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Posted Date: October 13th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-11116541/v1>

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Additional Declarations: No competing interests reported.

Large-Scale Group Decision-Making with Linear Uncertain Preference Relations Using Improved Density Peak Clustering and Two-Stage Feedback

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Abstract

This study investigates consensus reaching in large-scale group decision-making (LSGDM) with linear uncertain preference relations (LUPRs). A consensus model integrating dynamic trust updating, improved Density Peak Clustering (DPC), and a two-stage feedback mechanism is proposed. Trust relationships are updated according to preference similarities, while historical preference similarities and trust degrees are incorporated to construct a hybrid trust network. Based on the hybrid trust relationships, an improved DPC algorithm dynamically groups decision-makers, and the silhouette coefficient is used to determine the number of clusters. Decision-maker and subgroup weights are determined by considering credibility, hybrid trust degrees, and silhouette coefficients. The first feedback stage uses consensus contribution compensation to identify subgroups requiring adjustment, while the second uses a unit adjustment cost model to determine preference adjustments. The proposed model is evaluated through a case study on the prioritization of critical success factors in Integrated Project Delivery, together with sensitivity analysis, simulation experiments, and comparative analysis. The results show that the model can achieve high consensus success rates with limited preference adjustment across different decision settings, although consensus becomes more difficult to achieve when high consensus requirements are combined with large group sizes.

Keywords: Large-scale group decision-making, linear uncertain preference relations, dynamic social networks, density peak clustering, two-stage feedback mechanism

1 Introduction

Large-scale group decision-making (LSGDM) aims to coordinate the preferences of a large number of decision-makers (DMs) and obtain an acceptable collective solution. It generally involves at least 20 DMs and commonly includes subgroup clustering, consensus measurement, preference aggregation, and feedback adjustment [1, 2]. With the development of online platforms and social networks, DMs are increasingly connected through trust relationships that may change as their preferences

evolve during the consensus-reaching process (CRP). Therefore, how to represent dynamic trust relationships, effectively group DMs, and guide preference adjustment has become an important issue in LSGDM.

Preference information is a fundamental component of LSGDM. Different preference representation forms have been developed to describe DMs' judgments under uncertainty. Among them, linear uncertain preference relations (LUPRs) can represent uncertain pairwise comparisons through linear uncertain distributions and preserve both interval information and preference characteristics. In addition to preference information, social relationships among DMs have been widely incorporated into LSGDM models. Social network analysis provides a useful way to represent trust relationships and to support subgroup identification, weight determination, and consensus reaching.

Existing studies have investigated subgroup clustering from different perspectives. DMs can be clustered according to opinion similarity [3], social relationships [4], or multiple characteristics [5]. Methods such as fuzzy clustering [6], dynamic K-means [7], and three-way clustering [8] have been applied to identify subgroups in LSGDM. Density Peak Clustering (DPC) is also suitable for subgroup identification because it determines cluster centers according to local density and distance [9]. However, under some center-selection and assignment rules, high-density DMs may remain unassigned and isolated subgroups may occur, which can affect the completeness of the clustering result. In addition, the number of clusters usually needs to be determined appropriately for different group structures.

Trust relationships and historical information have also received increasing attention in dynamic social network environments. Recent studies have considered changes in trust relationships and the use of historical information during the CRP [10]. Since preference similarity may change after DMs adjust their opinions, trust relationships can also be updated accordingly. Meanwhile, historical preference similarities and trust degrees provide additional information for describing the relationships among DMs over multiple consensus rounds. Therefore, jointly considering current preference similarity, updated trust relationships, and their historical information can provide a basis for dynamic subgroup clustering.

Another important issue is the feedback mechanism used when the required consensus level has not been reached. Existing studies have developed minimum-adjustment strategies [11], cost-based mechanisms [12], and rule-based feedback methods [13]. However, directly adjusting all low-consensus DMs may lead to unnecessary preference changes. Moreover, subgroups with relatively high consensus levels can contribute to the overall consensus and may compensate, to some extent, for subgroups with lower consensus levels. After the subgroups requiring adjustment are identified, differences in DMs' credibility, trust relationships, weights, and previous adjustment magnitudes should also be considered when determining who should adjust their preferences.

To address these issues, this study proposes an LSGDM consensus model with LUPRs for dynamic social network environments. The model combines dynamic trust updating, historical preference similarities and trust degrees, an improved DPC algorithm, and a two-stage feedback mechanism. The main contributions are summarized as follows:

- 1) Trust relationships are dynamically updated according to preference similarities, and historical preference similarities and trust degrees are incorporated to construct a hybrid trust network. This hybrid trust network is updated during the CRP and provides the basis for subsequent clustering and weight determination.
- 2) An improved DPC algorithm is developed for dynamically grouping DMs. The assignment strategy is improved to deal with high-density DMs that cannot be assigned under conventional rules,

isolated one-member subgroups are reassigned, and the silhouette coefficient is used to determine the number of clusters within a predefined range.

3) DM and subgroup weights are determined by jointly considering DMs' credibility, hybrid trust degrees, and silhouette coefficients. The credibility of a DM is evaluated using consistency, information entropy, and uncertainty of the LUPR.

4) A two-stage feedback mechanism is developed. The first stage uses consensus contribution compensation to identify the subgroups that require adjustment, while the second stage uses a unit adjustment cost model to identify DMs requiring adjustment and determine their preference adjustments.

The remainder of this paper is organized as follows. Section 2 introduces the preliminary concepts. Section 3 presents the construction of the hybrid trust network and the improved DPC clustering method. Section 4 develops the proposed LSGDM consensus model. Section 5 presents the case study, and Section 6 reports the sensitivity analysis, simulation experiments, and comparative analysis. Finally, Section 7 concludes the paper.

2 Preliminaries

2.1 LSGDM

LSGDM is a methodology designed to address complex and critical decision-making problems by utilizing the collective judgments of a large number of DMs. Its primary goal is to find the optimal solution from a set of feasible alternatives through structured discussion and aggregation of individual preferences.

Let $M = \{e_1, e_2, \dots, e_m\}$, ($m \geq 20$) be the set of DMs, and $X = \{x_1, x_2, \dots, x_n\}$ be the set of alternatives. Let $W = \{\omega_1, \omega_2, \dots, \omega_m\}$ be the weighting vector defined on M , which satisfies $\sum_{l=1}^m \omega_l = 1$ and $0 \leq \omega_l \leq 1$ for all $l = 1, 2, \dots, m$. Each DM e_h provides an individual decision matrix $R_h = (r_{ij}^h)_{n \times n}$ to represent their preferences over the alternatives.

2.2 Linear uncertain preference relations

In LSGDM, linear uncertain preference relations (LUPRs) provide a flexible representation of DMs' judgments by simultaneously capturing interval uncertainty and internal preference distribution, thereby preserving more preference information than FPRs and IFPRs.

2.2.1 Uncertain measure

Definition 1 [14] An uncertainty space is defined as a triplet (Γ, L, M) , where Γ is a nonempty set, L is a σ -algebra over Γ , and M is an uncertain measure. An uncertain variable is a function ξ from this uncertainty space to the set of real numbers, such that the set $\{\xi \in B\}$ is an event for any Borel set B . The uncertainty distribution Φ of ξ is defined as:

$$\Phi(x) = M\{\xi \leq x\} \quad (1)$$

Let $\xi_1, \xi_2, \dots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \dots, \Phi_n$, respectively. Suppose the function $f(\xi_1, \xi_2, \dots, \xi_n)$ is strictly increasing with respect to $\xi_1, \dots, \xi_m$ and strictly decreasing with respect to $\xi_{m+1}, \dots, \xi_n$. Then the uncertain variable $\xi = f(\xi_1, \xi_2, \dots, \xi_n)$ has an inverse uncertainty distribution [15] given by:

$$\Psi^{-1}(\alpha) = f\left(\Phi_1^{-1}(\alpha), \dots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1 - \alpha), \dots, \Phi_n^{-1}(1 - \alpha)\right) \quad (2)$$

2.2.2 LUPRs with consistency analysis

Definition 2 [16] Let Φ_{ij} and Φ_{ij}^{-1} be the uncertain distribution function and its inverse function of the uncertain variable r_{ij} , respectively. If $\Phi_{ij}^{-1}(\alpha) + \Phi_{ji}^{-1}(1-\alpha) = 1$, $\Phi_{ii}^{-1}(\alpha) = 0.5$ holds for any $i, j \in N, 0 \leq \alpha \leq 1$ and r_{ij} satisfies a linear uncertain distribution:

$$\Phi_{ij}(x) = \begin{cases} 0, & \text{if } x \leq r_{ij}^-, \\ \frac{x - r_{ij}^-}{r_{ij}^+ - r_{ij}^-}, & \text{if } r_{ij}^- \leq x \leq r_{ij}^+, \\ 1, & \text{if } x \geq r_{ij}^+ \end{cases} \quad (3)$$

namely, $r_{ij} \sim \mathcal{L}[r_{ij}^-, r_{ij}^+]$, then the non-negative matrix $R = (r_{ij})_{n \times n}$ is an LUPR. The inverse distribution function of r_{ij} is $\Phi_{ij}^{-1}(\alpha) = (1 - \alpha)r_{ij}^- + \alpha r_{ij}^+$.

If $\Phi_{ij}^{-1}(\alpha) + 0.5 = \Phi_{ik}^{-1}(\alpha) + \Phi_{kj}^{-1}(\alpha)$ holds for any $\alpha, 0 \leq \alpha \leq 1$, then LUPR R is additively consistent, for all $i, k, j \in N$ and $i < k < j$. Accordingly, the consistency degree of LUPR R can be expressed as follows.

Definition 3 Let $R_h = (r_{ij}^h)_{n \times n}$ be an LUPR initially provided by e_h and $r_{ij}^h \sim \mathcal{L}[r_{ij}^{h-}, r_{ij}^{h+}]$. Let $\Phi_{ij}^{-1}(\alpha)$ be the inverse distribution function of r_{ij} . Based on Definition 2, the definition of the consistency degree $CD(e_h)$ of e_h is as follows:

$$CD(e_h) = 1 - \frac{4}{n(n-1)(n-2)} \sum_{\substack{i < k < j \\ i, k, j \in N}} \left| \Phi_{ik}^{-1}(\alpha) + \Phi_{kj}^{-1}(\alpha) - \Phi_{ij}^{-1}(1 - \alpha) - 0.5 \right|. \quad (4)$$

The following definition presents an alternative consistency measure based on priority weights.

Definition 4 [17] An LUPR $R = (r_{ij})_{n \times n}$ is additively consistent if there exists a nonnegative vector $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ such that

$$\Phi_{ij}^{-1}(\alpha) = \frac{n-1}{2}(\omega_i - \omega_j) + 0.5 \quad (i, j \in N, i < j) \quad (5)$$

For inconsistent LUPRs, Eq. (5) may not hold. Following [18], the priority weights can be derived by solving the following transformed optimization model:

$$\begin{aligned} & \text{Min} \sum_{i, j \in N} \delta_{ij} \\ & \text{s.t.} \begin{cases} \Phi_{ij}^{-1}(\partial) \leq \frac{n-1}{2}(\omega_i - \omega_j) + 0.5 + \delta_{ij}, \quad i, j \in N \\ \Phi_{ij}^{-1}(1 - \partial) \geq \frac{n-1}{2}(\omega_i - \omega_j) + 0.5 - \delta_{ij}, \quad i, j \in N \\ \sum_{i=1}^n \omega_i = 1, \quad 0 \leq \omega_i \leq 1, \quad i \in N \\ 0 \leq \partial \leq 1, \quad \delta_{ij} \geq 0, \quad i, j \in N \end{cases} \end{aligned} \quad (6)$$

2.3 Social network analysis (SNA)

Social network analysis (SNA) models DMs as nodes and directed trust relationships as edges [19]. The corresponding adjacency matrix records whether and to what degree one DM trusts another. Following [20, 21], this study considers complete direct trust relationships.

Definition 5 [21] A fuzzy sociometric matrix $T = (t_{ij})_{m \times m}$ on M is a relation in $M \times M$ with membership function $\mu_s : M \times M \rightarrow [0, 1]$, where $\mu_s(e_i, e_j) = t_{ij}$ denotes the trust degree assigned by e_i to e_j .

3 History-based dynamic clustering methods

3.1 Construction of a hybrid trust network incorporating historical data

3.1.1 Computation of preference similarity

As DMs' opinions evolve during the consensus-reaching process, preference similarities are dynamically updated and used to revise trust relationships. The preference similarity between DMs is calculated based on the LUPR distance as follows.

Definition 6 [18] Let $R_k = (r_{ij}^k)_{n \times n}$ and $R_l = (r_{ij}^l)_{n \times n}$ be two LUPRs, where the elements $r_{ij}^k \sim \mathcal{L}[r_{ij}^{-k}, r_{ij}^{+k}]$ and $r_{ij}^l \sim \mathcal{L}[r_{ij}^{-l}, r_{ij}^{+l}]$ are independently distributed. The distance $d(R_k, R_l)$ between R_k and R_l is defined as:

$$d(R_k, R_l) = \frac{1}{2n(n-1)} \sum_{i=1}^n \sum_{j=1}^n (|r_{ij}^{-k} - r_{ij}^{-l}| + |r_{ij}^{+k} - r_{ij}^{+l}|) \quad (7)$$

where $k, l = 1, 2, \dots, m$, and m is the number of DMs.

Furthermore, the similarity SD_{kl} between DMs e_k and e_l is denoted by:

$$SD_{kl} = 1 - d(R_k, R_l) \quad (8)$$

3.1.2 Trust network update and transforming directed trust into undirected trust

Trust and preference similarity are key drivers of DM interactions in social networks. Since trust evolves dynamically with preference similarity [10], we introduce a threshold-based trust updating mechanism. Given threshold β , trust increases when similarity exceeds the current trust level by more than β , decreases when it falls below it by more than β , and remains unchanged otherwise. The update rule is defined as follows:

$$\bar{t}_{kl} = \begin{cases} t_{kl} + (1 - t_{kl})(SD_{kl} - t_{kl} - \beta), & \text{if } SD_{kl} - t_{kl} > \beta \\ t_{kl}, & \text{if } |t_{kl} - SD_{kl}| \leq \beta \\ t_{kl} - t_{kl}(t_{kl} - SD_{kl} - \beta), & \text{if } t_{kl} - SD_{kl} > \beta \end{cases} \quad (9)$$

where t_{kl} , $\bar{t}_{kl}$, and SD_{kl} denote the initial trust degree, updated trust degree, and preference similarity between e_k and e_l , respectively.

Since preference similarity is undirected while trust is directed, directed trust must be converted into undirected trust before clustering. Following [22], the transformation preserves mutual acceptability between DMs and satisfies two conditions:

1. $\bar{\bar{t}}_{kl} \leq \min\{\bar{t}_{kl}, \bar{t}_{lk}\}$.
2. When $\min\{\bar{t}_{kl}, \bar{t}_{lk}\} = \min\{\bar{t}_{kh}, \bar{t}_{hk}\}$ and $\max\{\bar{t}_{kl}, \bar{t}_{lk}\} \geq \max\{\bar{t}_{kh}, \bar{t}_{hk}\}$, $\bar{\bar{t}}_{kl} \geq \bar{\bar{t}}_{kh}$.

Research indicates that the use of a T-norm operator can simultaneously satisfy these transformation conditions [23], thereby converting the directed trust relationships between DMs into an undirected form.

Accordingly, in this study, the transformation from the directed trust values $\bar{t}_{kl}$ (from DM e_k to DM e_l) and $\bar{t}_{lk}$ (from DM e_l to DM e_k) into the undirected trust values $\bar{\bar{t}}_{kl}$ and $\bar{\bar{t}}_{lk}$ is achieved by the following formula:

$$\bar{\bar{t}}_{kl} = \bar{\bar{t}}_{lk} = \frac{\bar{t}_{kl} \cdot \bar{t}_{lk}}{1 + (1 - \bar{t}_{kl})(1 - \bar{t}_{lk})} \quad (10)$$

3.1.3 Construction of a hybrid trust network incorporating historical data

Current clustering methods mainly rely on current-round trust relationships and preference similarity, while historical information is often ignored. To improve trust-evaluation stability, this study constructs a hybrid trust network under two assumptions: (1) trust relationships evolve with preference similarity during the CRP; and (2) integrating current trust, preference similarity, and historical information improves clustering and consensus robustness. Accordingly, the hybrid trust degree ST_{kl} between e_k and e_l is defined as follows:

$$ST_{kl} = \frac{2 \cdot SD_{kl} \cdot \bar{\bar{t}}_{kl}}{SD_{kl} + \bar{\bar{t}}_{kl}} \quad (11)$$

Accordingly, the hybrid trust distance d_{kl} is expressed as follows:

$$d_{kl} = 1 - ST_{kl} \quad (12)$$

Using Eq. (11), the hybrid trust matrix $HST = (ST_{kl})_{m \times m}$ is updated in each iteration, where $ST_{kl} \in [0, 1]$.

3.2 An improved DPC clustering algorithm

To partition DMs into homogeneous subgroups, this study adopts the Density Peak Clustering (DPC) method. According to [9], cluster centers are characterized by high local density and large distances from other cluster centers. The local density $\rho(e_k)$ of DM e_k is defined as :

$$\rho(e_k) = \sum_{l=1}^m \chi(d_{kl} - d_c) \quad (13)$$

where $\chi(x) = \begin{cases} 1, & \text{if } x < 0 \\ 0, & \text{otherwise} \end{cases}$ and d_c is the truncation distance.

The minimum distance $\Delta(e_k)$ from e_k to other DMs is defined as:

$$\Delta(e_k) = \begin{cases} \max_l(d_{kl}), & \text{if } \rho(e_k) \text{ has the highest local density} \\ \min_{l: \rho_l > \rho_k} (d_{kl}), & \text{otherwise} \end{cases} \quad (14)$$

The original DPC method [9] may fail when cluster centers are too close. Although [24] introduced a minimum distance threshold to address this issue, their method still requires each cluster center to have higher local density than its assigned points. Consequently, high-density points may remain unassigned, resulting in incomplete clustering, as shown in Example 1.

Example 1 For 20 DMs divided into three subgroups with $d_c = 2$ and $\kappa = 2.5$, the method in [24] identifies (e_{20}, e_{18}, e_{14}) as cluster centers. However, high-density points e_8 and e_{12} cannot be assigned because their densities exceed those of all selected centers, yielding incomplete clustering.

To overcome this limitation, the grouping strategy relaxes the density constraint by assigning DMs to the nearest higher-density center, or to the nearest equal-density center when necessary, and reassigning isolated one-member clusters. The silhouette coefficient is then used to determine the number of clusters within a predefined range. Suppose that DMs are divided into g subgroups, denoted as $G = \{sg_1, sg_2, \dots, sg_g\}$. The silhouette coefficient of DM e_k in subgroup sg_i is calculated as follows [9]:

$$I_1(e_k) = 1 - \frac{\sum_{e_l \in sg_i, l \neq k} ST_{kl}}{|sg_i| - 1} \quad (15)$$

$$I_2(e_k) = \min_{\substack{sg_j \in G, \\ sg_j \neq sg_i}} \left\{ \frac{\sum_{e_l \in sg_j} (1 - ST_{kl})}{|sg_j|} \right\} \quad (16)$$

$$sc(e_k) = \frac{I_2(e_k) - I_1(e_k)}{\max\{I_1(e_k), I_2(e_k)\}} \quad (17)$$

where $|sg_i|$ represents the number of members in subgroup sg_i , $I_1(e_k)$ denotes the average dissimilarity and distrust degree between e_k and other DMs in subgroup sg_i . $I_2(e_k)$ denotes the minimum of the average dissimilarity and distrust degree between e_k and DMs in any other subgroup. $sc(e_k)$ is the silhouette coefficient of e_k in subgroup sg_i , and $sc(e_k) \in [-1, 1]$.

To facilitate subsequent calculations, we process $sc(e_k)$ as follows:

$$sc'(e_k) = (sc(e_k) + 1)/2 \quad (18)$$

Clustering is performed based on the hybrid trust degree among DMs. The silhouette coefficient of subgroup sg_i is defined as the average silhouette coefficient of its members.

$$sc(sg_i) = \frac{\sum_{k \in sg_i} sc'(e_k)}{|sg_i|} \quad (19)$$

The final clustering result is determined by the solution with the maximum silhouette coefficient.

Algorithm 1 searches within a predefined range of cluster numbers and selects the clustering result with the highest silhouette coefficient. By integrating historical preference similarity and trust information, the resulting clustering structure is more stable and robust. The procedure is summarized in Algorithm 1.

Remark 1: The proposed clustering algorithm extends traditional DPC by incorporating historical similarity and trust information into the clustering process. It determines the number of clusters using the silhouette coefficient and improves robustness when handling isolated clusters and dynamic trust relationships.

4 An LSGDM consensus method

This section presents the proposed LSGDM consensus framework. After DM and subgroup weights are determined, subgroup and collective preferences are aggregated to measure consensus. If the threshold is not reached, a two-stage feedback mechanism is activated; otherwise, alternatives are ranked to obtain the final decision. The overall procedure is summarized in Algorithm 3.

Algorithm 1 Dynamic clustering algorithm based on historical data

Input: Preference matrices $R_k = (r_{ij}^k)_{n \times n}$, $k = 1, 2, \dots, m$, in round z ; initial trust matrix $T = (t_{ij})_{m \times m}$; parameters $[\eta_1, \eta_2]$, β , d_c , and κ .

Output: Clustering result.

Step 1: Calculate current preference similarities SD_{kl}^z using Eq. (8) and update trust degrees $\bar{t}_{kl}^z$ using Eq. (9).

Step 2: Compute historical preference similarity and historical trust degrees: $SD_{kl} = \frac{1}{z} \sum_{i=1}^z SD_{kl}^i$ and $\bar{t}_{kl} = \frac{1}{z} \sum_{i=1}^z \bar{t}_{kl}^i$, $k, l \in M$.

Step 3: Convert directed historical trust into undirected trust using Eq. (10), and calculate the hybrid trust degree and distance using Eqs. (11)–(12).

Step 4: Calculate the local density $\rho(e_k)$ and minimum distance $\Delta(e_k)$ of each DM using Eqs. (13)–(14), and sort DMs in descending order of $q(e_k) = \rho(e_k)\Delta(e_k)$.

Step 5: For each candidate cluster number $k_1 \in [\eta_1, \eta_2]$, select cluster centers in descending order of $q(e_k)$, subject to the distance threshold κ .

Step 6: Assign the remaining DMs to the nearest subgroup with higher local density; if no such subgroup exists, assign them to the nearest subgroup with no lower local density.

Step 7: Reassign any isolated one-member subgroup to the nearest eligible cluster center and remove the isolated subgroup.

Step 8: Calculate the subgroup silhouette coefficient using Eq. (19) for each k_1 .

Step 9: Output the clustering result with the maximum silhouette coefficient.

4.1 Determination of DM weights and subgroup weights

Section 3.2 clusters DMs into subgroups based on hybrid trust relationships. This section determines DM weights within subgroups and subgroup weights for consensus feedback.

DM weights are calculated by integrating preference credibility and social trust. Preference credibility is evaluated using three trust-risk factors: consistency, information entropy, and uncertainty.

Consistency: Consistency measures the logical reliability of a DM's preference information. For LUPRs, it is defined in Definition 2 and calculated by Eq. (4); a higher value indicates higher credibility.

Information entropy: Information entropy measures the quality of preference information [25]. For a LUPR, it is defined as follows:

Definition 7 [25] Let $R_k = (r_{ij}^k)_{n \times n}$ be an LUPR of DM e_k , then the information entropy $IE(e_k)$ of e_k can be expressed as:

$$IE(e_k) = -\frac{1}{\ln(n)} \sum_{i=1}^n \lambda_i \cdot \ln(\lambda_i) \quad (20)$$

where $\lambda_i = \frac{S_i}{\sum_{j=1}^n S_j}$ is the score proportion of alternative x_i , $S_i = \sum_{j \neq i}^n E(r_{ij}^k)$ is the comprehensive score of x_i , and $E(r_{ij}^k) = \frac{(r_{ij}^{-k} + r_{ij}^{+k})}{2}$ is the expected value of r_{ij} .

Clearly, $IE(e_k) \in [0, 1]$, and a larger $IE(e_k)$ indicates weaker discrimination among alternatives and thus lower credibility.

Uncertainty: Uncertainty reflects a DM's confidence and expertise. For LUPRs, a narrower distribution interval indicates higher confidence.

Definition 8 Let $R_k = (r_{ij}^k)_{n \times n}$ be an LUPR of DM e_k . Then, the uncertainty of R_k can be given as:

$$U(e_k) = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^n (r_{ij}^{+k} - r_{ij}^{-k}). \quad (21)$$

The credibility of a DM's preference information decreases as the uncertainty of their LUPR rises.

By integrating the three trust risk factors mentioned above, the credibility CR_k of e_k is formulated as follows:

$$CR_k = \left(\frac{CD(e_k) + 1 - U(e_k)}{2} \right)^{1+IE(e_k)} \quad (22)$$

The smaller the uncertainty $U(e_k)$ and information entropy $IE(e_k)$, and the larger the consistency degree $CD(e_k)$, the higher the credibility $CR(e_k)$ is.

Therefore, by integrating the personal credibility $CR(e_k)$ of DM e_k and their hybrid trust degree ST_{kl} , the global weight factor v_k can be calculated using the following formula:

$$v_k = CR_k + \frac{1}{m-1} \sum_{\substack{l=1 \\ l \neq k}}^m ST_{kl} \quad (23)$$

The corresponding individual global weights ω_k for DM e_k are as follows:

$$\omega_k = \frac{v_k}{\sum_{j=1}^m v_j} \quad (24)$$

The silhouette coefficient measures subgroup cohesion and consensus. A higher value indicates stronger internal consistency and thus justifies a larger subgroup weight. Combining the silhouette coefficient with member weights, the subgroup weight factor is defined as follows:

$$\omega_{sg_i} = \frac{1}{2} \cdot \left(\sum_{e_k \in sg_i} \omega_k + \frac{sc(sg_i)}{\sum_{sg_j \in G} sc(sg_j)} \right) \quad (25)$$

Let $\omega_{sg_i}^k$ denote the weight of DM e_k in subgroup sg_i , which can be calculated as:

$$\omega_{sg_i}^k = \frac{1}{2} \cdot \left(\frac{\omega_k}{\sum_{e_l \in sg_i} \omega_l} + \frac{sc'(e_k)}{\sum_{e_l \in sg_i} sc'(e_l)} \right) \quad (26)$$

4.2 Preference aggregation

By integrating the weight distribution of members within the subgroup sg_i and their individual preference matrices, the element $r_{ij}^{sg_i}$ of the comprehensive evaluation matrix $R_{sg_i} = (r_{ij}^{sg_i})_{n \times n}$ of the subgroup sg_i can be aggregated as follows:

$$r_{ij}^{sg_i} = \sum_{k \in sg_i} \omega_{sg_i}^k \cdot r_{ij}^k \quad (27)$$

Where $R_g = (r_{ij}^g)_{n \times n}$ is the global preference matrix, the element r_{ij}^g can be determined by:

$$r_{ij}^g = \sum_{sg_i \in G} \omega_{sg_i} \cdot r_{ij}^{sg_i} \quad (28)$$

4.3 Consensus level measurement

To prevent pseudo-consensus within subgroups, the consensus level of a subgroup sg_i is evaluated by measuring its internal and external ordinal consensus. Let $SCD_{sg_i}^{\text{int}}$ and $SCD_{sg_i}^{\text{ext}}$ denote the internal and external ordinal consensus levels of subgroup sg_i , respectively. These metrics can be calculated

as follows:

$$SCD_{sg_i}^{\text{int}} = \sum_{k \in sg_i} \omega_{sg_i}^k [1 - d(R_{sg_i}, R_k)] \quad (29)$$

$$SCD_{sg_i}^{\text{ext}} = 1 - d(R_{sg_i}, R_g) \quad (30)$$

By leveraging Eqs. (29) and (30), the subgroup-level consensus is computed as:

$$SCD_{sg_i} = (SCD_{sg_i}^{\text{int}} + SCD_{sg_i}^{\text{ext}}) / 2 \quad (31)$$

Subsequently, based on Eqs. (26) and (31), the overall consensus degree (GCD) is derived as follows:

$$GCD = \sum_{sg_i \in G} \omega_{sg_i} \cdot SCD_{sg_i}. \quad (32)$$

The GCD reflects the overall consensus level of the group, with $GCD \in [0, 1]$. A larger GCD indicates a higher degree of group consensus, and $GCD = 1$ represents perfect consensus among all subgroups. Since perfect consensus is difficult to achieve in practice due to differences in DMs' perspectives, backgrounds, and interests, an acceptable consensus threshold δ is usually predefined. If $GCD \geq \delta$, the group is regarded as having reached consensus; otherwise, feedback mechanisms are activated to improve the consensus level.

4.4 Feedback mechanism

In the CRP, the primary objective of the feedback mechanism is to facilitate the formation of group consensus by guiding DMs or subgroups to adjust their opinions. This mechanism generally consists of two phases: (1) identifying subgroups that require adjustment, and (2) performing opinion adjustments.

4.4.1 Identify low-consensus subgroups

Existing consensus models often neglect the compensatory effect between high- and low-consensus subgroups. When the consensus contribution of high-consensus subgroups exceeds a predefined threshold, the consensus requirement for low-consensus subgroups can be relaxed accordingly. As a result, some originally identified low-consensus subgroups may satisfy the adjusted threshold and be reclassified as consensus-achieved subgroups.

To incorporate this dynamic compensation mechanism, this study adopts a multi-round iterative consensus evaluation process. By continuously updating subgroup consensus levels and thresholds, the method identifies only those subgroups that genuinely hinder global consensus, providing effective targets for subsequent feedback adjustment.

Based on the above analysis, Algorithm 2 provides the basic iterative steps for identifying the low-consensus subgroups.

4.4.2 Identify the members requiring adjustment and modify their preferences

This study develops a cost-minimization feedback mechanism for low-consensus subgroups. Instead of adjusting all DMs, it selects the member with the lowest adjustment cost in each subgroup. The cost is determined by hybrid trust degree, DM weight, and historical adjustment amplitude, thereby promoting consensus while preserving opinion diversity and social trust. These three dimensions are defined as follows:

Algorithm 2 : Determine low-consensus subgroups

Input: Subgroup set $G = \{sg_i \mid i = 1, 2, \dots, g\}$; subgroup weight vector $W = \{\omega_{sg_i} \mid i = 1, 2, \dots, g\}$; consensus threshold δ .

Output: Low-consensus subgroup set G_l .

Step 1: Calculate SCD_{sg_i} using Eq. (31). If $SCD_{sg_i} \geq \delta$, assign sg_i to G_h ; otherwise, assign it to G_l .

Step 2: If $G_l = \emptyset$, output G_l and terminate. Otherwise, calculate the total consensus contribution of high-consensus subgroups: $CD_h = \sum_{sg_i \in G_h} \omega_{sg_i} SCD_{sg_i}$.

Step 3: Calculate the adjusted consensus threshold for low-consensus subgroups: $\tilde{\delta} = (\delta - CD_h) / \sum_{sg_i \in G_l} \omega_{sg_i}$.

Step 4: Move each $sg_i \in G_l$ satisfying $SCD_{sg_i} \geq \tilde{\delta}$ into G_h and remove it from G_l .

Step 5: Repeat Steps 2–4 until no subgroup can be moved from G_l to G_h .

Step 6: Output the final G_l .

Hybrid Trust Degree: A threshold ε is set for hybrid trust. If $ST_{kl} \geq \varepsilon$ and $d(R_g, R_l) < 1 - \delta$, the opinion of e_l is used as a reference for adjusting e_k ; otherwise, the average hybrid trust degree between e_k and other DMs is used. A higher hybrid trust degree implies a lower adjustment cost.

DMs' Weight: DMs with higher weights usually show stronger opinion persistence and therefore have higher unit adjustment costs.

Historical Adjustment Magnitude: A larger historical adjustment magnitude reduces a DM's willingness to adjust further, thereby increasing subsequent adjustment costs.

Based on the above principles, the adjustment cost $Unit_k$ for DM e_k can be calculated using the following formula:

$$Unit_k = \left(\frac{\omega_{sg_i}^k + adj(R_k, \overline{R_k})}{2} \right)^{\overline{ST_k}} \quad (33)$$

where $\overline{ST_k}$ represents the mean hybrid trust degree between DM e_k and the DMs trusted by e_k , $\overline{ST_k} = \frac{\sum_{e_l \in H_k} ST_{kl}}{|H_k|}$, $H_k = \{l \mid ST_{kl} \geq \varepsilon, d(R_g, R_l) < 1 - \delta, k, l \in m\}$, If $H_k = \emptyset$, $\overline{ST_k} = \frac{\sum_{l \in m, l \neq k} ST_{kl}}{m-1}$, $adj(R_k, \overline{R_k}) = d(R_k, R_k^1) + d(R_k^1, R_k^2) + \dots + d(R_k^{z-2}, \overline{R_k})$, which denotes the total adjustment amount from e_k 's original opinion (R_k) to their current opinion ($\overline{R_k}$). Where R_k^z denotes the opinion of e_k in the z -th round.

In each low-consensus subgroup, the DM who fails to satisfy the consensus threshold δ and has the lowest adjustment cost is selected for adjustment. To balance social acceptance and individual characteristics, the adjusted opinion is generated through trust-based fusion. If trusted DMs exist with trust degrees above ε , their opinions are weighted and fused with the target DM's original opinion; otherwise, the global opinion is used as the adjustment reference.

When $H_k \neq \emptyset$, the adjusted preference $\overline{r_{ij}^k}$ of DM e_k is expressed as follows:

$$\overline{r_{ij}^k} = \tau \cdot r_{ij}^k + (1 - \tau) \cdot \sum_{e_l \in H_k} (\omega_k^l \cdot r_{ij}^l) \quad (34)$$

where τ represents the weight given to retaining an individual's original preference in the CRP, $\tau = \frac{CR_k}{CR_k + (\sum_{e_l \in H_k} ST_{kl}) / |H_k|}$, $\omega_k^l = \frac{ST_{kl}}{\sum_{e_i \in H_k} ST_{ki}}$, $H_k = \{l \mid ST_{kl} \geq \varepsilon, d(R_g, R_l) < 1 - \delta, k, l \in m\}$.

When $H_k = \emptyset$, the adjusted preference $\overline{r_{ij}^k}$ of DM e_k is expressed as follows:

$$\overline{r_{ij}^k} = \tau \cdot r_{ij}^k + (1 - \tau) \cdot r_{ij}^g \quad (35)$$

where $R_g = (r_{ij}^g)_{n \times n}$ is the global preference matrix, $\tau = \frac{CR_k}{CR_k + (\sum_{l \in m, l \neq k} ST_{kl}) / (m-1)}$,

The main steps of the proposed LSGDM method are outlined in Algorithm 3 (Sections 4.1–4.4).

Algorithm 3 : Consensus reaching model for LSGDM with LUPRs

Input: Initial preference matrices $R_k = (r_{ij}^k)_{n \times n}$, $k = 1, 2, \dots, m$; initial trust matrix $T = (t_{ij})_{m \times m}$; parameters $\delta, \alpha, \varepsilon, \beta, [\eta_1, \eta_2], d_c, \kappa, \gamma$.

Output: Global preference matrix.

Step 1: Set $z = 1$ and calculate the credibility of each DM using Eqs. (4) and (20)–(22).

Step 2: Cluster DMs into subgroups using Algorithm 1.

Step 3: Calculate DM weights, subgroup weights, subgroup preference matrices, the global preference matrix, subgroup consensus levels, and the global consensus degree using Eqs. (15)–(32).

Step 4: If $GCD \geq \delta$, proceed to Step 7; otherwise, go to Step 5.

Step 5: Feedback adjustment:

Step 5.1: Identify low-consensus subgroups using Algorithm 2.

Step 5.2: Calculate adjustment costs using Eq. (33), identify the DMs requiring adjustment, and sort them in ascending order of adjustment cost.

Step 5.3: Adjust the selected DMs iteratively using Eqs. (34)–(35). After each adjustment, repeat Steps 2–4 to update clustering and consensus evaluation. If $GCD \geq \delta$, proceed to Step 7; otherwise, continue until all selected DMs have been adjusted, then proceed to Step 6.

Step 6: Set $z = z + 1$. If $z > \gamma$, go to Step 8; otherwise, return to Step 2.

Step 7: Output the global preference matrix and terminate the process.

Step 8: Terminate the process when the maximum number of iterations is exceeded.

4.5 Selection process

When the global consensus level reaches the predefined consensus threshold, the priority weights of each alternative can be derived using Eq. (6). Based on these weights, the alternatives can be ranked to select the optimal solution.

5 Case study

Integrated Project Delivery (IPD) emphasizes multi-stakeholder collaboration and information integration throughout the project lifecycle. Since project performance is influenced by multiple Critical Success Factors (CSFs), prioritizing CSFs can be formulated as an LSGDM problem.

Following AIA publications, 18 IPD-related CSFs are classified into five dimensions: organization and decision making, project management strategy, digital technologies, collaboration, and communication. The publicly available LSGDM dataset provided by Gong is adopted, which contains 20 LUPRs and the corresponding initial trust matrix provided by DMs. The corresponding dataset is publicly available at <https://doi.org/10.17632/3vhyvy5w3c.1>.

The model parameters are set as follows: $\delta = 0.915$, $d_c = 0.3$, $\kappa = 0.32$, $[\eta_1, \eta_2] = [4, 6]$, $\varepsilon = 0.7$, $\beta = 0.7$, $\gamma = 5$, $\vartheta = 0.8$, and $\alpha = 0.4$.

Each DM performs pairwise comparisons of the five dimensions using LUPRs. DM credibility is calculated using Eqs. (4) and (20)–(22).

Round 1 ($z = 1$):

Using Algorithm 1, the 20 DMs are clustered into five subgroups: $sg_1 = \{e_4, e_3, e_7, e_8, e_{12}\}$, $sg_2 = \{e_{17}, e_{11}, e_{20}\}$, $sg_3 = \{e_{13}, e_9, e_{16}, e_{19}\}$, $sg_4 = \{e_5, e_1, e_2\}$, and $sg_5 = \{e_{15}, e_6, e_{14}, e_{18}, e_{10}\}$. The silhouette coefficient selects five subgroups.

The global weights of DMs, subgroup weights, subgroup preference matrices, and global preference matrix are calculated using Eqs. (15)–(28). The subgroup consensus levels and global consensus degree are then obtained by Eqs. (29)–(32), as shown in Table 1.

Since the first-round GCD is 0.8882, which is lower than the predefined threshold $\delta = 0.915$, feedback adjustment is required. According to Algorithm 2, all five subgroups are initially identified as low-consensus subgroups. Based on the unit adjustment cost calculated by Eq. (33), the DMs

Table 1: First-round subgroup characteristics and global consensus

Subgroup	DMs	Silhouette coefficient	Subgroup Size	Subgroup Weight	Subgroup Consensus	GCD
sg_1	$e_4, e_3, e_7, e_8, e_{12}$	0.5784	5	0.2253	0.8989	0.8882
sg_2	e_{17}, e_{11}, e_{20}	0.6557	3	0.1904	0.8789	
sg_3	$e_{13}, e_9, e_{16}, e_{19}$	0.5588	4	0.1992	0.8926	
sg_4	e_5, e_1, e_2	0.5975	3	0.1855	0.8953	
sg_5	$e_{15}, e_6, e_{14}, e_{18}, e_{10}$	0.4571	5	0.1996	0.8738	

Table 2: Final subgroup characteristics and global consensus

Subgroup	DMs	Silhouette coefficient	Subgroup Size	Subgroup Weight	Subgroup Consensus	GCD
sg_1	$e_4, e_3, e_7, e_8, e_{12}, e_{18}$	0.5130	6	0.2704	0.9201	0.9152
sg_2	e_{11}, e_{13}, e_{16}	0.5129	3	0.1969	0.9295	
sg_3	$e_{20}, e_2, e_6, e_{14}, e_{15}, e_{17}, e_{19}$	0.4856	7	0.2891	0.9064	
sg_4	e_{10}, e_1, e_5, e_9	0.5886	4	0.2436	0.9050	

selected for adjustment are $\{e_5, e_4, e_{19}, e_6, e_{15}\}$ in ascending order of adjustment cost. After sequential adjustment, the GCD increases to 0.8972, but the consensus threshold is still not reached.

Round 2 ($z = 2$):

The second round of dynamic clustering and feedback adjustment is then conducted based on the updated hybrid trust network. Using Algorithm 1, the 20 DMs are re-clustered into five subgroups: $sg_1 = \{e_4, e_3, e_7, e_8, e_{12}\}$, $sg_2 = \{e_{11}, e_{13}, e_{16}\}$, $sg_3 = \{e_{20}, e_2, e_6, e_{14}, e_{15}, e_{17}, e_{19}\}$, $sg_4 = \{e_{10}, e_9\}$, and $sg_5 = \{e_5, e_1, e_{18}\}$. The corresponding subgroup consensus levels are 0.9065, 0.9023, 0.8918, 0.8824, and 0.8993, respectively, and the GCD reaches 0.8972. Since this value is still below the threshold $\delta = 0.915$, feedback adjustment is further required.

According to Algorithm 2, all five subgroups are again identified as low-consensus subgroups. The unit adjustment costs calculated by Eq. (33) indicate that the DMs requiring adjustment are $\{e_{12}, e_{13}, e_{20}, e_{10}, e_5\}$, with costs $Unit_{12} = 0.038$, $Unit_{13} = 0.045$, $Unit_{20} = 0.180$, $Unit_{10} = 0.235$, and $Unit_5 = 0.200$, respectively. Thus, the adjustment sequence is determined as $\{e_{12}, e_{13}, e_{20}, e_5, e_{10}\}$ in ascending order of unit adjustment cost.

After sequentially adjusting these DMs according to Eqs. (34) and (35), the GCD values increase to 0.8995, 0.9039, 0.9051, 0.9079, and 0.9121, respectively. Although the second-round adjustment increases the group consensus level, the final value remains slightly lower than the required threshold of 0.915. Therefore, the process proceeds to the third round of dynamic clustering and feedback adjustment.

Round 3 ($z = 3$):

In the third round, the dynamic compensation mechanism further distinguishes the subgroups that genuinely require adjustment. Subgroups sg_1 , sg_2 , and sg_5 first satisfy the consensus condition, and sg_4 is subsequently reclassified as a high-consensus subgroup after threshold compensation. Consequently, only sg_3 remains as the low-consensus subgroup. The adjustment cost calculation identifies e_{15} as the target DM, with $Unit_{15} = 0.044$. After adjusting the preference of e_{15} , the GCD reaches 0.9152, satisfying the predefined consensus threshold. The final subgroup characteristics and global consensus results are reported in Table 2.

Using Eq. (6), the final priority weights of the five alternatives are determined as 0.2824, 0.2565, 0.2101, 0.1169, and 0.1341, respectively. Therefore, the alternatives are ranked as $x_1(0.2824) > x_2(0.2565) > x_3(0.2101) > x_5(0.1341) > x_4(0.1169)$.

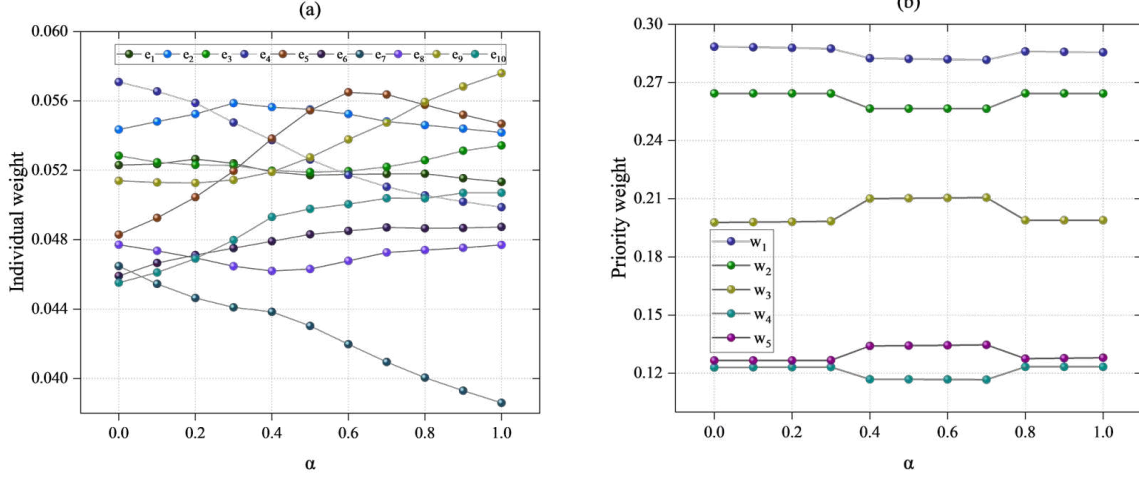


Fig. 1: Effect of risk attitude parameter on individual weight and priority weight

6 Performance evaluation and comparative analysis of the proposed model

This section first explores the influence of the risk attitude parameter through sensitivity analysis. Subsequently, it conducts simulation experiments and a comparative analysis to validate the effectiveness and feasibility of the proposed method.

6.1 Sensitivity analysis

To investigate the influence of risk attitude parameter on the model, we let α vary within the range $[0, 1]$ with a step size of 0.1. In the analysis of the impact of α on DMs' overall weights, we only select the top 10 DMs for analysis. In the example in Section 5, the priority weights produced by the model under different values of α are shown in Fig. 1.

As shown in Fig. 1(a), the risk attitude parameter α affects DM global weights, but no consistent trend is observed because its influence on the consistency level CD is non-monotonic. Nevertheless, Fig. 1(b) shows that the final ranking of alternatives remains stable. This is because the proposed weighting mechanism integrates group size, consensus level, trust relationships, and individual weights, reducing the impact of local parameter fluctuations. Thus, the model shows good robustness to changes in α .

6.2 Simulation analysis

This section evaluates the proposed model through numerical simulations using three indicators: adjusted distance (AD), consensus iteration rounds (Z) [26], and consensus success rate (P). The adjusted distance is defined as $AD = \frac{1}{2mn^2} \sum_{k=1}^m \sum_{i=1}^n \sum_{j=1}^n (|\hat{r}_{ij}^{k-} - r_{ij}^{k-}| + |\hat{r}_{ij}^{k+} - r_{ij}^{k+}|)$ [17].

Monte Carlo simulations are conducted following Algorithm 4. The tested parameter settings include consensus thresholds $\{0.88, 0.89, 0.90, 0.91, 0.92\}$, numbers of DMs $\{20, 30, 40, 50, 60\}$, and alternative numbers $\{5, 6, 7\}$. For each parameter combination, LUPR matrices and trust matrices are randomly generated to reflect data sparsity and preference uncertainty in practical scenarios. The consensus model is executed 1000 times, and the average results are used for performance evaluation.

Other parameters are set as follows: $\alpha = 0.4$, $\varepsilon = 0.3$, $\beta = 0.7$, $[\eta_1, \eta_2] = [5, 10]$, $d_c = 0.3$, $\kappa = 0.5$, and $\gamma = 10$. The simulation procedure is shown in Algorithm 4, and the results are reported in Figs. 2–4.

Algorithm 4 Simulation algorithm

Input: Consensus thresholds $\{0.88, 0.89, 0.90, 0.91, 0.92\}$; DM counts $\{20, 30, 40, 50, 60\}$; numbers of alternatives $\{5, 6, 7\}$; risk attitude parameter α ; hybrid trust degree threshold ε ; trust relationship update threshold β ; maximum number of consensus iterations γ ; range of cluster numbers $[\eta_1, \eta_2]$; cutoff distance d_c ; distance threshold κ ; number of simulation runs $R = 1000$.

Output: Average adjusted distance AD , average number of consensus iterations Z , and consensus success rate P .

Step 1: Construct all parameter combinations (δ, m, n) from the consensus thresholds, DM counts, and numbers of alternatives.

Step 2: For each parameter combination (δ, m, n) , set the simulation counter $r = 1$.

Step 3: Randomly generate m LUPRs $R_k = (r_{ij}^k)_{n \times n}$, $k = 1, 2, \dots, m$, and an initial trust matrix $T = (t_{ij})_{m \times m}$.

Step 4: Run Algorithm 3 once using the generated preference matrices and trust matrix. Record the adjusted distance, the number of consensus iterations, and whether the consensus threshold is reached.

Step 5: Set $r = r + 1$. If $r \leq R$, return to Step 3; otherwise, proceed to Step 6.

Step 6: For the current parameter combination, calculate the average adjusted distance AD , the average number of consensus iterations Z , and the consensus success rate P over the R simulation runs.

Step 7: If untested parameter combinations remain, select the next combination and return to Step 2; otherwise, output all simulation results and terminate.

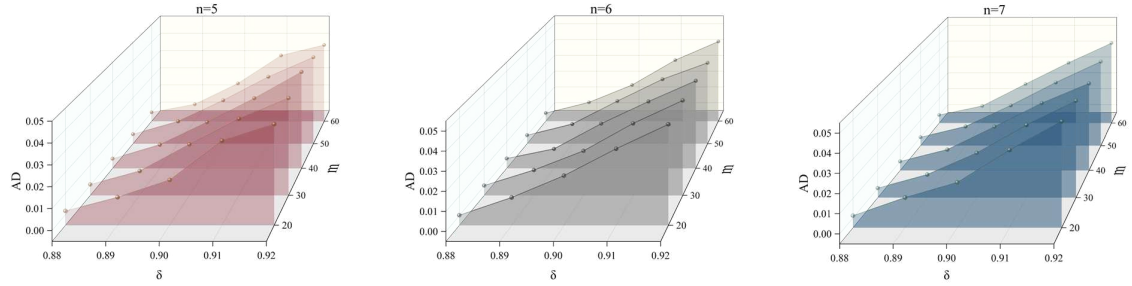


Fig. 2: Effect of parameters m , δ , and n on AD

6.2.1 Adjustment Distance analysis

The AD is used to measure the overall extent of the DMs' preference adjustments, where a smaller value indicates higher efficiency of the model in guiding consensus. As shown in Fig. 2, compared with the scale of DMs (m) and the number of alternatives (n), the AD is more sensitive to changes in the consensus threshold (δ). With an increase in δ , the AD shows an upward trend. In contrast, variations in m and n have a weak impact on the AD . This indicates that the model maintains good adjustment efficiency even when the scale of DMs and the number of alternatives change.

6.2.2 Consensus iteration rounds analysis

The value of Z represents the convergence efficiency of the model in reaching consensus. Fig. 3 shows that the Z is jointly influenced by δ and m , as δ and m increase, the Z also rises, this is because higher consensus requirements and larger-scale groups demand more rounds of coordination. However, the impact of n on the Z is relatively limited, which indicates that the model can adapt to different complexity levels of decision scenarios.

6.2.3 Consensus success rate analysis

The value of P is a key indicator for evaluating the model's reliability. As shown in Fig. 4, when $\delta \leq 0.90$, the model can achieve a consensus success rate close to 100% across different parameter settings. When $\delta = 0.91$, a slight decrease in the success rate only occurs in the extreme configuration

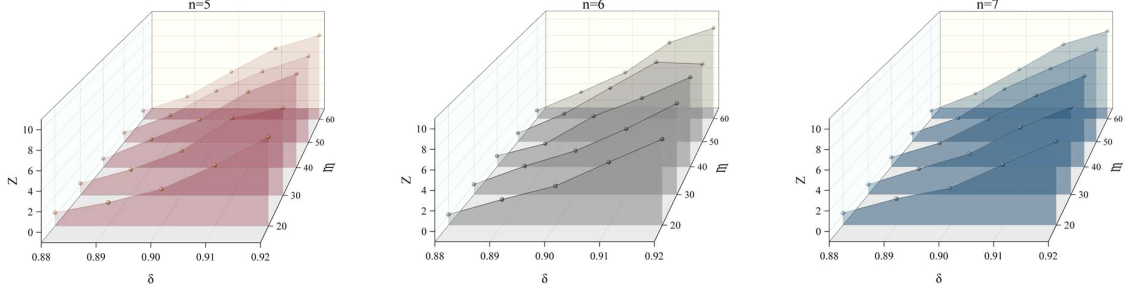


Fig. 3: Effect of parameters m , δ , and n on Z

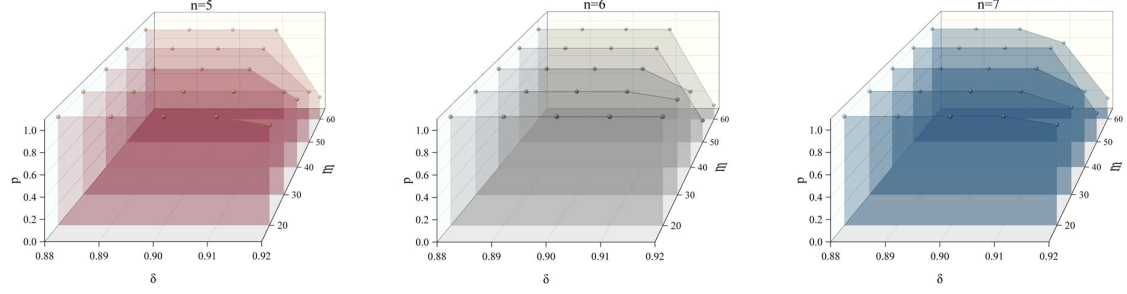


Fig. 4: Effect of parameters m , δ , and n on P

($n = 7$, $m = 60$). When δ is increased to 0.92, the model only performs well for small-scale groups ($m \leq 40$). This indicates that the model has applicability limits for scenarios where high consensus requirements and large-scale groups coexist.

6.2.4 Overall performance

The model demonstrates good effectiveness and efficiency under conventional and high consensus requirements. When $\delta \leq 0.91$, the consensus success rate remains above 95%, while AD stays below 0.04, indicating that high consensus can be achieved with limited opinion adjustment. This performance mainly benefits from the hybrid trust network and the two-stage feedback mechanism, which narrows the adjustment range through dynamic consensus compensation and improves feedback efficiency by selecting low-cost DMs based on mixed trust. Historical data further smooths trust fluctuations and stabilizes the CRP.

In addition, changes in n have only a limited influence on AD , Z , and P , indicating that the proposed model maintains relatively stable performance across different numbers of alternatives. However, when $\delta = 0.92$ and $m > 40$, the consensus success rate decreases, indicating greater difficulty in reaching consensus when high consensus requirements are combined with large group sizes.

6.3 Comparative analysis

6.3.1 Quantitative comparison analysis

To validate the effectiveness of the proposed method, this section compares it with the approach of [17]. Given that the two methods employ different techniques for determining priority weights, the comparative experiment adopts the same method as that used by [17] to ensure consistency. The results in Table 3 show that the proposed method produces the same ranking and requires the same number of iterations as the reference method in the considered scenario, demonstrating consistency between the two methods.

Table 3: Quantitative comparison analysis

Method	Adjustment rules	Initial GCD	Final GCD	Iteration	Ranking
Method in [17]	Cost-minimization & dual-management adjustment	0.876	0.900	2	$x_2 > x_1 > x_3 > x_4 > x_5$
The proposed method	Cost-aware trust-based adjustment with dual management	0.888	0.902	2	$x_2 > x_1 > x_3 > x_4 > x_5$

Table 4: Qualitative comparative analysis

Reference	Dynamic trust network	Historical preference info.	Historical trust info.	Unit adjustment costs	Dynamic clustering
Gong et al.[17]	No	No	No	Yes	No
Peng et al.[10]	Yes	Yes	No	No	Yes
Li et al.[8]	Yes	No	No	No	No
Guo et al.[22]	No	No	No	Yes	No
The proposed	Yes	Yes	Yes	Yes	Yes

6.3.2 Qualitative comparative analysis

This study compares the proposed method with SNA-based approaches from five aspects: dynamic trust network, historical preference information, historical trust information, unit adjustment cost, and dynamic clustering.

Although existing studies integrate trust relationships and preference information into social networks [27, 28], most treat trust as static and cannot capture its evolution during the CRP. Since DMs tend to trust peers with similar opinions, the proposed method dynamically updates trust according to preference similarity and incorporates historical preference similarity and trust degree to construct a hybrid trust network. This design better represents trust evolution and preference–trust interaction during the CRP. The qualitative comparison results are summarized in Table 4.

Table 4 shows that the proposed method jointly supports dynamic trust updating, historical preference and trust information, dynamic clustering, and unit adjustment costs. It therefore combines the partially dynamic mechanisms of existing approaches with cost-aware feedback and risk attitudes.

7 Conclusions

This study proposes an LSGDM consensus model with LUPRs for dynamic social network environments. The model updates trust relationships according to current preference similarities and incorporates historical preference similarities and trust degrees to construct a hybrid trust network. Based on the hybrid trust relationships, an improved DPC algorithm is used to dynamically group DMs, while a two-stage feedback mechanism first identifies low-consensus subgroups through consensus contribution compensation and then determines preference adjustments using a unit adjustment cost model. In this way, the clustering and feedback processes are jointly updated during the consensus-reaching process.

The case study, sensitivity analysis, simulation experiments, and comparative analysis evaluate the performance of the proposed model under different decision settings. In the simulation experiments, when the consensus threshold satisfies $\delta \leq 0.91$, the consensus success rate remains above 95%, while the adjusted distance stays below 0.04. Changes in the number of alternatives have only a limited influence on the adjusted distance, the number of consensus iterations, and the consensus success rate. However, when $\delta = 0.92$ and the number of DMs exceeds 40, the consensus success rate decreases, indicating greater difficulty in reaching consensus when high consensus requirements are combined with large group sizes.

Future research can extend the proposed model to incomplete trust relationships, ultra-large-scale groups, and heterogeneous preference information. Further studies may also examine its performance using additional real-world decision-making data.

Data availability. The dataset used in this study is publicly available at <https://doi.org/10.17632/3vhyvy5w3c.1>.

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Acknowledgements. This research was supported by the National Natural Science Foundation of China (Nos. 62466048, 62363016, and 62566050), the Natural Science Foundation of Qinghai Province (No. 2025-ZJ-994M), and the Scientific Research Innovation Capability Support Project for Young Faculty of Universities (No. SRICSPYF-BS2025007).

Author contributions. X.L.: Conceptualization, Methodology, Supervision, Writing–review and editing. K.L.: Conceptualization, Methodology, Software, Formal analysis, Writing–original draft. F.L.: Methodology, Validation, Investigation, Writing–review and editing. Z.Y.: Methodology, Validation, Supervision, Writing–review and editing. Z.C.: Methodology, Formal analysis, Writing–review and editing.

Competing interests. The authors declare no competing interests.