

Supplementary Information

A lost sarcophagus constrains the closure sequence of the Menkaure pyramid at Giza

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S1 Purpose

This supplement exposes the numerical inputs, deterministic transformations and evidential classification used in the manuscript. The main test uses Perring's explicitly stated dimensions for a reconstructed intermediate architectural state. All numerical inputs are unchanged in this revision.

'Figs. 7, 8, and 9, represent the state of the Portcullis and of the Passage previous to its being finally closed up. As the Sarcophagus must have been already deposited, the small Passage, 2 feet 4 inches high, by 2 feet 11½ inches wide, might, Mr. Perring conjectures, have been left for the admission of the body in a wooden case.' [Perring, printed p. 8, description of Plate IV]. This wording binds state, height and width in one source while explicitly marking the proposed bodily use as Perring's conjecture.

S2 Frozen dimensional ledger

Variable	Value	State	Use
Coffer length	96 in	Measured 1837	Run-length condition
Coffer transverse sides	37 × 35 in	Measured 1837	Minimum thickness
Raw passage height	57 in	Pre-restriction state	Installation envelope
Intermediate width	35.5 in	Perring p. 8 state	Aperture width
Intermediate height	28 in	Perring p. 8 state	Aperture height
Built-up run	201 in	Separate p. 9 record	Conditional sensitivity only
M&R section	93 × 90 cm; variable height	Later reconstruction	Sensitivity only
Petrie channel	35.4–35.6 × 35.6 in	Post-extraction survey	Reconciliation only

S3 Source-state separation

State	Source basis	Admissible claim
Open / installation	Perring raw envelope and sequence	Coffer introduced before restriction
Intermediate aperture	Perring p. 8 description and Plate IV	Principal opening model
Built-up run	Perring p. 9 dimensional table	Separate record; overlap unresolved
Reopened / extraction	Vyse's removal narrative	Recorded reopening interventions
Later / reconstructed	Petrie survey; M&R reconstruction	Reconciliation and sensitivity only

S4 Orientation-independent proof

For the rigid rectangular model with outer dimensions $96 \times 37 \times 35$ in, the opening must contain the central section when the chest's centre crosses its plane. In a unit in-plane direction u expressed in the coffer's axes, the full central chord is $c(u) = \min_i(a_i/|u_i|)$, omitting zero denominators. Since every $|u_i| \leq 1$, $c(u)$ is at least 35 in. Perring's 28 in opening height therefore excludes passage by 7 in independently of orientation or opening thickness.

The idealized open-topped rectangular chest retains all twelve outer edges. For its outer cuboid B , five-faced shell S and opening plane P , the section satisfies $\text{conv}(S \cap P) = B \cap P$. A convex rectangular opening containing the shell section must contain the same convex hull. This statement assumes an intact rectangular shell; it does not reconstruct the detailed carving, damage or irregular surfaces of the lost historical object.

With assumed adverse ± 0.5 in rounding, the minimum coffer edge is 34.5 in and the opening height is 28.5 in, leaving 6.0 in (15.24 cm) of exclusion. Equal symmetric error would have to reach $\delta = 3.5$ in on both dimensions before the inequality could close. This is a sensitivity scenario, not an empirical confidence interval.

Perring separately reports a 201 in built-up run on p. 9. Only if field or archival registration shows that the p. 8 aperture continued through that run does the long-prism calculation apply: the coffer's 108.7 in space diagonal is shorter than 201 in, so the object would have to lie wholly within the restriction at some instant.

S5 Controls and sensitivity

The intermediate aperture gives $+0.5$ in width relative to the coffer's 35 in side but -7 in height relative to its minimum edge. Even adverse ± 0.5 in rounding leaves -6.0 in. The height exclusion is the principal result and needs no passage length. The 994 versus 1,295 in² projection comparison is conditional on continuity between Perring's p. 8 aperture and p. 9 built-up run. Maragioglio and Rinaldi's later section and Petrie's post-extraction survey are not used to establish the intermediate opening's geometry.

S6 Falsification conditions

Field target	Supporting observation	Contradicting or limiting observation
Intermediate opening	Plate IV and text describe a single reconstructable 28×35.5 in opening.	Dimensions belong to different features or incompatible stages.
Opening and run registration	Locate any common clear section and establish its continuous extent.	Insufficient continuous overlap prevents the long-prism test; it does not affect the thin-opening proof.
Ramp and block interface	Bearing surfaces and block positions match the relation shown in the historical record.	Registered fabric contradicts the proposed relation between ramp and blocks.
Extraction interventions	Removal features can be located consistently with the narrative.	Positive evidence places the removed obstruction elsewhere; missing scars alone are inconclusive.
Later survey registration	Later dimensions are tied to identified post-removal or reconstructed states.	Evidence establishes the same section and unchanged geometry before and after extraction.
Recovered coffer	Secure identification and preserved original surfaces support the recorded dimensions and rectangular model.	Reliable incompatible geometry requires recalculation; later damage or incomplete remains may prevent a test.

S7 Deterministic audit outputs

Output	Value	Role
Intermediate width clearance	$+0.500$ in	Width comparison only
Intermediate height clearance	-7.000 in	Principal result
Adverse-rounding exclusion	6.000 in / 15.24 cm	Principal robustness
Projection-area deficit	-301 in ² / -23.24%	Conditional sensitivity
Height exclusion	17.78 cm / 20.00%	Principal result
Run/space-diagonal ratio	1.850	Conditional on record continuity
Raw height minus coffer side	$+20.000$ in	Pre-restriction control

S8 Machine-readable files

Supplementary Data 1 contains the dimensional ledger in CSV format. Supplementary Software 1 independently recomputes the numerical results and assertions. Supplementary Software 2 regenerates Figures 1–3 and the six equation images; it does not generate manuscript prose. The historical basis and model limitations are explained in Methods 2.1–2.5 and the Discussion of the revised manuscript.