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Optimal channel strategy under brand offline encroachment with online coupons

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Abstract: The recent slowdown in online retail sales growth has compelled brands to reassess the value of offline channels. This study examines a brand's motivation for offline channel encroachment, emphasizing how operational costs and coupon promotions influence channel strategy and profit allocation across supply chain members. Incorporating coupons into a model of offline encroachment provides new insights into coupon-based promotion. Results show that establishing an offline channel does not necessarily benefit the brand. A dual-channel strategy enhances the brand profit only when operational costs remain sufficiently low, although this condition often harms the retailer. Notably, encroachment may still benefit the retailer if the brand adopts a low wholesale price. With coupons, the brand channel decision depends on the interaction between operational cost and coupon sensitivity. When consumers exhibit strong sensitivity to coupons, their introduction can discourage the brand from entering the offline market, even when operational cost is zero. Another counterintuitive outcome is that online coupons may reduce offline demand without diminishing brand profit. A mutually beneficial outcome can emerge when the brand and retailer coordinate on the channel decision.

Keywords: Offline encroachment; Coupons; Operating cost; Coupon sensitivity

1. Introduction

During the past decade, e-commerce platforms have experienced rapid expansion in online retailing, prompting numerous traditional brands, including ERKE, Walmart, and Zara, to develop online channels. However, recent years have witnessed a noticeable deceleration in online retail growth. The global growth rate of e-commerce sales declined from 25.4% in 2018 to 7.2% in 2024. This development has led many online-oriented brands to redirect attention toward offline channels, adopting dual-channel strategies to reach end consumers (Gao et al., 2021). This transformation spans multiple industries, including electronics, fashion apparel, and household appliances. Brands such as Honor, Samsung, and Amazon have increasingly emphasized offline expansion. In 2024, the home appliance sector of JD achieved an eightfold increase in offline sales volume and a fifteenfold rise in store visits relative to the previous year. Offline channel encroachment can substantially extend a brand product reach. As stated by Lu, President of Xiaomi, the company intends to open 3,000 brick-and-mortar stores within three years. Consequently, dual-channel distribution through offline encroachment has become a key strategic decision for numerous brands. Nevertheless, such entry may provoke online retailers, decrease their market share, and lower online sales, potentially creating channel conflict (Tsay and Agrawal, 2004; Cai, 2010).

As e-commerce has grown, competition among online retailers has intensified. To attract consumers, promotional strategies have shifted from simple price wars to diversified schemes, including Billion-Dollar Subsidies, Pinduoduo group-buying programs, and Amazon Lightning-deals. In parallel, JD.com has introduced Brand-Day events to enhance the consumer experience. Among various promotional instruments, coupon promotions offered via digital platforms have become particularly popular. For example, brands such as Adidas and Hotwind routinely employ coupon strategies in daily operations. Compared with offline channels, online coupons are less costly to distribute and more convenient for consumers to access. When consumers derive greater benefit from e-platform coupons, offline demand may weaken. Therefore, coupons not only attract consumers but also enhance the competitiveness of online retailers, thereby influencing a brand offline channel performance.

In practice, due to limited experience in direct sales, incomplete logistics networks,

and constrained consumer reach, brands often face lower sales efficiency than online retailers. Additionally, the impact of online coupons on market demand creates further barriers to offline encroachment. For instance, Walmart closed more than 80 stores in China between 2016 and 2020, partly because online coupon promotions drew significant consumer traffic. Thus, channel operating cost and online coupons represent crucial determinants in brand offline channel encroachment. Although prior studies have investigated supply-chain channel strategies extensively, the combined effects of channel choice, operating cost, and platform coupons remain insufficiently explored, necessitating further study.

Given the above context, this paper examines how platform coupons influence a brand channel strategy. Specifically, we address the following research questions: (1) Does establishing an offline channel always benefit the brand and harm the retailer? (2) How do e-platform coupons influence channel strategy equilibrium, and can coupon introduction lead to losses for the brand? (3) Can e-platform coupons effectively deter brand encroachment?

To address these questions, the supply chain model comprising a brand, a retailer, and an e-platform is constructed. The retailer purchases products from the brand and sells them via the e-platform, which offers coupons to consumers. The brand sets a wholesale price and decides whether to encroach by establishing an offline channel. First, the effect of operating costs on the brand channel strategy is analyzed as a benchmark. Next, we examine how coupons shape strategic interaction between the brand and retailer under encroachment, with particular attention to the relationship between operating cost and coupon sensitivity. Finally, by comparing scenarios with and without coupons, channel strategic preferences and potential consensus among the brand, retailer, and e-platform are identified.

Our analysis yields several key insights:

(1) Contrary to intuition, encroachment can benefit the retailer when the brand channel operating cost is moderate. Although the offline channel diverts part of the online sales volume, the brand may lower the wholesale price at the threshold between dual-channel and single-channel strategies under encroachment threat, thereby improving retailer profitability.

(2) Coupons can mitigate the negative impact of encroachment on the retailer. When operating cost is low, encroachment increases the wholesale price and reduces online demand, harming the retailer. However, high coupon sensitivity can erode the brand cost advantage and deter offline entry. This indicates that coupons function not only as a sales instrument but also as a strategic lever against offline encroachment.

(3) Interestingly, coupons may reduce offline demand while unexpectedly increasing brand profit. They do not necessarily harm the brand and may even ease channel conflict. When both coupon sensitivity and operating cost increase within a certain range, it becomes easier for both parties to reach agreement on a channel strategy, creating a win-win outcome.

The remainder of this paper is organized as follows. Section 2 reviews related literature. Section 3 describes the model setup and demand specification. Section 4 analyzes channel strategies and optimal decisions with and without coupons. Section 5 examines the effects of operating cost and coupon sensitivity on profits and strategic preferences. Section 6 concludes and suggests directions for future research.

2. Related Literature

The literature relevant to this study falls into two streams: channel encroachment and coupon promotion. We clarify the connections between our work and these areas below.

Extensive research has examined channel encroachment. Influencing factors have been analyzed from multiple perspectives, including customer preferences (Dong et al., 2025), information asymmetry (Hu et al., 2025), and firm behavior (Wang et al., 2024). Gao et al. (2021) explored how interaction between the double-marginalization effect and cost-signal distortion affects manufacturer dual-channel operation. Using a bilateral vertical supply chain model, Sun et al. (2021) analyzed how cost reduction motivates suppliers to enter the retail market and alters profit distribution. Zhang et al. (2022) examined the relationship between RFID tag costs and manufacturer encroachment, finding that the cost threshold for encroachment increases with RFID tag prices when RFID is deployed in the direct channel. Gupta et al. (2024) demonstrated that cost learning may eventually make encroachment profitable for suppliers. These studies highlight the critical role of cost considerations in channel decisions. Although traditional views suggest that encroachment heightens manufacturer-retailer

competition, some studies find mutual benefit (Xia & Niu, 2020). In platform supply chains, Hou et al. (2023) investigated the interaction between encroachment and blockchain adoption in combating counterfeits. Zhang et al. (2025) studied channel structure under private-brand encroachment and showed that private-brand entry can lower wholesale prices. In contrast, our research emphasizes the joint effect of operating cost and coupon promotion on encroachment. Although offline channels can raise consumer valuation to some degree, brands may face operational inefficiencies compared with online direct sales. Our findings indicate that offline encroachment does not universally benefit the brand or harm the retailer; instead, the outcome depends on the interaction between operating cost and coupon sensitivity.

This study also relates to literature on coupon promotion, which broadly encompasses two streams: coupon-promotion strategies (Pal and Sana 2025; Wan and Fan 2025) and impact of coupons on consumer choices (Nayal et al. 2025). Early research by Raju et al. (1994) used a random choice model to analyze three coupon types, concluding that on-pack coupons can secure greater market share than peel-off coupons. More recently, Bauner et al. (2019) examined coupon strategies for manufacturers and retailers where national brands compete with private labels, and found that higher functional differentiation motivates manufacturers to raise coupon face value. Wang et al. (2020) compared discount and coupon promotions, showing that when the share of well-informed consumers is low, discounts attract more potential buyers and improve profits relative to coupons. Li et al. (2022) developed a game-theoretic model to identify optimal coupon strategies in omnichannel supply chains, comparing four coupon distribution modes in buy-online-pick-up-in-store and offline contexts. Li et al. (2024) further modeled coupon competition between an online platform and third-party seller under commission-based partnerships. A small set of studies has examined consumer preference heterogeneity. For instance, Luo et al. (2017) investigated optimal e-coupon strategies for omnichannel retailers considering channel-preference heterogeneity. Empirically, Duan et al. (2022) showed that consumer perception of online coupons can offset negative effects of unfavorable reviews on sales. Ram et al. (2024) used machine learning to predict redemption probability for social coupons in the food sector, revealing that targeting highly active social-media users

with short-validity coupons increases redemption. Existing work primarily focuses on how coupons improve business performance and shape consumer decisions. Notably, despite evidence that coupons help online retailers attract consumers, limited theoretical research has addressed the interaction between coupon promotions and brand offline encroachment.

Bridging these two research streams, this work focuses on brand channel strategies and retailer order decisions in scenarios with and without coupons. The analysis reveals the mechanism linking coupon promotions and channel strategies, while also examining preference alignment and strategic consensus between retailers and brands regarding promotion and channel decisions. The results show that coupons function not only as a promotional instrument but also as an effective deterrent to encroachment. Meanwhile, supply chain profit maximization can be achieved through coordinated use of coupon promotions and offline channel establishment. These findings provide actionable managerial insights for firms engaged in supply chain channel management and product promotion.

3. Model setup

3.1 Model description

Consider a supply chain composed of a brand, a retailer, and an e-commerce platform (e-platform), where the brand distributes products through two channels (see Figure 1). One is the online channel, in which the retailer obtains products from the brand at wholesale price w and sells quantity q_r through the e-platform. The other is the offline direct channel established by the brand, which sells quantity q_m . However, compared with the retailer, the brand has lower channel efficiency in offline retailing and incurs an operating cost c . To isolate key factors, the production cost and retailer sales cost are normalized to zero. In addition, the e-platform charges the retailer a fee rate γ per unit sold and, to counter the brand's offline channel and strengthen profits, provides online consumers with coupon value f . All parties are risk-neutral and maximize their own profits.

Without loss of generality, the base demand is normalized to one. Consumer valuation for the product in the offline direct channel is v , uniformly distributed on $[0,1]$ and equals θv when purchased through the online channel. The parameter $\theta \in$

$[0,1]$ captures valuation differences between online and offline channels. Thus, a consumer's utility from the offline channel is $U_m = v - p_m$, where p_m is the price set by the brand. Likewise, $U_r = \theta v - p_r + \beta f$ denotes consumer utility from the online channel when the e-platform offers coupons, where p_r is the retailer price and β represents coupon sensitivity. To ensure that the utility from a unit coupon exceeds that from a unit price reduction, β is set to be greater than one.

Interactions among the brand, retailer, and e-platform follow a Stackelberg game structure. First, given operating cost c , the brand sets the wholesale price w and the e-platform decides whether to offer coupon value f . Next, the retailer chooses the online channel quantity q_r . After observing q_r , the brand determines the direct channel quantity q_m if it engages in encroachment. Finally, the market clears and all supply chain members obtain their respective profits.

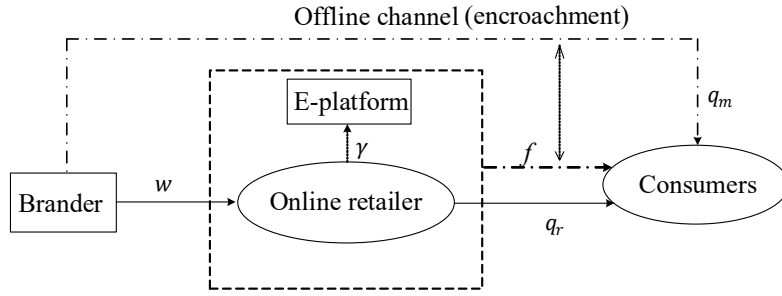


Figure 1. Supply chain model

3.2 Demand characterization

In this section, we propose market demand regarding the introduction of the coupon and then focus on quantity decisions in both channels.

With the offline encroachment motivation of the brand, with coupons, the consumer utility obtained from both channels is U_r and U_m given in the model description. Thus, the offline channel will be weakly dominated by the online channel if product valuation v satisfies $\{v | \theta v - p_r + \beta f \geq v - p_m \& \theta v - p_r + \beta f \geq 0\}$. Accordingly, as v satisfies $\{v | v - p_m \geq \theta v - p_r + \beta f \& v - p_m \geq 0\}$, the offline channel is more popular with consumers. With a market size of one, the demand functions are formulated as

$$q_m = \begin{cases} 1 - \frac{p_m - p_r + \beta f}{1 - \theta}, & p_m \geq \frac{p_r - \beta f}{\theta} \\ 1 - p_m, & \text{otherwise} \end{cases} \quad (1)$$

$$q_r = \begin{cases} \frac{p_m - p_r + \beta f}{1 - \theta} - \frac{p_r - \beta f}{\theta}, & p_m \geq \frac{p_r - \beta f}{\theta} \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

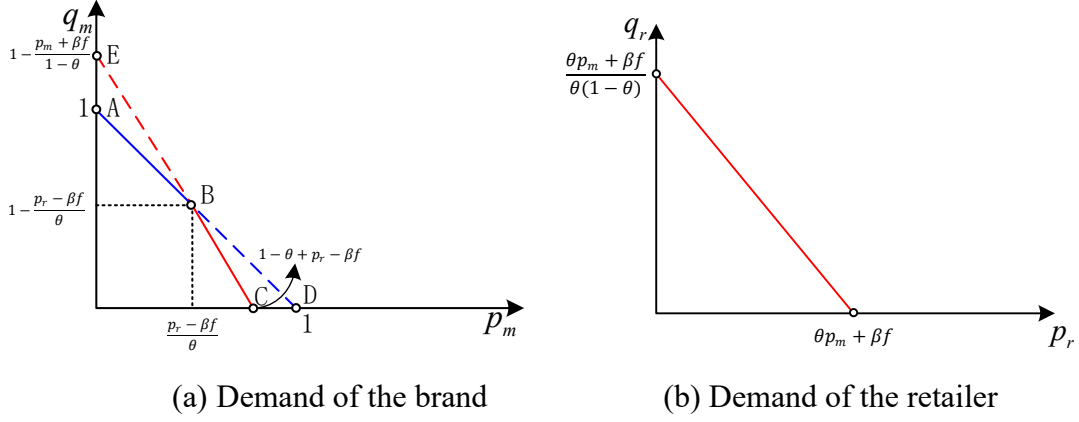


Figure 2. Demand functions of offline and online channel.

As shown in Figure 2(a), the brand's demand becomes more price elastic when p_m exceeds $\frac{p_r - \beta f}{\theta}$ because higher pricing leads some consumers to migrate to the online channel from the offline channel, as shown in Figure 2(b). As $p_m < \frac{p_r - \beta f}{\theta}$, a lower price offered by the brand attracts all consumers in the offline channel and results in $q_r = 0$, which requires $\theta > 1$. Obviously, this case rarely occurs in practice. Hence, this paper focuses only on the case where $q_r \neq 0$.

According to demand functions (1) and (2), the inverse demand functions are therefore given as

$$p_m = 1 - q_m - q_r \theta \quad (3)$$

$$p_r = (1 - q_m - q_r) \theta + f \beta \quad (4)$$

Correspondingly, the inverse demand functions without coupons can be obtained easily, i.e., $p_r = (1 - q_m - q_r) \theta$ and $p_m = 1 - q_m - q_r \theta$. By comparison, coupons provided by the e-platform disrupt the market equilibrium and promote channel demand to reach a new equilibrium.

4. Channel strategies regarding the introduction of coupons

In this section, we first analyze the benchmark model in the absence of coupons and then, considering e-platform behavior, further discuss the complex case with coupons.

4.1 Benchmark

In the benchmark case, e-platform behavior does not offer coupons to consumers, and the brand may encroach on the market by setting up the offline channel, which is denoted by a superscript “ $\leftarrow$ ”.

4.1.1 Member decision

In such a case, we work backward. At the final stage, given a wholesale price w and the retailer's order quantity q_r , the brand chooses the order quantity q_m to maximize his profit:

$$\max_{q_m} \underbrace{wq_r}_{\text{online channel}} + \underbrace{(1 - q_m - q_r\theta - c)q_m}_{\text{offline channel}} \quad (5)$$

Thus,

$$q_m = \left(\frac{1 - \theta q_r - c}{2}\right)^+ = \begin{cases} \frac{1 - \theta q_r - c}{2}, & \text{if } q_r < \frac{1 - c}{\theta} \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

Accounting for the brand's reaction, the retailer's profits are expressed as

$$\pi_r(q_r) = \begin{cases} (1 - \gamma)((1 - q_m(q_r) - q_r)\theta - w)q_r, & \text{if } q_r < \frac{1 - c}{\theta} \\ (1 - \gamma)((1 - q_r)\theta - w)q_r, & \text{if } q_r \geq \frac{1 - c}{\theta} \end{cases} \quad (7)$$

There are two expressions of the retailer's profits with respect to the threshold value $q_r = \frac{1 - c}{\theta}$. The retailer chooses order quantities q_r to maximize his profit. To obtain the optimal q_r^* , we divide the retailer's order problem into three cases.

Case 1: As $q_r < \frac{1 - c}{\theta}$, this order quantity $q_r = \frac{\theta + c\theta - 2w}{2(2 - \theta)\theta}$ is optimal only when the wholesale price w satisfies $\bar{T}_1$.

$$\bar{T}_1 = \left\{ w \mid \frac{\theta + c\theta - 2w}{2(2 - \theta)\theta} < \frac{1 - c}{\theta}, w < \frac{(4 - 3\theta)(1 + c) + 2w}{2(2 - \theta)} \right\} \quad (8)$$

where $w < \frac{(4 - 3\theta)(1 + c) + 2w}{2(2 - \theta)}$ comes from $w < p_m$, which ensures that the retailer acquires products from the brand rather than the offline channel.

Case 2: If the order quantity q_r satisfies $q_r > \frac{1 - c}{\theta}$, with $q_m = 0$ and $p_m = 0$, the offline channel disappears. $q_r = \frac{\theta - w}{2\theta}$ will be optimal if and only if the wholesale price w satisfies $\bar{T}_2$.

$$\bar{T}_2 = \left\{ w \mid \frac{\theta - w}{2\theta} > \frac{1 - c}{\theta} \right\} \quad (9)$$

Case 3: If the brand sets the wholesale price w in neither (8) nor (9), the optimal order quantity q_r exists when the wholesale price w charged by the brand belongs to condition (10)

$$\bar{T}_2 = \left\{ w \mid \frac{2w - \theta - c\theta}{2(\theta - 2)\theta} \geq \frac{1 - c}{\theta}, \frac{\theta - w}{2\theta} \leq \frac{1 - c}{\theta}, w < \frac{(4 - 3\theta)(1 + c) + 2w}{2(2 - \theta)} \right\} \quad (10)$$

With conditions (8), (9) and (10), when the first-order conditions of equation (7) are solved, we can obtain the retailer's optimal order strategy, as shown in Lemma 1.

Lemma 1. If coupons are not introduced, the optimal order quantities for the brand and retailer exist:

$$(\bar{q}_m^*, \bar{q}_r^*) = \begin{cases} (\frac{1-c}{2}, 0), \text{ if } w \in [\frac{c\theta+\theta}{2}, +\infty) \\ (\frac{4+2w+c(\theta-4)-3\theta}{8-4\theta}, \frac{2w-\theta-c\theta}{2(\theta-2)\theta}), \text{ if } w \in [\frac{(3-c)\theta-4(1-c)}{2}, \frac{c\theta+\theta}{2}) \\ (0, \frac{1-c}{\theta}), \text{ if } w \in [\theta-2+2c, \frac{(3-c)\theta-4(1-c)}{2}) \\ (0, \frac{\theta-w}{2\theta}), \text{ if } w \in (0, \theta-2+2c) \end{cases}.$$

Lemma 1 indicates the optimal strategies of the brand and the retailer. From Lemma 1, it follows that the brand order quantity directly depends on the retailer, implying that the retailer can, to some extent, influence brand encroachment by adjusting the order quantity q_r . However, it should be noted that the retailer's order quantity is tied to the brand wholesale price. Accordingly, the brand can directly control the retailer's order through the wholesale price and then indirectly regulate its own order quantity q_m and channel strategy. In this scenario without coupons, the brand clearly dominates the distribution channels in the supply chain.

Furthermore, anticipating this response, the brand sets the wholesale price according to the level of operating cost in order to maximize profit. For simplicity, the symbols “ D ”, “ S^T ” and “ S ” denote the dual-channel strategy, the single-channel strategy with threat, and the single-channel strategy, respectively. Thus, we present the brand optimal wholesale pricing strategy in Proposition 1.

Proposition 1. Given the operation cost c , thresholds $\bar{c}_1$ and $\bar{c}_2$ exist, and the following wholesale price strategies of the brand exist:

(1) Dual channel region (D). When $0 < c \leq \bar{c}_1$, the brand prefers the dual channel strategy and sets $\bar{w}^{D*} = \frac{\theta(8-(5+c)\theta)}{2(8-5\theta)}$, which decreases with c , where $\bar{c}_1 = \frac{8-5\theta}{8-3\theta}$;

(2) Single-channel region with threat (S^T). When $\bar{c}_1 < c \leq \bar{c}_2$, the brand prefers the dual channel strategy and sets $\bar{w}^{S^T*} = \frac{3\theta-4+c(4-\theta)}{2}$, which increases with c , where

$$\bar{c}_2 = \frac{4-\theta}{4};$$

(3) Single channel region (S). When $\bar{c}_2 < c$, the threat of encroachment disappears.

The brand uses the online channel alone and sets $\bar{w}^{S*} = \frac{\theta}{2}$, which is independent of c .

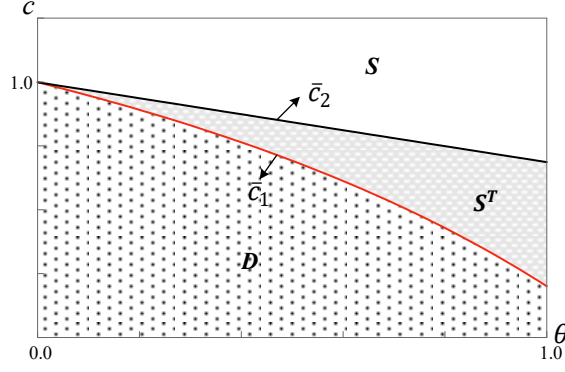


Figure 3. Region of channel strategy.

Proposition 1 and Figure 3 indicate the impacts of c and θ on the wholesale price and channel strategy. Significantly, the optimal wholesale prices in response to c differ among the three regions. The reason can be explained as follows. In the dual channel strategy, a higher operating cost produces a lower wholesale price, namely, $\frac{\partial \bar{w}^D}{\partial c} < 0$, which implies that the offline channel can produce a lower marginal profit and that the brand prefers to cut the wholesale price to sell more products through the online channel ($\frac{\partial \bar{q}_r^{D*}}{\partial c} > 0$). However, in the single-channel strategy with the threat, with the encroachment threat, the retailer tends to place a conservative order quantity. The higher the operating cost is, the less harm the threat will do to the retailer, which encourages the brand to raise the price of the product for more profits. Therefore, the wholesale price increases, and the online quantity decreases in terms of the operating cost ($\frac{\partial \bar{w}^{S^T}}{\partial c} > 0$, $\frac{\partial \bar{q}_r^{S^T}}{\partial c} < 0$). In the single channel region, the operating cost is large enough that the brand gives up the threat and relies solely on the online channel, which indicates that the operating cost no longer affects the wholesale price.

Thus, as c increases, the inefficiency of the offline channel is gradually avoided by dynamically adjusting the wholesale price, which reduces the burden of the brand on the operating cost. In addition, it is worth noting that the single channel region increases with the valuation difference θ in Figure 3. This is because consumers can obtain more utility from a higher valuation difference in the online channel, which enhances the efficiency of the online channel. In this case, with a high operating cost, the brand can benefit more relative to the single-channel strategy with threat.

Corollary 1. *If $0 < c \leq \bar{c}_1$, the brand tends not to close the online channel through a higher wholesale price and would not use the offline channel alone.*

Corollary 1 proposes an interesting view. The retailer will withdraw from the market

with $q_r = 0$ if the wholesale price w is more than or equal to $\frac{c\theta + \theta}{2}$. Although a monopoly is obtained by the brand when only the offline channel is used, a decline in profit will occur relative to adopting a dual-channel approach. In detail, the online channel operates with higher efficiency and, for each unit sold offline, the associated cost can be lower. Nonetheless, this factor explains only part of the outcome, since the remaining effect results from double marginalization. Relative to the offline channel alone, the dual channel strategy improves the overall sales volume, e.g., $\bar{q}_r^{D*} + \bar{q}_m^{D*} > \bar{Q}^1$, and then reduces the impact of double marginalization, which produces more profit for the brand. Thus, as $0 < c < \bar{c}_1$, the brand will set $w = \frac{\theta(8-(5+c)\theta)}{2(8-5\theta)}$ and maintain the dual-channel strategy.

4.1.2 Channel equilibrium and implications

The channel equilibrium is formally introduced in this subsection, and the manner in which the operating cost alters the equilibrium strategy under the scenario without coupons is described. The equilibrium results are presented in proposition 2.

Proposition 2. *If the coupon is not introduced, the channel equilibrium exists, which is described in Table 1.*

Table 1. Outcomes of the quantity game between brand and retailer

	Impact of the operating cost c on the equilibrium outcome		
	$0 < c \leq \bar{c}_1$	$\bar{c}_1 < c \leq \bar{c}_2$	$\bar{c}_2 \leq c$
	(D)	(S ^T)	(S)
$\bar{w}$	$\frac{\theta(8-(5+c)\theta)}{2(8-5\theta)}$	$\frac{3\theta-4+c(4-\theta)}{2}$	$\frac{\theta}{2}$
$\bar{q}_r$	$\frac{2c}{8-5\theta}$	$\frac{1-c}{\theta}$	$\frac{1}{4}$
$\bar{q}_m$	$\frac{(8-5\theta)-(8-3\theta)c}{2(8-5\theta)}$	0	0
$\bar{\pi}_r$	$\frac{2c^2(1-\gamma)(2-\theta)\theta}{(8-5\theta)^2}$	$\frac{(1-c)^2(1-\gamma)(2-\theta)}{2\theta}$	$\frac{(1-\gamma)\theta}{16}$
$\bar{\pi}_m$	$\frac{8+c^2(8-\theta)-5\theta}{4(8-5\theta)}$	$\frac{(1-c)(c(4-\theta)-4+3\theta)}{2\theta}$	$\frac{\theta}{8}$
$\bar{\pi}_p$	$\frac{2c^2\gamma(2-\theta)\theta}{(8-5\theta)^2}$	$\frac{\gamma(1-c)^2(2-\theta)}{2\theta}$	$\frac{\gamma\theta}{16}$

Proposition 2 shows that operating cost plays a crucial part in shaping channel equilibrium and the resulting channel choice. In general, the preferred arrangement tends to be either a dual-channel system or a single-channel system. However, three

¹ $\bar{Q} = \frac{1-c}{2}$ denotes the quantity under the offline channel alone.

channel operating strategies exist, which depend on the operating cost c . First, when $0 < c \leq \bar{c}_1$, the offline channel operates efficiently, the brand values market encroachment, and a dual-channel approach is adopted. Second, when the operating cost reaches a moderate level, the optimal choice becomes a single-channel strategy with an encroachment threat, which appears counterintuitive. Under this condition, even though the brand does not enter the market ($q_m = 0$), the incentive to encroach remains and serves to pressure the retailer to modify the order ($\frac{1-c}{\theta} < \frac{2c}{8-5\theta}$). Finally, when the operating cost exceeds the threshold $\bar{c}_2$, for the retailer, lower channel efficiency turns encroachment into an incredible threat, which inspires the brand to rely only on the online channel for selling products.

Corollary 2. *If $0 < c \leq \bar{c}_1$, the following results exist:*

- (1) *The brand always prefers to encroach, i.e., $\bar{\pi}_m^{D*} > \bar{\pi}_m^{S*}$;*
- (2) *There exists a threshold value $\bar{c}_1^N$ such that $\bar{\pi}_r^D > \bar{\pi}_r^S$ if $\bar{c}_1^N < c \leq \bar{c}_1$ and $\bar{\pi}_r^D < \bar{\pi}_r^S$ otherwise. where $\bar{c}_1^N = \frac{(8-5\theta)\sqrt{4-2\theta}}{4(4-2\theta)}$.*

Corollary 2 indicates the impacts of the brand's encroachment on the profits of all parties. When $0 < c \leq \bar{c}_1$, the offline channel is highly efficient, from which the brand can obtain an extra profit. Thus, the brand is always better off with encroachment. For the retailer, as $0 < c < \bar{c}_1^N$, with a higher wholesale price, the marginal revenue and order quantity of the retailer are further squeezed by encroachment ($\frac{\partial \bar{w}^{D*}}{\partial c} < 0$), which hurts the retailer. However, when c exceeds $\bar{c}_1^N$, the retailer can also benefit from encroachment. This change arises because encroachment helps mitigate double marginalization, allowing the retailer to gain from a reduced wholesale price. As a result, the performance variation of the e-platform aligns with that of the retailer. Overall, operating cost exerts a key influence on both encroachment decisions and double marginalization. Building on Proposition 1, Corollary 3 is further derived.

Corollary 3. *The losses of the retailer and e-platform caused by brand encroachment decrease with the valuation difference θ , i.e., $\frac{\partial \bar{c}_1^N(\theta)}{\partial \theta} < 0$.*

In corollary 3, the threshold value $\bar{c}_1^N(\theta)$ decreases when θ is near one, which means that, for the retailer, the harm of encroachment is alleviated to some extent. With a high valuation difference, the online channel shows a strong competitive edge and narrows the comparative advantage of the brand. Specifically, the order quantity of the online channel increases as θ increases, whereas the order quantity of the offline channel shows a downward trend, namely, $\frac{\partial \bar{q}_r^{D*}}{\partial \theta} > 0$ and $\frac{\partial \bar{q}_m^{D*}}{\partial \theta} < 0$. Therefore,

removing the valuation gap among consumers would help the retailer defend against the brand's encroachment. In reality, channel differences are inherent, and raising consumers' valuation of online products through pricing strategies or ordering decisions is challenging. Consequently, when brand encroachment incentives exist, the core managerial challenge for retailers lies in mitigating the adverse influence of encroachment on their profitability.

4.2. Channel strategies in the introduction of coupons

This situation in which the e-platform joins the three-party game by providing coupons to consumers is examined, and the influence of the coupon policy on the brand's channel decision is analyzed.

4.2.1 Member decision with coupons

In such a case, by applying backward induction, we first analyze the retailer's order strategy with coupons. Therefore, given the inverse demand functions (3) and (4), the brand decides the order quantity q_m by $\max_{q_m} wq_r + (1 - q_m - q_r\theta - c)q_m$, which yields $q_m = \frac{1 - \theta q_r - c}{2}$.

Obviously, it is similar to the case of no coupon in which the order quantity q_m depends on the retailer order quantity q_r . When $q_r < \frac{1-c}{\theta}$, the retailer determines the order quantity q_r by $\max_{q_r} (1 - \gamma)((1 - q_m - q_r)\theta + f\beta - w)q_r$. We can easily obtain the optimal order quantity $q_r = \frac{2f\beta + \theta + c\theta - 2w}{2(2-\theta)\theta}$. If $q_r > \frac{1-c}{\theta}$, $q_m = 0$; therefore, $\max_{q_r} (1 - \gamma)((1 - q_r)\theta + f\beta - w)q_r$ yields the optimal order quantity $q_r = \frac{f + \theta - w}{2\theta}$. Next, based on the interaction between the brand and the retailer regarding order quantity, and consistent with the benchmark setting, the problem can be classified into three cases:

Case 1: The optimal order quantity of the retailer is $q_r^* = \frac{2f\beta + \theta + c\theta - 2w}{2(2-\theta)\theta}$, and the order quantity q_m follows the line segment BC if and only if the wholesale price w and the coupon value f satisfy T_1 .

$$T_1 = \left\{ (w, f) \mid \frac{2f\beta + \theta + c\theta - 2w}{2(2-\theta)\theta} < \frac{1-c}{\theta}, w < \frac{4+4c+2w-2f\beta-3\theta-3c\theta}{8-4\theta} \right\} \quad (11)$$

Case 2: The optimal order quantity of the retailer is $q_r^* = \frac{1-c}{\theta}$, and the order quantity $q_m = 0$ is located at point C if and only if the wholesale price w and the coupon value f satisfy T_2 .

$$T_2 = \left\{ (w, f) \mid \frac{f\beta + \theta - w}{2\theta} \leq \frac{1-c}{\theta}, \frac{2f\beta + \theta + c\theta - 2w}{2(2-\theta)\theta} \geq \frac{1-c}{\theta}, w < \frac{1}{2}(2 + w - f\beta - \theta) \right\} \quad (12)$$

Case 3: The optimal order quantity of the retailer is $q_r^* = \frac{f\beta + \theta - w}{2\theta}$ if and only if the wholesale price w and the coupon value f satisfy T_3 .

$$T_3 = \left\{ (w, f) \mid \frac{f\beta + \theta - w}{2\theta} > \frac{1-c}{\theta} \right\} \quad (13)$$

Through the above constraints, the optimal strategies of the retailer and brand can be derived, as stated in Lemma 2, which assists in explaining how the ordering decision changes when the e-platform engages in supply chain operation through the coupon strategy.

Lemma 2. *If coupons are introduced, given the wholesale price w and the coupon value f , the optimal order quantity for the brand and retailer is as follows:*

$$(q_m^*, q_r^*) = \begin{cases} \left(\frac{4+2w-2f\beta+c(\theta-4)-3\theta}{8-4\theta}, \frac{2f\beta+\theta+c\theta-2w}{2(2-\theta)\theta} \right), & \text{if } f \in \left(0, \frac{4-3\theta-4c+\theta c+2w}{2\beta} \right] \\ \left(0, \frac{1-c}{\theta} \right), & \text{if } f \in \left(\frac{4-3\theta-4c+\theta c+2w}{2\beta}, \frac{2-2c-\theta+w}{\beta} \right] \\ \left(0, \frac{f\beta+\theta-w}{2\theta} \right), & \text{if } f \in \left(\frac{2-2c-\theta+w}{\beta}, +\infty \right) \end{cases}.$$

Lemma 2 presents the brand and retailer ordering strategies. Compared with Lemma 1, two insights emerge. First, once coupons are introduced, the retailer's order quantity becomes directly influenced by the coupon value rather than the wholesale price. This implies that coupon strategies indirectly adjust the offline channel order quantity through the retailer's ordering decision and further affect the brand channel strategy. Moreover, given coupon sensitivity β , Lemma 2 shows that the retailer places a larger order when the e-platform sets a higher coupon value. This occurs because the online channel provides additional coupon value f , enabling consumers to obtain higher utility from the retailer, which shifts demand by reducing offline purchases and increasing online purchases. Consequently, the retailer must place more orders to satisfy market demand.

On the basis of the above statements, considering the retailer's reaction, the e-platform decides its coupon value f as the solution to $\max_f \gamma((1 - q_m^* - q_r^*)\theta + f\beta - w)q_r^* - f q_r^*$. Next, the brand determines his wholesale price w by $\max_w w q_r^* + (1 - q_m^* - q_r^*\theta - c)q_m^*$. The equilibrium outcomes of the e-platform and brand are presented in proposition 3.

Proposition 3. *There are threshold values β_1 , β_2 and β_3 for the coupon sensitivity β and c_1 , c_2 , c_3 and c_4 for the operation cost c , and the following equilibria of the*

brand and e-platform exist:

(1) If $0 < \beta \leq \beta_1$, the equilibria are

$$(w^*, f^*) = \begin{cases} \left(\frac{\theta(k_1 - \theta c)}{2k_1}, \frac{2c(\beta\gamma - 1)(2 - \theta)\theta}{\beta k_1} \right), & \text{if } 0 < c \leq c_1 \\ ((2\beta\gamma - 4)(1 - c) + \theta, \frac{(4\beta\gamma - 4 - \theta)(1 - c)}{2\beta}), & \text{if } c_1 < c \leq c_2, \text{ where } \beta_1 = \frac{5}{4\gamma}, \\ \left(\frac{\theta}{2}, \frac{\theta(\beta\gamma - 1)}{4\beta - 2\beta^2\gamma} \right), & \text{if } c_2 < c \end{cases}$$

$$c_1 = \frac{k_1(k_2 + \theta)}{k_1 k_2 + \theta^2}, \quad c_2 = \frac{8 - 4\beta\gamma - \theta}{4(2 - \beta\gamma)}, \quad k_1 = 16 + 4\beta\gamma(\theta - 2) - 9\theta, \quad k_2 = 8 - 2\beta\gamma(2 - \theta) - 3\theta;$$

(2) If $\beta_1 < \beta \leq \beta_2$, the equilibria are

$$(w^*, f^*) = \begin{cases} \left(\frac{\theta(k_1 - \theta c)}{2k_1}, \frac{2c(\beta\gamma - 1)(2 - \theta)\theta}{\beta k_1} \right), & \text{if } 0 < c \leq c_3 \\ \left(\frac{\theta}{2}, \frac{\theta(\beta\gamma - 1)}{4\beta - 2\beta^2\gamma} \right), & \text{if } c_3 < c \leq c_4 \\ ((2\beta\gamma - 4)(1 - c) + \theta, \frac{(4\beta\gamma - 4 - \theta)(1 - c)}{2\beta}), & \text{if } c_4 < c \leq c_2 \\ \left(\frac{\theta}{2}, \frac{\theta(\beta\gamma - 1)}{4\beta - 2\beta^2\gamma} \right), & \text{if } c_2 < c \end{cases}, \text{ where } \beta_2 = \frac{3}{2\gamma},$$

$$c_3 = \frac{2(2 - \beta\gamma)k_1 - \theta\sqrt{2k_1(4\beta\gamma - 5)(2 - \beta\gamma)}}{(32 + 8\beta\gamma(\theta - 2) - 10\theta)(2 - \beta\gamma)}, \quad c_4 = \frac{(k_1 + \theta)(\beta\gamma - 2) - \sqrt{(\beta\gamma - 2)(2\beta\gamma - 3)\theta^3/2}}{(k_1 + 3\theta)(\beta\gamma - 2)};$$

(3) If $\beta_2 < \beta \leq \beta_3$, the equilibria are

$$(w^*, f^*) = \begin{cases} \left(\frac{\theta(k_1 - \theta c)}{2k_1}, \frac{2c(\beta\gamma - 1)(2 - \theta)\theta}{\beta k_1} \right), & \text{if } 0 < c \leq c_3 \\ \left(\frac{\theta}{2}, \frac{\theta(\beta\gamma - 1)}{4\beta - 2\beta^2\gamma} \right), & \text{if } c_3 < c \end{cases}, \text{ where } \beta_3 = \frac{4 - \theta}{2\gamma};$$

(4) If $\beta_3 < \beta$, $\forall c(0, +\infty)$, the equilibria are $w^* = \frac{\theta}{2}$ and $f^* = \frac{\theta(\beta\gamma - 1)}{4\beta - 2\beta^2\gamma}$.

Proposition 3 provides the optimal pricing strategy of the brand and the coupon strategy of the e-platform. Compared with the benchmark case, it is observed that the pricing decision is influenced not only by the operating cost but also by the sensitivity of the coupon. In contrast to proposition 1, four distinct cases arise when coupons are introduced. Specifically, if $0 < \beta \leq \beta_1$, the brand sells products through a dual channel strategy with $0 < c \leq c_1$. As c increases and is less than c_2 , the dual channel strategy is shifted to the single channel strategy with the threat. Once $c_2 < c$, a higher operation cost eliminates the brand's encroachment threat, and the brand relies only on the online channels to sell products. When $\beta_1 < \beta \leq \beta_2$, only when the cost is high does the brand prefer the online channel strategy. Most interestingly, when $c_3 < c \leq c_4$, the brand sets the wholesale price $w^* = \frac{\theta}{2}$ and adopts the online channel alone. As β increases, the demand in the offline channel decreases, whereas that in the online

channel increases, which, with a relatively high cost, leads to a loss of the brand's profit under the dual channel strategy. In the case of $\beta_2 < \beta \leq \beta_3$, as $0 < c \leq c_3$, the brand encroaches and sets $w^* = \frac{\theta(k_1 - \theta c)}{2k_1}$; otherwise, he employs the online channel strategy.

If $\beta_3 < \beta$, the wholesale price $w^* = \frac{\theta}{2}$ and coupon value $f^* = \frac{\theta(\beta\gamma - 1)}{4\beta - 2\beta^2\gamma}$ are set by the brand and retailer, respectively, and the brand never encroaches on the market regardless of the operating cost.

Through the above analysis, we can further obtain corollary 4.

Corollary 4. *Under the dual channel strategy, the wholesale price w^* decreases and the coupon strategy f^* increases with the degree of coupon sensitivity β .*

Corollary 4 illustrates the pricing behavior of the brand and the coupon decision of the e-platform in relation to coupon sensitivity. When coupon sensitivity is high, consumers are willing to pay a higher premium for each coupon provided by the platform, which markedly increases online demand. As a result, in order to claim a greater share of profits from the retailer, the e-platform tends to adopt a higher coupon level. This positive coupon policy strongly encourages some consumers to shift from the offline channel to the online channel, thereby weakening the offline channel advantage in the encroachment setting. Consequently, to prevent profit reduction, brands need to place greater emphasis on the online channel due to the lower wholesale price.

4.2.2 Channel equilibrium and implications with coupons

Compared with the benchmark, in this case, the equilibrium outcomes show relative complexity, as stated in proposition 4.

Proposition 4. *If the coupons are introduced, there are two threshold values $\underline{\beta}$ and $\bar{\beta}$ for the coupon sensitivity β , and the channel equilibrium is described in Table 2 and Table 3.*

Table 2. Decisions of supply chain members.

	w	q_m	q_r	f	Channel Strategy
$(0, c_1]$	$\frac{\theta(k_1 - \theta c)}{2k_1}$	$\frac{(1 - c)k_1 - 2\theta c}{2}$	$\frac{2c}{k_1}$	$\frac{2c(\beta\gamma - 1)(2 - \theta)\theta}{\beta k_1}$	D
$(\underline{\beta}, \beta_1]$	$(2\beta\gamma - 4)(1 - c) + \theta$	0	$\frac{1 - c}{\theta}$	$\frac{(4\beta\gamma - 4 - \theta)(1 - c)}{2\beta}$	S^r
$(c_2, +\infty)$	$\frac{\theta}{2}$	0	$\frac{1}{4(2 - \beta\gamma)}$	$\frac{\theta(\beta\gamma - 1)}{4\beta - 2\beta^2\gamma}$	S
$(0, c_3]$	$\frac{\theta(k_1 - \theta c)}{2k_1}$	$\frac{(1 - c)k_1 - 2\theta c}{2}$	$\frac{2c}{k_1}$	$\frac{2c(\beta\gamma - 1)(2 - \theta)\theta}{\beta k_1}$	D

$(\beta_1, \beta_2]$	$(c_3, c_4]$	$\frac{\theta}{2}$	0	$\frac{1}{4(2-\beta\gamma)}$	$\frac{\theta(\beta\gamma-1)}{4\beta-2\beta^2\gamma}$	S
	$(c_4, c_2]$	$(2\beta\gamma-4)(1-c)+\theta$	0	$\frac{1-c}{\theta}$	$\frac{(4\beta\gamma-4-\theta)(1-c)}{2\beta}$	S^T
	$(c_2, +\infty)$	$\frac{\theta}{2}$	0	$\frac{1}{4(2-\beta\gamma)}$	$\frac{\theta(\beta\gamma-1)}{4\beta-2\beta^2\gamma}$	S
$(\beta_2, \beta_3]$	$(0, c_3]$	$\frac{\theta(k_1-\theta c)}{2k_1}$	$\frac{(1-c)k_1-2\theta c}{2}$	$\frac{2c}{k_1}$	$\frac{2c(\beta\gamma-1)(2-\theta)\theta}{\beta k_1}$	D
	$(c_3, +\infty)$	$\frac{\theta}{2}$	0	$\frac{1}{4(2-\beta\gamma)}$	$\frac{\theta(\beta\gamma-1)}{4\beta-2\beta^2\gamma}$	S
$(\beta_3, \bar{\beta})$	$\forall c$	$\frac{\theta}{2}$	0	$\frac{1}{4(2-\beta\gamma)}$	$\frac{\theta(\beta\gamma-1)}{4\beta-2\beta^2\gamma}$	S

Table 3. Profits of supply chain members.

		π_r	π_m	π_P	Channel strategy
$(\underline{\beta}, \beta_1]$	$(0, c_1]$	$\frac{2c^2(1-\gamma)(2-\theta)\theta}{k_1^2}$	$\frac{(1-c)^2k_1+4\theta c^2}{4k_1}$	$\frac{2c^2(2-\beta\gamma)(2-\theta)\theta}{\beta k_1^2}$	D
	$(c_1, c_2]$	$\frac{(1-c)^2(1-\gamma)(2-\theta)}{2\theta}$	$\frac{(1-c)((8+2\beta\gamma\theta-4\beta\gamma-3\theta)(c-1)+2\theta)}{2\theta}$	$\frac{(1-c)^2((2+\theta)(1-\beta\gamma)+2)}{2\beta\theta}$	S^T
	$(c_2, +\infty)$	$\frac{(1-\gamma)\theta}{16(\beta\gamma-2)^2}$	$\frac{\theta}{16-8\beta\gamma}$	$\frac{\theta}{32\beta-16\beta^2\gamma}$	S
$(\beta_1, \beta_2]$	$(0, c_3]$	$\frac{2c^2(1-\gamma)(2-\theta)\theta}{k_1^2}$	$\frac{(1-c)^2k_1+4\theta c^2}{4k_1}$	$\frac{2c^2(2-\beta\gamma)(2-\theta)\theta}{\beta k_1^2}$	D
	$(c_3, c_4]$	$\frac{(1-\gamma)\theta}{16(\beta\gamma-2)^2}$	$\frac{\theta}{16-8\beta\gamma}$	$\frac{\theta}{32\beta-16\beta^2\gamma}$	S
	$(c_4, c_2]$	$\frac{(1-c)^2(1-\gamma)(2-\theta)}{2\theta}$	$\frac{(1-c)((8+2\beta\gamma\theta-4\beta\gamma-3\theta)(c-1)+2\theta)}{2\theta}$	$\frac{(1-c)^2((2+\theta)(1-\beta\gamma)+2)}{2\beta\theta}$	S^T
	$(c_2, +\infty)$	$\frac{(1-\gamma)\theta}{16(\beta\gamma-2)^2}$	$\frac{\theta}{16-8\beta\gamma}$	$\frac{\theta}{32\beta-16\beta^2\gamma}$	S
$(\beta_2, \beta_3]$	$(0, c_3]$	$\frac{2c^2(1-\gamma)(2-\theta)\theta}{k_1^2}$	$\frac{(1-c)^2k_1+4\theta c^2}{4k_1}$	$\frac{2c^2(2-\beta\gamma)(2-\theta)\theta}{\beta k_1^2}$	D
	$(c_3, +\infty)$	$\frac{(1-\gamma)\theta}{16(\beta\gamma-2)^2}$	$\frac{\theta}{16-8\beta\gamma}$	$\frac{\theta}{32\beta-16\beta^2\gamma}$	S
$(\beta_3, \bar{\beta})$	$\forall c$	$\frac{(1-\gamma)\theta}{16(\beta\gamma-2)^2}$	$\frac{\theta}{16-8\beta\gamma}$	$\frac{\theta}{32\beta-16\beta^2\gamma}$	S

From the e-platform's viewpoint, under the coupon strategy, as a part of the supply chain, the e-platform has a significant effect on the brand's channel preferences. However, considering the cost derived from the coupon strategy, the e-platform benefits from the coupon strategy regardless of whether encroachment occurs or not, if and only if $\beta \in (\underline{\beta}, \bar{\beta})$; otherwise, it suffers. This is because, on the basis of $\frac{\partial q_r^i}{\partial \beta} \geq 0 (i \in \{D, S^T, S\})$, low coupon sensitivity means inefficiency and that the impact on demand is not significant when coupons are introduced. Thus, when the coupon sensitivity β is

lower than the threshold $\underline{\beta}$, the benefits generated by the increased demand cannot offset the coupon cost, which discourages the platform from coupons. On the other hand, higher coupon efficiency leads to a more aggressive coupon strategy, as noted in proposition 3, although this results in increased expenses for the e-platform. However, under the dual-channel structure, the profits extracted from the retailer may decline because of encroachment. Thus, for the platform, $\beta < \bar{\beta}$ ensures that there is an external surplus in the interaction between the coupon cost and the profits. In addition, proposition 4 further indicates that the retailer cannot consistently induce the e-platform to counter the offline channel by relying on the coupon strategy. For the brand, the introduction of coupons evidently disrupts the brand's channel strategy and performance, which is related not only to the operating cost c but also to the interaction between the operating cost c and the coupon sensitivity. Specifically, as $\beta \in (\underline{\beta}, \beta_1]$, the channel strategy is similar to the basic scenario; encroaching on the market arises with a smaller operation cost. The change occurs at $\beta > \beta_1$. When $\beta \in (\beta_1, \beta_2]$, the part of the dual channel strategy and the single channel with the threat is attacked by the single channel strategy, which is explained in proposition 3. With β increasing, the dual channel strategy is adopted if $0 < c < c_3$; otherwise, the brand gives up encroachment. Furthermore, only a single strategy exists in the case of $\beta_3 < \beta < \bar{\beta}$. From the above analysis, we learn that, as β increases, introducing coupons strengthens the online channel and weakens the offline channel in competitiveness, which also implies that the offline channel is not used alone.

Corollary 5. *When $\underline{\beta} < \beta \leq \beta_1$, there exists a threshold value $c_1^F \in (0, c_1)$,*

- (1) *If $0 < c < c_1^F$, $\pi_r^D < \pi_r^S$ and $\pi_r^D > \pi_r^S$ otherwise;*
- (2) *The coupon strategy reduces the damage caused by encroachment to the retailer, i.e., $c_1^F < \bar{c}_1^N$.*

Corollary 5 demonstrates the impact of the brand's encroachment on the retailer's profit under the introduction of coupons. Similar to the results under the benchmark, if the channel efficiency is high, i.e., $0 < c < c_1^F$, the brand's behavior will hurt the retailer. However, unlike the benchmark scenario, corollary 5 (2) reveals the benefit of the introduction of coupons in reducing the damage of the encroachment, which suggests that, in some cases, the retailer should actively cooperate with the e-platforms to narrow the scope of the damage.

5. Role of operating cost and coupons

In this section, we focus on the effects of operation costs and coupons on the channel strategy preferences and profits of chain members. With numerical analysis, we employ the following parameters: $\beta = 6.54$, $\theta = 0.9$, and $\gamma = 0.2$.

5.1 Impact of operating cost and coupons on performance

In this subsection, we investigate the differences in chain members' profits between the benchmark and the case of coupons.

Corollary 6. (1) *The introduction of coupons weakens the motivation behind the brand encroachment, i.e., $\bar{c}_1 < c_1$, and the brand prefers not to encroach as β increases;*

(2) *The introduction of coupons improves the profits of the manufacturer, retailer and total profit of the supply chain.*

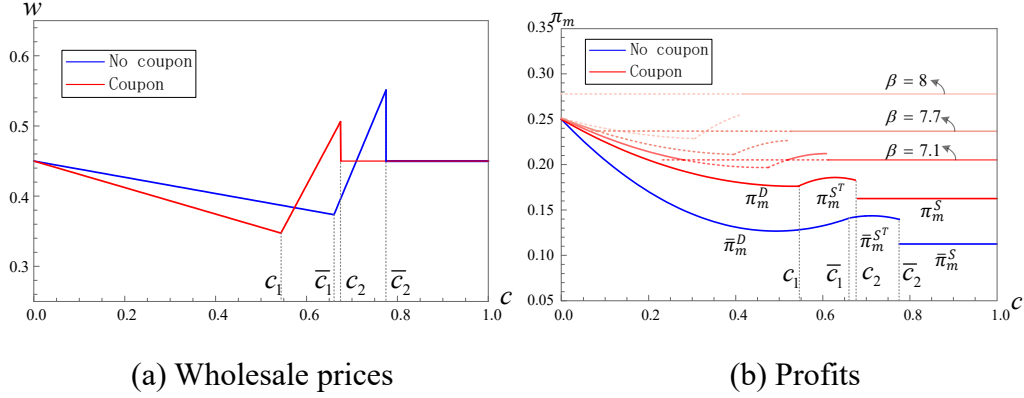


Figure 4. Wholesale prices and profits of brander.

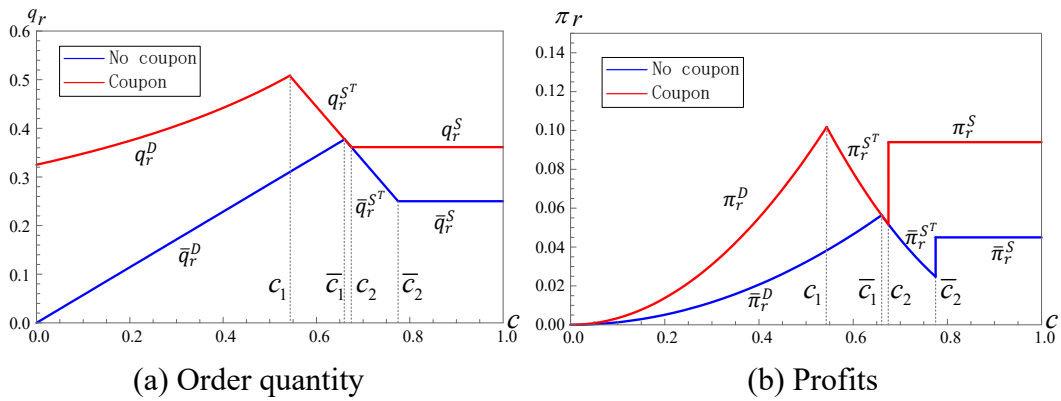


Figure 5. Order quantities and profits of retailer.

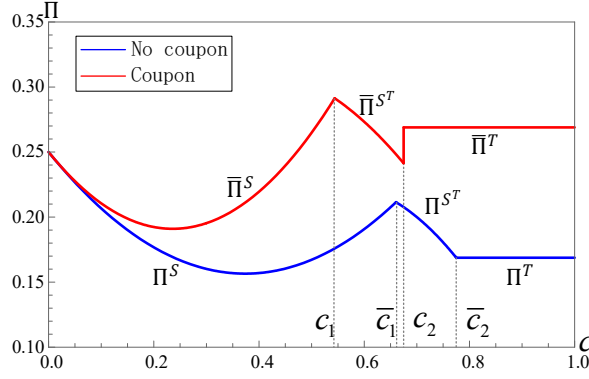


Figure 6. Total profits of supply chain.

Corollary 6 and Figure 4 show the trends in wholesale prices and brand profits. The effect of c on the wholesale price and profits is not monotonic. When operating cost is small, the offline channel exhibits a relative efficiency advantage, motivating the brand to restrict online sales through a higher wholesale price. As c increases, the brand instead incentivizes the retailer to sell more products by reducing the wholesale price, as illustrated in Figure 4(a). Regarding profits, a higher operating cost reduces the margin of the brand, so in Figure 4(b) and Figure 5(b), brand profit gradually declines, and retailer profit increases under the dual-channel strategy. Under the single-channel-with-threat strategy, as c rises, a higher wholesale price yields a higher margin for the brand and leads to a lower retailer order quantity, as shown in Figure 5(a). Consequently, brand profit increases and retailer profit decreases. Once the encroachment threat vanishes, the decisions and profits of both parties become independent of c .

Although coupons exert a negative effect on offline demand, one might expect coupon strategies to burden the brand by lowering profits. However, the opposite result emerges. With coupons, the brand sets a lower offline quantity and wholesale price relative to the no-coupon case in Figure 4(a), but compensates by increasing sales through the online channel. As a result, brand total profit exceeds that in the no-coupon scenario. The retailer also benefits from a higher order quantity, and the brand is better off through wholesale transactions. Thus, coupons may not harm the brand. Moreover, $\bar{c}_1 < c_1$ arises because coupon introduction enhances online-channel efficiency, inducing the brand to encroach only when operating cost is extremely small. In addition, as β increases, Figure 4(b) shows that the brand gradually abandons encroachment even when operating cost is very small or zero. This indicates that coupons can serve as a tool to curb encroachment.

Since coupons boost online sales, the retailer gains substantially from increased demand. At the same time, wholesale trade with the retailer yields greater benefit for

the brand than in the no-coupon case. Overall, under coupon strategies, total supply-chain revenue becomes significantly higher than in the benchmark, as shown in Figure 6. This result arises from the spillover effect generated by consumer responses to coupons.

5.2 Channel strategy preference and strategy consensus

As stated above, we analyze the channel strategy preferences of the brand summarized in Table 1 and Table 2. In this part, we explore how the brand's and the retailer's strategy preferences are affected by the interaction between the operating cost and the coupon sensitivity, as presented in corollary 7 and characterized in Figure 7.

Corollary 7. (1) When $\beta \in (0, \underline{\beta})$ and $(\bar{\beta}, +\infty)$, the e-platform prefers the absence of coupons; on this basis, a threshold value $\bar{c}_2^N$ exists:

If $0 < c < \bar{c}_1^N$ and $\bar{c}_2^N \leq c$, the retailer prefers the single-channel strategy;

If $\bar{c}_1^N \leq c < \bar{c}_1$, the retailer prefers the dual channel strategy;

if $\bar{c}_1 \leq c < \bar{c}_2^N$, the retailer prefers the single-channel strategy with threat;

(2) When $\beta \in (\underline{\beta}, \bar{\beta})$, the e-platform prefers the introduction of coupons. On this basis,

threshold values $\bar{\beta}$ and c_2^F exist; when $\underline{\beta} < \beta < \bar{\beta}$:

if $0 < c \leq c_1^F$ and $c_2^F < c$, the retailer prefers the single-channel strategy;

If $c_1^F < c \leq c_1$, the retailer prefers the dual channel strategy;

If $c_1 < c \leq c_2^F$, the retailer prefers the single-channel strategy with the threat;

When $\beta \in (\bar{\beta}, +\infty)$, $\forall c \in (0, +\infty)$, the retailer always prefers the single-channel strategy.

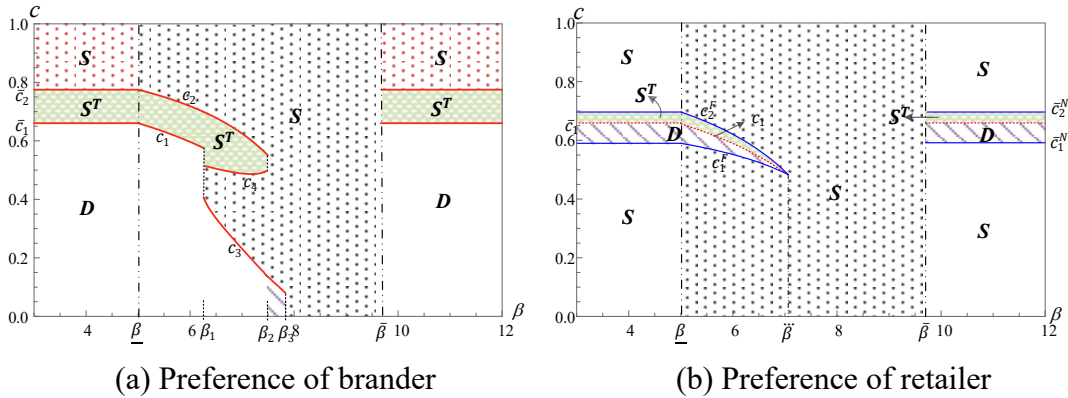


Figure 7. Channel strategy preference of the brander and retailer.

Corollary 7 and Figure 7 describe the channel strategy preference of the retailer. In the absence of coupons, the operating cost satisfying $0 < c < \bar{c}_1^N$ reflects greater efficiency in offline sales, more products will be sold by the offline channel, and the brand controls the online sales by a higher wholesale price, which lowers the retailer's profit compared with that under the single-channel strategy. Most dubiously, with $\bar{c}_1^N \leq$

$c < \bar{c}_1$, the dual channel strategy is optimal for the retailer in Figure 7(b). At the beginning of encroachment, because a higher c results in a lower wholesale price, a higher order quantity will be placed by the retailer, which has positive effects on the retailer's profit. When $\bar{c}_1 \leq c < \bar{c}_2^N$, the retailer benefits from the greater threat caused by the lower wholesale price. As the operating cost exceeds $\bar{c}_2^N$, both parties show a similar preference for a single channel.

Next, as coupons are introduced, for both parties, there are significant differences in the channel strategy preference. In Figure 7(a), the dual channel strategy, single channel strategy with the threat and single channel strategy in turn are adopted by the brand as the operating cost increases, which requires the coupon sensitivity to remain at a low level ($\underline{\beta} < \beta < \beta_1$). However, when the coupon sensitivity and the operating cost increase, in the offline channel, a lower demand caused by the coupon and a higher channel operating cost place a burden on the brand. Notably, compared with Figure 3, the dual channel strategy gradually loses advantages, and only the single channel strategy exists as β increases, which implies that the brand's channel strategy depends on the interaction between the operating cost and the coupon sensitivity.

For the retailer, under the dual channel strategy, if $c_1^F < c \leq c_1$, the brand tends to charge a lower wholesale price for the retailer than under the single channel strategy does, which produces extra revenue. Similarly, under the single-channel strategy with the threat, if $c_1 < c \leq c_2^F$, the retailer benefits more from a lower wholesale price. However, when the coupon sensitivity satisfies $\bar{\beta} < \beta < \bar{\beta}$, the single-channel strategy has greater operation efficiency. In this case, only the single-channel strategy can make the retailer better off regardless of the operating cost.

After deriving the strategy preferences of the brand and the retailer, we observe that the brand optimal channel strategy often harms the retailer. Therefore, it is necessary to examine whether the brand and the retailer can reach a strategy consensus in which both parties agree and achieve a win-win outcome. The results are provided in Corollary 8.

Corollary 8. (1) *The strategy consensuses of the brand and the retailer are listed in Table 4;*

(2) *Under these consensus conditions, both the brand and the retailer can obtain Pareto improvements through the introduction of coupons.*

Table 4. The strategy consensuses of the brand and retailer.

$\beta \notin (\underline{\beta}, \bar{\beta})$	Operating cost c					
	$(\bar{c}_1^N, \bar{c}_1)$		$(\bar{c}_1, \bar{c}_2^N)$		$(\bar{c}_2, 1)$	
	D		S^T		S	
	(c_1^F, c_1)	(c_1, c_2^F)	(c_3, c_4)	$(c_3, 1)$	$(c_2, 1)$	$(0, 1)$
$\beta \in (\underline{\beta}, \beta_1)$	D	S^T	—	—	S	—
$\beta \in (\beta_1, \ddot{\beta})$	—	S^T	S	—	S	—
$\beta \in (\ddot{\beta}, \beta_2)$	—	—	—	—	S	—
$\beta \in (\beta_2, \beta_3)$	—	—	—	S	—	—
$\beta \in (\beta_3, \bar{\beta})$	—	—	—	—	—	S

Corollary 8 identifies the ranges in which the brand and retailer reach consensus regarding the adoption of coupons. Without coupons, the offline channel and the encroachment threat provide the brand with profit opportunities. Only when operating cost remains at a moderate level does the retailer benefit from the brand channel decisions. With high operating cost, both parties agree on adopting a single-channel strategy. When coupons are introduced, the consensus between the two parties shifts gradually toward the single-channel strategy as both coupon sensitivity and operating cost increase. Specifically, for any given operating cost, a higher coupon sensitivity broadens the range within which both parties prefer the single-channel strategy, clearly indicating that both can obtain a Pareto improvement under consensus, meaning that win-win outcomes are more likely to be achieved.

When the brand channel decision comes at the expense of retailer profit, the resulting zero-sum competition intensifies channel conflict and undermines supply chain stability. Therefore, identifying conditions in which both parties share the same strategy preference under the interaction between operating cost and coupons not only improves individual payoffs but also enhances overall supply chain performance and supports sustainable operation.

6. Conclusion

For many brands, establishing an offline channel has become an important strategic direction, although it may negatively affect the existing online channel. Additionally, operating cost and online coupons influence the brand channel decisions. Accordingly, a benchmark model without coupons is first constructed, followed by a model in which

coupons are introduced by the platform. By examining the effects of operating cost and coupons on channel strategy, several conclusions and managerial implications are obtained.

(1) The brand may not consistently favor establishing an offline channel. When operating costs are relatively low, a dual-channel strategy may increase brand profit, although it may also reduce retailer profit. When operating costs are relatively high, offline expansion may impose a burden on the brand. These findings suggest that firms may need to avoid blindly pursuing offline entry and should pay closer attention to channel operation efficiency to reduce the risk of losses.

(2) A potentially counterintuitive insight is that encroachment may benefit the retailer when brand operating cost is at an intermediate level, possibly because the brand may set a lower wholesale price around the switching point between the dual-channel and single-channel-with-threat strategies. This suggests that firms should not assume that encroachment necessarily harms them and may instead need to evaluate competitive entry objectively.

(3) Coupon introduction may help deter the brand from pursuing offline entry even when operating cost is zero, and may help mitigate retailer losses caused by encroachment, provided that coupon sensitivity is sufficiently high. In practice, offering coupons may not only promote sales but may also serve as a competitive strategy against offline entry.

(4) An interesting result is that online coupons tend to reduce offline demand but may not necessarily reduce brand profit, suggesting that coupons may alleviate channel conflict without necessarily imposing a substantial financial burden on the brand. In addition, when both coupon sensitivity and operating cost are relatively high, both parties may be more likely to reach consensus on channel strategy and achieve a mutually beneficial outcome.

Although coupons are commonly viewed solely as sales-promotion tools, and encroachment is often assumed to generate additional profits, our findings show that coupons can serve as an effective mechanism to limit encroachment threats, achieve channel coordination, and improve supply chain performance. This study updates the conventional understanding of operating cost and coupon roles.

However, our work also has several limitations. First, the analysis and conclusions of this study rely on analytical derivations from mathematical models, whereas channel encroachment and coupon distribution are widely observed in practice. Future research could further examine the relationship between coupon distribution and channel

operations based on firm-level data and practical case studies. Second, this study focuses exclusively on the scenario in which coupons are provided by the platform, whereas in business practice, there are scenarios in which online retailers independently provide coupons. Future research could explore brand manufacturers' channel strategies when coupons are provided by online retailers. Third, the model in this study assumes symmetric information; however, in reality, information asymmetry regarding demand, consumer preferences, and other factors is pervasive. Future research could incorporate asymmetric information factors, such as demand or consumer preferences, to provide a more comprehensive understanding of how asymmetric information influences coupon distribution and channel strategy selection.

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