

Sampling Limits on Recovering Finite-Depth Double-Diffusive Interface Structure from Hydrographic Profiles

Sriram Kalathoor

sriram2@gatech.edu

Georgia Institute of Technology

Research Article

Keywords: double diffusion, salt fingers, hydrographic sampling, interface structure, Ice-Tethered Profiler, GO-SHIP

Posted Date: October 15th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-11118018/v1>

License: © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Additional Declarations: No competing interests reported.

Sampling Limits on Recovering Finite-Depth Double-Diffusive Interface Structure from Hydrographic Profiles

Sriram P. Kalathoor^{1*}

^{1*}Daniel Guggenheim School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, Georgia, USA.

Corresponding author(s). E-mail(s): sriram2@gatech.edu;

Abstract

Hydrographic profiles resolve only a sparse projection of a spatially organized double-diffusive interface. We quantify which scalar-interface properties survive that projection by sampling four controlled finite-depth salt-finger simulations with vertical profiles and section-like station arrays. The simulation labels are fixed by prescribed initial spectra, while the measured outputs are salinity-gradient width, gradient localization and entropy, temperature–salinity correlation, and horizontal interface scale. Ice-Tethered Profiler, Argo, and GO-SHIP records are screened independently with TEOS-10 Turner angle and buoyancy frequency to establish their thermohaline regimes and sampling characteristics. The retained ITP windows are predominantly diffusive-convective rather than salt-fingering-favorable, demonstrating why profile morphology cannot establish double-diffusive branch by itself. In the synthetic profiles, median salinity-gradient-width error remains at or below 0.229 for spacing bins wholly below one-half of the native interface width, but is at least 0.561 once spacing equals or exceeds that width. In section-like sampling, the exact dominant mode is recovered in 0.822 of deterministic phase evaluations when station spacing is no greater than one-half of the target wavelength and in none of the evaluations at coarser spacing. Retaining one, three, and five of nine aligned profiles gives median interface-width errors of 0.289, 0.150, and 0.093 and mean prescribed-family contrast-sign retention of 0.693, 0.792, and 0.856. These are deterministic information-retention measures, not predictive accuracies. The results provide scale-aware limits for observing scalar-interface structure; heat, salt, and buoyancy flux remain unobserved without additional velocity or microstructure measurements.

Keywords: double diffusion; salt fingers; hydrographic sampling; interface structure; Ice-Tethered Profiler; GO-SHIP

1 Introduction

Temperature and salinity profiles are central to identifying thermohaline structure in the ocean, but a profile samples only one vertical line through a spatially extended flow. This limitation is especially important for double diffusion. The local ordering of temperature, salinity, and density gradients determines whether salt fingering or diffusive convection is favorable, while the nonlinear interface may also vary horizontally and evolve over a finite vertical extent (Ruddick 1983; Schmitt 1994; Radko 2013). A sharp scalar gradient can therefore be physically informative without uniquely describing the surrounding interface.

Ocean observing systems span markedly different geometries. Ice-Tethered Profilers provide vertically dense records beneath Arctic pack ice (Krishfield et al. 2008; Toole et al. 2011), Argo supplies globally distributed but horizontally asynchronous autonomous profiles (Roemmich et al. 2009, 2019), and GO-SHIP sections combine high-quality vertical casts with finite station spacing (Sloyan et al. 2019). These systems have revealed double-diffusive staircases and their regional variability (Timmermans et al. 2008; Shibley et al. 2017; van der Boog et al. 2021). Their sampling, however, does not directly reveal which features of a spatial interface were lost between casts or levels.

Resolved simulations provide a controlled way to measure that loss because both the complete field and its sparse projection are available. Direct simulations have established nonlinear salt-finger evolution, secondary instability, and transport in idealized domains (Simeonov et al. 2009; Traxler et al. 2011). The present study uses four finite-depth salt-finger histories whose initial horizontal spectra are prescribed. We do not infer a transport pathway from hydrographic morphology. Instead, we ask how accurately sparse profiles retain scalar-interface measurements and how section-like sampling recovers a known horizontal scale.

Three distinctions control the analysis. First, the simulation families are defined before the flow evolves, using the imposed spectrum and random realization; observed morphology is an outcome rather than a class definition. Second, all sampling results are reported relative to a resolved scale: vertical spacing is normalized by the native salinity-gradient width, and station spacing is normalized by the native dominant wavelength. Model coordinates are not equated with $\bar{d}$, meters, or kilometers. Third, the combinations of profiles and sampling phases are deterministic reductions of four histories. They quantify information retention but cannot support an out-of-sample classification accuracy.

The study addresses four questions. Which thermohaline regimes are represented by the selected real-profile windows? How does vertical coarsening alter local scalar-interface measurements? How does station spacing and sampling phase alter horizontal-scale recovery? How rapidly do measurements from spatial subsets approach the corresponding nine-profile summary? Together, the answers define what sparse hydrography can recover about interface structure and what remains outside the measurement.

2 Source Fields and Sampling Methods

2.1 Finite-depth salt-finger simulations

The source fields solve the incompressible Boussinesq equations for velocity $\mathbf{u} = (u, v, w)$, pressure p , temperature T , and salinity S ,

$$\nabla \cdot \mathbf{u} = 0, \quad (1)$$

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \text{Pr} \nabla^2 \mathbf{u} + \text{Pr} b \mathbf{e}_z, \quad (2)$$

$$\partial_t T + \mathbf{u} \cdot \nabla T = \nabla^2 T, \quad (3)$$

$$\partial_t S + \mathbf{u} \cdot \nabla S = \tau \nabla^2 S, \quad (4)$$

with the linear nondimensional buoyancy $b = T - S$. The control parameters are $\text{Pr} = 7$, $\tau = 0.01$, and density ratio $R_\rho = 1.2$. The temperature jump is $\Delta T = 1$ and the salinity jump is $\Delta S = \Delta T / R_\rho$. Hence $1 < R_\rho < 1/\tau$, placing the modeled interface in the salt-fingering range.

The domain is $(L_x, L_y, L_z) = (164.3674, 82.1837, 164.3674)$ and the grid contains $384 \times 192 \times 960$ finite-volume cells, giving $\Delta x = \Delta y \simeq 0.428$ and $\Delta z \simeq 0.171$. Horizontal boundaries are periodic. The vertical boundaries are impermeable, the tangential velocity uses the model's free-slip condition, and both scalars have zero normal gradient. Symmetric cosine relaxation layers occupy the upper and lower $24.6551 = 0.15L_z$ of the domain. Within them, velocity relaxes toward zero and the scalars relax toward their undisturbed two-layer profiles with time scale 5. The interface is centered at $z_0 = L_z/2$, has hyperbolic-tangent thickness $\delta = 3$, and is displaced by a horizontal field $\eta(x, y)$ of amplitude $A = 0.70$. Localized scalar and velocity perturbations have amplitude 0.04.

The calculations use Oceananigans (Ramadhan et al. 2020; Wagner et al. 2025), centered second-order advection, molecular viscosity $\nu = \text{Pr}$, scalar diffusivities $(\kappa_T, \kappa_S) = (1, \tau)$, and no subgrid closure. A nonhydrostatic pressure projection enforces incompressibility. The nondimensional time step is 0.0015 and all histories extend to $t = 60$. Mid- y scalar sections are saved every 0.25. Coordinates, times, and simulated scalar measures are nondimensional unless stated otherwise.

The four input-defined histories are summarized in Table 1. H uses a high-wavenumber annulus, L uses the low-wavenumber spectrum, and M1 and M2 use the same mixed low/high spectrum with independent phase and noise realizations. In normalized Fourier indices, the low component spans approximately $5 \leq k_x \leq 16$ and $2 \leq |k_y| \leq 8$, while the high component spans approximately $7 \leq k_x \leq 24$ and $3 \leq |k_y| \leq 12$. The mixed displacement is $0.2\mathcal{L} + 0.8\mathcal{H}$. No later measurement is used to assign these symbols.

Figure 1 shows the salinity fields at $t = 45$. The nine vertical lines mark the aligned profile positions used in the subset analysis. The strong differences in vertical reach are visible, but those differences neither define nor relabel the prescribed families.

2.2 Profile and section reductions

At $t = 15, 30, 45, 57.75$, and 60, nine profiles are sampled at common x positions across the mid- y section. The positions are evenly spaced between 8% and 92% of the

Table 1: Input-defined simulation histories. All parameters not listed in the last two columns are identical.

Symbol	Prescribed spectrum	Random realization	Role in sampling analysis
H	High-wavenumber annulus $\mathcal{H}$	First	Shorter-scale source field
L	Low-wavenumber spectrum $\mathcal{L}$	First	Broader-scale source field
M1	$0.2\mathcal{L} + 0.8\mathcal{H}$	First	Mixed-spectrum source field
M2	$0.2\mathcal{L} + 0.8\mathcal{H}$	Independent second	Realization sensitivity

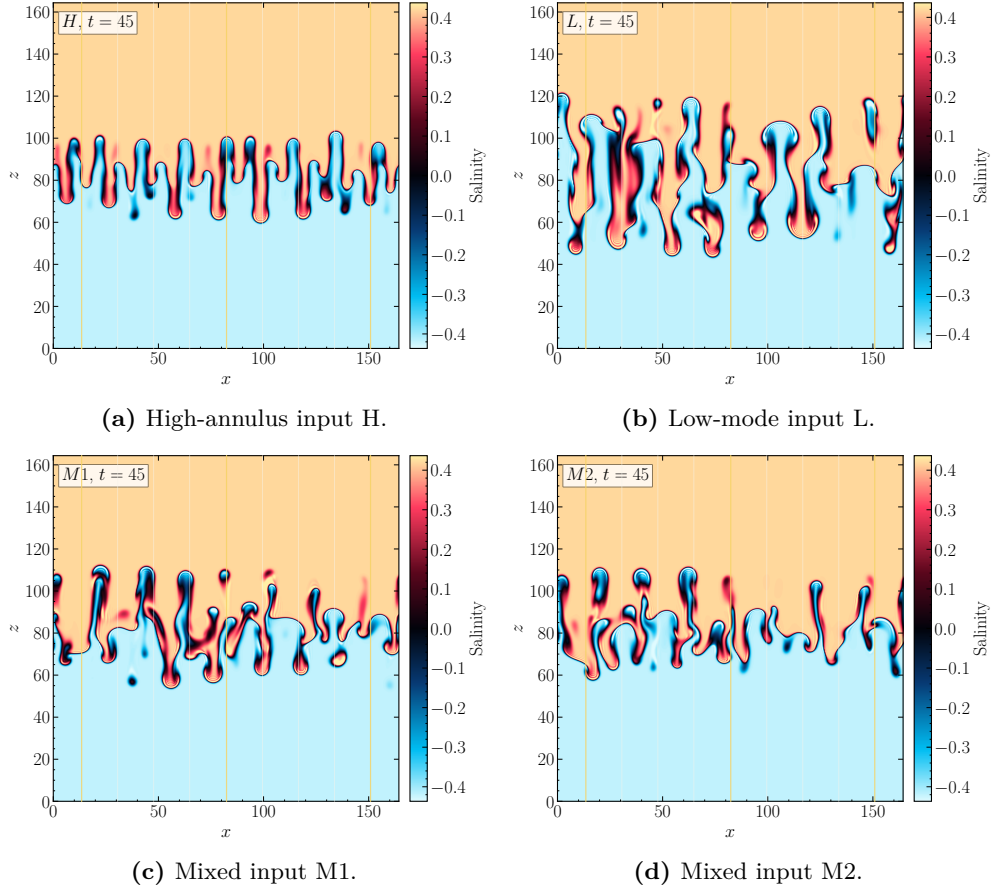


Fig. 1: Mid- y salinity fields at $t = 45$ for the four prescribed input spectra. Pale vertical lines show all nine profile positions; gold lines identify the first, middle, and last positions. The common color scale preserves differences between histories.

horizontal span, corresponding to $x = 13.48\text{--}150.88$. This placement avoids duplicating the periodic seam while covering most of the domain. Every nonempty subset of these nine positions is evaluated in the spatial-retention analysis. Dependence on the initial placement is addressed separately by the section analysis, which sweeps every distinct native-grid offset for each station spacing.

Figure 2 illustrates the reduction from a resolved section to vertical casts. The synthetic profiles retain the native 960 vertical levels before controlled coarsening. They do not reproduce platform-specific noise or sensor response. The analysis isolates information lost through spatial sampling.

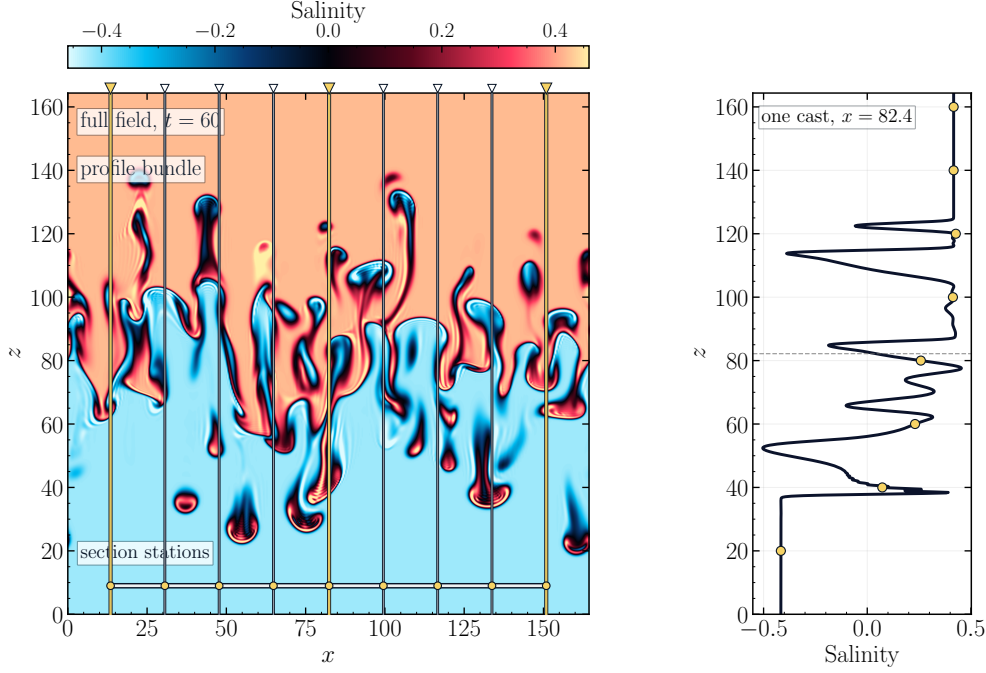


Fig. 2: Reduction of a resolved mid- y salinity field to aligned vertical profiles and section-like stations. The inset compares one native profile with a deliberately coarse sample of the same profile.

For a sampled salinity profile, define the nonnegative gradient density

$$g_S(z) = \left| \frac{\partial S}{\partial z} \right|, \quad p_S(z) = \frac{g_S(z)}{\int g_S dz}, \quad z_c = \int z p_S(z) dz. \quad (5)$$

If Q_q denotes the q -quantile of the cumulative gradient weight, the salinity-gradient interquartile width is

$$W_S = Q_{0.75} - Q_{0.25}. \quad (6)$$

The central and outer fractions in the synthetic coordinates are

$$F_c = \int_{|z-z_c| \leq 20} p_S(z) dz, \quad F_o = \int_{|z-z_c| \geq 40} p_S(z) dz. \quad (7)$$

For discrete gradient weights p_i over N retained intervals, normalized gradient entropy is

$$H_S = -\frac{\sum_{i=1}^N p_i \ln p_i}{\ln N}. \quad (8)$$

The profile temperature–salinity correlation is

$$r_{TS} = \frac{\sum_i (T_i - \bar{T})(S_i - \bar{S})}{\sqrt{\sum_i (T_i - \bar{T})^2 \sum_i (S_i - \bar{S})^2}}. \quad (9)$$

These quantities measure scalar-interface structure. None contains velocity and none is a heat, salt, or buoyancy flux.

Vertical coarsening resamples each synthetic profile at uniform spacings of 1, 2, 5, 10, and 20 model units. Each imposed spacing is divided by the native W_S of that same profile,

$$R_z = \frac{\Delta z_s}{W_S}. \quad (10)$$

Width error is $|W_S^{(s)} - W_S|/|W_S|$. Errors in F_c , F_o , and H_S are absolute differences on their unit intervals; the absolute error in r_{TS} is divided by its full range of two.

For section-like sampling, a salinity-gradient-weighted center $\eta(x)$ is calculated in every native column. With Fourier power $P_m = |\hat{\eta}_m|^2$, the dominant nonzero mode and wavelength are

$$m_* = \arg \max_{m \geq 1} P_m, \quad \lambda_* = \frac{L_x}{m_*}. \quad (11)$$

Uniform periodic arrays of 8, 12, 16, 24, 32, 48, 64, 96, 128, and 192 stations are applied. For each station count, every distinct offset on the native 384-point grid is evaluated. The horizontal sampling ratio is

$$R_x = \frac{\Delta x_s}{\lambda_*}. \quad (12)$$

Exact recovery means that the sampled track and native track have the same dominant integer mode.

2.3 Deterministic spatial-subset measures

The nine profiles within a history are spatial samples of one field, not independent realizations. Their combinations are therefore used only for deterministic sensitivity. For metric q , history c , time t , and subset A , let $\tilde{q}_{c,t,A}$ be the subset median and $\tilde{q}_{c,t,9}$

the full nine-profile median. Width retention is measured by

$$E_W(c, t, A) = \frac{|\widetilde{W}_{c,t,A} - \widetilde{W}_{c,t,9}|}{|\widetilde{W}_{c,t,9}|}. \quad (13)$$

Bounded metrics use the normalizations defined above.

For a prescribed input-family pair (a, b) , a contrast sign is retained when

$$\text{sign}(\widetilde{q}_{a,t,A} - \widetilde{q}_{b,t,A}) = \text{sign}(\widetilde{q}_{a,t,9} - \widetilde{q}_{b,t,9}). \quad (14)$$

H and L are their individual histories; M is the mean of M1 and M2 at the same time and aligned subset. All subset combinations are retained in the denominator. The resulting fractions describe the selected four histories and five times. They are not confidence levels or estimates of performance on a new realization. The nine-profile endpoint is exact by construction.

2.4 Observational regime screen

The observational products serve two roles: they establish realistic profile and section sampling characteristics, and they demonstrate that scalar morphology must be interpreted within a thermohaline regime. The retained products are 166 upper-interface windows from ITP 84, 36 accepted Argo interface windows obtained after screening 54 profile records from 22 source NetCDF files, and 108 usable CTD windows from the 109-station GO-SHIP I06S section. Thus, the Argo counts refer to 36 profile windows and 22 files, not 22 unique profiles.

Practical Salinity and in-situ temperature are converted to TEOS-10 Absolute Salinity and Conservative Temperature using pressure, longitude, and latitude (IOC, SCOR and IAPSO 2010). Turner angle Tu , stability ratio, and buoyancy frequency squared N^2 are evaluated between adjacent levels. Stable intervals with $45^\circ < Tu < 90^\circ$ are salt-fingering-favorable, and stable intervals with $-90^\circ < Tu < -45^\circ$ are diffusive-convection-favorable (Ruddick 1983). Intervals with $N^2 \leq 0$ are recorded separately as gravitationally unstable. Remaining intervals with $N^2 > 0$ are stable and outside the two double-diffusive sectors.

The gradient-active subset contains intervals for which the Absolute-Salinity-gradient magnitude is at least 10% of its maximum within the retained window. Regime fractions are weighted by pressure-interval thickness so that irregular vertical spacing does not give broad and narrow intervals equal weight. The screen establishes local favorability rather than realized instability, flux, or causal attribution.

3 Results and Discussion

3.1 The observing products do not represent one double-diffusive branch

The regime screen separates instrument format from physical analogue. In the gradient-active windows, the median stable- N^2 fraction is 1.000 for ITP, 0.861 for the Argo

sample, and 1.000 for I06S (Figure 3 and Table 2). The median salt-fingering-favorable fraction is 0.000, 0.159, and 0.000, respectively, while the corresponding diffusive-convection-favorable fractions are 0.846, 0.021, and 0.232.

The distributions are broader than the medians alone. Salt-fingering-favorable intervals occupy more than half of the active span in 13 of 36 Argo windows and 25 of 108 I06S windows. Diffusive-convection-favorable intervals occupy more than half of the active span in 161 of 166 ITP windows. The ITP population is therefore an opposite-branch example relative to the salt-finger simulations. Its dense vertical sampling remains useful, but its scalar widths and entropies cannot validate the simulated salt-finger structures.

This result changes the role of the real data. The observations are not pooled into one morphology distribution and compared with the simulations. ITP supplies dense-profiler sampling in a predominantly diffusive-convective setting. Argo supplies irregular routine-profile sampling and quality-control context. I06S supplies dense vertical casts at finite horizontal station spacing. A closer salt-fingering comparison is restricted to physically screened favorable windows, and even those windows do not provide transport without velocity or microstructure information.

The observing geometries also differ within their own physical coordinates (Figure 4). ITP 84 has a median pressure spacing of 1.0 dbar, while the Argo and I06S medians are 1.98 and 2.0 dbar. The 108 usable I06S windows extend across 4540 km, with a median positive station spacing of 55.4 km and substantial local departures caused by repeated casts and changes in station placement. These pressure and distance scales describe the measurements only; they are not conversions of the nondimensional model coordinates.

Table 2: Observational window counts, median pressure spacing, and median pressure-weighted active-regime fractions.

Source	Profiles	$\widetilde{\Delta p}$ (dbar)	Stable N^2 fraction	Salt finger fraction	Diffusive conv. fraction
ITP 84	166	1.00	1.000	0.000	0.846
Argo sample	36	1.98	0.861	0.159	0.021
GO-SHIP I06S	108	2.00	1.000	0.000	0.232

3.2 Vertical spacing controls interface-width fidelity

The salinity-gradient width is the most sampling-sensitive retained profile quantity. Across ratio bins wholly below $R_z = 0.5$, the largest median width error is 0.229 (Figure 5a). In the first bin with $R_z \geq 1$, the median error is 0.561. It increases to 1.039 for a median ratio of 1.657 and to 4.824 in the coarsest aggregate bin, whose median ratio is 5.563. Once sample spacing approaches the native interface width, a width inferred from the resampled gradient can describe the sampling grid more strongly than the original transition.

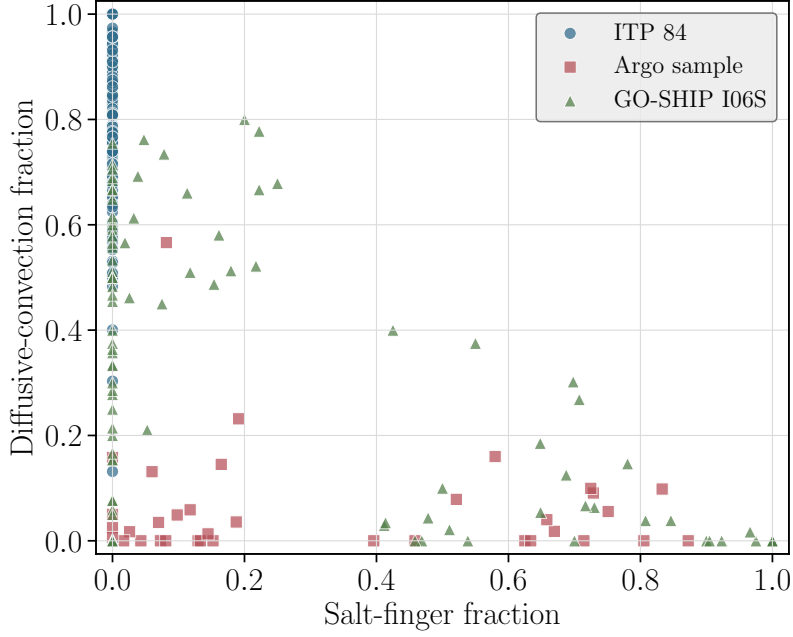


Fig. 3: Pressure-weighted thermohaline regime fractions in the salinity-gradient-active portion of each retained observational window. Each point is one profile window. The ITP population lies primarily in the diffusive-convection-favorable sector, whereas the Argo and I06S samples contain broader mixtures of stable regimes.

The behavior is not monotonic at every ratio because profile shape, sample phase, and interpolation interact. For example, the median width error in the $0.75 \leq R_z < 1$ bin is lower than in the preceding bin. The main transition is nevertheless clear: every bin below 0.5 has a median width error below 0.23, whereas the bins at and above one begin above 0.56 and become much larger with coarsening.

Bounded quantities respond differently (Figure 5b). Median entropy error grows from 0.028 in the finest bin to 0.166 in the $1 \leq R_z < 1.5$ bin, but decreases in the coarsest bin. Central and outer fractions can also remain numerically close under severe coarsening because their fixed, wide integration windows saturate near zero or one. Their small errors therefore do not establish preserved fine-scale structure. Width is the clearest measure of whether the interface itself is vertically resolved.

These results yield a dimensionless sampling criterion rather than a platform conversion. The synthetic spacings of 1–20 model units are never equated with observed pressure spacing. A field program can apply the same ratio only after estimating an interface width in its own physical coordinate.

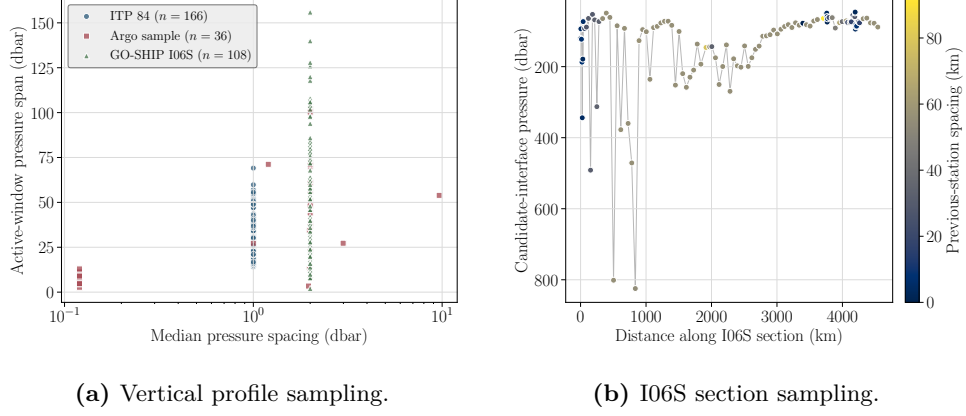


Fig. 4: Sampling geometry of the retained observational windows in physical units. Panel (a) relates median pressure spacing to the pressure span over which salinity gradients are active. Panel (b) shows candidate-interface pressure along I06S; marker color gives the distance from the preceding station. The panels establish observational context and do not assign oceanic scales to the simulations.

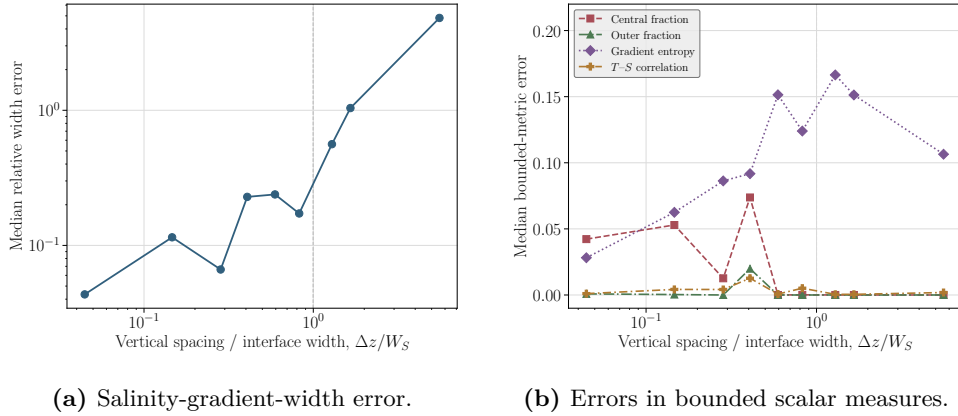


Fig. 5: Vertical information loss as sample spacing approaches the native salinity-gradient width. Points are medians within ratio bins and line styles distinguish the measured quantity.

3.3 Horizontal scale recovery requires spacing and phase control

The horizontal phase sweep contains 3,100 deterministic case-time-offset evaluations. Among the 1,180 evaluations with $R_x \leq 0.5$, the native dominant mode is recovered in 0.822; among the 1,920 evaluations with $R_x > 0.5$, it is recovered in none (Figure 6).

Two samples per target wavelength are therefore necessary in this suite, consistent with the Nyquist limit, but they are not sufficient for every multi-scale field.

The shortfall below $R_x = 0.5$ is concentrated in the mixed inputs. H and L recover the exact mode in every evaluated bin with median $R_x \leq 0.333$. M1 recovers 0.781, 0.531, and 0.788 in its three finest ratio bins, while M2 recovers 1.000, 0.878, and 0.673. Unresolved neighboring modes can alias into the retained spectrum, and shifting the station array changes their sampled phases. The target wavelength alone therefore provides the necessary spacing scale, while the surrounding spectral population controls the remaining ambiguity.

Continuous section measures show the same loss of information without requiring an exact-mode decision (Figure 7). At median $R_x = 0.333$, the median relative interface-amplitude error is 0.025 and the median absolute low-mode-fraction error is 0.013. At $R_x = 0.667$, these errors rise to 0.111 and 0.069; at $R_x = 1$, they reach 0.222 and 0.714. The 90th-percentile envelopes are broad even at finer spacing because the same array spacing samples different phases and neighboring modes across the four histories and five times. These envelopes summarize deterministic perturbations and are not sampling-confidence intervals.

This result replaces a universal profile-count threshold. A fixed count has no transferable meaning without the domain and target wavelength. Section design should instead state station spacing relative to the horizontal scale that must be recovered and should test more than one station offset when multi-scale structure is expected.

3.4 Spatial coverage improves metric and contrast retention gradually

Retaining more aligned profiles progressively brings the subset median closer to the nine-profile median (Figure 8a and Table 3). For W_S , the median normalized error is 0.289 with one profile, 0.221 with two, 0.150 with three, and 0.093 with five. The corresponding 90th-percentile errors are 1.033, 1.101, 0.746, and 0.557. A small subset can therefore improve the typical estimate while still leaving substantial location-sensitive errors in the upper tail.

Entropy and correlation are more stable under spatial subsetting. Median entropy error decreases from 0.026 for one profile to 0.012 for three and 0.006 for five; median normalized correlation error decreases from 0.006 to 0.002 and 0.001. Central and outer fractions have small medians but appreciable 90th-percentile errors, again reflecting saturation for many profiles and localized departures for a smaller set. No one profile count stabilizes every scalar measure uniformly.

The input-family contrast test gives the same gradual result (Figure 8b). Averaged over H–L, H–M, and L–M comparisons, the fraction of full-sample contrast signs retained is 0.693 with one profile, 0.792 with three, and 0.856 with five. At five profiles, the individual fractions are 0.856, 0.850, and 0.864. The nine-profile value is one by construction. Small reversals between neighboring subset sizes are possible because an even-sized median averages two central order statistics while an odd-sized median uses one; changing which spatial samples straddle the center can locally increase an error.

The subset calculations do not show that two, three, or five profiles classify a new interface. H and L each have one realization, and M has two. Every subset reuses the

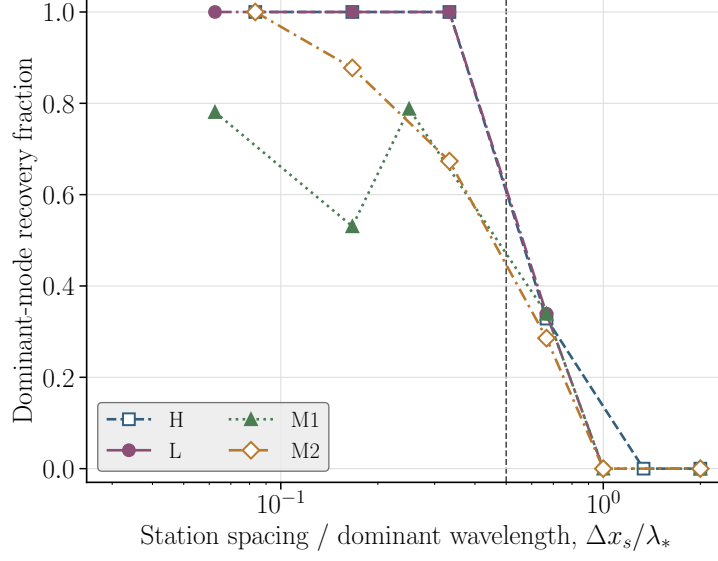


Fig. 6: Exact dominant-mode recovery versus station spacing normalized by the native dominant wavelength. Each curve summarizes all selected times and sampling offsets for one input-defined history. The dashed vertical line marks $\Delta x_s/\lambda_* = 0.5$. Fractions summarize deterministic reductions of the four histories rather than independent survey outcomes.

same spatial profiles, times, and histories. Independent realizations in every prescribed input family would be required for a predictive classifier and an out-of-sample accuracy.

Table 3: Selected deterministic profile-subset results. Width and entropy columns report median normalized errors relative to the nine-profile median. Contrast columns report the fraction of aligned subsets retaining the full-sample sign.

Profiles	Width error	Entropy error	H–L retained	L–M retained
1	0.289	0.026	0.693	0.698
2	0.221	0.022	0.750	0.784
3	0.150	0.012	0.788	0.814
5	0.093	0.006	0.856	0.864
9	0.000	0.000	1.000	1.000

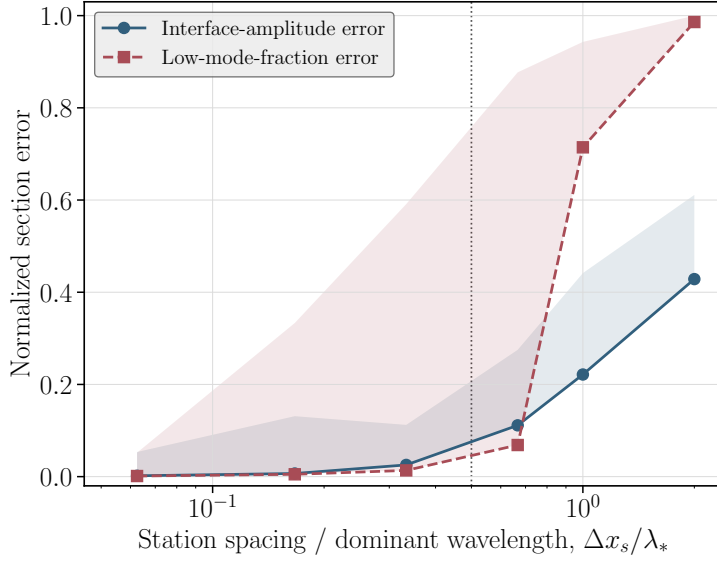
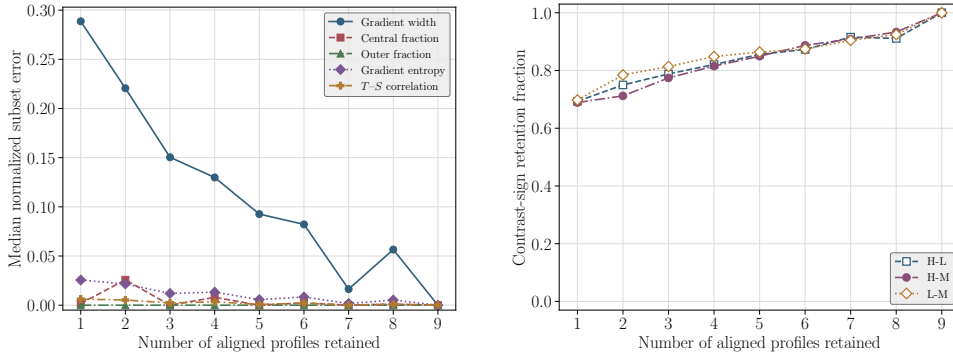


Fig. 7: Continuous section errors versus station spacing normalized by the native dominant wavelength. Curves show medians across the deterministic case–time–offset sweep; shading extends from the median to the 90th percentile. The dotted line marks $\Delta x_s/\lambda_* = 0.5$. Evaluation counts are not interpreted as independent samples.



(a) Errors relative to the nine-profile median. (b) Prescribed-family contrast-sign retention.

Fig. 8: Deterministic retention under spatial subsetting of the nine aligned profiles. Curves summarize all combinations at five selected times; their dependent combination counts are not treated as independent sample sizes.

3.5 Implications for hydrographic design

The vertical and horizontal results concern different information. A vertically dense isolated cast can resolve local gradient width but cannot determine a section-scale wavelength. A horizontally dense section can recover an interface track while still misrepresenting each local gradient if its vertical spacing is too coarse. Spatial profile subsets can improve robust summaries of local metrics, but the number required depends on horizontal heterogeneity. Sampling guidance must therefore identify the intended quantity before specifying a profile count.

For local scalar-interface width, the relevant ratio is $\Delta z_s/W_S$; the present histories show a clear deterioration as this ratio reaches one. For a dominant section scale, the relevant ratio is $\Delta x_s/\lambda_*$; values above one half cannot recover the target mode in the present sweep. Multi-scale fields require additional margin below the Nyquist limit because aliasing and station phase remain important. These criteria are portable because they compare sampling to a target scale within the same coordinate system, not because the nondimensional simulations have been assigned an oceanic length.

The observational screen adds a prior physical question: is the sampled window favorable to the double-diffusive branch under discussion? The dense ITP windows demonstrate that excellent vertical resolution does not make an opposite-branch profile a salt-finger analogue. The Argo and I06S samples contain salt-fingering-favorable windows, but a Turner-angle screen indicates local susceptibility only. Heat, salt, or buoyancy exchange requires additional measurements or a separately validated relationship between morphology and flux.

The source fields are also deliberately limited. They share one density ratio, diffusivity ratio, interface thickness, amplitude, and domain, and contain only four histories. The scale-normalized loss curves are therefore controlled sensitivity results rather than a global ocean observing prescription. Their strongest use is methodological: state the physical regime, normalize sampling by the feature to be recovered, distinguish local and horizontal information, and preserve the dependence structure of synthetic observations.

4 Conclusions

Sparse temperature–salinity measurements retain selected aspects of a resolved finite-depth double-diffusive interface, but the retained information depends on both physical regime and scale-relative sampling. TEOS-10 screening shows that the selected ITP windows are predominantly diffusive-convective, while the Argo and I06S samples contain mixtures of stable sectors. Similar scalar morphology across these products cannot establish a common double-diffusive branch.

Vertical interface-width estimates remain comparatively faithful when the imposed spacing is well below the native salinity-gradient width and become quantitatively unreliable as spacing reaches that width. Horizontal dominant-mode recovery has a complementary limit: station spacing must not exceed half the target wavelength, and multi-scale fields can require finer spacing because aliasing depends on spectral content and station phase.

Spatially distributed profiles improve full-sample metric and prescribed-family contrast retention, but the improvement is gradual and metric dependent. One, three, and five profiles reduce the median width error from 0.289 to 0.150 and 0.093, while mean contrast-sign retention rises from 0.693 to 0.792 and 0.856. These values describe deterministic subsets of four known histories. They do not measure predictive classification.

The resulting observing hierarchy is quantity specific. Local interface structure requires vertical resolution relative to interface width. Horizontal organization requires station spacing relative to wavelength. Thermohaline interpretation requires an independent regime screen. Transport remains outside the scope of scalar morphology alone. These boundaries make sparse-profile evidence more useful by stating precisely which parts of a resolved interface it can and cannot support.

Statements and Declarations

Funding. This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Competing interests. The author declares no competing interests.

Author contributions. Sriram P. Kalathoor performed the conceptualization, methodology, software development, data curation, analysis, visualization, and writing.

Data availability. The finite-depth simulation products used to construct the synthetic observations are derived from the archived Zenodo record (Kalathoor 2026). The observational inputs are publicly available from the ITP 84 archive (<https://scienceweb.whoi.edu/itp/data/itpsys84/itp84final.zip>), the Argo GDAC (<https://data-argo.ifremer.fr/>), and the CCHDO I06S cruise record (<https://cchdo.ucsd.edu/cruise/33RR20080204>). Derived observability tables underlying the figures and summary tables are available from the corresponding author on reasonable request.

Use of generative AI and AI-assisted technologies. During preparation of this work, AI-assisted tools were used for language editing, code review, and manuscript organization. The author reviewed and edited the resulting material and takes full responsibility for the content of this article.

References

- van der Boog CG, Koetsier JO, Dijkstra HA, et al (2021) Global dataset of thermohaline staircases obtained from Argo floats and ice-tethered profilers. *Earth System Science Data* 13:43–61. <https://doi.org/10.5194/essd-13-43-2021>
- IOC, SCOR and IAPSO (2010) The International Thermodynamic Equation of Seawater–2010: Calculation and Use of Thermodynamic Properties. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO, URL https://www.teos-10.org/pubs/TEOS-10_Manual.pdf
- Kalathoor SP (2026) Finite-depth salt-finger simulations and derived analysis data. <https://doi.org/10.5281/zenodo.20806266>, URL <https://doi.org/10.5281/zenodo.20806266>

- Krishfield R, Toole J, Proshutinsky A, et al (2008) Automated ice-tethered profilers for seawater observations under pack ice in all seasons. *Journal of Atmospheric and Oceanic Technology* 25(11):2091–2105. <https://doi.org/10.1175/2008JTECHO587.1>
- Radko T (2013) *Double-Diffusive Convection*. Cambridge University Press
- Ramadhan A, Wagner GL, Hill C, et al (2020) Oceananigans.jl: Fast and friendly geophysical fluid dynamics on GPUs. *Journal of Open Source Software* 5(53):2018. <https://doi.org/10.21105/joss.02018>
- Roemmich D, Johnson G, Riser S, et al (2009) The Argo program: Observing the global oceans with profiling floats. *Oceanography* 22(2):34–43. <https://doi.org/10.5670/oceanog.2009.36>
- Roemmich D, Alford MH, Claustre H, et al (2019) On the future of Argo: A global, full-depth, multi-disciplinary array. *Frontiers in Marine Science* 6:439. <https://doi.org/10.3389/fmars.2019.00439>
- Ruddick BR (1983) A practical indicator of the stability of the water column to double-diffusive activity. *Deep Sea Research Part A Oceanographic Research Papers* 30(10):1105–1107. [https://doi.org/10.1016/0198-0149\(83\)90063-8](https://doi.org/10.1016/0198-0149(83)90063-8)
- Schmitt RW (1994) Double diffusion in oceanography. *Annual Review of Fluid Mechanics* 26(1):255–285. <https://doi.org/10.1146/annurev.fl.26.010194.001351>
- Shibley NC, Timmermans ML, Carpenter JR, et al (2017) Spatial variability of the Arctic Ocean’s double-diffusive staircase. *Journal of Geophysical Research: Oceans* 122(2):980–994. <https://doi.org/10.1002/2016JC012419>
- Simeonov JA, Stern ME, Radko T (2009) Direct numerical simulation of 3d salt fingers: From secondary instability to chaotic convection. <https://doi.org/10.48550/arXiv.0910.2434>, URL <https://arxiv.org/abs/0910.2434>, arXiv:0910.2434
- Sloyan BM, Wanninkhof R, Kramp M, et al (2019) The global ocean ship-based hydrographic investigations program (GO-SHIP): A platform for integrated multidisciplinary ocean science. *Frontiers in Marine Science* 6:445. <https://doi.org/10.3389/fmars.2019.00445>
- Timmermans ML, Toole J, Krishfield R, et al (2008) Ice-tethered profiler observations of the double-diffusive staircase in the Canada Basin thermocline. *Journal of Geophysical Research: Oceans* 113:C00A02. <https://doi.org/10.1029/2008JC004829>
- Toole J, Krishfield R, Timmermans ML, et al (2011) The ice-tethered profiler: Argo of the Arctic. *Oceanography* 24(3):126–135. <https://doi.org/10.5670/oceanog.2011.64>
- Traxler AL, Stellmach S, Garaud P, et al (2011) Dynamics of fingering convection. Part 1. Small-scale fluxes and large-scale instabilities. *Journal of Fluid Mechanics*

677:530–553. <https://doi.org/10.1017/S002211201100098X>

Wagner GL, Silvestri S, Constantinou NC, et al (2025) High-level, high-resolution ocean modeling at all scales with Oceananigans. <https://doi.org/10.48550/arXiv.2502.14148>, URL <https://arxiv.org/abs/2502.14148>, arXiv:2502.14148