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Stationarity-Based Reformulation of Fibonacci-Based Nearest-Level Modulation for a Nine-Level Asymmetric Cascaded H-Bridge Inverter

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Abstract This paper revisits the Fibonacci-series-based nearest-level modulation (F-NLM) proposed for a nine-level asymmetric cascaded H-bridge inverter with a 1:3 dc-source ratio. The published procedure perturbs four switching angles sequentially and recalculates total harmonic distortion (THD), reaching its final set after 60 reported iterations. The unresolved question is whether those angles remain independent at an interior stationary solution, or whether the THD objective itself imposes a lower-dimensional structure. Inverse mapping of the reported angles first reveals an effective scalar region near $q \approx 0.2368$. It is then proved that every interior stationary point of the analytical THD criterion used in the original search satisfies $\sin \alpha_i = (i - 0.5)q$, so that the stationary multi-angle problem reduces to a scalar fixed-point condition. An unrestricted four-angle minimization with 400 random initializations independently recovers the low-THD stationary solution $q^* \approx 0.236386$, with THD = 8.902299%. The finite Fibonacci ratio $q_F = 13/55 = 0.236364$ is adopted as a rational surrogate, generating $\{6.7872^\circ, 20.7656^\circ, 36.2215^\circ, 55.8196^\circ\}$ and an analytical THD of 8.902300%. Independent 50-Hz PSIM simulations yield 8.9023% for the deterministic formulation and 8.9033% for the published iterative angles. The near-identical distortion values are evidence of equivalence. The contribution is therefore not an additional THD reduction, but the characterization and dimensional reduction of the interior stationary set; 13/55 provides a direct Fibonacci realization for the investigated nine-level case. The result is restricted to the analytical THD functional and waveform assumptions considered in this study.

Keywords asymmetric cascaded H-bridge · Fibonacci sequence · nearest-level modulation · low-switching-frequency modulation · harmonic distortion · deterministic synthesis

1 Introduction

Multilevel voltage-source converters synthesize stepped output voltages from multiple dc levels and are established in medium- and high-power conversion because additional voltage levels can reduce device voltage stress, output-voltage distortion, and filtering requirements [1, 2]. Among the classical families, the cascaded H-bridge (CHB) is especially attractive when modularity and independently supplied cells are compatible with the application [3, 4]. Its asymmetric variants deliberately use unequal dc sources to increase the number of attainable voltage levels without a proportional increase in cell count [5].

Converter topology defines the voltage states that can be synthesized, whereas modulation determines when and how those states are used. This distinction is central in multilevel conversion because the modulation law directly affects harmonic distribution, switching activity, losses, and waveform quality [6–8]. Recent studies in Electrical Engineering likewise show that changes in modulation strategy can modify harmonic content, switching-frequency requirements, and frequency-dependent converter behavior even when the power stage is unchanged [11, 12].

A related converter-design example by El Kattel et al. [9] shows that the modulation and duty-cycle law can define distinct operating regions of an isolated three-phase dc-dc converter without changing the basic power stage. Although

that topology differs from the CHB considered here, the methodological point is relevant: the switching law can alter converter behavior even when the hardware structure is retained.

Oliveira's doctoral thesis [10] provides a broader switched-converter perspective. Using PWM command, the converter is divided into operating regions that differ in duty-cycle range and in the time and number of switches conducting simultaneously. Although the topology is a three-phase isolated dc-dc converter rather than a CHB, the analysis is conceptually relevant here because it shows that a modulation law organizes temporal conduction states and operating regions, rather than being merely a list of isolated switching angles.

For CHB converters, the modular voltage synthesis makes this topology particularly convenient for isolating modulation effects. CHB structures have been extensively reviewed for medium-voltage and renewable-energy applications [3, 4], and asymmetric source arrangements have been used to obtain more voltage levels with reduced structural complexity [5]. Within Electrical Engineering, Francis and Meganathan [13] experimentally studied an asymmetric CHB with a trinary source progression and compared NLM, staircase, and carrier-based modulation. Related non-carrier formulations for asymmetric CHB structures have also been reported [14]. The present study adopts the smaller two-cell 1:3 configuration

because it produces nine levels while keeping the switching-angle geometry analytically transparent.

For low-switching-frequency operation, nearest-level modulation (NLM) maps a sinusoidal reference to the nearest admissible converter level and is attractive because of its simple decision logic and low switching activity [15, 16]. The apparent simplicity of NLM does not remove design freedom: modifying rounding rules, decision thresholds, or transition instants changes the staircase geometry and therefore the spectrum. This observation motivates a large body of NLM variants and is the starting point for the deterministic reformulation developed here.

1.1 Related literature

Castellain [17] treats Staircase/SHE for modular multilevel conversion, where p switching angles control the fundamental component together with up to $p-1$ harmonic constraints, producing nonlinear transcendental systems solved iteratively. Although SHE differs from NLM, the example shows how low-frequency modulation becomes an angle-search problem.

NLM research has followed several directions. Hu and Jiang [18] increased the effective number of output levels in MMCs while preserving the low-switching-frequency character of NLM; Xiong et al. [19] modified switching intervals for CHB converters; Nguyen and Kwak [20] targeted output quality; and Wang et al. [21] introduced a flexible formulation trading switching activity against capacitor-voltage balancing. Variable-rounding strategies [22], variable-structure NLM [23], level-enhanced nearest-level control [24] and NLC rotation strategies [25] confirm that substantial harmonic changes follow from reformulating the level-selection rule rather than the hardware. Surveys of low-switching-frequency modulation reach the same conclusion: waveform quality depends on a small set of discrete decisions [7, 8, 16, 26], a burden often treated as a search over modulation variables [27].

Fibonacci structures had already appeared in power electronics before the F-NLM formulation considered here: as a search procedure reducing comparisons in matrix-converter modulation [28], as efficient search or recursive structures in signal processing and control [29, 30], through fixed-index quotient properties underlying finite-ratio constructions [31], and as switched-capacitor conversion ratios [32, 33] and converter-modeling procedures [34]. Because Fibonacci can play such distinct roles, its role in a modulation algorithm must be identified explicitly rather than invoked numerically.

Within this context, Kumar et al. [35] proposed a Fibonacci-series-based NLM (F-NLM) for a nine-level 1:3 asymmetric CHB. Starting from conventional NLM, the four positive-quarter-wave switching angles are reduced sequentially by 0.1° , 0.2° , 0.3° , and 0.5° , respectively. After each accepted change, the analytical THD is recalculated. The reported significant states culminate at iteration 60 with $\{6.8^\circ, 20.8^\circ, 36.3^\circ, 56.0^\circ\}$ and analytical THD of 8.9028%. The authors then verify the modulation in MATLAB/Simulink and report, for a resistive load, a fundamental-voltage increase from 157.7 V to 163.1 V and a simulated THD decrease from 8.34% to 7.67%.

The analytical and simulation figures in Kumar et al. [35] must be distinguished. The 9.3706% to 8.9028% values belong to the analytical THD criterion used during the angle search, whereas 8.34% to 7.67% are MATLAB/Simulink results obtained from their circuit model. The present study preserves that distinction: published analytical angles are compared analytically, while a separate PSIM model is used as an independent numerical verification environment.

Luo et al. [36] later identified the implementation burden of the Fibonacci-based NLM algorithm as the inverter level count increases, citing computing power, storage and memory. The present paper does not address arbitrary-level scalability, but that observation sharpens a more basic question: in the published nine-level case, are repeated multi-angle objective evaluations structurally necessary at all?

Ghasemian et al. [39] recently established that, for a prescribed fundamental component, the minimum-THD multilevel waveform is a nearest-level waveform with active switching angles $\theta_k = \arcsin[(2k-1)L/(2A)]$. Under $q = L/A$, this is the same angle family as Eq. (2). The design question here is different: Kumar's F-NLM search does not prescribe the fundamental component, so the present work characterizes the stationary set of that unconstrained THD objective and the scalar condition that selects its interior stationary roots.

1.2 Motivation and contributions

The answer obtained here can be stated at the outset. Kumar et al. [35] treat four switching angles as independent variables and reach their final set after 60 reported iterations. This paper proves that every interior stationary point of the THD criterion they adopt satisfies $\sin(\alpha_i) = (i - 0.5)q$, so that the stationary problem is scalar rather than four-dimensional. The remainder of this section states why the question matters and what is not being claimed. The published procedure manipulates four angles independently, but the final solution was never examined to determine whether those values actually span a four-dimensional stationary set. Since conventional NLM already follows a common sinusoidal law, the final angles may contain a latent scalar relation; if such a relation follows from the THD objective itself, the search can be reduced at the level of the optimization problem rather than only reinterpreted after the fact.

The significance of this question is not tied to the absolute runtime of a 60-step offline search, which is computationally modest on modern hardware. The issue is whether repeated objective-function evaluations are mathematically necessary at all. If the stationarity conditions already constrain an interior solution to a scalar trajectory, then the sequential multi-angle search is structurally redundant at the design stage even when its execution time is small.

Answering this question requires separating four issues: (i) the structure visible in the reported angles, (ii) the stationarity structure of the objective function, (iii) the role of a finite Fibonacci ratio as a direct rational representation, and (iv) whether the resulting formulation preserves the electrical result in an independent simulation environment.

Accordingly, this study does not seek an additional THD reduction beyond the reference result. Near-identical electrical results are the verification target because they indicate that the

direct formulation reaches the same stationary region without the sequential search. To isolate this structural property, the analytical and PSIM comparisons retain the idealized staircase assumptions and the same THD objective used in the reference formulation. Hardware-specific effects such as dead time, semiconductor voltage drops, source imbalance, driver delay, and measurement noise are therefore outside the present scope because they would introduce variables unrelated to the mathematical hypothesis.

The main contributions are:

- An inverse mapping of the published final switching angles showing that the reported solution is numerically consistent with a one-parameter sinusoidal family, while explicitly accounting for the 0.1° reporting granularity of the source data.
- A characterization of the stationary set of the analytical THD criterion adopted in the reference study: every interior stationary point satisfies $\sin \alpha_i = (i - 0.5)q$, reducing the multi-angle stationarity system to a scalar fixed-point condition. Unlike the prescribed-fundamental result in [39], the fundamental component here is an outcome of the unconstrained F-NLM objective.
- An unrestricted four-angle numerical minimization that independently recovers the same low-THD stationary solution, together with the finite Fibonacci ratio $q_F = 13/55$ as a close rational surrogate that preserves the mathematical link to the original F-NLM construction.
- An analytical and PSIM comparison demonstrating that the direct formulation reproduces the final iterative operating region while bypassing the offline sequential angle-search stage for the investigated nine-level converter.

1.3 Paper organization

Section 2 defines the nine-level CHB, conventional NLM, and the iterative F-NLM problem. Section 3 develops the inverse scalar mapping, the Fibonacci rational representation, the one-dimensional THD landscape, the direct threshold realization, and the stationarity theorem that establishes the scalar structure of the adopted THD criterion. Section 4 describes the matched PSIM verification. Section 5 compares analytical and numerical results and discusses the structural, computational, and scope implications. Section 6 concludes the paper.

2 Background and Problem Formulation

2.1 Nine-level asymmetric CHB

The investigated converter follows the same nine-level 1:3 asymmetric CHB configuration used by Kumar et al. [35]. The CHB family is well established for modular multilevel synthesis [3, 4], while asymmetric dc-source ratios can increase the number of attainable levels with a small number of cells [5, 13, 14]. Two single-phase full-bridge cells are connected in series. The lower-voltage cell is supplied by V_{dc} and the higher-voltage cell by $3V_{dc}$. Each bridge can contribute a positive, zero, or negative voltage; therefore, the series sum can realize $0, \pm V_{dc}, \pm 2V_{dc}, \pm 3V_{dc}$, and $\pm 4V_{dc}$. Figure 1 presents a deliberately simplified voltage-synthesis view of the topology so that the paper focuses on modulation rather than device-level drawing details.

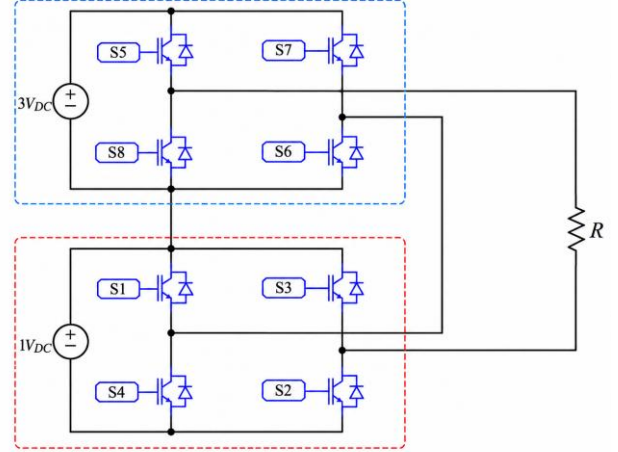


Fig. 1 Conceptual structure of the investigated nine-level asymmetric CHB. Two cascaded H-bridge cells are supplied by V_{dc} and $3V_{dc}$, yielding nine output levels

The switching-state map is reproduced in Table 1 because it is shared by every compared modulation strategy. Thus, changes in output voltage arise only from transition timing. The two zero states are retained separately for positive and negative half cycles, which keeps the switching logic symmetric while preserving the same zero output voltage.

Table 1 Switching states of the nine-level CHB (1 = ON, 0 = OFF).

Lvl.	S1	S2	S3	S4	S5	S6	S7	S8
0+	1	0	1	0	1	0	1	0
+1	1	1	0	0	1	0	1	0
+2	0	0	1	1	1	1	0	0
+3	1	0	1	0	1	1	0	0
+4	1	1	0	0	1	1	0	0
0-	0	1	0	1	0	1	0	1
-1	0	0	1	1	0	1	0	1
-2	1	1	0	0	0	0	1	1
-3	0	1	0	1	0	0	1	1
-4	0	0	1	1	0	0	1	1

2.2 Conventional NLM

Under quarter-wave symmetry, four transition angles in the interval 0° – 90° define the complete nine-level staircase. Conventional NLM changes level at the midpoint between adjacent voltage states, consistent with the nearest-level interpretation widely used in multilevel converters [15, 16, 18–25]. If the positive reference peak is normalized to n level increments, the switching angles reported by Kumar et al. [35] are

$$\alpha_i = \sin^{-1} \left(\frac{i - 0.5}{n} \right), \quad i = 1, 2, \dots, n \quad (1)$$

For $n = 4$, the four normalized decision thresholds are 0.5, 1.5, 2.5, and 3.5 when the sinusoidal reference has peak amplitude 4. The quarter-wave geometry is shown in Fig. 2. Defining $q_0 = 1/n = 0.25$ makes explicit the scalar multiplying every transition index without introducing any new modulation freedom:

$$\alpha_i(q) = \sin^{-1}[(i - 0.5)q] \quad (2)$$

The exact q_0 angles are $\{7.1808^\circ, 22.0243^\circ, 38.6822^\circ, 61.0450^\circ\}$. Kumar et al. report the same conventional operating point rounded to one decimal degree as $\{7.2^\circ, 22.0^\circ, 38.7^\circ, 61.1^\circ\}$. The distinction is small electrically but must be preserved when THD is compared at four or more decimal places.

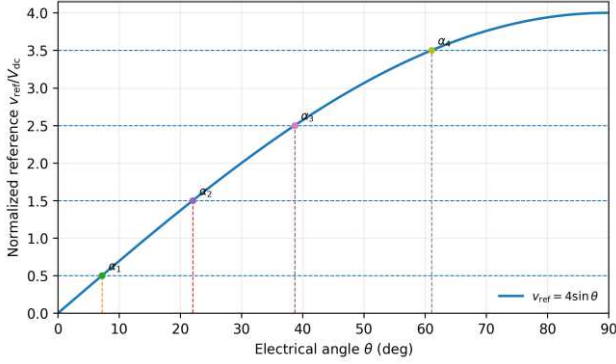


Fig. 2 Quarter-wave geometry of conventional NLM for the normalized nine-level reference. The transition thresholds are 0.5, 1.5, 2.5, and 3.5

2.3 Iterative Fibonacci-based NLM

The F-NLM algorithm begins from the conventional angles and perturbs them sequentially. Angle α_1 is decreased in 0.1° steps while THD improves. When further decrease is no longer accepted, the search advances to α_2 using 0.2° steps, then to α_3 using 0.3° steps, and finally to α_4 using 0.5° steps. The sequence of decrements is associated with Fibonacci numbers by Kumar et al. [35]. The significant reported iterations are reproduced in Table 2.

Table 2 Significant iterations reported for the F-NLM search [35].

Iter.	α_1 ($^\circ$)	α_2 ($^\circ$)	α_3 ($^\circ$)	α_4 ($^\circ$)	THD (%)
1	7.20	22.00	38.70	61.10	9.3706
17	7.00	21.50	37.30	61.10	9.3059
32	7.00	21.50	37.50	56.60	8.9717
50	6.90	20.90	36.10	56.60	8.9157
60	6.80	20.80	36.30	56.00	8.9028

For n switching angles, the analytical distortion criterion used to guide the reference search is given by Kumar et al. [35]. Related analytical formulations of THD in multilevel inverters with unequal dc sources are discussed more generally by Farokhnia et al. [38]; here, however, the exact functional adopted by Kumar et al. is retained to ensure a matched comparison.

$$\text{THD} = 100 \sqrt{\frac{\frac{\pi^2 n^2}{8} - \frac{\pi}{4} \sum_{i=0}^{n-1} (2i+1) \alpha_{i+1}}{[\sum_{i=1}^n \cos(\alpha_i)]^2} - 1} \quad (3)$$

Equation (3) is used in this paper for every analytical comparison and, in Section 3.7, as the objective whose stationary set is analyzed. This prevents an important category error: the analytical values in Table 2 must not be mixed with the circuit-simulation THD reported by Kumar et al. Because the final angle set is the output of a sequential four-variable procedure, the natural next question is whether those four coordinates remain independent at a stationary solution of this same criterion.

2.4 Problem statement

Let $A_K = \{\alpha_{1,K}, \alpha_{2,K}, \alpha_{3,K}, \alpha_{4,K}\}$ denote the final published F-NLM set. If A_K is genuinely four-dimensional, inversion of each angle through the NLM sinusoidal relation should generally produce distinct effective scalars. Conversely, if all four inversions produce nearly the same value, then the final result lies close to the one-dimensional manifold defined by Eq. (2). The problem can therefore be tested directly from published data, without modifying the original optimization objective and without assuming a Fibonacci ratio in advance.

This distinction is central to the logic of the paper. The first step is diagnostic: recover the hidden scalar, if it exists. Only after that step is the Fibonacci sequence reintroduced to determine whether a natural finite ratio family occupies the same numerical region. The final step is verification: implement conventional NLM, the final published angles, and the deterministic ratio in the same PSIM plant and determine whether their analytical relationship survives circuit-level simulation.

3 Deterministic Mathematical Reformulation

3.1 Inverse mapping of the published solution

Inverting Eq. (2) for each final published angle defines the scalar coordinate

$$q_{i,K} = \frac{\sin(\alpha_{i,K})}{i - 0.5} \quad (4)$$

Table 3 Inverse-mapped scalar values from the final angles of Kumar et al. [35].

i	$\alpha_{i,K}$ ($^\circ$)	$q_{i,K}$
1	6.8	0.236808
2	20.8	0.236738
3	36.3	0.236805
4	56.0	0.236868

The inverse mapping gives q_i values clustered around $q_{K,\text{mean}} \approx 0.2368$; the computed values are listed in Table 3 to six decimal places only to facilitate numerical reproduction. Their full range is approximately 0.055% of the mean. These digits should not be interpreted as measurement precision because Kumar et al. report the source angles only to 0.1° . If that one-decimal reporting is interpreted as a $\pm 0.05^\circ$ granularity, q_1 alone varies by approximately $\pm 0.73\%$, which is much larger than the observed inter-angle dispersion. The concentration is therefore treated as structural evidence, not as a high-precision estimate of an underlying physical constant.

Figure 3 quantifies the inverse-mapped dispersion at the five significant iterations reported by Kumar et al. [35]. The relative full range $(q_{\text{max}} - q_{\text{min}})/q_{\text{mean}}$ evolves from 0.371% at iteration 1 to 3.156%, 2.394%, and 1.930% at iterations 17, 32, and 50, then collapses to 0.055% at iteration 60. Re-evaluating Eq. (3) at the same five angle sets gives 9.370551%, 9.305879%, 8.971740%, 8.915702%, and 8.902832% THD, reproducing Table 2 to four decimal places. The published states therefore show a temporary departure from the scalar relation followed by sharp reconcentration at the final low-THD state, consistent with the stationarity result of Section 3.7.

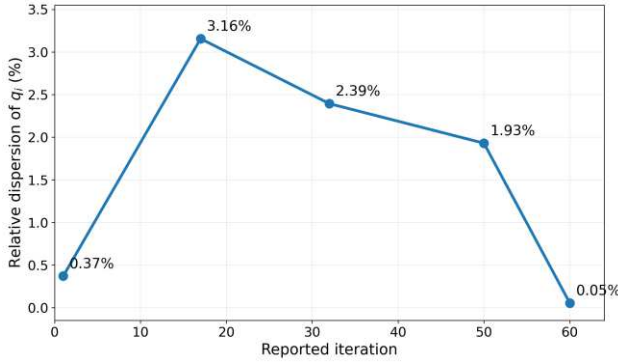


Fig. 3 Relative full-range dispersion of the inverse-mapped scalar coordinates q_i at the five significant iterations reported by Kumar et al. [35]. Only published iteration states are used; the final state reconcentrates to approximately 0.055% dispersion

3.2 One-parameter reconstruction test

A structural observation becomes stronger if removing the residual dispersion does not degrade the objective that generated the original solution. Accordingly, all four angles are reconstructed from the single mean $q_{K,\text{mean}}$ through Eq. (2). This yields $\{6.7999^\circ, 20.8061^\circ, 36.2999^\circ, 55.9774^\circ\}$. Using Eq. (3), the reconstructed set produces 8.902739% THD, whereas the rounded published final angles produce 8.902832%.

The difference is only 0.000092 percentage point and, notably, favors the exact one-parameter reconstruction. Therefore, the small residual differences among $q_{1,K}$ to $q_{4,K}$ do not encode a measurable THD benefit under the original analytical criterion. This test is important because it separates a numerical coincidence from a structural reduction: if the residual four-dimensional freedom were essential, forcing all angles onto one scalar would be expected to worsen the optimized criterion.

3.3 Fibonacci-based direct synthesis

After the one-dimensional region has been identified independently from the published switching angles, the Fibonacci sequence can be revisited without circular reasoning. The purpose is no longer to make Fibonacci itself prove the dimensional reduction, but to determine whether a compact finite ratio from the same sequence can represent the scalar region revealed by the published result.

Because the original F-NLM procedure is explicitly based on the Fibonacci sequence, successive ratios starting from $F_7 = 13$ are informative: $13/21 = 0.619048$, $13/34 = 0.382353$, and $13/55 = 0.236364$. These values follow the familiar progression associated with inverse powers of the golden ratio. More generally, for the classical Fibonacci sequence, fixed-gap ratios satisfy

This limiting behavior is consistent with the classical Fibonacci-golden-ratio relationship discussed by Benavoli et al. [29] and with the fixed-index quotient property described by Falcon [31]. Table 4 lists the three finite ratios used here and their corresponding limiting relations.

$$\frac{F_n}{F_{n+k}} \rightarrow \varphi^{-k}, \quad k = 1, 2, 3, \dots \quad (5)$$

Table 4 Successive finite Fibonacci ratios starting from $F_7 = 13$ and their limiting relations.

Ratio	Value	Limiting relation
$\frac{F_7}{F_8} = \frac{13}{21}$	0.619048	$\varphi^{-1} \approx 0.618034$
$\frac{F_7}{F_9} = \frac{13}{34}$	0.382353	$\varphi^{-2} \approx 0.381966$
$\frac{F_7}{F_{10}} = \frac{13}{55}$	0.236364	$\varphi^{-3} \approx 0.236068$

Among these compact successive ratios, $13/55$ is the first to enter the 0.236 numerical scale independently revealed by the inverse mapping. It is adopted here as a finite Fibonacci rational representative, not as a uniquely privileged constant and not as a proof of optimality. Section 3.7 subsequently shows that the low-THD stationary scalar of the adopted objective lies extremely close to this finite ratio.

$$q_F = \frac{F_7}{F_{10}} = \frac{13}{55} = 0.23636364 \quad (6)$$

The four switching angles then follow directly from the one-parameter NLM relation:

$$\alpha_{i,F} = \sin^{-1} \left[(i - 0.5) \left(\frac{13}{55} \right) \right], \quad i = 1, \dots, 4 \quad (7)$$

$$\alpha_F = \{6.7872^\circ, 20.7656^\circ, 36.2215^\circ, 55.8196^\circ\} \quad (8)$$

No THD minimization is used to choose q_F in this construction. The finite ratio is retained because it preserves the Fibonacci origin of the published method while providing a compact rational representation of the recovered scalar region. The stationarity theorem in Section 3.7 provides the underlying reduction principle; 13/55 should therefore be interpreted as a rational surrogate of the low-THD stationary scalar, not as the mathematical source of the one-dimensionality.

3.4 One-dimensional THD landscape

With Eq. (2), the four-angle analytical criterion becomes THD(q). The exact conventional condition is $q_0 = 0.25$, the recovered final iterative condition is represented by $q_{K,\text{mean}} \approx 0.236805$, and the deterministic Fibonacci choice is $q_F = 13/55$. Figure 4 shows the resulting curve. The rounded conventional set reported by Kumar et al. gives 9.3706%, whereas the exact q_0 family gives 9.3637%; this small distinction is retained throughout the paper.

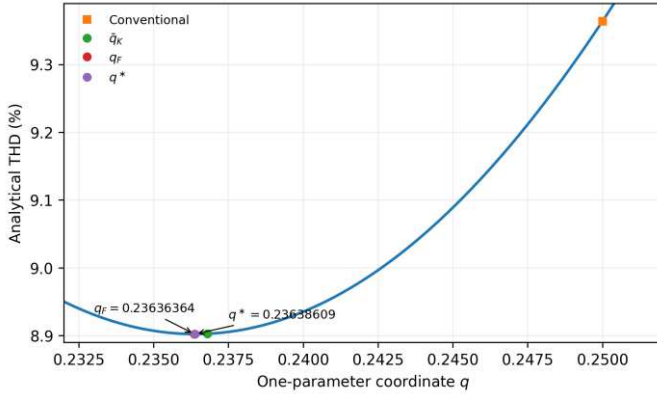


Fig. 4 Analytical THD(q) landscape obtained with Eq. (3) and the one-parameter family in Eq. (2). The point q^* is shown only as a posteriori one-dimensional minimum

Only after q_F is fixed is the one-dimensional curve minimized numerically as a posterior check. The low-THD minimum occurs at $q^* \approx 0.23638609$ with $\text{THD}(q^*) \approx 8.90229885\%$. The relative difference $|q_F - q^*|/q^*$ is approximately 0.0095%. This proximity is noteworthy, but it is not the selection mechanism. At this stage q^* is simply the minimum of the scalar curve; Section 3.7 establishes that every interior stationary point of the original multi-angle criterion must lie on the same one-parameter family and then compares this low-THD solution with an unrestricted four-angle numerical search.

3.5 Effective reference-scaling interpretation

The scalar q has a conventional interpretation that must be made explicit. If N is the number of positive switching transitions and m is an effective sinusoidal amplitude scaling, comparison of the conventional normalized-reference definition with Eq. (2) gives Eq. (9).

$$q = \frac{1}{Nm} \quad (9)$$

For $N = 4$, q_K , mean corresponds to $m_{\text{eff}} \approx 1.05572$ and q_F corresponds to $m_F \approx 1.05769$. Therefore, q is not proposed as a new independent modulation degree of freedom. Rather, it is the coordinate that reveals that the final output of a four-angle iterative procedure is almost equivalent to a common reference scaling.

This interpretation also defines the novelty boundary. The contribution is the stationarity-based reduction of the published unconstrained THD problem and the direct Fibonacci representation for the investigated nine-level case; it is not the invention of the scalar q itself, nor a claim that 13/55 is a universal optimum for NLM.

3.6 Direct threshold realization

The structural simplification is particularly transparent in the implementation domain. With a sinusoidal reference whose peak is normalized to 4, the decision threshold associated with α_i is $T_i = 4\sin(\alpha_i)$. Conventional NLM therefore uses $\{0.5, 1.5, 2.5, 3.5\}$. The final Kumar angles correspond to $\{0.473616, 1.420428, 2.368053, 3.316150\}$. For the deterministic formulation, substitution of Eq. (7) yields

$$T_{i,F} = 4(i - 0.5) \left(\frac{13}{55} \right) \quad (10)$$

which gives $\{0.472727, 1.418182, 2.363636, 3.309091\}$. Hence, once 13/55 is specified, even the inverse trigonometric calculation can be bypassed in a threshold implementation: each decision boundary follows from a simple arithmetic expression.

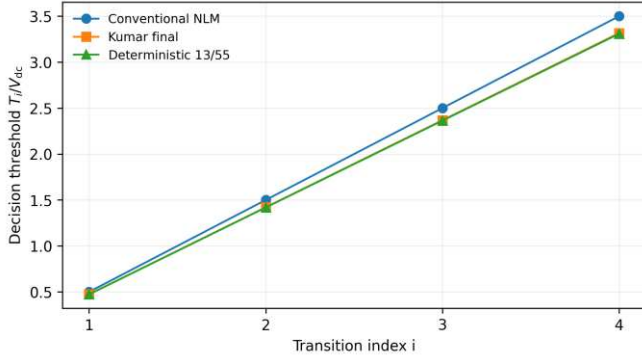


Fig. 5 Decision thresholds for conventional NLM, the final iterative F-NLM angles of Kumar et al. [35], and the deterministic $q_F = 13/55$ formulation

3.7 Stationarity of the analytical THD criterion

Subsections 3.1 and 3.2 showed empirically that the published final F-NLM angles lie close to the one-parameter family of Eq. (2). The present subsection establishes the stronger result: for the analytical THD functional of Eq. (3), the one-parameter structure is a necessary property of every interior stationary point.

For n positive-quarter-wave transition angles $\alpha = [\alpha_1, \dots, \alpha_n]^T$, expressed in radians, define the two angle-dependent aggregates

$$C(\alpha) = \sum_{i=1}^n \cos(\alpha_i), \quad (11)$$

$$S(\alpha) = \sum_{i=1}^n (2i-1) \alpha_i$$

$$U(\alpha) = \frac{\pi^2 n^2}{8} - \frac{\pi}{4} S(\alpha) - C^2(\alpha) \quad (12)$$

With these definitions, Eq. (3) can be written in the compact form shown below. Because the criterion is positive and $C > 0$ in the admissible interior domain, minimizing THD is equivalent to minimizing D .

$$D(\alpha) = \frac{U(\alpha)}{C^2(\alpha)} \quad (13)$$

$$THD(\alpha) = 100\sqrt{D(\alpha)}$$

Theorem 1 (One-parameter structure of interior stationary points). Let $\alpha \in (0, \pi/2)^n$ be an interior stationary point of the analytical distortion criterion in Eq. (3), i.e., $\partial D/\partial \alpha_i = 0$ for $i = 1, \dots, n$. Then there exists a scalar $q > 0$ such that

$$\sin(\alpha_i) = (i - 0.5)q, \quad i = 1, \dots, n \quad (14)$$

and therefore α belongs to the one-parameter family of Eq. (2).

Proof. From Eq. (11), the required derivatives are

$$\begin{aligned} \frac{\partial S}{\partial \alpha_i} &= 2i - 1, & \frac{\partial C}{\partial \alpha_i} &= -\sin(\alpha_i), \\ \frac{\partial U}{\partial \alpha_i} &= -\frac{\pi}{4}(2i - 1) + 2C\sin(\alpha_i) \end{aligned} \quad (15)$$

Applying the quotient rule to $D = U/C^2$ yields

$$\frac{\partial D}{\partial \alpha_i} = \frac{C \frac{\partial U}{\partial \alpha_i} + 2U\sin(\alpha_i)}{C^3} \quad (16)$$

Since $C > 0$, stationarity requires the numerator to vanish, which gives

$$2(C^2 + U)\sin(\alpha_i) = \frac{\pi}{4}(2i - 1)C \quad (17)$$

Because C and U are scalar functionals of the complete angle vector, they have the same value for every index i . Equation (14) follows immediately with

$$q = \frac{(\pi/4)C}{C^2 + U} = \frac{C}{\pi n^2/2 - S} \quad (18)$$

The second equality follows from $C^2 + U = \pi^2 n^2/8 - (\pi/4)S$. Thus every interior stationary point of Eq. (3) is constrained to the sinusoidal one-parameter family. ■

An independent derivation reaches the same angle geometry. Ghasemian et al. [39] prove that, for a prescribed fundamental component, the minimum-THD waveform is nearest-level and satisfies $\theta_k = \arcsin[(2k-1)L/(2A)]$, which becomes Eq. (2) under $q = L/A$. Theorem 1 addresses the complementary unconstrained problem: it is a necessary stationarity condition for Kumar's THD objective and yields the scalar fixed-point equation, including multiple interior roots. Unlike the prescribed-fundamental problem, the present formulation does not select q through a target fundamental component; q must additionally satisfy the unconstrained stationarity fixed-point condition.

A physical reading of q can also be obtained. For the normalized staircase described by Eq. (3), the peak fundamental component and total mean-square output voltage can be expressed as $V_{1,pk}$ and V_{rms}^2 , respectively, in Vdc-based normalized units. Substitution into Eq. (18) gives the physical interpretation below. At a stationary point, q is therefore not a free additional degree of freedom; it is fixed by the waveform that it generates.

$$\begin{aligned} V_{1,pk} &= \frac{4}{\pi}C, & V_{rms}^2 &= n^2 - \frac{2}{\pi}S, \\ q &= \frac{V_{1,pk}}{2V_{rms}^2} = \frac{V_{1,rms}}{\sqrt{2}V_{rms}^2} \end{aligned} \quad (19)$$

Substituting Eq. (2) into Eq. (18) reduces the n -angle stationarity system to a scalar fixed-point condition. For $n = 4$, this condition has more than one interior root. The low-THD root is $q^* = 0.23638609$, giving $THD = 8.90229885\%$, whereas another interior stationary root occurs near $q =$

0.28349655 and gives approximately 12.94% THD. Accordingly, no uniqueness claim is made; the relevant solution is the low-THD stationary root.

As an independent four-angle numerical confirmation, Eq. (3) was minimized over the four ordered angles using 400 randomly initialized Nelder-Mead searches. The best solution obtained was $\{6.78783^\circ, 20.76765^\circ, 36.22553^\circ, 55.82761^\circ\}$ with THD = 8.90229885%. Inverse mapping of this solution gives q_i values equal to q^* within numerical tolerance (full relative dispersion below $10^{-5}\%$). This numerical result corroborates the low-THD stationary solution but is not presented as an analytical proof of global optimality over all boundary cases.

The theorem changes the interpretation of the published data. The final reconcentration of the inverse-mapped q_i values near 0.2368 is the behavior expected when a search approaches an interior stationary point of Eq. (3). The residual 0.055% dispersion in Table 3 is smaller than the variation induced by the one-decimal reporting granularity of the published angles and therefore should not be overinterpreted as a separate four-dimensional feature.

The Fibonacci ratio $q_F = 13/55$ differs from the low-THD stationary scalar q^* by approximately 0.0095%. Their analytical THDs differ by about 1.3×10^{-6} percentage point. Thus, after Theorem 1 establishes the scalar structure, 13/55 can be interpreted cleanly as a compact rational Fibonacci surrogate of the low-THD stationary coordinate while preserving the sequence-based origin of the published F-NLM method.

4 Numerical Verification

4.1 Verification strategy

The numerical stage provides an independent circuit-level path for testing the analytical conclusions. PSIM evaluates the switching-state circuit directly rather than reusing Eq. (3), so agreement between the analytical formulation and the circuit simulation constitutes a controlled cross-check. Because the research question concerns whether different angle-generation procedures synthesize the same ideal staircase, the verification intentionally retains ideal switching devices and matched operating conditions. Hardware-specific nonidealities are deferred to implementation-focused studies, where they become part of the research question rather than confounding variables.

Three cases are therefore implemented on one common PSIM power stage: (i) exact conventional NLM, (ii) the final F-NLM angle set reported by Kumar et al. [35], and (iii) the deterministic $q_F = 13/55$ formulation. The topology, sources, switching-state map, load, reference amplitude, fundamental frequency, and THD measurement procedure remain unchanged. This is consistent with controlled-comparison practice in which only the mathematical formulation under investigation is allowed to vary [37].

4.2 PSIM model and common conditions

The PSIM model uses normalized dc sources of 1 V and 3 V, giving the same 1:3 asymmetry as the reference converter. A 10- Ω resistive load is connected to the output. Ideal switching devices are used because losses are not part of the objective.

The sinusoidal reference has a peak magnitude of 4 and a fundamental frequency of 50 Hz, reproducing the operating frequency used in the reference simulations. The THD analyzer is configured with 50 Hz as the fundamental frequency.

Table 5 Common PSIM verification conditions.

Parameter	Value
Converter	Single-phase 9-level asymmetric CHB
DC-source ratio	1:3
Normalized sources	1 V and 3 V
Reference	± 4 V sinusoid
Fundamental frequency	50 Hz
Load	$R = 10 \Omega$
Switch model	Ideal
THD measurement	PSIM, 50-Hz fundamental
Acquisition window	One 20-ms fundamental period

4.3 Modulation cases

The conventional case uses thresholds $\{0.5, 1.5, 2.5, 3.5\}$. The iterative-reference case uses the final published angles $\{6.8^\circ, 20.8^\circ, 36.3^\circ, 56.0^\circ\}$, implemented through the equivalent thresholds $\{0.473616, 1.420428, 2.368053, 3.316150\}$. The deterministic case uses Eq. (10) directly. No numerical optimization or THD feedback is executed inside PSIM for q_F .

Because all three implementations share the same switching-state table, a change of threshold modifies only the instant at which the converter moves between adjacent voltage levels. This is the desired isolation: if the iterative and deterministic cases produce nearly the same RMS voltage and THD, the agreement can be attributed to their nearly coincident transition geometry rather than to differences in the plant.

4.4 Evaluation metrics

The principal metric is voltage THD because it is the objective used by Kumar et al. to guide the iterative search. Total RMS output voltage and RMS load current are retained as complementary quantities so that a decrease in normalized THD is not interpreted without considering the accompanying change in fundamental-voltage utilization. Waveform plots with the common sinusoidal reference provide a qualitative check of the staircase timing.

Weighted THD and an extended table of individual harmonics are intentionally not introduced as additional optimization objectives. They are useful in broader modulation comparisons, but they would answer a different research question. The present paper is deliberately restricted to testing whether the published THD-guided iterative result can be represented and reproduced deterministically.

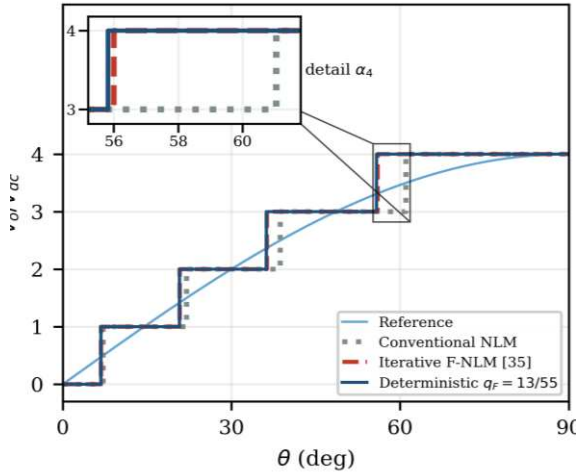


Fig. 6 Quarter-wave staircase of the three evaluated angle sets against the common 50-Hz sinusoidal reference, with a detail of the fourth transition; the iterative and deterministic waveforms are visually indistinguishable over the full quarter cycle

5 Results and Discussion

5.1 Analytical comparison

Table 6 Analytical comparison of the evaluated angle sets.

Method	Angles (deg)	V1/Vdc	THD (%)
Conv. rounded	7.20, 22.00, 38.70, 61.10	4.0527	9.3706
Exact q_0	7.1808, 22.0243, 38.6822, 61.0450	4.0539	9.3637
Kumar final	6.80, 20.80, 36.30, 56.00	4.1927	8.902832
1-D $q_{K,mean}$	6.7999, 20.8061, 36.2999, 55.9774	4.1930	8.902739
Proposed 13/55	6.7872, 20.7656, 36.2215, 55.8196	4.1973	8.902300

Table 6 first confirms that the conventional NLM result depends slightly on whether the one-decimal angles printed by Kumar et al. or the exact $q_0 = 0.25$ values are used. The resulting THDs, 9.3706% and 9.3637%, differ by 0.0069 percentage point. This difference is not a physical change in NLM and is therefore treated only as a rounding effect.

The more consequential comparison is between the final iterative solution and the exact one-dimensional reconstruction. Their normalized fundamental components are 4.1927Vdc and 4.1930Vdc, respectively, while their THDs are 8.902832% and 8.902739%. Thus, collapsing the final angle set onto a single q coordinate preserves both the fundamental component and the distortion criterion. This result is the analytical evidence that the original four-angle procedure converged to an effectively one-dimensional operating region.

The proposed $q_F = 13/55$ set gives 8.902300% analytical THD and $V1/Vdc = 4.1973$. Relative to the final Kumar set, the THD difference is 0.000532 percentage point. This near-

identity is the expected result rather than a marginal performance gain: it verifies that the direct Fibonacci representation reaches the same low-THD stationary region that the iterative procedure searched for.

5.2 Fundamental-voltage utilization

THD is a ratio normalized by the fundamental component, so the increase in $V1$ must be reported rather than hidden. Relative to exact conventional NLM, the final Kumar set increases the normalized fundamental component by approximately 3.42%, while the deterministic set increases it by approximately 3.54%. Kumar et al. observed the same qualitative behavior in their MATLAB/Simulink resistive-load case, where the fundamental voltage increased from 157.7 V to 163.1 V as THD decreased.

The physical reason is geometric. The optimized/deterministic transitions occur slightly earlier than the conventional midpoint transitions, so the waveform spends more time at higher staircase levels during the positive quarter cycle and, by symmetry, at more negative levels during the negative quarter cycle. The area projected onto the fundamental sinusoid therefore increases. This should be described as greater utilization of the available dc-source voltage, not as voltage boost, because neither source magnitude nor converter topology is changed.

5.3 PSIM results

Table 7 PSIM results under identical 50-Hz conditions.

Method	Vrms (V)	Irms (A)	THD (%)	Δ THD
Conventional	2.8789	0.2879	9.3602	—
Kumar final	2.9759	0.2976	8.9033	-4.88%
Deterministic	2.9782	0.2978	8.9023	-4.89%

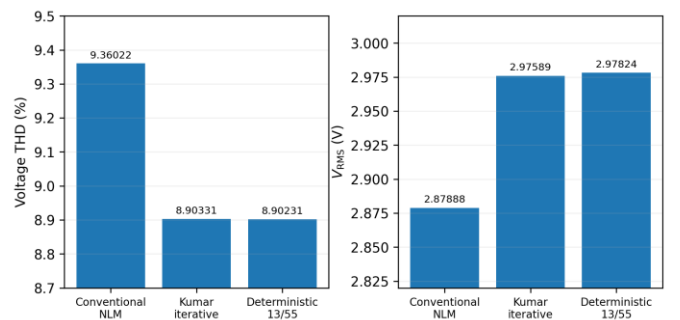


Fig. 7 PSIM comparison of voltage THD and total RMS output voltage under identical 50-Hz, 10- Ω conditions

Conventional NLM produces 9.3602% THD. Substituting the final published Kumar angles reduces the PSIM result to 8.9033%, while the deterministic q_F formulation gives 8.9023%. Relative to conventional NLM, these correspond to reductions of approximately 4.881% and 4.892%, respectively. The iterative and deterministic cases differ by

only about 0.001 percentage point; this difference is not claimed as an electrical improvement, but as circuit-level evidence that the deterministic construction reproduces the searched operating region.

The RMS quantities show the same equivalence. The final Kumar set gives approximately 2.9759 V and 0.29759 A, whereas q_F gives 2.9782 V and 0.29782 A. The voltage difference is approximately 0.079%. Because the load is purely resistive and fixed at 10Ω , the current follows the same relative change. These data independently reproduce the analytical conclusion that the two switching laws occupy essentially the same region.

Table 8 Agreement between analytical and PSIM voltage THD.

Case	Analytical (%)	PSIM (%)	Difference (pp)
Exact conventional q_0	9.3637	9.3602	-0.0035
Kumar final	8.9028	8.9033	+0.0005
Deterministic 13/55	8.9023	8.9023	<0.0001

The cross-check in Table 8 is useful because the analytical expression and PSIM are independent calculation paths. At the precision shown, the deterministic case gives the same THD to four decimal places, while the final Kumar angles differ by about 0.0005 percentage point; the conventional case differs by approximately 0.0035 percentage point. No interpretation is attached to differences below the reported numerical resolution. The qualitative trend also agrees with the MATLAB/Simulink results of Kumar et al. [35], where moving from conventional NLM to the final F-NLM angles likewise increases voltage utilization while reducing THD; absolute values are not compared because the source scale and the reported voltage metric differ.

5.4 Meaning of the one-parameter structure

The published trajectory provides empirical evidence for the structure established by Theorem 1. Its relative q_i dispersion increases from 0.371% at the rounded conventional state to 3.156% at iteration 17, then contracts to 2.394%, 1.930%, and finally 0.055% at iterations 32, 50, and 60. This final reconcentration is consistent with the necessary stationary condition: an interior stationary point of Eq. (3) must satisfy one common scalar relation, although intermediate coordinate-search states need not do so.

The scalar q is inherent in NLM geometry and is not introduced here as a new degree of freedom. A prescribed-fundamental minimum-THD formulation independently reaches the same angle family [39]. The distinct result here is that the stationary set of the unconstrained THD functional used by Kumar et al. is confined to that family and that its fixed-point condition selects the interior stationary roots. Thus, the published four-coordinate search is structurally redundant for locating an interior stationary solution of that criterion, even though its intermediate states are not scalar.

This distinction parallels the type of contribution emphasized by Menzinger et al. [37]: two formulations may synthesize practically equivalent electrical outputs while differing materially in mathematical representation and computational path. Here, the stronger result is that the reduction is not only empirical. Theorem 1 provides the mechanism: the gradient equations share the same global aggregates C and U , forcing every stationary angle to satisfy the same proportional sine law.

5.5 Computational interpretation

The phrase 'search elimination' must still be interpreted carefully. The 60 reported iterations in Kumar et al. occur during design-stage selection of the angles; they do not imply that a converter using the final stored angles executes 60 THD calculations every fundamental cycle. Consequently, no $60\times$ online speedup is claimed.

At the design stage, Theorem 1 reduces the stationary problem from n angle coordinates to the scalar fixed-point condition of Eqs. (2) and (18). For $n = 4$, this is a 75% reduction in design-variable dimensionality. The published THD values also show diminishing returns: 85.3% of the total reported reduction is obtained by iteration 32 and 97.2% by iteration 50; the last ten reported iterations contribute only 2.8%. The final rounded-angle value, 8.902832%, is only 0.000533 percentage point above the low-THD stationary value 8.902299%. Once $q_F = 13/55$ is selected as the rational surrogate, Eq. (10) gives the thresholds directly without an iterative THD-guided angle-search stage. The computational contribution is therefore reduced design dimensionality and search structure, not online runtime speed.

5.6 Scope and limitations

Four limitations define the conclusions. First, Theorem 1 applies only to interior stationary points of the THD functional in Eq. (3) under the waveform assumptions adopted in this study; no claim is made beyond this analytical setting. Second, it addresses an objective in which the fundamental component is not prescribed; prescribed-fundamental minimum-THD formulations [39] answer a different design question despite sharing the same angle family. Third, $q_F = 13/55$ is a close surrogate for the investigated nine-level case, not a universal ratio, and the unrestricted numerical search is corroborative rather than a proof of global optimality over all boundary cases. Fourth, the PSIM verification intentionally uses an ideal power stage and excludes semiconductor drops, dead time, source variation, controller latency, and measurement effects.

6 Conclusion

This paper has shown that the near one-dimensional structure observed in the final Fibonacci-based NLM angles of Kumar et al. is not merely an empirical feature of one reported solution. Inverse mapping first reveals q_i values clustered near 0.2368, but the central result is Theorem 1: every interior stationary point of the analytical THD criterion used in the reference search necessarily satisfies $\sin \alpha_i = (i - 0.5)q$. The stationary multi-angle problem is therefore constrained to a scalar sinusoidal family for any n within the class described by that functional.

For the investigated nine-level case, the low-THD scalar stationary solution is $q^* \approx 0.236386$ with $\text{THD} \approx 8.902299\%$. An independent unrestricted four-angle numerical minimization recovers the same solution within numerical tolerance. The finite Fibonacci ratio $13/55 = 0.236364$ is then interpreted as a compact rational surrogate that preserves the sequence-based origin of the published method; it produces 8.902300% analytical THD. Independent 50-Hz PSIM results give 8.9023% for the deterministic formulation and 8.9033% for the final published iterative angles.

The near-identical analytical and PSIM distortion values are therefore the intended verification result rather than a claim of further THD improvement. The contribution is the analytical identification of the stationary-set dimensionality, its application to the reinterpretation of the published four-angle search, and a direct Fibonacci-based realization for the considered converter. The theorem is specific to the THD functional analyzed here, $13/55$ is not a universal optimum, and the numerical four-angle search is corroborative rather than a proof of global optimality over all boundary cases.

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Statements and Declarations

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Author Contributions

A.W.S.B. conceived the study, developed the mathematical formulation, performed the analytical investigation and simulations, prepared the figures, and wrote the original manuscript. S.V.G.O. supervised the research, contributed to methodology and technical interpretation, and critically reviewed the manuscript. D.G.C. contributed to technical validation, critical review of the methodology and results, and manuscript revision. All authors reviewed and approved the final manuscript.

Data Availability

The datasets generated and analyzed during the current study, including analytical calculations and simulation results, are available from the corresponding author upon reasonable request. Benchmark data used for comparison were obtained from the published study by Kumar et al. cited in the manuscript.

Competing Interests

The authors declare no competing interests.

Use of Generative AI

The core research question, methodological choices, mathematical formulation, and scientific interpretation were developed integrally by the authors. Leading the conceptual and mathematical development. During manuscript preparation, OpenAI ChatGPT and Anthropic Claude were used as assistive tools for language editing, structural organization, support and verification, and preparation of data plots. All AI-assisted outputs were critically evaluated, corrected where necessary, and approved by the authors, who take full responsibility for the final manuscript.

References

- Rodriguez J, Lai J-S, Peng FZ (2002) Multilevel inverters: a survey of topologies, controls, and applications. *IEEE Trans Ind Electron* 49(4):724–738. <https://doi.org/10.1109/TIE.2002.801052>
- Franquelo LG, Rodriguez J, Leon JI, Kouro S, Portillo R, Prats MAM (2008) The age of multilevel converters arrives. *IEEE Ind Electron Mag* 2(2):28–39. <https://doi.org/10.1109/MIE.2008.923519>
- Malinowski M, Gopakumar K, Rodriguez J, Perez MA (2010) A survey on cascaded multilevel inverters. *IEEE Trans Ind Electron* 57(7):2197–2206. <https://doi.org/10.1109/TIE.2009.2030767>
- Hameed SR, AL-Mhana TH (2024) Cascaded H-bridge multilevel inverter: review of topologies and pulse width modulation. *Tikrit J Eng Sci* 31(1):138–151. <https://doi.org/10.25130/tjes.31.1.12>
- Saeedian M, Adabi J, Hosseini SM (2017) Cascaded multilevel inverter based on symmetric-asymmetric DC sources with reduced number of components. *IET Power Electron* 10(12):1468–1478. <https://doi.org/10.1049/iet-pel.2017.0039>
- Leon JI, Kouro S, Franquelo LG, Rodriguez J, Wu B (2016) The essential role and the continuous evolution of modulation techniques for voltage-source inverters in the past, present, and future power electronics. *IEEE Trans Ind Electron* 63(5):2688–2701. <https://doi.org/10.1109/TIE.2016.2519321>
- Edpuganti A, Rathore AK (2015) A survey of low switching frequency modulation techniques for medium-voltage multilevel converters. *IEEE Trans Ind Appl* 51(5):4212–4228. <https://doi.org/10.1109/TIA.2015.2437351>
- Antônio-Ferreira A, Collados-Rodríguez C, Gomis-Bellmunt O (2018) Modulation techniques applied to medium voltage modular multilevel converters for renewable energy integration: A review. *Electr Power Syst Res* 155:21–39. <https://doi.org/10.1016/j.epsr.2017.08.015>
- El Kattel MB, Mayer R, Oliveira SVG (2018) Conversor CC–CC abaixador elevador trifásico isolado em alta frequência. *Eletrônica de Potência* 23(4):454–465. <https://doi.org/10.18618/REP.2018.4.0004>
- Oliveira SVG (2006) Conversor CC-CC elevador trifásico com isolamento em alta frequência. Doctoral thesis, Universidade Federal de Santa Catarina (UFSC), Florianópolis, Brazil <https://repositorio.ufsc.br/handle/123456789/88722>
- Nguyen-Vinh Q, Le T-L (2025) Optimal frequency modulation of carrier waves and its application to induction motor drive systems. *Electr Eng* 107:6791–6803. <https://doi.org/10.1007/s00202-024-02887-7>
- Zeng R, Shi W, Guo H (2026) Influence of modulation strategies on busbar inductance in motor controllers under thermomagnetic conditions. *Electr Eng* 108:252. <https://doi.org/10.1007/s00202-026-03610-4>
- Francis R, Meganathan D (2019) A dual-mode cascaded H-bridge multilevel inverter for improving THD. *Electr Eng* 101:225–237. <https://doi.org/10.1007/s00202-019-00768-y>
- Palle K, Vattikonda S, Vangala SS, Tammali K, Bhanuchandar A, Mohandas A (2022) A generalized non-carrier modulation technique for an asymmetric source configuration of single-phase CHB-MLI topology using PLECS tool. In: 2022 IEEE 2nd International Conference on Sustainable Energy and Future Electric Transportation (SeFeT), pp 1–5. <https://doi.org/10.1109/SeFeT55524.2022.9909071>
- Deng Y, Harley RG (2015) Space-vector versus nearest-level pulse width modulation for multilevel converters. *IEEE Trans Power Electron* 30(6):2962–2974. <https://doi.org/10.1109/TPEL.2014.2331687>

16. Kumar AR, Deepa T, Padmanaban S, Kothari DP (2019) A guide to nearest level modulation and selective harmonics elimination modulation scheme for multilevel inverters. In: 2019 Innovations in Power and Advanced Computing Technologies (i-PACT), pp 1–8. <https://doi.org/10.1109/i-PACT44901.2019.8960205>
17. Castellain DG (2024) Estratégias de modulação aplicadas a Conversor CA-CA Modular Multinível. Dissertation, Universidade do Estado de Santa Catarina (UDESC), Joinville, Brazil https://www.udesc.br/arquivos/cct/id_cpmenu/729/Daniel_Tese_1740591_7950361_729.pdf
18. Hu P, Jiang D (2015) A level-increased nearest level modulation method for modular multilevel converters. IEEE Trans Power Electron 30(4):1836–1842. <https://doi.org/10.1109/TPEL.2014.2325875>
19. Xiong C-L, Wu X-J, Diao F, Feng X-Y (2016) Improved nearest level modulation for cascaded H-bridge converter. Electron Lett 52(8):648–650. <https://doi.org/10.1049/el.2015.4220>
20. Nguyen MH, Kwak S (2020) Nearest-level control method with improved output quality for modular multilevel converters. IEEE Access 8:110237–110250. <https://doi.org/10.1109/ACCESS.2020.3001587>
21. Wang W, Ma K, Cai X (2021) Flexible nearest level modulation for modular multilevel converter. IEEE Trans Power Electron 36(12):13686–13696. <https://doi.org/10.1109/TPEL.2021.3089706>
22. Yin J, Leon JI, Perez MA, Franquelo LG, Marquez A, Li B, Vazquez S (2021) Variable rounding level control method for modular multilevel converters. IEEE Trans Power Electron 36(4):4791–4801. <https://doi.org/10.1109/TPEL.2020.3020941>
23. Sarwer Z, Sarwar A, Zaid M, Hussan MR, Tariq M, Alamri B, Alahmadi A (2021) Implementation of a novel variable structure nearest level modulation on cascaded H-bridge multilevel inverter. IEEE Access 9:133974–133986. <https://doi.org/10.1109/ACCESS.2021.3113704>
24. Gobburi HB, Borghate VB, Meshram PM (2023) A level enhanced nearest level control for modular multilevel converter without using sensors. IEEE Trans Ind Appl 59(4):4364–4374. <https://doi.org/10.1109/TIA.2023.3270113>
25. Gobburi HB, Borghate VB, Patro SK, Meshram PM (2025) An improved NLC rotation strategy for MMC. Int J Circuit Theory Appl 53:7233–7245. <https://doi.org/10.1002/cta.4483>
26. Sharma S, Kumar J, Sirohi V, Jat GL (2022) Performance evaluation of multilevel inverters for low switching frequency modulation techniques. In: 2022 IEEE Delhi Section Conference (DELCON), pp 1–6. <https://doi.org/10.1109/DELCON54057.2022.9753311>
27. Bhowmick S, Vasu R, Chattopadhyay SK, Chakraborty C (2022) A particle swarm optimization based hybrid pulse width modulation strategy for single DC source per phase fed binary asymmetric cascaded H-bridge photovoltaic inverter. In: 2022 IEEE 1st Industrial Electronics Society Annual On-Line Conference (ONCON), pp 1–6. <https://doi.org/10.1109/ONCON56984.2022.10126691>
28. Verma S, Agarwal V (2012) Realization of Fibonacci search algorithm for single phase matrix converter. In: 2012 IEEE 5th India International Conference on Power Electronics (IICPE), pp 1–4. <https://doi.org/10.1109/IICPE.2012.6450423>
29. Benavoli A, Chisci L, Farina A (2009) Fibonacci sequence, golden section, Kalman filter and optimal control. Signal Process 89(8):1483–1488. <https://doi.org/10.1016/j.sigpro.2009.02.003>
30. Assimakis N, Adam M, Triantafyllou C (2013) Lainiotis filter, golden section and Fibonacci sequence. Signal Process 93(4):721–730. <https://doi.org/10.1016/j.sigpro.2012.09.014>
31. Falcon S (2016) The k-Fibonacci difference sequences. Chaos Solitons Fractals 87:153–157. <https://doi.org/10.1016/j.chaos.2016.03.038>
32. Sundararaman K, Elavarasu R, Sivanand R (2020) A switched capacitor hybrid Fibonacci converter with reduced switches for LED applications. In: 2020 International Conference on Power, Energy, Control and Transmission Systems (ICPECTS), pp 1–6. <https://doi.org/10.1109/ICPECTS49113.2020.9337023>
33. Eguchi K, Hirata S, Shimoji M, Zhu H (2012) Design of a step-up/step-down k (=2,3,...) Fibonacci dc-dc converter designed by switched-capacitor techniques. In: 2012 Fifth International Conference on Intelligent Networks and Intelligent Systems, pp 170–173. <https://doi.org/10.1109/ICINIS.2012.67>
34. Khoudiri A, Guesmi K, Hamzaoui A, Essounbouli N, Mahi D (2013) Modeling of DC-DC converters based on Fibonacci-Horner theorem. In: 3rd International Conference on Systems and Control, pp 140–145. <https://doi.org/10.1109/ICoSC.2013.6750849>
35. Kumar AR, Padmanaban S, Subudhi PS, Blaabjerg F, Dhanamjayulu C (2020) A Fibonacci series based nearest level modulation scheme for multilevel inverters. In: 2020 IEEE 21st Workshop on Control and Modeling for Power Electronics (COMPEL), pp 1–5. <https://doi.org/10.1109/COMPEL49091.2020.9265866>
36. Luo D, Lin D, Zhang W, Lian W (2024) A novel jittered-carrier phase-shifted sine pulse width modulation for cascaded H-bridge converter. J Eng 2024:e12391. <https://doi.org/10.1049/tje2.12391>
37. Menzinger JS, Otero Llosa AH, Liberado EV, dos Santos WM (2026) Comparative evaluation of SVPWM duty-cycle calculation methods for two-level and quasi-two-level flying-capacitor inverters. J Control Autom Electr Syst. <https://doi.org/10.1007/s40313-026-01321-6>
38. Farokhnia N, Vadizadeh H, Fathi SH, Anvariash F (2011) Calculating the formula of line-voltage THD in multilevel inverter with unequal DC sources. IEEE Trans Ind Electron 58(8):3359–3372. <https://doi.org/10.1109/TIE.2010.2089938>
39. Ghasemian A, Mohamadian S, Buccella C, Cecati C (2025) Analyzing the relationship between minimum THD and nearest level waveforms in multilevel inverters. In: 2025 International Conference on Clean Electrical Power (ICCEP). <https://doi.org/10.1109/ICCEP65222.2025.11143663>