the evaluation of structural load-bearing performance, precise stiffness prediction, and coupled finite element simulations for extreme cold engineering and cryogenic service equipment.

Declaration of Conflicting Interests

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Data availability

The datasets generated during and analysed during the current study are available from the corresponding author on reasonable request.

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