

Understanding ENSO asymmetry with a reduced stochastic model: deterministic nonlinearity and multiplicative noise

Cristian Martinez-Villalobos

`cristian.martinez.v@uai.cl`

Universidad Adolfo Ibanez <https://orcid.org/0000-0001-7057-5553>

Boris Dewitte

Universidad Católica del Norte: Universidad Catolica del Norte

Soong-Ki Kim

Pohang University of Science and Technology

Malte F. Stuecker

University of Hawai'i at Mānoa: University of Hawai'i at Manoa

Research Article

Keywords: ENSO asymmetry, reduced stochastic model, nonlinear dynamics, multiplicative noise, Fokker-Planck equation, CMIP6

Posted Date: October 13th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-11087240/v1>

License:   This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Understanding ENSO asymmetry with a reduced stochastic model: deterministic nonlinearity and multiplicative noise

Cristian Martinez-Villalobos^{1,2*}, Boris Dewitte^{3,4,5}, Soong-Ki Kim⁶ and Malte F. Stuecker^{7,8}

¹Faculty of Engineering and Sciences, Universidad Adolfo Ibáñez, Santiago, Chile.

²Data Observatory Foundation, Santiago, Chile.

³Center for Ecology and Sustainable Management of Oceanic Islands (ESMOI), Departamento de Biología Marina, Facultad de Ciencias del Mar, Universidad Católica del Norte, Coquimbo, Chile.

⁴Centro de Estudios Avanzados en Zonas Áridas (CEAZA), Coquimbo, Chile.

⁵CECI, Université de Toulouse III, CERFACS/CNRS/IRD, Toulouse, France.

⁶Division of Environmental Science and Engineering, Pohang University of Science and Technology, Pohang, South Korea.

⁷International Pacific Research Center (IPRC), School of Ocean and Earth Science and Technology (SOEST), University of Hawai‘i at Mānoa, Honolulu, HI, USA.

⁸Department of Oceanography, School of Ocean and Earth Science and Technology (SOEST), University of Hawai‘i at Mānoa, Honolulu, HI, USA.

*Corresponding author(s). E-mail(s): cristian.martinez.v@uai.cl;

Abstract

El Niño Southern Oscillation (ENSO) exhibits pronounced asymmetry, with stronger warm events than cold events, yet the mechanisms underlying this non-Gaussian behavior remain debated. Here we develop a

minimal stochastic framework to diagnose the origin of ENSO asymmetry using daily sea surface temperature (SST) and zonal wind anomalies in the equatorial Pacific. Starting from a two-variable SST-wind system estimated from observations, we derive a reduced one-dimensional stochastic model for the Niño 3 SST index, in which nonlinear deterministic drift and state-dependent stochastic forcing emerge naturally. This reduction enables analytical solutions for the stationary probability density function under four model configurations, isolating the roles of nonlinear dynamics and stochastic forcing. We show that nonlinear deterministic drift is the leading contributor of ENSO skewness in this daily reduced framework, generating asymmetric Niño 3 distributions even under additive noise, while state-dependent noise acts as a secondary modifier, primarily affecting the probability of extreme warm events. The resulting model provides a mechanistic interpretation of ENSO statistics in terms of phase-space structure and effective SST damping. Extending this analysis to CMIP6 simulations, we show that intermodel differences in ENSO asymmetry are related, to leading order, to differences in effective SST damping and SST variance. These results highlight the central role of nonlinear deterministic processes in shaping ENSO asymmetry within a framework estimated at daily resolution and provide a tractable basis for interpreting non-Gaussian climate variability.

Keywords: ENSO asymmetry, reduced stochastic model, nonlinear dynamics, multiplicative noise, Fokker–Planck equation, CMIP6

1 Introduction

El Niño–Southern Oscillation (ENSO) arises from coupled interactions between the tropical Pacific Ocean and atmosphere, in which sea surface temperature (SST) anomalies and large-scale winds evolve together through well-established feedbacks. At leading order, positive SST anomalies in the eastern equatorial Pacific reduce the zonal SST gradient, weaken the Walker circulation and the equatorial trade winds, thereby suppressing upwelling and enhancing local warming; negative anomalies act in the opposite direction[1–3]. This two-way interaction between SST and winds, known as the Bjerknes feedback[1, 2], provides a mechanism for the growth of anomalies and underlies the amplitude and persistence of ENSO events[4]. A fundamental feature of ENSO variability is its pronounced asymmetry: warm events (El Niño) tend to be stronger and more spatially extensive than cold events (La Niña), leading to positively skewed SST distributions in the eastern Pacific[5–7]. Despite decades of study, the dynamical origin of this asymmetry remains debated, with proposed explanations invoking nonlinear deterministic feedbacks[8, 9], state-dependent atmospheric noise[7, 10], or a combination of both. A key unresolved question is how these contributions can be disentangled within a unified and interpretable framework.

Disentangling the respective roles of nonlinear deterministic feedbacks and stochastic atmospheric forcing is difficult because these coupled feedbacks operate in a system that is inherently multiscale. While ENSO is often characterized as an interannual mode, its evolution is continuously influenced by higher frequency atmospheric variability[11–13], including intraseasonal wind bursts, synoptic disturbances, and rapid air-sea flux fluctuations[14]. These fast processes act as stochastic forcing on the ocean, modulating the growth, decay, and phase transitions of SST anomalies. How such short-time-scale variability projects onto, is rectified by, and feeds back onto the large-scale, low frequency structure of ENSO remains incompletely understood[13, 15–17]. In practice, ENSO variability is commonly summarized using monthly SST indices and smoothed monthly diagnostics, which emphasize its seasonal-to-interannual character[18, 19] and filter out much of the faster atmospheric variability[20]. While this coarse-graining is appropriate for isolating the seasonal-to-interannual ENSO signal, it also suppresses the event-scale structure of the fast atmospheric variability through which the ocean is forced. Consequently, within monthly-mean frameworks, the contribution of fast atmospheric variability to ENSO asymmetry is represented in a temporally coarse-grained form, often through state-dependent or multiplicative noise parameterizations[7, 10, 13, 15, 21]. The resulting partition between resolved deterministic drift and unresolved stochastic forcing can therefore depend on temporal resolution.

That difficulty motivates a closer examination of ENSO dynamics at shorter time scales. At daily resolution, SST anomalies respond to both slowly varying coupled feedbacks and rapidly fluctuating atmospheric forcing, particularly through zonal wind variability along the equator[22]. From this perspective, ENSO can be viewed as the outcome of a reduced set of coupled variables, SST and winds, subject to both deterministic feedbacks and stochastic forcing. In classical conceptual models[23, 24], however, this interaction is mediated by subsurface ocean heat content, typically represented by sea surface height (SSH) or thermocline depth, which acts as an intermediary between wind forcing and SST. Such upper-ocean variables provide important memory and are central to recharge-discharge descriptions of ENSO evolution[23, 24]. In stripped-down recharge-discharge formulations, low-frequency thermocline and heat-content anomalies are closely linked to equatorial wind forcing through ocean adjustment[25, 26]. This motivates examining SST-wind coupling directly as a complementary reduced representation. Using winds as an explicit model variable also allows part of the fast atmospheric variability to be represented directly through the wind field and its coupling to SST, while the remaining unresolved component enters as stochastic forcing. The daily sampling preserves the observed separation between relatively rapid wind adjustment and much slower SST evolution[20, 27], providing the empirical basis for the fast-variable reduction developed below. The present SST-wind formulation is not intended as a dynamical replacement for that degree of freedom. Rather, we ask whether explicitly retaining an upper-ocean state variable

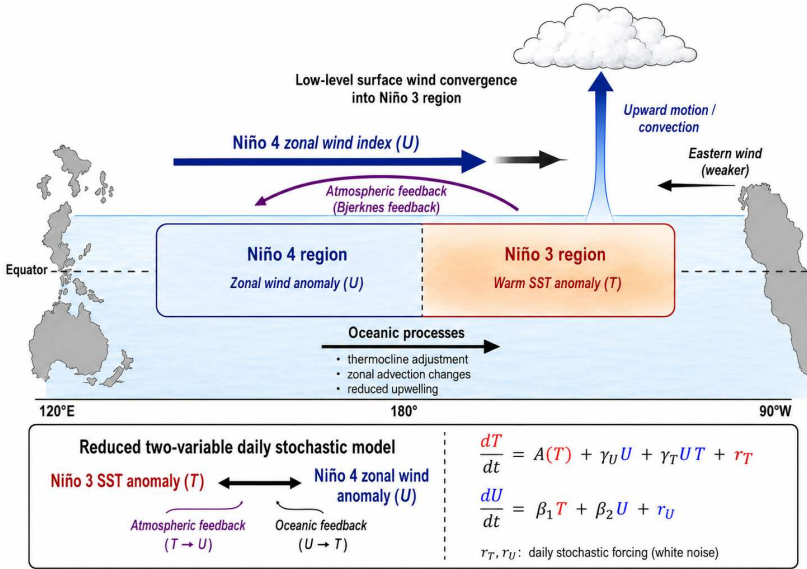
4 *ENSO asymmetry in a reduced stochastic model*

Fig. 1 Schematic of the reduced two-variable daily stochastic model used in this study. Niño 3 SST anomalies (T) and Niño 4 zonal wind anomalies (U) are represented as adjacent equatorial-Pacific indices coupled through atmospheric feedback from (T) to (U) and oceanic feedback from (U) to (T). Westerly Niño 4 zonal wind anomalies favor low-level convergence, convection, and warm SST anomalies in the Niño 3 region through thermocline adjustment, horizontal and vertical advection, entrainment, and mixing, while (r_T) and (r_U) denote daily stochastic forcing. Schematic generated with ChatGPT image generation (OpenAI) and edited by the authors.

materially changes the statistical attribution of Niño 3 SST asymmetry considered here, a question tested directly with an SSH-augmented formulation in Supplementary Text 2.

Such a reduced description fits naturally within the broader hierarchy of conceptual and stochastic ENSO models, including recharge-oscillator, delayed-oscillator, and linear inverse modeling frameworks, which distill the coupled dynamics into a small number of interacting degrees of freedom[3, 4, 23, 24, 28–34]. Observed ENSO statistics, particularly for eastern Pacific SST anomalies, exhibit clear non-Gaussian features, including positive skewness and an enhanced likelihood of extreme warm events[5, 6, 35], sometimes referred to as “super El Niños”. These features suggest that nonlinear processes, either in the deterministic dynamics[6, 36–39], the stochastic forcing[7, 10, 11, 13], or both, play a role in shaping ENSO variability. While nonlinear reduced models have been shown to reproduce ENSO asymmetry and regime behavior[6, 24, 38, 40], their implications are often assessed through trajectories or simulated event statistics, rather than through closed-form stationary distributions that isolate the contribution of individual drift and noise terms. In low-dimensional stochastic systems, the deterministic drift (the state-dependent mean tendency) and stochastic forcing structure determine

the stationary probability distribution through the associated Fokker-Planck equation[41–45], providing an explicit, though not necessarily simple, link between dynamical mechanisms and long-term statistics. This interpretation isolates the respective roles of nonlinear deterministic dynamics and stochastic structure in shaping ENSO asymmetry.

To examine this question, here we use a minimal coupled framework at daily resolution, in which SST anomalies interact directly with equatorial zonal wind variability. The framework considers both linear and nonlinear deterministic terms, as well as additive and state-dependent stochastic forcing in the resulting reduced formulations, while remaining intentionally low dimensional. Our objective is not to construct a new predictive model of ENSO, but to use a deliberately simple stochastic-dynamical system as a diagnostic tool. This daily perspective complements traditional low-frequency and monthly-mean analyses of ENSO dynamics[7, 10, 18, 23, 29]. Section 2 describes the observational and CMIP6 datasets. Section 3 introduces the two-variable daily SST-wind model, while Sections 4 and 5 examine its deterministic phase-space structure and joint SST-wind statistics. Section 6 reduces the coupled system to an effective one-dimensional SST equation, in which state-dependent stochastic forcing emerges from the elimination of the wind variable rather than being imposed ad hoc. Section 7 derives the stationary distributions of four reduced model configurations and uses them to separate the contributions of nonlinear drift and state-dependent noise to the asymmetry of the Niño 3 SST distribution. Section 8 then applies the resulting effective-damping framework to CMIP6 simulations.

2 Data and Methods

To implement this daily framework, we use daily sea surface temperature (SST) from the NOAA Daily Optimum Interpolation SST product (OISST v2[46]) at $0.25^\circ \times 0.25^\circ$ resolution, and daily 10 m zonal wind from the NCEP-NCAR Reanalysis 1[47] on its native Gaussian grid (approximately 2° resolution) over the period 1982–2019. The Niño3 SST index ($5^\circ S - 5^\circ N$, $210^\circ - 270^\circ E$) and the Niño4 zonal wind index ($5^\circ S - 5^\circ N$, $160^\circ - 210^\circ E$) are calculated as area averages over their respective regions[18]. Leap days are removed, so that each year contains 365 days. For each index, anomalies are calculated relative to the 1982–2019 climatology by removing a smooth seasonal cycle obtained from the daily mean annual cycle fitted with the first two Fourier harmonics (annual and semiannual components). The resulting anomaly time series are then linearly detrended over the full period.

In addition to these surface fields, we use daily sea surface height (SSH) from the Mercator Ocean GLORYS2V4 reanalysis[48, 49] as an upper-ocean proxy for thermocline and heat content variations in the tropical Pacific[50, 51]. The archive used here spans 1993–2017. SSH anomalies are computed consistently with SST and wind, with removal of the seasonal cycle and linear detrending applied prior to analysis.

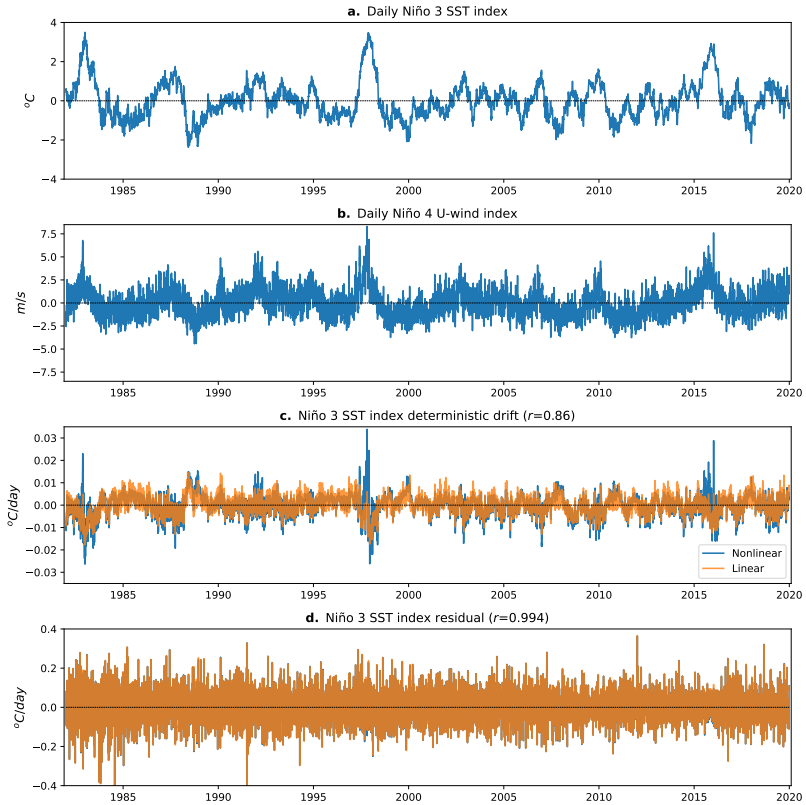


Fig. 2 **a.** **(b.)** Daily Niño 3 (Niño 4) SST (U-wind) index. **c.** Daily deterministic drift ($A(T)+\gamma_U U+\gamma_T UT$) on the Niño 3 SST index equation estimated in the linear and nonlinear models during 1982–2019. This is evaluated using the observed daily Niño 3 SST (T ; panel a) and Niño 4 U-wind (U ; panel b) indices. **d.** Residual r_T estimated from the linear and nonlinear models. In **c** and **d** titles the parentheses denote the correlation coefficient between the respective time series.

Beyond these observational and reanalysis datasets, we also analyze historical simulations from 23 CMIP6 models[52]: ACCESS-CM2, ACCESS-ESM1-5, BCC-CSM2-MR, BCC-ESM1, CMCC-CM2-SR5, CMCC-ESM2, CNRM-CM6-1, CNRM-ESM2-1, CanESM5, EC-Earth3, EC-Earth3-Veg, GFDL-CM4, GFDL-ESM4, HadGEM3-GC31-LL, IPSL-CM5A2-INCA, IPSL-CM6A-LR, KIOST-ESM, MIROC6, MPI-ESM1-2-HR, MPI-ESM1-2-LR, MRI-ESM2-0, NESM3, and UKESM1-0-LL. For consistency across models, daily SST and near-surface zonal wind (uas) are first bilinearly regridded to a common $2.5^\circ \times 2.5^\circ$ latitude-longitude grid before regional indices are computed. Niño3 SST and Niño4 zonal wind indices, hereafter referred to as T and U respectively, are then constructed analogously to the observational definitions.

3 Model formulation

We start with a minimal daily stochastic model describing the interaction between eastern equatorial Pacific SST anomalies and western-central equatorial Pacific zonal wind anomalies, represented by the Niño3 SST index (T) and the Niño4 zonal wind index (U). This choice is motivated by the canonical ENSO feedback loop[2–4] and by the longitudinal structure of equatorial atmospheric adjustment to tropical heating[53–56]. In response to SST-induced diabatic heating anomalies, the large-scale atmospheric response projects onto an equatorial Kelvin wave to the east and a pair of Rossby gyres to the west, as in the classical Gill-Matsuno response and related simple atmospheric models for ENSO[53, 54, 57]. For ENSO growth and asymmetry, the western response is particularly consequential because it organizes zonal wind anomalies over the western-central equatorial Pacific, where wind-stress forcing efficiently drives oceanic adjustments involving equatorial wave dynamics, thermocline-slope variations, zonal advection, upwelling, and entrainment processes[23, 24, 58–61]. These oceanic adjustments subsequently project onto eastern Pacific SST anomalies. Consistent with this view, we represent the atmosphere by a single wind index in Niño 4, capturing the dominant equatorial wind-stress forcing region[22–24, 58, 62], and the ocean surface state by a single SST index in Niño 3, capturing a robust and widely used measure of eastern/central equatorial Pacific SST variability that is strongly coupled to basin-scale wind anomalies and widely used to characterize ENSO amplitude and phase[18].

We emphasize that this deliberately reduced two-index representation does not explicitly represent many processes known to shape ENSO evolution and extremes, including meridional and off-equatorial Pacific air-sea coupling and extratropical precursors, such as the Pacific Meridional Modes and the seasonal footprinting mechanism[33, 63–74], as well as interactions with other modes of variability and ocean basins[33, 75, 76]. Because the model is fitted to observed SST and wind indices, the influence of these processes may still be present indirectly in the time series and in the estimated statistics; what is not represented is their role as separate dynamical degrees of freedom. Our objective here is not to provide a dynamically complete model, but rather to isolate the minimal equatorial coupling required to diagnose how deterministic and stochastic mechanisms generate non-Gaussianity in the Niño3 region (T). Figure 1 illustrates this reduction: the oceanic branch of the coupling is represented by the influence of Niño 4 zonal wind anomalies on the Niño 3 SST tendency, while the atmospheric branch is represented by the tendency of Niño 3 SST anomalies to organize Niño 4 wind anomalies.

To represent this minimal equatorial coupling, the model is given by

$$\begin{aligned}\frac{dT}{dt} &= A(T) + \gamma_U U + \gamma_T UT + r_T \\ \frac{dU}{dt} &= \beta_1 T + \beta_2 U + r_U.\end{aligned}\tag{1}$$

Within this reduced framework, the model terms represent, in aggregate, key ENSO processes: **i** local and nonlocal SST damping by surface heat fluxes and oceanic adjustment, summarized by $A(T)$ [24, 77, 78]; **ii** linear wind forcing of SST through wind-stress-driven changes in thermocline structure, horizontal and vertical advection, entrainment, and mixing, summarized by $\gamma_U U$ [24, 58, 61, 79]; **iii** state-dependent wind-SST coupling, whereby the same wind anomaly can have a different SST tendency depending on the background ENSO state, for example due to nonlinear advection, state-dependent mixing, or mixed-layer depth variations, summarized by $\gamma_T UT$ [36, 37, 39, 78]; and **iv** the linear tendency for SST anomalies to organize wind anomalies via the Bjerknes feedback, summarized by $\beta_1 T$ in the U tendency equation, together with atmospheric damping summarized by $\beta_2 U$ [2, 3, 54, 58]. Because our focus is on diagnosing the origin of non-Gaussianity and asymmetry in T , we keep the U dynamics linear and allow the dominant sources of nonlinearity to enter through the T tendency; we tested nonlinear extensions of the U equation, with negligible impact on the behavior and statistics of T .

In the fitted system, U influences T asymmetrically. The interaction term $\gamma_T UT$ has the same sign when both $U > 0$ and $T > 0$ and when both $U < 0$ and $T < 0$, so for a positive fitted value of γ_T it reinforces both warm-westerly and cold-easterly configurations. Because T and U are correlated in the fitted system, as expected from the observed covariability between zonal wind anomalies and ENSO evolution[13, 80, 81], these same-sign configurations are dynamically relevant. The linear term $\gamma_U U$, however, breaks that symmetry and, with the fitted parameters (see below), yields stronger net reinforcement of the warm-westerly configuration. Finally, the terms $r_T(t)$ and $r_U(t)$ denote the residuals of the respective equations, which we parameterize below as stochastic forcing.

With this general structure in place, we next consider a linear and a nonlinear version of the temperature equation. In the linear case,

$$\begin{aligned} A(T) &= \alpha_1^L T \\ \gamma_U &= \gamma_U^L \\ \gamma_T &= 0 \\ r_T(t) &= r_T^L(t). \end{aligned} \quad (\text{linear}) \quad (2)$$

In the nonlinear case,

$$\begin{aligned} A(T) &= \alpha_0^N + \alpha_1^N T + \alpha_2^N T^2 + \alpha_3^N T^3 \\ \gamma_U &= \gamma_U^N \\ \gamma_T &= \gamma_T^N \\ r_T(t) &= r_T^N(t). \end{aligned} \quad (\text{nonlinear}) \quad (3)$$

The parameters of both formulations are calculated using a standard discrete Euler forward differencing scheme[82] with time step δt of 1 day. Specifically, $\frac{dT}{dt} \approx \frac{T(t+\delta t) - T(t)}{\delta t} = A(T(t)) + \gamma_U U(t) + \gamma_T U(t)T(t) + r_T(t)$, and analogous for U . This estimation procedure is consistent with the Itô interpretation of the stochastic differential equation[42, 82]. Because the stochastic forcing in the fitted two-variable system is represented here with a state-independent covariance, the Itô and Stratonovich drifts coincide at this stage. The distinction becomes relevant for the reduced SST equation developed later in Section 6, where the stochastic forcing becomes state dependent; see Supplementary Text 1. As shown in Fig. 2c and 2d, the linear and nonlinear fits remain strongly correlated in their day-to-day deterministic tendencies ($r = 0.86$), while their residuals are nearly identical ($r = 0.994$), indicating that the nonlinear terms modify the structure of the resolved drift without substantially altering the unresolved residual variability. The corresponding fitted T-residual noise amplitudes, $\sqrt{B_{11}^2 + B_{12}^2}$, are also nearly identical, $0.07019^\circ C day^{-1/2}$ for the linear model and $0.07014^\circ C day^{-1/2}$ for the nonlinear model (Table 1).

This fitted formulation also exhibits a clear timescale separation between T , U and their respective residual terms r_T , r_U . The e-folding decorrelation timescale of T is approximately 353 days, whereas r_T decorrelates on timescales of order 2 days. Similarly, the e-folding decorrelation timescale of U is approximately 12.9 days, much longer than the approximately 0.8-day decorrelation timescale of r_U . This hierarchy supports modeling the residuals stochastically[27] and provides the empirical basis for the reduction of the system to an effective one-dimensional description for T (see Section 6).

To represent these residuals stochastically, we adopt a formulation that differs from the one commonly used in stochastic recharge-oscillator models. Typically, in these models stochastic forcing is introduced either only in the T equation or as independent noise terms in each equation[24, 45, 83]. These choices are often made for simplicity and analytical tractability. Here, however, we estimate the dynamical system directly from daily observations, and the resulting residual tendencies exhibit a well-defined covariance structure. We model the residual vector $\mathbf{r}(t) = (r_T(t), r_U(t))^T$ using a coupled stochastic forcing formulation,

$$\begin{aligned} r_T(t) &= B_{11}\eta_1(t) + B_{12}\eta_2(t) \\ r_U(t) &= B_{21}\eta_1(t) + B_{22}\eta_2(t), \end{aligned} \quad (4)$$

where η_1 and η_2 are independent centered Gaussian white noise processes satisfying $\langle \eta_1(t) \rangle = \langle \eta_2(t) \rangle = 0$, $\langle \eta_1(t)\eta_2(t') \rangle = 0$ and $\langle \eta_1(t)\eta_1(t') \rangle = \langle \eta_2(t)\eta_2(t') \rangle = \delta(t - t')$. This construction allows the residuals in the T and U equations to be correlated. The coefficient matrix $\mathbf{B}$ with coefficients B_{11} , B_{12} , B_{21} , B_{22} is calculated as $\mathbf{B} = \mathbf{Q}^{1/2}$, where $\mathbf{Q}$ is estimated as $\mathbf{Q} = \langle \mathbf{r}(t)\mathbf{r}^T(t) \rangle \delta t$ [29, 84], with $\delta t = 1 \text{ day}$, where $\mathbf{r}$ is the vector $\mathbf{r} = (r_T(t), r_U(t))^T$. All estimated parameters are listed in Table 1.

Table 1 Estimated parameters of the fitted two-variable stochastic models and of the corresponding reduced one-dimensional SST equations. The variables are the daily Niño 3 SST anomaly T in $^{\circ}\text{C}$ and the daily Niño 4 zonal wind anomaly U in m s^{-1} . Linear-model coefficients that are absent by construction are shown as zero. The dash indicates that the parameter is not defined for that formulation.

Parameter	Symbol	Linear	Nonlinear
<i>Original two-variable drift parameters</i>			
Constant SST drift ($^{\circ}\text{C day}^{-1}$)	α_0	0	-9.99×10^{-4}
Linear SST drift (day^{-1})	α_1	-5.276×10^{-3}	-5.444×10^{-3}
Quadratic SST drift ($(^{\circ}\text{C})^{-1} \text{ day}^{-1}$)	α_2	0	-2.46×10^{-4}
Cubic SST drift ($(^{\circ}\text{C})^{-2} \text{ day}^{-1}$)	α_3	0	-1.29×10^{-4}
Linear wind forcing of SST ($^{\circ}\text{C day}^{-1} (\text{m s}^{-1})^{-1}$)	γ_U	2.568×10^{-3}	2.200×10^{-3}
State-dependent wind-SST interaction ($\text{day}^{-1} (\text{m s}^{-1})^{-1}$)	γ_T	0	1.563×10^{-3}
SST forcing of Niño 4 zonal wind ($(\text{m s}^{-1}) \text{ day}^{-1} (^{\circ}\text{C})^{-1}$)	β_1	1.021×10^{-1}	1.021×10^{-1}
Linear damping of Niño 4 zonal wind (day^{-1})	β_2	-1.106×10^{-1}	-1.106×10^{-1}
<i>Two-variable stochastic forcing</i>			
Noise amplitude from η_1 into T equation ($^{\circ}\text{C day}^{-1/2}$)	B_{11}	7.018×10^{-2}	7.013×10^{-2}
Noise amplitude from η_2 into T equation ($^{\circ}\text{C day}^{-1/2}$)	B_{12}	-1.368×10^{-3}	-1.430×10^{-3}
Noise amplitude from η_1 into U equation ($(\text{m s}^{-1}) \text{ day}^{-1/2}$)	B_{21}	-1.368×10^{-3}	-1.430×10^{-3}
Noise amplitude from η_2 into U equation ($(\text{m s}^{-1}) \text{ day}^{-1/2}$)	B_{22}	5.610×10^{-1}	5.610×10^{-1}
<i>Effective one-dimensional drift after adiabatic elimination</i>			
Effective constant SST drift ($^{\circ}\text{C day}^{-1}$)	$\tilde{\alpha}_0$	0	-9.61×10^{-4}
Effective linear SST drift (day^{-1})	$\tilde{\alpha}_1$	-2.906×10^{-3}	-3.381×10^{-3}
Effective quadratic SST drift ($(^{\circ}\text{C})^{-1} \text{ day}^{-1}$)	$\tilde{\alpha}_2$	0	1.197×10^{-3}
Effective cubic SST drift ($(^{\circ}\text{C})^{-2} \text{ day}^{-1}$)	$\tilde{\alpha}_3$	0	-1.29×10^{-4}
<i>Effective one-dimensional stochastic forcing after adiabatic elimination</i>			
Additive noise amplitude multiplying η_1 ($^{\circ}\text{C day}^{-1/2}$)	G_1	7.015×10^{-2}	7.011×10^{-2}
Additive part of second noise amplitude ($^{\circ}\text{C day}^{-1/2}$)	G_2	1.166×10^{-2}	9.730×10^{-3}
Multiplicative coefficient of first noise amplitude ($\text{day}^{-1/2}$)	E_1	0	-2.0×10^{-5}
Multiplicative coefficient of second noise amplitude ($\text{day}^{-1/2}$)	E_2	0	7.927×10^{-3}
Variance-matched amplitude for $G_{1N} + E_1 T$ ($^{\circ}\text{C day}^{-1/2}$)	S_1	–	7.011×10^{-2}
Variance-matched amplitude for $G_{2N} + E_2 T$ ($^{\circ}\text{C day}^{-1/2}$)	S_2	–	1.218×10^{-2}

To assess whether this minimal two-variable formulation omits an essential upper-ocean degree of freedom, we also tested an extended formulation including a thermocline proxy (sea surface height; SSH)[50, 51, 85] in the spirit of recharge-discharge oscillator models[23, 24, 51]. The results were qualitatively and quantitatively very similar for the statistical diagnostics considered here. This sensitivity test should not be interpreted as a formal reduction of a three-variable recharge-discharge system to the present $T-U$ model, nor as evidence that upper-ocean recharge dynamics are dynamically unimportant. Rather, it

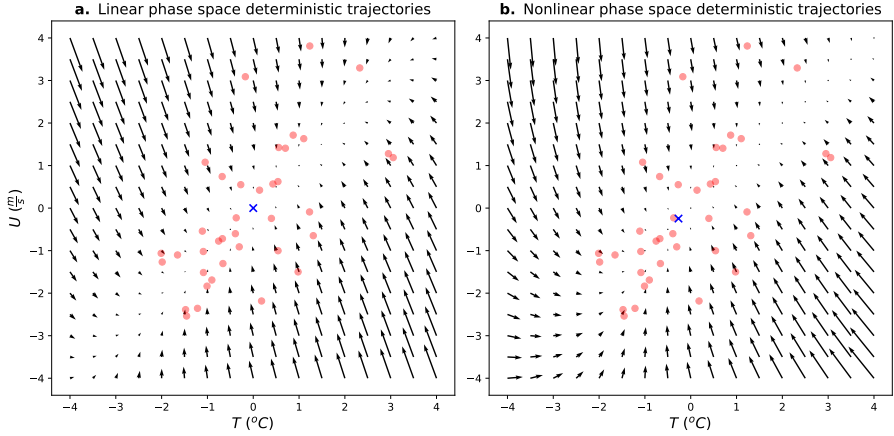


Fig. 3 Phase space (arrows) of the deterministic daily tendency field for the daily Niño 3 SST index (T) and Niño 4 U-wind index (U) in the **a** linear and **b** nonlinear models. Arrows are computed from the fitted daily tendencies dT/dt ($^{\circ}C day^{-1}$) and dU/dt ($ms^{-1} day^{-1}$). For aesthetic purposes, the T direction is exaggerated 10 times relative to the U direction. For reference, the red dots show the observed T and U values on Jan 1st of each year (1982–2019), and the blue crosses show the fixed point in each case.

shows that explicitly adding Niño 3 SSH does not materially alter the central linear-versus-nonlinear contrast in the Niño 3 SST statistics examined here. Accordingly, the simpler $T - U$ formulation is sufficient for the linear-versus-nonlinear comparison pursued here. Additional details are provided in Supplementary Text 2.

4 Deterministic phase-space structure of the SST-wind system

With this minimal coupled framework, we now examine the deterministic backbone of the coupled system defined by equation 1, with the linear and nonlinear parameterizations given in equations 2 and 3. Throughout this section we temporarily ignore the stochastic residuals r_T and r_U in order to isolate the intrinsic phase-space structure[86, 87] implied by the fitted drift terms, $A(T) + \gamma_U U + \gamma_T UT$ in the T tendency and $\beta_1 T + \beta_2 U$ in the U tendency. Our goal is to understand how the deterministic dynamics organize trajectories in $((T, U))$ space, and whether asymmetries between warm and cold states are already encoded in the estimated drift structure prior to the inclusion of stochastic forcing.

4.1 Fixed points and local stability

To begin, fixed points (T^*, U^*) of the deterministic system satisfy

$$\frac{dT}{dt} = 0, \quad \frac{dU}{dt} = 0, \quad (5)$$

with the deterministic tendencies given by equation 1. A fixed point represents a state in phase space where, in the absence of stochastic forcing, the system would remain indefinitely. Fixed points define the organizing centers of the dynamics. Their stability determines whether small perturbations decay (stable equilibrium), grow (unstable equilibrium), or oscillate while decaying or growing[88, 89]. In practice, the behavior near the fixed point governs the geometry of trajectories in phase space and sets the deterministic backbone upon which stochastic fluctuations act.

To assess local stability, we linearize the system around a fixed point[88, 90]. This is done using the Jacobian matrix, defined as the matrix of first partial derivatives of the vector field:

$$J(T, U) = \begin{pmatrix} \frac{\partial}{\partial T} \left(\frac{dT}{dt} \right) & \frac{\partial}{\partial U} \left(\frac{dT}{dt} \right) \\ \frac{\partial}{\partial T} \left(\frac{dU}{dt} \right) & \frac{\partial}{\partial U} \left(\frac{dU}{dt} \right) \end{pmatrix}. \quad (6)$$

The Jacobian measures how small perturbations in T and U modify their tendencies. Evaluated at a fixed point, it provides the linear approximation to the dynamics in its neighborhood.

Specifically, if we define small perturbations as

$$\delta T = T - T^*, \quad \delta U = U - U^*, \quad (7)$$

then to first order,

$$\frac{d}{dt} \begin{pmatrix} \delta T \\ \delta U \end{pmatrix} = J(T^*, U^*) \begin{pmatrix} \delta T \\ \delta U \end{pmatrix}. \quad (8)$$

Thus, near a fixed point, the nonlinear system behaves approximately like a linear system governed by the matrix $J(T^*, U^*)$. The eigenvalues of this matrix determine whether perturbations decay (negative real part), grow (positive real part), or oscillate while decaying or growing (complex eigenvalues, with the sign of the real part setting the growth or decay rate)[88, 89].

4.1.1 Linear case

Applying this framework to the linear formulation (equations 1 and 2), the system is linear in (T, U) . For the fitted parameters used here, this equilibrium is located at the origin $(T^*, U^*) = (0, 0)$ (blue cross in Fig. 3a) and is locally stable. The Jacobian matrix is constant and given by

$$J_L = \begin{pmatrix} \alpha_1^L & \gamma_U^L \\ \beta_1 & \beta_2 \end{pmatrix} \approx \begin{pmatrix} -5.276 \times 10^{-3} & 2.568 \times 10^{-3} \\ 1.021 \times 10^{-1} & -1.106 \times 10^{-1} \end{pmatrix}. \quad (9)$$

Because all coefficients are constant, this Jacobian is identical everywhere in phase space. Its eigenvalues determine the stability of the system globally. For the fitted parameters, the eigenvalues of J_L are real and negative, implying a stable node. These eigenvalues correspond to e-folding timescales of approximately 352 days and 8.8 days, respectively. The deterministic system exhibits two-timescale relaxation toward $T^* = 0$, $U^* = 0$, with a slow coupled SST-wind mode and a fast mode dominated by wind adjustment.

Importantly, in this linear formulation, the Jacobian J_L plays the same role as the operator M in a classical linear inverse model (LIM) framework[20, 29, 84, 91, 92]. In LIM studies, stability and oscillatory behavior are routinely diagnosed through the eigenvalues of M or of the corresponding finite-time propagator[20, 84, 93]. The corresponding eigenmodes are the principal oscillation patterns (POPs) introduced by Hasselmann (1988)[91]. Here, J_L is precisely that operator for the two-variable linear system.

This linear operator structure also makes clear that, because $\frac{\partial}{\partial T} \left(\frac{dT}{dt} \right) = \alpha_1^L$ is constant, the local growth rate of SST anomalies does not depend on the sign or amplitude of T . Thus, at the deterministic level, the linear model contains no intrinsic preference for warm or cold anomalies.

4.1.2 Nonlinear case

By contrast, under equation 3, the derivatives entering the Jacobian depend on (T, U) because of the quadratic, cubic, and interaction terms. Local stability, however, is determined by evaluating the Jacobian at the fixed point. For the fitted parameters used here, the nonlinear formulation also admits a single fixed point near the origin (blue cross in Fig. 3b), and it is locally stable. Evaluated at that fixed point $(T^*, U^*) \approx (-0.267^\circ\text{C}, -0.246 \text{ m s}^{-1})$, the Jacobian is

$$J_N(T^*, U^*) = \begin{pmatrix} \alpha_1^N + 2\alpha_2^N T^* + 3\alpha_3^N (T^*)^2 + \gamma_T^N U^* & \gamma_U^N + \gamma_T^N T^* \\ \beta_1 & \beta_2 \end{pmatrix} \quad (10)$$

$$\approx \begin{pmatrix} -5.725 \times 10^{-3} & 1.783 \times 10^{-3} \\ 1.021 \times 10^{-1} & -1.106 \times 10^{-1} \end{pmatrix}.$$

For the fitted parameters, the equilibrium is also a stable node. Its eigenvalues are real and negative, corresponding to e-folding timescales of approximately 249 days and 8.9 days, respectively. As in the linear system, the slower mode represents coupled SST-wind adjustment, while the faster mode is dominated by wind adjustment. However, unlike the linear case, the Jacobian now varies with state. In particular, the effective local damping or growth rate of SST anomalies, $\frac{\partial}{\partial T} \left(\frac{dT}{dt} \right)$, depends on both T and U . Consequently, the restoring strength differs between warm and cold states and between different wind regimes. Even though the number of fixed points does not change, the curvature of the flow field and the local growth rates do. This distinction between constant and state-dependent Jacobian structure underlies the phase-space asymmetries discussed below.

4.2 Deterministic phase-space geometry and squared-amplitude growth

Figure 3 summarizes how the fitted deterministic drift terms organize trajectories in (T, U) space, and how this organization differs between the linear

and nonlinear formulations. The observed January 1 states (red dots) provide an observational reference for the region of (T, U) phase space sampled by the system, without implying that the deterministic trajectories reproduce the observed event evolution. In the linear model (Fig. 3a), the vector field and representative trajectories are consistent with a constant Jacobian: trajectories relax smoothly toward the fixed point, the flow is symmetric under the simultaneous sign reversal $(T, U) \rightarrow (-T, -U)$, and warm-westerly and cold-easterly configurations are treated as mirror-image counterparts. Because the eigendirection structure is uniform across phase space, the deterministic geometry is globally consistent with a stable two-timescale relaxation system.

In contrast, the nonlinear model (Fig. 3b) shows a clear reconfiguration of the flow geometry. Trajectory curvature differs between quadrants, and effective damping and growth tendencies vary with both the sign and amplitude of T and U . In particular, regions with positive T and westerly wind anomalies ($U > 0$), which reinforce eastern Pacific warming [24, 58, 60], exhibit reduced effective damping or enhanced growth relative to their cold-easterly counterparts. These asymmetries in the flow follow directly from the state dependence introduced by the nonlinear drift: even though the equilibrium remains locally stable, the nonlinear terms modify how trajectories explore phase space before returning to the fixed point.

To quantify where deterministic dynamics amplify or damp SST anomalies, we complement the phase-space view with a scalar squared-amplitude growth diagnostic based on the evolution of T^2 , which captures whether anomalies of either sign are amplified or decay toward equilibrium. Using the identity

$$\frac{d}{dt}(T^2) = 2T \frac{dT}{dt}, \quad (11)$$

with dT/dt given by equations 1 and 2 in the linear case and equations 1 and 3 for the nonlinear case, a positive value implies that anomalies grow in magnitude (warm anomalies become warmer and cold anomalies become colder), while a negative value implies that anomalies of either sign decay back toward the equilibrium state. The heatmaps in Fig. S2 visualize $\frac{d}{dt}(T^2)$ across (T, U) space. In the linear case (Fig. S2a), substituting Eq. 2 implies that $\frac{d}{dt}(T^2)$ contains only quadratic and bilinear terms in T and U , and is invariant under the simultaneous sign reversal $(T, U) \rightarrow (-T, -U)$. Accordingly, the heatmap shows a smooth structure, consistent with the constant Jacobian: warm-westerly and cold-easterly mirror states exhibit the same deterministic squared-amplitude growth.

In the nonlinear case (Fig. S2b), the higher-order terms in Eq. 3 introduce additional contributions to $\frac{d}{dt}(T^2)$ that make amplification strongly quadrant-dependent. The heatmap shows enhanced growth in warm states under westerly wind anomalies ($U > 0$), which reinforce eastern Pacific warming, and stronger damping over much of the remaining phase space. This behavior is directly tied to the fact that the effective local damping or growth rate of SST anomalies varies with state. In particular, the quantity

$$\frac{\partial}{\partial T} \left(\frac{dT}{dt} \right) \quad (12)$$

is no longer constant under equation 3, so the local restoring strength can differ between warm and cold states and between different wind regimes, even though the equilibrium itself remains locally stable.

To clarify the heatmap structure, Fig. S3 provides slices at fixed values of U (e.g., $U = -8, -4, 0, 4, 8 \text{ m s}^{-1}$) of $\frac{dT}{dt}$ as a function of T . In the linear model, these curves are parabolic with a U -dependent linear term: changing U shifts the vertex of the parabola, but the same-sign warm-westerly and cold-easterly configurations remain mirror counterparts under $(T, U) \rightarrow (-T, -U)$. The nonlinear slices, however, display pronounced asymmetry between warm and cold branches. For positive U (westerly wind anomalies), amplification of positive T exceeds that of negative T but the details depend on the magnitude of U : for $U = 4 \text{ m s}^{-1}$, positive growth occurs mainly for moderately warm anomalies, while cold anomalies are damped throughout; for $U = 8 \text{ m s}^{-1}$, both warm and cold anomalies can exhibit positive growth, but the warm branch is substantially stronger and broader, with a much larger maximum at positive T . For negative U (easterly wind anomalies), the behavior is shifted in the opposite direction: positive T anomalies are strongly damped, while only weak positive growth appears for modest negative T . At sufficiently large $|T|$, the curves bend downward, indicating nonlinear saturation and strong damping of large anomalies. These slices make explicit how the nonlinear, state-dependent Jacobian structure produces quadrant-dependent growth.

5 Joint statistics of the SST-wind system

The deterministic phase-space analysis in Section 4 revealed clear structural differences between the linear and nonlinear formulations. Although both systems admit a single stable fixed point near the origin, the nonlinear model introduces state-dependent curvature in the flow, modifying the effective local amplification and decay properties across quadrants of (T, U) space. The natural next question is whether these geometric differences translate into measurable differences in the statistical structure of the coupled system once stochastic forcing is included. We examine the joint probability distribution of daily Niño 3 SST anomalies T and Niño 4 zonal wind anomalies U generated by the full two-dimensional stochastic integrations.

The fitted linear (eqs. 1 and 2) and nonlinear (eqs. 1 and 3) models are integrated at daily resolution using an Euler forward scheme with time step $\delta t = 1$ day and the empirically estimated residual covariance structure (Eq. 4) appropriate for each case. The resulting long simulations allow robust estimation of the joint probability density in (T, U) space. To quantify sampling variability, each model integration is divided into 200 epochs, each of equal length to the length of the observational time series (38 years of daily data). These epochs provide, at every bin ($0.5^\circ\text{C} \times 0.5^\circ\text{m s}^{-1}$; Fig.4), a distribution of joint probabilities that defines the 5th-95th percentile range expected under

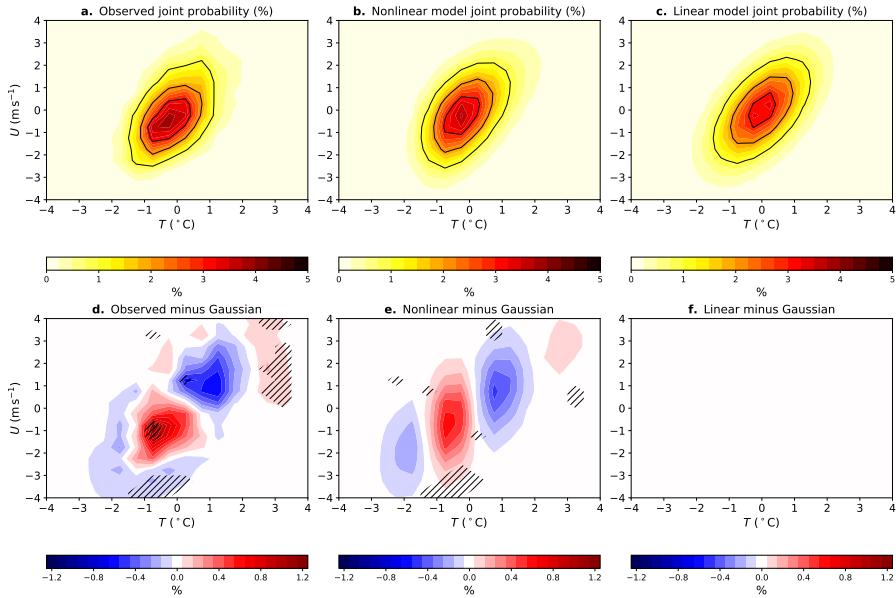


Fig. 4 Joint probability distributions of daily Niño 3 SST index (T) and Niño 4 zonal wind index (U) in **a.** observations, **b.** the nonlinear model, and **c.** the linear model. The contours show the percentage of events occurring within bins of width $0.5^\circ\text{C} \times 0.5\text{ m s}^{-1}$. **d.** Difference between the observed joint probability distribution and a joint Gaussian distribution with the same mean and covariance matrix. Hatched areas indicate bins where the observed joint probability lies outside the 5th–95th percentile range of joint distributions generated by the linear model, estimated from 200 model epochs of the same length as the observations (38 years of daily data). **e.** Difference between the nonlinear-model joint probability distribution and a joint Gaussian distribution with the same mean and covariance matrix as the nonlinear model. Hatched areas indicate bins where the observed joint distribution lies outside the 5th–95th percentile range generated by the nonlinear model. **f.** Difference between the linear-model joint probability distribution and a joint Gaussian distribution with the same mean and covariance matrix as the linear model. As expected for the linear Gaussian system, the difference is indistinguishable from zero at the plotted contour interval.

each model given finite sampling. We use these epochs to test whether observed departures from Gaussian structure are statistically consistent with the linear or nonlinear dynamics.

Figure 4a shows the observed joint probability distribution of T and U . The dominant feature is the positive covariance between eastern Pacific SST anomalies and central Pacific zonal wind anomalies, expressed as an elongated cloud tilted along the positive diagonal. This bulk covariance structure reflects the coupled nature of ENSO dynamics[2, 80, 81]. However, this covariance structure alone does not uniquely determine the full joint distribution, and different systems with the same covariance may exhibit distinct higher-order structure[5, 43, 90].

The nonlinear model (Fig. 4b) reproduces the overall tilt and concentration of probability in (T, U) space. The linear model (Fig. 4c), by construction, generates a strictly joint-Gaussian distribution: the contours are elliptical and

entirely determined by the covariance matrix and additive Gaussian noise[29, 34, 42]. Both models capture the bulk covariance structure, but only the linear model is constrained to remain exactly Gaussian. Whether additional higher-order structure is present beyond this shared covariance is assessed next.

To isolate higher-order structure, we subtract from each joint distribution a joint Gaussian with the same mean and covariance matrix. Figure 4d shows the observed joint probability minus its Gaussian reference. The resulting pattern reveals systematic regions of excess and deficit probability relative to an elliptical Gaussian distribution with identical second moments. These departures are not randomly distributed in (T, U) space. Instead, they exhibit a coherent quadrant structure: enhanced probability is concentrated preferentially in regions corresponding to warm SST anomalies accompanied by westerly wind anomalies (top of the first quadrant with large positive T and large positive U), while a pronounced positive anomaly is also centered at moderate negative T and moderate negative U , and a clear deficit appears at moderate T and positive U , with additional excess probability in the far warm-westerly tail and compensating deficits in other regions. This organization indicates that the system spends disproportionately more time in dynamically amplified warm-westerly states than would be expected under a purely linear Gaussian framework. Such structured deviations are precisely what one expects from state-dependent amplification mechanisms identified in the deterministic phase-space analysis (see section 4).

These departures are statistically significant. The hatched regions in Fig. 4d indicate bins where the observed joint probability lies outside the 5th-95th percentile range of joint distributions generated by the linear model, based on the 200 observational-length epochs. Because the linear model generates an exactly joint-Gaussian distribution, its expected Gaussian-subtracted field is identically zero as confirmed by the linear-model-minus-Gaussian field in Fig. 4f, which is indistinguishable from zero at the plotted contour interval; the relevant question is whether finite-sample realizations of a linear system can nevertheless produce apparent departures like those seen in observations. It is unlikely that the observed non-Gaussian structure can be attributed to finite-sample variability of a linear system with the same covariance. The linear formulation reproduces second moments but does not capture the organized redistribution of probability across quadrants.

Applying the same Gaussian subtraction diagnostic to the nonlinear model (Fig. 4e) shows that nonlinear drift terms generate non-Gaussian joint structure in (T, U) space with a spatial organization similar to observations. Regions of excess probability align with the dynamically favored quadrants implied by the nonlinear flow geometry, and the magnitude of deviations is comparable. When the observed joint distribution is evaluated against the sampling envelope of the nonlinear model, much of the observed structure falls within the variability range of the nonlinear integrations. This indicates that, despite some discrepancies outside the nonlinear model's finite-sample envelope, the

deterministic nonlinearities introduced in the T equation provide a substantially improved representation of the main features of the observed joint non-Gaussianity in the daily-resolved observations.

6 Adiabatic elimination of the wind: equivalent 1-D SSTA equation

To make the full stochastic system explicit, we combine equations (1) and (4) to obtain the coupled stochastic model

$$\begin{aligned}\frac{dT}{dt} &= A(T) + \gamma_U U + \gamma_T UT + B_{11}\eta_1(t) + B_{12}\eta_2(t) \\ \frac{dU}{dt} &= \beta_1 T + \beta_2 U + B_{21}\eta_1(t) + B_{22}\eta_2(t).\end{aligned}\tag{13}$$

Within this full stochastic system, the main difference between the linear and nonlinear formulations lies in how $A(T)$ is parameterized (Eqs. 2 and 3), and in the presence of the interaction term $\gamma_T UT$, which vanishes in the linear case ($\gamma_T = 0$). The coefficients B_{ij} are only slightly different for the linear and nonlinear cases (Table 1). As discussed earlier, the system exhibits a clear separation of timescales between both the dynamical variables (T , U) and their associated stochastic contributions. In addition, the two dynamical variables exhibit markedly different timescales: the Niño 3 SST index T decorrelates on a timescale of approximately one year, while the Niño 4 zonal wind index U decorrelates on a timescale of roughly two weeks. This contrast reflects the rapid atmospheric adjustment compared to the much slower evolution of the oceanic SST anomalies.

The presence of this timescale separation allows the wind variable to be eliminated in favor of the slower SST variable using the method of adiabatic elimination (i.e., fast-variable elimination under time-scale separation, implemented here as a leading-order quasi-steady approximation; [42, 94–96]). The broader separation between a slowly evolving SST mode and more rapidly adjusting coupled variables has a long history in ENSO theory, including the slow SST mode and fast-wave limit discussed by [97]. Similar reductions have long been used in intermediate-complexity coupled ocean-atmosphere models, where atmospheric variables are written diagnostically in terms of SST anomalies (e.g., [3, 4, 58, 70]). Because the wind variable adjusts rapidly, its tendency can be approximated as zero on the timescale of SST evolution. Setting $\frac{dU}{dt} \approx 0$ yields

$$U \approx -\frac{\beta_1}{\beta_2} T - \frac{B_{21}}{\beta_2} \eta_1 - \frac{B_{22}}{\beta_2} \eta_2.\tag{14}$$

Substituting this expression into the SST equation leads to a single stochastic equation governing the Niño 3 SST index. Although the wind variable has

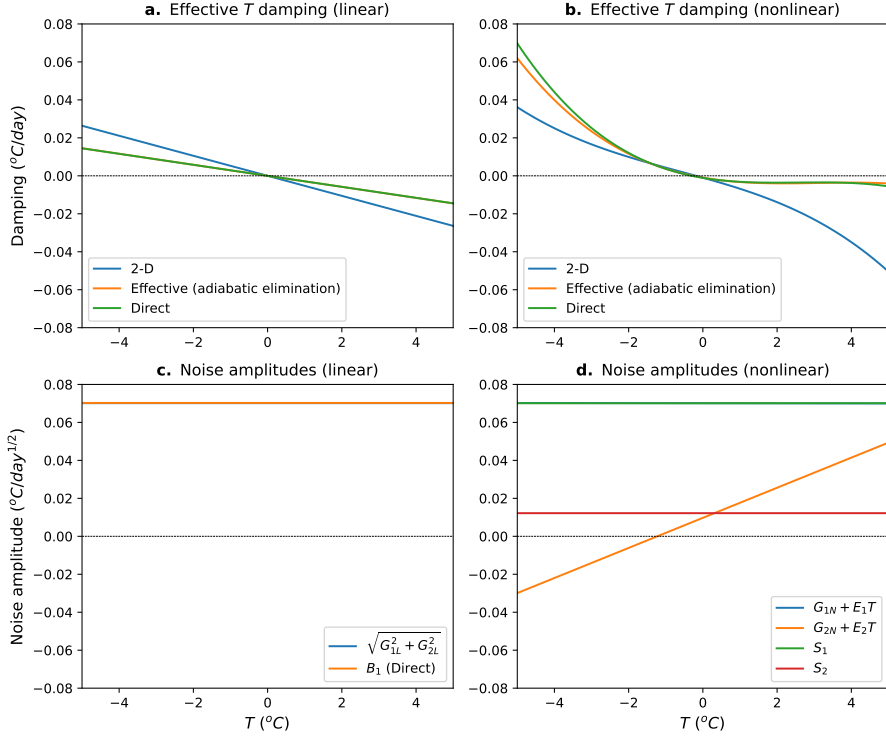


Fig. 5 Different Niño 3 index dampings in the **a.** linear and **b.** nonlinear case. In both cases, these plots (**a** and **b**) compare the damping in the two-variable model ($\alpha_1^L T$ in the linear case, and $\alpha_0^N + \alpha_1^N T + \alpha_2^N T^2 + \alpha_3^N T^3$ in the nonlinear case), with the effective damping after adiabatic elimination of U ($\tilde{\alpha}_1^L T$ in the linear case, and $\tilde{\alpha}_0^N + \tilde{\alpha}_1^N T + \tilde{\alpha}_2^N T^2 + \tilde{\alpha}_3^N T^3$ in the nonlinear case), and the dampings calculated directly from an assumed univariate model for T ($\hat{\alpha}_1^L T$ in the linear case, and $\hat{\alpha}_0^N + \hat{\alpha}_1^N T + \hat{\alpha}_2^N T^2 + \hat{\alpha}_3^N T^3$ in the nonlinear case). The similarity between the "effective" and "direct" estimations provides evidence on the usefulness of the simple two-variable model. **c.** Estimation of the noise amplitudes after adiabatic elimination of U ($G_{1L}\eta_1 + G_{2L}\eta_2$) and direct estimation assuming just one noise process ($B_1\eta$). What is displayed in the figure are the root-mean-squared amplitudes ($\sqrt{G_{1L}^2 + G_{2L}^2}$ and B_1). Note that in our estimation $G_{1L} \gg G_{2L}$. As can be observed, the noise amplitude does not depend on the state of the system T in this case. **d.** Same as **c.** but in the nonlinear case. In this case, the noise amplitude can depend on the state of the system T , so we display $G_{1N} + E_1 T$ and $G_{2N} + E_2 T$ as a function of T . We also display the equivalent noise amplitudes (S_1 and S_2) if we simply ignored the noise state-dependence. These are estimated as $S_1 = \sqrt{G_{1N}^2 + E_1^2 \langle T^2 \rangle}$, and $S_2 = \sqrt{G_{2N}^2 + E_2^2 \langle T^2 \rangle}$. This is justified by calculating the equation for the second moment of T ($\langle T^2 \rangle$) and imposing that both processes yield the same T variance (e.g., Martínez-Villalobos et al., 2018). Note that the first CAM noise amplitude ($G_{1N} + E_1 T \approx S_1$) has negligible state-dependency.

been eliminated explicitly, its dynamical influence remains embedded in the resulting coefficients. The reduced equation takes the form

$$\frac{dT}{dt} = A_{eff}(T) + (G_1 + E_1 T)\eta_1(t) + (G_2 + E_2 T)\eta_2(t), \quad (15)$$

with coefficients E_i and G_i given by

$$\begin{aligned} G_1 &= B_{11} - \gamma_U \frac{B_{21}}{\beta_2} \\ E_1 &= -\gamma_T \frac{B_{21}}{\beta_2} \\ G_2 &= B_{12} - \gamma_U \frac{B_{22}}{\beta_2} \\ E_2 &= -\gamma_T \frac{B_{22}}{\beta_2}. \end{aligned} \tag{16}$$

These expressions apply to both linear and nonlinear cases, with the corresponding parameter values. In particular, in the linear formulation $\gamma_T = 0$, implying $E_1 = E_2 = 0$, and the stochastic forcing remains purely additive. In the nonlinear formulation, however, the interaction term ($\gamma_T UT$) introduces state-dependent noise amplitudes, leading to correlated additive-multiplicative (CAM) stochastic forcing in the reduced equation[7, 43, 98, 99].

In ENSO conceptual modeling, CAM-type stochastic forcing terms —i.e., stochastic terms with both an additive and a multiplicative component, such as $(1 + BT)\eta$ [10] or $(G + ET)\eta$ [7]— are often introduced heuristically into the governing equations, rather than derived explicitly from a reduced coupled system, to represent unresolved atmospheric variability such as westerly wind bursts or intraseasonal disturbances (e.g., [7, 10, 24, 100–102]). In several studies these terms are introduced as phenomenological parameterizations of such variability, often without an accompanying uncorrelated additive noise component (e.g., [10, 100, 102]). The present derivation provides a complementary and more mechanistic perspective: the CAM structure arises here naturally from the dynamics once the wind variable is eliminated and the empirically estimated residual covariance of the coupled system is retained (see also [44, 98]). The same type of multiplicative noise that is often inserted phenomenologically in conceptual ENSO models can be traced back to the interaction between SST anomalies, atmospheric variability, and the correlated stochastic forcing implied by the observed residual tendencies, thereby clarifying its dynamical origin within the coupled system.

The same elimination also modifies the deterministic drift of the reduced equation. Eliminating the wind variable alters the effective damping experienced by SST anomalies, producing a new drift term A_{eff} . The functional form of this term depends on whether the underlying coupled system is linear or nonlinear. For the linear formulation,

$$A_{eff}^L(T) = \tilde{\alpha}_1^L T, \tag{17}$$

where

$$\tilde{\alpha}_1^L = \alpha_1^L - \frac{\beta_1}{\beta_2} \gamma_U^L. \quad (18)$$

For the nonlinear formulation, the effective drift retains a cubic polynomial form,

$$A_{eff}^N(T) = \tilde{\alpha}_0^N + \tilde{\alpha}_1^N T + \tilde{\alpha}_2^N T^2 + \tilde{\alpha}_3^N T^3, \quad (19)$$

with coefficients

$$\begin{aligned} \tilde{\alpha}_0^N &= \alpha_0^N + \frac{1}{2}(E_1 G_1 + E_2 G_2) \\ \tilde{\alpha}_1^N &= \alpha_1^N - \frac{\beta_1}{\beta_2} \gamma_U^N + \frac{1}{2}(E_1^2 + E_2^2) \\ \tilde{\alpha}_2^N &= \alpha_2^N - \frac{\beta_1}{\beta_2} \gamma_T^N \\ \tilde{\alpha}_3^N &= \alpha_3^N. \end{aligned} \quad (20)$$

The additional terms involving E_i and G_i arise from the multiplicative noise contributions generated during the elimination procedure (see Supplementary Text 1 for the full derivation). Physically, they reflect the fact that the atmospheric forcing modifies the effective SST dynamics through both deterministic feedbacks and noise-state interactions, including the noise-induced drift (also known as Itô correction), associated with state-dependent stochastic forcing[42, 44, 82]. As a simple consistency check, the fitted reduced nonlinear drift also approximately satisfies the stationary first-moment constraint expected for a centered SST anomaly distribution, $\langle A_{eff}^N(T) \rangle \simeq 0$; see Supplementary Text 3 for details.

This reduced description of the effective SST dynamics also receives empirical support in Figure 5. Panels (a) and (b) compare the SST effective damping inferred directly from the original two-variable model (Eqs. 1-3) with the effective damping predicted after adiabatic elimination (Eqs. 17-20) and with a "direct" estimation of the damping from SST data, obtained by fitting univariate stochastic models for T directly to the SST tendency data. Specifically, for the linear and nonlinear cases we estimate

$$\frac{dT}{dt} = \hat{\alpha}_1^L T + r_{T,dir}^L, \quad \frac{dT}{dt} = \hat{\alpha}_0^N + \hat{\alpha}_1^N T + \hat{\alpha}_2^N T^2 + \hat{\alpha}_3^N T^3 + r_{T,dir}^N, \quad (21)$$

where the coefficients $\hat{\alpha}_i$ are obtained by regression of the observed SST tendency onto the corresponding powers of T , with their numerical values listed in Table S1. The close agreement among the last two estimates (for example, in the linear case, $\tilde{\alpha}_1^L = -2.906 \times 10^{-3} \text{ day}^{-1}$ after adiabatic elimination and $\hat{\alpha}_1^L = -2.899 \times 10^{-3} \text{ day}^{-1}$ from the direct univariate fit) indicates that the coupled two-variable system reproduces the effective SST dynamics with high fidelity. Although the wind variable is essential for representing the coupled

dynamics, its influence on SST variability can largely be captured through modifications of the SST damping itself.

Panels 5c and 5d show the corresponding stochastic amplitudes. In the linear case the effective noise amplitude (Eqs. 15-16) is independent of the state of the system, and the root-mean-square amplitude obtained from the elimination procedure closely matches that estimated directly from the residual of the direct univariate linear model $r_{T,\text{dir}}^L$, whose standard deviation defines the amplitude B_1 shown in panel 5c. In the nonlinear case the stochastic forcing becomes state dependent, and there is no equally simple direct estimate of its amplitude from a univariate SST equation alone, due to the CAM terms ($E_i T$, Eqs. 15-16). The reduced model yields two stochastic components, $(G_{1N} + E_1 T)\eta_1$ and $(G_{2N} + E_2 T)\eta_2$, shown in panel 5d. The first of these exhibits negligible state dependence (Fig. 5d), so it can be approximated as additive noise, while the second shows a clear state dependence. For reference, we also display the variance-matched constants $S_1 = \sqrt{G_{1N}^2 + E_1^2 \langle T^2 \rangle}$ and $S_2 = \sqrt{G_{2N}^2 + E_2^2 \langle T^2 \rangle}$ that would replace these amplitudes if the state dependence were ignored. The resulting structure, with one approximately additive noise process and one CAM noise process, is consistent with the form used in linear and nonlinear univariate CAM noise models[43, 44, 98, 103]. Note that simple ENSO conceptual models often dispense with the additional additive noise term[10, 45, 83, 100], although the absence of this additional term is not always derived or independently justified.

These results show that the two-variable stochastic model not only provides a consistent representation of the observed joint statistics of SST and wind anomalies, but also yields a reduced univariate SST equation whose dynamical structure closely matches that inferred directly from SST observations applied to a univariate T model. In particular, the reduced equation highlights two mechanisms capable of generating asymmetry in SST variability: nonlinear deterministic drift and CAM noise. Because the reduced model is one-dimensional, its stationary probability distribution can be obtained analytically[42–44, 104, 105]. In the following section we exploit this property to assess the relative contributions of the nonlinear drift and multiplicative noise to the observed ENSO asymmetry.

7 Relative roles of deterministic nonlinearities and CAM noise

7.1 Model setup

The reduced equation identifies two candidate mechanisms of asymmetry in the Niño 3 distribution: nonlinear deterministic drift and state-dependent stochastic forcing in the form of CAM noise. In this section, we use analytical stationary distributions of the reduced model to quantify the relative importance of these two mechanisms in shaping the asymmetry of the probability distribution of the Niño 3 index.

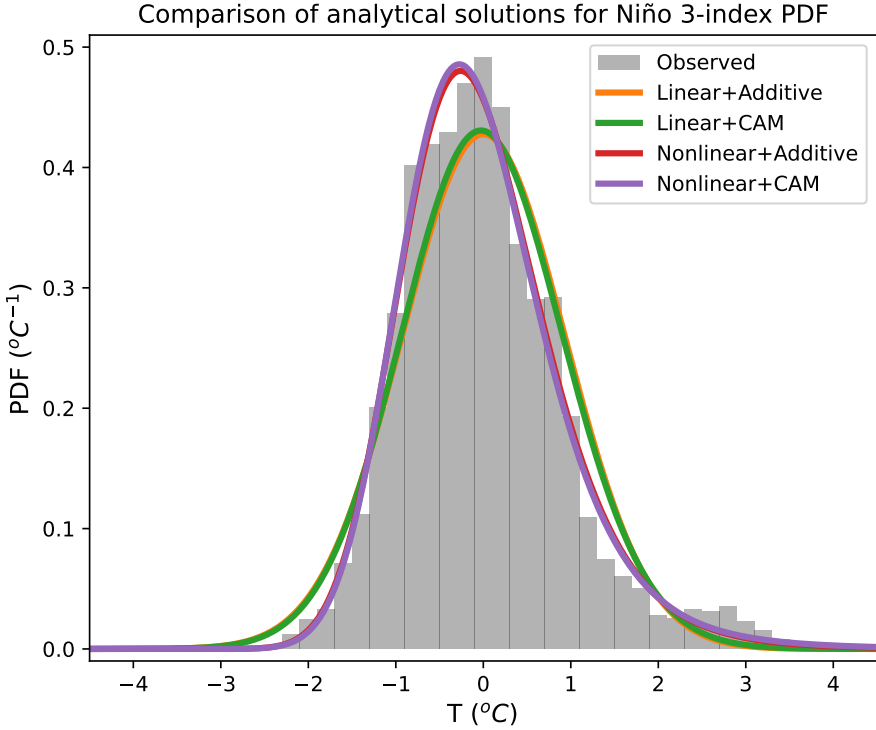


Fig. 6 Analytical stationary probability density functions of the Niño 3 SST index for the four reduced stochastic models. The curves correspond to the stationary solutions in Eqs. (23)–(26): **Linear+additive**, **Linear+CAM**, **Nonlinear+additive**, and **Nonlinear+CAM**. The observed daily Niño 3 SST anomaly distribution is shown for comparison using a bin width of 0.2°C . The **Linear+additive** model captures the variance but remains Gaussian, while the nonlinear-drift models reproduce the observed positive skewness more closely. The corresponding standard deviation and skewness are $[\sigma = 0.925^{\circ}\text{C}, S = 0.761]$ for observations, $[\sigma = 0.933^{\circ}\text{C}, S = 0.000]$ for **Linear+additive**, $[\sigma = 0.934^{\circ}\text{C}, S = 0.058]$ for **Linear+CAM**, $[\sigma = 0.914^{\circ}\text{C}, S = 0.780]$ for **Nonlinear+additive**, and $[\sigma = 0.930^{\circ}\text{C}, S = 0.945]$ for **Nonlinear+CAM**. Tail behavior is shown in more detail in Fig. S4.

To this end we consider four reduced stochastic models that isolate different combinations of these mechanisms. The models are constructed using the coefficients obtained from the adiabatic elimination procedure and the noise amplitudes shown in Figs. 5c,d (Eq. 16). They all use equation 15 as a starting point, but with different choices for the deterministic drift and stochastic forcing terms. These configurations separate the mathematical contributions of nonlinear drift and state-dependent noise to the stationary distribution. In the reduced nonlinear system, however, these contributions are not physically independent: the SST-wind interaction contributes both to the effective nonlinear drift and to the state-dependent noise generated by the reduction.

The first model, **Nonlinear+CAM**, corresponds closely to the effective stochastic equation obtained from the nonlinear two-variable system (Eq. 15)

and retains both deterministic nonlinearities and state-dependent noise. In practice, the only modification relative to the reduced equation derived in the previous section (Eq. 15) is that the amplitude of the first noise process ($G_{1N} + E_1T$) is replaced by the root-mean-square-equivalent constant value S_1 , with $S_1 = \sqrt{G_{1N}^2 + E_1^2\langle T^2 \rangle}$, where G_{1N} denotes the coefficient G_1 obtained from the nonlinear formulation of the two-variable system (and analogously for G_{2N} below; see caption of Fig. 5 for details). This simplification is justified by the empirically negligible state dependence of this term ($G_{1N} + E_1T \approx S_1$; Fig. 5d), and numerical experiments confirm that replacing $G_{1N} + E_1T$ by S_1 has no noticeable effect on the resulting stationary distributions (top panel of Fig. S4).

The second model, **Nonlinear+additive**, retains the nonlinear deterministic drift but removes the state dependence of the stochastic forcing by replacing the state-dependent amplitude $G_{2N} + E_2T$ with a constant amplitude S_2 , with $S_2 = \sqrt{G_{2N}^2 + E_2^2\langle T^2 \rangle}$ (see caption of Fig. 5 for details). Since the state dependence of this component is substantial (Fig. 5d), comparing this model with the previous one allows us to quantify the extent to which CAM noise contributes to ENSO asymmetry and extreme event probabilities.

The third model, **Linear+additive**, corresponds to the exact reduced equation obtained from the linear two-variable system (Eqs. 15-17) and provides the Gaussian baseline expected from a linear Ornstein-Uhlenbeck process. Finally, the fourth model, **Linear+CAM**, introduces state-dependent noise into an otherwise linear deterministic system by replacing “by hand” G_{2L} with the state-dependent expression ($G_{2N} + E_2T$), used in the **Nonlinear+CAM** model. Although similar formulations have been used previously as minimal models for generating non-Gaussian climate variability and ENSO asymmetry[7, 43, 99, 103–105], in the present framework such a formulation does not arise from a systematic reduction of the two-variable system, because the $\gamma_T UT$ interaction term that generates state-dependent stochastic forcing also contributes a nonlinear term ($-\frac{\beta_1}{\beta_2}\gamma_T^N T^2$) to the deterministic drift. In contrast to asymptotic reductions in the strong time-scale separation limit, where such nonlinear drift contributions may be neglected[98], here they are retained because they play a central role in shaping the effective SST dynamics (Fig. 5b). The Linear+CAM configuration is introduced here as a diagnostic benchmark to assess the role of multiplicative noise in isolation and to test whether multiplicative noise alone can reproduce the observed asymmetry of the Niño 3 index distribution.

Mathematically, the four models are written as

$$\begin{aligned}
 \frac{dT}{dt} &= \tilde{\alpha}_0^N + \tilde{\alpha}_1^N T + \tilde{\alpha}_2^N T^2 + \tilde{\alpha}_3^N T^3 + S_1\eta_1 + (G_{2N} + E_2T)\eta_2 && \text{Nonlinear+CAM} \\
 \frac{dT}{dt} &= \tilde{\alpha}_0^N + \tilde{\alpha}_1^N T + \tilde{\alpha}_2^N T^2 + \tilde{\alpha}_3^N T^3 + S_1\eta_1 + S_2\eta_2 && \text{Nonlinear+additive} \\
 \frac{dT}{dt} &= \tilde{\alpha}_1^L T + G_{1L}\eta_1 + G_{2L}\eta_2 && \text{Linear+additive}
 \end{aligned}$$

$$\frac{dT}{dt} = \tilde{\alpha}_1^L T + G_{1L} \eta_1 + (G_{2N} + E_2 T) \eta_2 \quad \text{Linear+CAM} \quad (22)$$

7.2 Analytical stationary distributions

An important advantage of writing the models in this reduced form is that their stationary probability distributions can be obtained analytically from the corresponding Fokker-Planck equation [42–44, 98, 104, 105]. This allows the influence of deterministic nonlinearities and state-dependent noise to be evaluated directly from the structure of the stationary distributions, without relying on long numerical integrations.

The analytical stationary distributions for the four models are given by (see Supplementary Text 4 for details of the derivation)

$$p_s(T) = N_1 \exp(\delta_0^N T) \exp(\delta_1^N T^2) \exp\left(\delta_2^N \tan^{-1}\left(\frac{G_{2N} + E_2 T}{S_1}\right)\right) \times \left[1 + \left(\frac{G_{2N} + E_2 T}{S_1}\right)^2\right]^{\delta_3^N} \quad \text{Nonlinear+CAM} \quad (23)$$

$$p_s(T) = \left[\frac{2N_2}{S_1^2 + S_2^2}\right] \exp(\kappa_0^N T) \exp(\kappa_1^N T^2) \exp(\kappa_2^N T^3) \exp(\kappa_3^N T^4) \quad \text{Nonlinear+additive} \quad (24)$$

$$p_s(T) = \left[\frac{2N_3}{G_{1L}^2 + G_{2L}^2}\right] \exp(\kappa_1^L T^2) \quad \text{Linear+additive} \quad (25)$$

$$p_s(T) = N_4 \exp\left(\delta_2^L \tan^{-1}\left(\frac{G_{2N} + E_2 T}{G_{1L}}\right)\right) \left[1 + \left(\frac{G_{2N} + E_2 T}{G_{1L}}\right)^2\right]^{\delta_3^L} \quad \text{Linear+CAM,} \quad (26)$$

with N_i constants chosen so that each density integrates to unity, and where

$$\delta_0^N = \frac{2\tilde{\alpha}_2^N}{E_2^2} - \frac{4\tilde{\alpha}_3^N G_{2N}}{E_2^3} \quad (27)$$

$$\delta_1^N = \frac{\tilde{\alpha}_3^N}{E_2^2} \quad (28)$$

$$\delta_2^N = \frac{2\tilde{\alpha}_0^N}{S_1 E_2} - \frac{2\tilde{\alpha}_1^N G_{2N}}{S_1 E_2^2} + \frac{2\tilde{\alpha}_2^N}{S_1 E_2^3} (G_{2N}^2 - S_1^2) - \frac{2\tilde{\alpha}_3^N G_{2N}}{S_1 E_2^4} (G_{2N}^2 - 3S_1^2) \quad (29)$$

$$\delta_3^N = \frac{\tilde{\alpha}_1^N}{E_2^2} - \frac{2\tilde{\alpha}_2^N G_{2N}}{E_2^3} + \frac{\tilde{\alpha}_3^N}{E_2^4} (3G_{2N}^2 - S_1^2) - 1 \quad (30)$$

$$\kappa_0^N = \frac{2\tilde{\alpha}_0^N}{S_1^2 + S_2^2} \quad (31)$$

$$\kappa_1^N = \frac{\tilde{\alpha}_1^N}{S_1^2 + S_2^2} \quad (32)$$

$$\kappa_2^N = \frac{2\tilde{\alpha}_2^N}{3(S_1^2 + S_2^2)} \quad (33)$$

$$\kappa_3^N = \frac{\tilde{\alpha}_3^N}{2(S_1^2 + S_2^2)} \quad (34)$$

$$\kappa_1^L = \frac{\tilde{\alpha}_1^L}{G_{1L}^2 + G_{2L}^2} \quad (35)$$

$$\delta_2^L = -\frac{2\tilde{\alpha}_1^L G_{2N}}{G_{1L} E_2^2} \quad (36)$$

$$\delta_3^L = \frac{\tilde{\alpha}_1^L}{E_2^2} - 1. \quad (37)$$

The numerical values of the coefficients appearing in Eqs. 27-37 are listed in Table S1. Although not obvious from the functional forms above, in the limit $E_2 \rightarrow 0$ the CAM-noise stationary distributions reduce to their additive-noise counterparts, so that **Nonlinear+CAM** converges to **Nonlinear+additive** and **Linear+CAM** converges to **Linear+additive**. This limit is derived explicitly in Supplementary Text 5. In addition, the **Linear+additive** model is recovered as a special case of the **Nonlinear+additive** formulation when the nonlinear drift coefficients vanish ($\tilde{\alpha}_0^N = \tilde{\alpha}_2^N = \tilde{\alpha}_3^N = 0$), with the remaining linear drift and additive-noise coefficients identified with their linear case values. Similarly, the **Linear+CAM** model is recovered from the **Nonlinear+CAM** formulation under the same linear-drift conditions, with the remaining coefficients taken as those of the **Linear+CAM** benchmark.

A particularly useful special case is the **Linear+additive** model, in which the stationary distribution is Gaussian with mean zero and standard deviation

$$\sigma = \sqrt{\frac{G_{1L}^2 + G_{2L}^2}{-2\tilde{\alpha}_1^L}} = 0.933^\circ C. \quad (38)$$

For this expression to define a finite stationary variance, the denominator must be positive, requiring $\tilde{\alpha}_1^L < 0$. Using Eq. 18 and the zonal wind damping coefficient $\beta_2 < 0$ (Eq. 13), this condition can be written as $\alpha_1^L \beta_2 - \beta_1 \gamma_U^L > 0$ (i.e., the product of the damping coefficients has to exceed the product of the feedback coefficients), linking the finite Gaussian variance to the balance between damping and coupled SST-wind feedbacks in the original two-variable linear system (Eq. 13). The resulting value closely matches the observed daily Niño 3 index standard deviation ($0.925^\circ C$). This agreement indicates that linear stochastic dynamics capture the variance, or overall amplitude, of Niño 3 SST index variability.

7.3 Relative contribution of nonlinear deterministic dynamics and CAM noise

Against this Gaussian baseline, Figure 6 compares the analytical probability density functions obtained from the four models with the observed Niño 3 index probability distribution. The **Linear+additive** case reproduces the variance but fails to capture the positive skewness evident in the observations. Introducing state-dependent noise into this linear framework (**Linear+CAM**) produces only modest departures from Gaussian behavior. As shown more clearly in the tail and cumulative-distribution panels of Fig. S4, including the warm-tail exceedance probability $1 - CDF$, CAM noise alone within this linear-drift benchmark leads to relatively small changes in the probabilities of extreme events and does not reproduce the observed asymmetry of the distribution.

In contrast, introducing deterministic nonlinearities into the drift term substantially alters the shape of the stationary distribution. The **Nonlinear+additive** model already produces a strongly asymmetric distribution with an enhanced warm tail, reproducing much of the observed positive skewness of the Niño 3 index. The comparison shows that deterministic nonlinearities associated with the reduced SST dynamics (which incorporate SST-wind coupling through the reduced coefficients) are the dominant mechanism shaping the asymmetric structure of the Niño 3 distribution within the reduced framework estimated from daily data. Including state-dependent noise in the nonlinear system (**nonlinear+CAM**) further modifies the distribution, primarily by slightly increasing the probability of extreme warm events and slightly decreasing the probability of extreme cold events relative to the **Nonlinear+additive** cases. These differences are negligible in the central part of the distribution but become modestly more visible in the tails (Fig. S4).

These comparisons indicate that the asymmetric structure of the Niño 3 distribution arises primarily from deterministic nonlinearities in the reduced SST dynamics, while state-dependent stochastic forcing plays a secondary role that mainly affects the probabilities of the most extreme events.

This two-mechanism perspective is complementary to the recharge-oscillator synthesis of Vialard et al., 2025[24], in which ENSO skewness varies jointly with deterministic quadratic nonlinearity and the amplitude of state-dependent stochastic forcing. The present framework differs in that the drift and noise structures are diagnosed from a daily SST-wind system and subsequently separated through the reduced-model hierarchy. Levine et al., 2016[10] similarly showed analytically, within a recharge-oscillator framework, that ENSO asymmetry depends on the amplitude of state-dependent stochastic forcing.

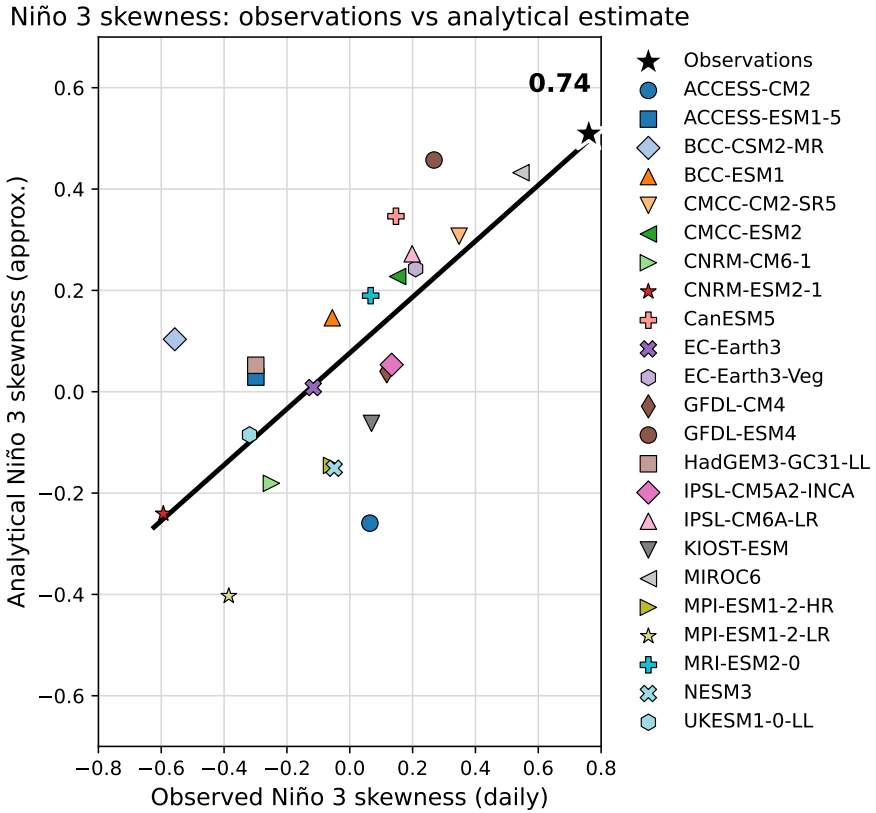


Fig. 7 Relationship between the daily Niño 3 SST skewness diagnosed from observations and CMIP6 historical simulations and the analytical skewness approximation derived from the reduced stochastic model (Eq. 39). The plotted "analytical" skewness corresponds to the leading-order moment-closure estimate, which uses the model-diagnosed Niño 3 variance and the diagnosed effective nonlinear drift coefficients. The solid line shows the least-squares fit including both models and observations, and the number reported ($r = 0.74$) corresponds to the Pearson correlation coefficient. When the correlation is computed across models only, excluding the observational point, the correlation coefficient is $r = 0.68$.

8 Effective damping as a framework for interpreting ENSO skewness across models

The analytical framework developed in the previous sections suggests that the asymmetry of eastern Pacific SST variability arises primarily from the structure of the deterministic drift in the reduced stochastic equation for T . In particular, the nonlinear terms that emerge after adiabatic elimination of the wind variable modify the effective damping experienced by SST anomalies (e.g., Fig. 5b). As shown above, this provides a mechanism through which a positively skewed stationary distribution of T can arise even under additive stochastic forcing. If this interpretation is relevant beyond observations, then

differences in ENSO skewness across climate models should be interpretable, at least to leading order, in terms of differences in effective SST damping ($\tilde{\alpha}_i$ terms) and in the amplitude of stochastic forcing (S_i terms). In this section, we test this hypothesis by comparing the skewness of daily Niño 3 SST anomalies in CMIP6 simulations with an analytical estimate of skewness derived from the reduced stochastic model.

8.1 Analytical approximation of Niño 3 index skewness from the reduced stochastic model

To construct an analytical approximation for Niño 3 skewness from the reduced stochastic model, we first identify the formulation that best captures the asymmetry while remaining mathematically simple. Among the models considered, the **Nonlinear+CAM** formulation provides the most complete representation of the reduced dynamics, including both nonlinear deterministic drift and CAM noise arising from the adiabatic elimination of the wind variable. However, the results above show that the simpler **Nonlinear+Additive** model already reproduces almost all of the asymmetry of the Niño 3 index. In that formulation, the nonlinear deterministic drift is retained, while the stochastic forcing is treated as additive.

The ability of this simpler model to capture the skewness of the more complete **Nonlinear+CAM** reduced system suggests that the dominant source of asymmetry, within this daily-resolved reduced framework, lies in the nonlinear structure of the deterministic drift rather than in the state dependence of the stochastic forcing. Because the **Nonlinear+Additive** formulation retains the essential nonlinear feedbacks while being mathematically simpler (Eq. 22), it provides a useful framework for examining how the skewness of the SST distribution arises from the reduced stochastic dynamics.

Within the **Nonlinear+Additive** framework, the SST tendency is governed by the effective nonlinear drift obtained after adiabatic elimination of the wind variable $A_{eff}^N(T) = \tilde{\alpha}_0^N + \tilde{\alpha}_1^N T + \tilde{\alpha}_2^N T^2 + \tilde{\alpha}_3^N T^3$, together with additive stochastic forcing (Eq. 22) that maintains the variance of the SST anomalies. Although the stationary probability density function can be obtained analytically, closed-form expressions for its moments are not generally available. We use the moment equations associated with this reduced stochastic model and approximate the skewness coefficient of the stationary SST anomaly distribution by closing the first three moment equations using a low-order cumulant closure, in which fourth- and higher-order cumulants are neglected while retaining the leading skewness contribution [106, 107] (see Supplementary Text 6 for details). Retaining the leading terms yields the analytical approximation

$$S_0 \approx -\frac{\tilde{\alpha}_0^N + 3\tilde{\alpha}_2^N \sigma^2}{\sigma (\tilde{\alpha}_1^N + 10\tilde{\alpha}_3^N \sigma^2)}, \quad (39)$$

where σ denotes the standard deviation of the Niño 3 SST anomaly. Because this approximation relies on a low-order cumulant closure for higher-order

moments, S_0 should be interpreted as a leading-order diagnostic rather than an accurate prediction of the skewness coefficient S . Its value lies in making explicit how the different components of the effective drift and the variance of the system combine to produce skewness.

Equation 39 shows that the skewness of the SST distribution is not determined by a single parameter but rather by a balance among several contributions. The coefficients $\tilde{\alpha}_0^N$ and $\tilde{\alpha}_2^N$ contribute to the asymmetry of the effective drift, whereas terms $\tilde{\alpha}_1^N T$ and $\tilde{\alpha}_3^N T^3$ are antisymmetric in T and therefore contribute to the restoring tendency but do not by themselves generate skewness. Note that in a linear system, the counterparts of $\tilde{\alpha}_0$ and $\tilde{\alpha}_2$ are identically zero, which, as expected, implies no skewness for a linear system driven by additive noise. The variance σ^2 , which is maintained by the additive stochastic forcing, also enters the expression. Although additive noise by itself does not generate skewness, the magnitude of the variance influences how strongly the nonlinear deterministic terms shape the stationary distribution.

8.2 Intermodel relationship between effective damping and ENSO skewness

The analytical approximation derived above provides a direct link between the skewness of the Niño 3 index and the effective drift parameters of the reduced system, raising the question of whether this relationship can help organize differences in ENSO asymmetry across climate models. To explore this, we analyze daily Niño 3 SST anomalies from the historical simulations of 23 CMIP6 models[52] together with observational estimates. For each dataset, the skewness of the Niño 3 index is computed directly from the SST time series, and the coefficients of the effective drift ($\tilde{\alpha}_i^N$) are diagnosed from the fitted **Non-linear+Additive** reduced model (and include the Niño 4 zonal wind anomaly effect). The reduced model is fitted independently to each observational or CMIP6 dataset; no observational drift coefficients are imposed on the climate models. The standard deviation σ of the SST anomaly is also computed for each dataset. These quantities are then used to evaluate the analytical skewness approximation given above. The resulting analytical skewness coefficient estimate is compared with the skewness coefficient measured directly from the SST time series.

Figure 7 summarizes this analytical-versus-diagnosed skewness comparison across the CMIP6 ensemble together with the observational estimate. The comparison reveals a clear relationship between the skewness estimated from the analytical approximation and the skewness diagnosed directly from the SST time series across models. The scatter around this relationship is expected given that the analytical expression is a rough approximation based on a low-order cumulant closure for higher-order moments. Despite this spread, the analytically estimated and measured skewness coefficients remain positively correlated across the CMIP6 ensemble ($r = 0.74$ when the observational estimate is included and $r = 0.68$ across models only). Across models, differences in ENSO skewness are related, to leading order, to differences in the effective

nonlinear drift structure of the reduced SST dynamics, as well as to the amplitude of the stochastic forcing, which influences the skewness coefficient value through its effect on the SST variance (Eq. 39). Because the drift coefficients and SST variance covary across models, however, Fig. 7 does not isolate their individual contributions to the intermodel spread in skewness.

9 Discussion

The results presented here provide a minimal dynamical interpretation of ENSO asymmetry based on a simple stochastic representation of the coupled SST-wind system, in which eastern equatorial Pacific SST anomalies and central-western equatorial zonal wind anomalies evolve together through air-sea coupling. In the linear formulation (Eqs. 1 and 2), the dynamics are governed by a single stable equilibrium and a constant Jacobian matrix. Stability is uniform across phase space and the system behaves analogously to a linear Ornstein-Uhlenbeck process, similar to the dynamics implied by a classical Linear Inverse Model (LIM). In this linear framework, the coupling is still present, but it does not produce any state dependence in the local stability properties of the system. Stochastic forcing simply excites fluctuations around the equilibrium and the stationary probability distribution is Gaussian, implying no deterministic mechanism that favors El Niño (warm) over La Niña (cold) states. By contrast, the nonlinear formulation (Eqs. 1 and 3) modifies the geometry of the deterministic dynamics. Although the equilibrium remains locally stable, the Jacobian becomes state dependent and the effective damping varies across phase space. This nonlinear drift changes which regions of SST-wind phase space are more favorable to amplification or damping, leading to a deformation of the trajectory structure relative to the linear case. In particular, the quadrant with positive SST anomalies and westerly wind anomalies becomes more favorable to growth than its cold counterpart, consistent with the known role of westerly wind anomalies in promoting El Niño growth through reduced upwelling, thermocline adjustments, and related coupled feedbacks[17, 58, 60, 61, 79, 81].

This dynamical interpretation complements a number of previous studies based on monthly data, which have represented aspects of wind-forcing influence primarily through state-dependent stochastic atmospheric forcing[7, 10, 11, 13, 102], including formulations motivated by westerly wind bursts[10, 100, 101] and, more recently, an intermittent non-Gaussian CAM representation of such bursts[108]. Here, however, the empirical data-driven estimation of the coupled daily SST-wind system indicates that an asymmetry associated with warm-westerly states is already evident in the deterministic part of the dynamics in the daily-resolved system, implying that part of the warm-cold asymmetry of Niño 3 variability is already encoded in the deterministic drift of the coupled SST-wind system.

To interpret the statistical consequences of this dynamical structure, a key step is to reduce the coupled system to an effective one-dimensional

stochastic equation for the Niño 3 SSTA index. This reduction exploits one of the main advantages of working with daily data: the clear separation of timescales between relatively fast atmospheric wind variability (decorrelation timescale of about two weeks) and the slower evolution of SST (decorrelation timescale of about 1 year). Under this timescale separation, zonal wind anomalies can be adiabatically eliminated and expressed as a function of SST anomalies and stochastic forcing. This reduction yields a simplified stochastic equation for the Niño 3 SSTA index that modifies and reorganizes the deterministic nonlinearities of the original coupled system into an effective SST drift while also incorporating the influence of wind variability through effective stochastic terms and through changes in the effective damping structure. Notably, the resulting equation contains correlated additive-multiplicative (CAM) noise[7, 43, 44, 98, 99, 103]. Whereas in a number of previous stochastic ENSO frameworks multiplicative noise is introduced as a useful phenomenological representation of unresolved atmospheric variability (e.g., [10, 45, 83, 100–102]), here the CAM structure emerges naturally from the elimination of the fast atmospheric variable. The coefficients of the different system terms, including both the effective drift and stochastic terms, can be explicitly linked to the coupling terms in the original two-variable system, providing a direct dynamical interpretation of the state-dependent noise as well as of the modified effective SST damping.

Within this reduced framework, the comparison of stochastic model configurations allows the relative roles of deterministic nonlinearities and state-dependent noise to be isolated. Across the four variants examined (**Nonlinear + CAM**, **Nonlinear + Additive**, **Linear + Additive**, **Linear + CAM**), the results consistently indicate that deterministic nonlinear drift plays the primary role in generating the skewness of the Niño 3 SST index, consistent with previous analyses of nonlinear stochastic recharge-oscillator models[45]. More broadly, Geng and Jin (2023)[109] similarly found in an intermediate coupled model that deterministic nonlinear dynamics can generate ENSO diversity, while stochastic forcing broadens the range of conditions over which that diversity occurs. Even in the presence of purely additive noise, the nonlinear drift reproduces much of the observed asymmetry of the SST anomaly distribution. State-dependent stochastic forcing modifies the probability of extreme events and slightly alters the tails of the distribution, but its influence on the overall skewness is comparatively small. By contrast, the linear system produces nearly Gaussian behavior even when multiplicative noise is included. Across these configurations, the geometry of the deterministic dynamics, rather than the detailed structure of stochastic forcing, largely determines the emergence of Niño 3 asymmetry within the reduced framework considered here.

These findings also clarify the relationship between deterministic nonlinearities and state-dependent stochastic forcing in conceptual ENSO models. In several previous stochastic formulations, CAM noise has been invoked as a mechanism for generating non-Gaussian SST statistics[7, 10, 24, 83, 100, 103, 105]. Many of these formulations are best understood as useful reduced

representations of unresolved atmospheric variability, particularly when working with monthly data[10, 102]. In the present daily data-driven framework, the reduction of the coupled SST-wind system shows that the same underlying coupling that generates state-dependent noise in the reduced equation also modifies the effective deterministic drift and damping of SST anomalies (Fig. 5). This point is important for interpreting the relation between daily and monthly perspectives. At daily resolution, part of the asymmetry can be identified explicitly in the deterministic component of the coupled dynamics. After temporal coarse-graining, however, some of that same influence may instead appear as state-dependent stochastic forcing from the perspective of a monthly model, because the fast coupled variability is no longer resolved explicitly[20, 41, 98, 99, 110]. More generally, temporally coarse-graining does not commute with nonlinear multiplication: for example, $\langle UT \rangle \neq \langle U \rangle \langle T \rangle$, leaving unresolved covariance terms whose effects require closure in a temporally coarse-grained model. Our results do not imply that deterministic nonlinearities must dominate ENSO asymmetry at monthly timescales. Rather, they show that, for the empirically estimated daily SST-wind system examined here, the coupled dynamics include a deterministic asymmetric component whose role can be separated from the residual stochastic forcing before coarse-graining. In that sense, deterministic nonlinearities and multiplicative noise should not always be interpreted as fully distinct physical ingredients, but may partly reflect different reduced descriptions of the same underlying coupled variability. At monthly resolution, the corresponding deterministic and stochastic parameters would therefore need to be estimated or closed at that temporal scale. The daily coefficients nevertheless provide information about the sign, state dependence, and relative importance of the resolved couplings before coarse-graining.

Beyond this conceptual point, the reduced SST equation also provides a useful framework for interpreting differences in ENSO asymmetry across comprehensive climate models. Within this formulation, the skewness estimate derived from the reduced model should be understood as a leading-order approximation, based on moment closure assumptions, that provides guidance on how the reduced SST dynamics organize ENSO asymmetry across models. Applying this insight to historical CMIP6 simulations reveals a positive relationship between the skewness analytically estimated from the reduced model and the skewness diagnosed directly from model output. Although the relationship is necessarily approximate, since the skewness estimate is based on a low-order cumulant closure rather than an exact closed-form expression, the correlation suggests that intermodel differences in ENSO asymmetry may be organized, to leading order, by differences in the effective reduced dynamics of SST anomalies, including both the asymmetric components of the drift and the overall restoring strength of the system. More broadly, this result suggests that skewness-based metrics[5, 111, 112] are necessary but not sufficient for evaluating ENSO asymmetry in climate models. The fact that the CMIP6 models

generally underestimate the observed Niño 3 skewness (Fig. 7) already indicates a mismatch in the simulated balance of processes. However, agreement in skewness alone would still not be sufficient to establish that this balance is realistic, because similar skewness values can arise from different combinations of effective drift structure and variance. In this sense, the reduced dynamical framework developed here provides a more process-oriented complement to skewness-based model evaluation. The effective coefficients should be interpreted as aggregate diagnostics of each model's reduced coupled process balance rather than as one-to-one measures of particular mixed-layer or oceanic processes.

Several limitations remain. The model is intentionally minimal, as in other low-order conceptual ENSO frameworks[23, 24, 45, 113], consisting of two dynamical variables chosen to isolate the coupled SST-wind mechanism emphasized here. Important processes such as thermocline variability, ocean wave adjustment, spatial SST structure, and seasonal phase-locking are not represented explicitly, nor are interactions with variability outside the equatorial Pacific, including midlatitude precursors such as the Pacific Meridional Modes[63, 64, 67, 70, 73, 114–116]. Similarly, the one-dimensional reduction is designed to diagnose stationary SST statistics rather than the full ENSO spectrum or phase-transition dynamics, which depend on degrees of freedom and memory not represented explicitly in the reduced SST equation. These omissions define the scope of the present analysis, rather than undermining its usefulness. Precisely because the framework is reduced, it allows the roles of deterministic nonlinear drift and state-dependent stochastic forcing to be isolated and interpreted more clearly than in a more complete but less transparent system. The observational coefficients are also estimated from a 38-year record using a particular combination of SST and wind products, so their precise values may be sensitive to finite sampling and dataset choice. The qualitative mechanism and the consistency among the coupled, reduced, and direct SST formulations provide more robust points of comparison than any single fitted coefficient. The results identify a physically interpretable reduced mechanism through which ENSO asymmetry can arise, rather than as a complete description of all processes contributing to ENSO variability in nature.

Despite these limitations, even a very low-order stochastic representation of the coupled SST-wind system can provide useful insight into the mechanisms underlying ENSO statistics. The analysis suggests that the asymmetry of eastern Pacific SST variability may arise primarily from nonlinear deterministic feedbacks embedded in the effective drift of the daily two-variable system examined here, which favor stronger amplification of warm-westerly states than their cold counterparts, while stochastic forcing acts mainly to populate the resulting probability distribution. More broadly, the value of this type of simple, interpretable framework is not only that it reproduces non-Gaussian ENSO statistics, but that it helps identify which aspects of asymmetry are already encoded in the resolved coupled dynamics and which remain in the residual stochastic forcing.

References

- [1] Bjerknes, J. A possible response of the atmospheric Hadley circulation to equatorial anomalies of ocean temperature. *Tellus* **18**, 820–829 (1966).
- [2] Bjerknes, J. ATMOSPHERIC TELECONNECTIONS FROM THE EQUATORIAL PACIFIC. *Monthly Weather Review* **97**, 163–172 (1969).
- [3] Neelin, J. D. *et al.* ENSO theory. *Journal of Geophysical Research: Oceans* **103**, 14261–14290 (1998).
- [4] Battisti, D. S. & Hirst, A. C. Interannual Variability in a Tropical Atmosphere–Ocean Model: Influence of the Basic State, Ocean Geometry and Nonlinearity. *Journal of the Atmospheric Sciences* **46**, 1687–1712 (1989).
- [5] Burgers, G. & Stephenson, D. B. The “normality” of El Niño. *Geophysical Research Letters* **26**, 1027–1030 (1999).
- [6] Takahashi, K. & Dewitte, B. Strong and moderate nonlinear El Niño regimes. *Climate Dynamics* **46**, 1627–1645 (2016).
- [7] Martinez-Villalobos, C., Newman, M., Vimont, D. J., Penland, C. & David Neelin, J. Observed El Niño–La Niña Asymmetry in a Linear Model. *Geophysical Research Letters* **46**, 9909–9919 (2019).
- [8] Okumura, Y. M. & Deser, C. Asymmetry in the Duration of El Niño and La Niña. *Journal of Climate* **23**, 5826–5843 (2010).
- [9] Kondrashov, D., Kravtsov, S., Robertson, A. W. & Ghil, M. A Hierarchy of Data-Based ENSO Models. *Journal of Climate* **18**, 4425–4444 (2005).
- [10] Levine, A., Jin, F. F. & McPhaden, M. J. Extreme Noise–Extreme El Niño: How State-Dependent Noise Forcing Creates El Niño–La Niña Asymmetry. *Journal of Climate* **29**, 5483–5499 (2016).
- [11] Kug, J.-S., Jin, F.-F., Sooraj, K. P. & Kang, I.-S. State-dependent atmospheric noise associated with ENSO. *Geophysical Research Letters* **35** (2008).
- [12] Gushchina, D. & Dewitte, B. Intraseasonal Tropical Atmospheric Variability Associated with the Two Flavors of El Niño. *Monthly Weather Review* **140**, 3669–3681 (2012).
- [13] Capotondi, A., Sardeshmukh, P. D. & Ricciardulli, L. The Nature of the Stochastic Wind Forcing of ENSO. *Journal of Climate* **31**, 8081–8099 (2018).

- [14] Sura, P., Newman, M. & Alexander, M. A. Daily to Decadal Sea Surface Temperature Variability Driven by State-Dependent Stochastic Heat Fluxes. *Journal of Physical Oceanography* **36**, 1940–1958 (2006).
- [15] Perez, C. L., Moore, A. M., Zavala-Garay, J. & Kleeman, R. A Comparison of the Influence of Additive and Multiplicative Stochastic Forcing on a Coupled Model of ENSO. *Journal of Climate* **18**, 5066–5085 (2005).
- [16] Kleeman, R. Stochastic theories for the irregularity of ENSO. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* **366**, 2509–2524 (2008).
- [17] Yu, S. & Fedorov, A. V. The Essential Role of Westerly Wind Bursts in ENSO Dynamics and Extreme Events Quantified in Model “Wind Stress Shaving” Experiments. *Journal of Climate* **35**, 7519–7538 (2022).
- [18] Trenberth, K. E. The Definition of El Niño. *Bulletin of the American Meteorological Society* **78**, 2771–2778 (1997).
- [19] Hanley, D. E., Bourassa, M. A., O’Brien, J. J., Smith, S. R. & Spade, E. R. A Quantitative Evaluation of ENSO Indices. *Journal of Climate* **16**, 1249–1258 (2003).
- [20] Penland, C. A stochastic model of IndoPacific sea surface temperature anomalies. *Physica D: Nonlinear Phenomena* **98**, 534–558 (1996).
- [21] Zavala-Garay, J., Moore, A. M., Perez, C. L. & Kleeman, R. The Response of a Coupled Model of ENSO to Observed Estimates of Stochastic Forcing. *Journal of Climate* **16**, 2827–2842 (2003).
- [22] Sarachik, E. S. & Cane, M. A. *The El Niño-Southern Oscillation Phenomenon* (Cambridge University Press, Cambridge, 2010).
- [23] Jin, F.-F. An Equatorial Ocean Recharge Paradigm for ENSO. Part I: Conceptual Model. *Journal of the Atmospheric Sciences* **54**, 811–829 (1997).
- [24] Vialard, J. *et al.* The El Niño Southern Oscillation (ENSO) Recharge Oscillator Conceptual Model: Achievements and Future Prospects. *Reviews of Geophysics* **63**, e2024RG000843 (2025).
- [25] Jin, F.-F. An Equatorial Ocean Recharge Paradigm for ENSO. Part II: A Stripped-Down Coupled Model. *Journal of the Atmospheric Sciences* **54**, 830–847 (1997).
- [26] Thual, S., Dewitte, B., Ayoub, N. & Thual, O. An Asymptotic Expansion for the Recharge–Discharge Model of ENSO. *Journal of Physical*

- Oceanography* **43**, 1407–1416 (2013).
- [27] Hasselmann, K. Stochastic climate models Part I. Theory. *Tellus* **28**, 473–485 (1976).
- [28] Suarez, M. J. & Schopf, P. S. A Delayed Action Oscillator for ENSO. *Journal of the Atmospheric Sciences* **45**, 3283–3287 (1988).
- [29] Penland, C. & Sardeshmukh, P. D. The Optimal Growth of Tropical Sea Surface Temperature Anomalies. *Journal of Climate* **8**, 1999–2024 (1995).
- [30] Moore, A. M. & Kleeman, R. Stochastic Forcing of ENSO by the Intraseasonal Oscillation. *Journal of Climate* **12**, 1199–1220 (1999).
- [31] Chen, N. & Majda, A. J. Simple stochastic dynamical models capturing the statistical diversity of el niño southern oscillation. *Proceedings of the National Academy of Sciences* **114**, 1468–1473 (2017).
- [32] Izumo, T., Colin, M., Jin, F.-F. & Pagli, B. The hybrid recharge delayed oscillator: A more realistic el niño conceptual model. *Journal of Climate* **37**, 2765–2787 (2024).
- [33] Zhao, S. *et al.* Explainable El Niño predictability from climate mode interactions. *Nature* **630**, 891–898 (2024).
- [34] Martinez-Villalobos, C. *et al.* A Low-Order Data-Driven Model of ENSO Diversity. *Geophysical Research Letters* **52**, e2025GL118649 (2025).
- [35] Santoso, A., Mcphaden, M. J. & Cai, W. The Defining Characteristics of ENSO Extremes and the Strong 2015/2016 El Niño. *Reviews of Geophysics* **55**, 1079–1129 (2017).
- [36] An, S.-I. & Jin, F.-F. Nonlinearity and Asymmetry of ENSO. *Journal of Climate* **17**, 2399–2412 (2004).
- [37] Su, J. *et al.* Causes of the El Niño and La Niña Amplitude Asymmetry in the Equatorial Eastern Pacific. *Journal of Climate* **23**, 605–617 (2010).
- [38] Takahashi, K., Karamperidou, C. & Dewitte, B. A theoretical model of strong and moderate El Niño regimes. *Climate Dynamics* **52**, 7477–7493 (2019).
- [39] Srinivas, G. *et al.* Dominant contribution of atmospheric nonlinearities to ENSO asymmetry and extreme El Niño events. *Scientific Reports* **14**, 8122 (2024).

- [40] Bianucci, M., Capotondi, A., Mannella, R. & Merlino, S. Linear or Nonlinear Modeling for ENSO Dynamics? *Atmosphere* **9** (2018).
- [41] Sura, P., Newman, M., Penland, C. & Sardeshmukh, P. Multiplicative Noise and Non-Gaussianity: A Paradigm for Atmospheric Regimes? *Journal of the Atmospheric Sciences* **62**, 1391–1409 (2005).
- [42] Gardiner, C. W. *Stochastic Methods: A Handbook for the Natural and Social Sciences*, vol. 13 of *Springer Series in Synergetics* (Springer, Berlin, Heidelberg, 2009), 4 edn.
- [43] Sardeshmukh, P. D. & Sura, P. Reconciling Non-Gaussian Climate Statistics with Linear Dynamics. *Journal of Climate* **22**, 1193–1207 (2009).
- [44] Majda, A. J., Franzke, C. & Croomelin, D. Normal forms for reduced stochastic climate models. *Proceedings of the National Academy of Sciences* **106**, 3649–3653 (2009).
- [45] An, S.-I., Kim, S.-K. & Timmermann, A. Fokker–Planck dynamics of the El Niño–Southern Oscillation. *Scientific Reports* **10**, 16282 (2020).
- [46] Reynolds, R. W. *et al.* Daily High-Resolution-Blended Analyses for Sea Surface Temperature. *Journal of Climate* **20**, 5473–5496 (2007).
- [47] Kalnay, E. *et al.* The NCEP/NCAR 40-Year Reanalysis Project. *Bulletin of the American Meteorological Society* **77**, 437–472 (1996).
- [48] Lellouche, J.-M. *et al.* Evaluation of global monitoring and forecasting systems at Mercator Océan. *Ocean Science* **9**, 57–81 (2013).
- [49] Garric, G. *et al.* Performance and quality assessment of the global ocean eddy-permitting physical reanalysis GLORYS2V4. In *EGU General Assembly Conference Abstracts*, 18776 (2017).
- [50] Rebert, J. P., Donguy, J. R., Eldin, G. & Wyrtki, K. Relations between sea level, thermocline depth, heat content, and dynamic height in the tropical Pacific Ocean. *Journal of Geophysical Research: Oceans* **90**, 11719–11725 (1985).
- [51] Meinen, C. S. & McPhaden, M. J. Observations of Warm Water Volume Changes in the Equatorial Pacific and Their Relationship to El Niño and La Niña. *Journal of Climate* **13**, 3551–3559 (2000).
- [52] Eyring, V. *et al.* Overview of the Coupled Model Intercomparison Project Phase 6 (CMIP6) experimental design and organization. *Geoscientific Model Development* **9**, 1937–1958 (2016).

- [53] Gill, A. E. Some simple solutions for heat-induced tropical circulation. *Quarterly Journal of the Royal Meteorological Society* **106**, 447–462 (1980).
- [54] Zebiak, S. E. A Simple Atmospheric Model of Relevance to El Niño. *Journal of the Atmospheric Sciences* **39**, 2017–2027 (1982).
- [55] Neelin, J. D. On the Interpretation of the Gill Model. *Journal of the Atmospheric Sciences* **46**, 2466–2468 (1989).
- [56] Yu, J.-Y. & Neelin, J. D. Analytic Approximations for Moist Convectively Adjusted Regions. *Journal of the Atmospheric Sciences* **54**, 1054–1063 (1997).
- [57] Matsuno, T. Quasi-Geostrophic Motions in the Equatorial Area. *Journal of the Meteorological Society of Japan. Ser. II* **44**, 25–43 (1966).
- [58] Zebiak, S. E. & Cane, M. A. A Model El Niño–Southern Oscillation. *Monthly Weather Review* **115**, 2262–2278 (1987).
- [59] Picaut, J., Masia, F. & du Penhoat, Y. An Advective-Reflective Conceptual Model for the Oscillatory Nature of the ENSO. *Science* **277**, 663–666 (1997).
- [60] McPhaden, M. J. Genesis and Evolution of the 1997–98 El Niño. *Science* **283**, 950–954 (1999).
- [61] An, S.-I. & Jin, F.-F. Collective Role of Thermocline and Zonal Advective Feedbacks in the ENSO Mode. *Journal of Climate* **14**, 3421–3432 (2001).
- [62] Clarke, A. J. *An Introduction to the Dynamics of El Nino and the Southern Oscillation* (2008).
- [63] Vimont, D. J., Wallace, J. M. & Battisti, D. S. The Seasonal Footprinting Mechanism in the Pacific: Implications for ENSO. *Journal of Climate* **16**, 2668–2675 (2003).
- [64] Chiang, J. C. H. & Vimont, D. J. Analogous Pacific and Atlantic Meridional Modes of Tropical Atmosphere–Ocean Variability. *Journal of Climate* **17**, 4143–4158 (2004).
- [65] Chang, P. *et al.* Pacific meridional mode and El Niño–Southern Oscillation. *Geophysical Research Letters* **34** (2007).
- [66] Alexander, M. A., Vimont, D. J., Chang, P. & Scott, J. D. The Impact of Extratropical Atmospheric Variability on ENSO: Testing the Seasonal Footprinting Mechanism Using Coupled Model Experiments. *Journal of*

- Climate* **23**, 2885–2901 (2010).
- [67] Zhang, H., Clement, A. & Nezio, P. D. The South Pacific Meridional Mode: A Mechanism for ENSO-like Variability. *Journal of Climate* **27**, 769–783 (2014).
- [68] Di Lorenzo, E. *et al.* ENSO and meridional modes: A null hypothesis for Pacific climate variability. *Geophysical Research Letters* **42**, 9440–9448 (2015).
- [69] McGregor, S., Timmermann, A., Jin, F.-F. & Kessler, W. S. Charging El Niño with off-equatorial westerly wind events. *Climate Dynamics* **47**, 1111–1125 (2016).
- [70] Martinez-Villalobos, C. & Vimont, D. J. An Analytical Framework for Understanding Tropical Meridional Modes. *Journal of Climate* **30**, 3303–3323 (2017).
- [71] Stuecker, M. F. Revisiting the Pacific Meridional Mode. *Scientific Reports* **8**, 3216 (2018).
- [72] Amaya, D. J. The Pacific Meridional Mode and ENSO: A Review. *Current Climate Change Reports* **5**, 296–307 (2019).
- [73] Dewitte, B. *et al.* The ENSO-induced South Pacific Meridional Mode. *Frontiers in Climate* **4** (2023).
- [74] Martinez-Villalobos, C., Dewitte, B., Garreaud, R. D. & Loyola, L. Extreme coastal El Niño events are tightly linked to the development of the Pacific Meridional Modes. *npj Climate and Atmospheric Science* **7**, 123 (2024).
- [75] Cai, W. *et al.* Pantropical climate interactions. *Science* **363**, eaav4236 (2019).
- [76] Kim, J.-W. & Yu, J.-Y. Single- and multi-year ENSO events controlled by pantropical climate interactions. *npj Climate and Atmospheric Science* **5**, 88 (2022).
- [77] Jin, F.-F., Kim, S. T. & Bejarano, L. A coupled-stability index for ENSO. *Geophysical Research Letters* **33** (2006).
- [78] Guan, C., McPhaden, M. J., Wang, F. & Hu, S. Quantifying the Role of Oceanic Feedbacks on ENSO Asymmetry. *Geophysical Research Letters* **46**, 2140–2148 (2019).
- [79] Jin, F.-F. & An, S.-I. Thermocline and Zonal Advective Feedbacks Within the Equatorial Ocean Recharge Oscillator Model for ENSO.

- Geophysical Research Letters* **26**, 2989–2992 (1999).
- [80] Rasmusson, E. M. & Carpenter, T. H. Variations in Tropical Sea Surface Temperature and Surface Wind Fields Associated with the Southern Oscillation/El Niño. *Monthly Weather Review* **110**, 354–384 (1982).
- [81] Vecchi, G. A. & Harrison, D. E. Tropical Pacific Sea Surface Temperature Anomalies, El Niño, and Equatorial Westerly Wind Events. *Journal of Climate* **13**, 1814–1830 (2000).
- [82] Ewald, B. & Penland, C. Numerical Generation of Stochastic Differential Equations in Climate Models. In *Handbook of Numerical Analysis*, vol. 14, 279–306 (Elsevier, 2009).
- [83] Levine, A. F. Z. & Jin, F.-F. Noise-Induced Instability in the ENSO Recharge Oscillator. *Journal of the Atmospheric Sciences* **67**, 529–542 (2010).
- [84] Penland, C. Random Forcing and Forecasting Using Principal Oscillation Pattern Analysis. *Monthly Weather Review* **117**, 2165–2185 (1989).
- [85] Xue, Y., Leetmaa, A. & Ji, M. ENSO Prediction with Markov Models: The Impact of Sea Level. *Journal of Climate* **13**, 849–871 (2000).
- [86] Dommenges, B. D. & Al-Ansari, M. Asymmetries in the ENSO phase space. *Climate Dynamics* **60**, 2147–2166 (2023).
- [87] Priya, P. & Dommenges, D. ENSO phase space dynamics in CMIP models. *Climate Dynamics* **63**, 270 (2025).
- [88] Dijkstra, H. A. *Nonlinear Climate Dynamics* (Cambridge University Press, Cambridge, 2013).
- [89] Strogatz, S. H. *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering* (CRC Press, Boca Raton, 2018), 2 edn.
- [90] Sura, P. & Hannachi, A. Perspectives of Non-Gaussianity in Atmospheric Synoptic and Low-Frequency Variability. *Journal of Climate* **28**, 5091–5114 (2015).
- [91] Hasselmann, K. PIPs and POPs: The reduction of complex dynamical systems using principal interaction and oscillation patterns. *Journal of Geophysical Research* **93** (1988).
- [92] Penland, C. & Magorian, T. Prediction of Niño 3 Sea Surface Temperatures Using Linear Inverse Modeling. *Journal of Climate* **6**, 1067–1076 (1993).

- [93] Thompson, C. J. & Battisti, D. S. A Linear Stochastic Dynamical Model of ENSO. Part I: Model Development. *Journal of Climate* **13**, 2818–2832 (2000).
- [94] Majda, A. J., Timofeyev, I. & Vanden Eijnden, E. Models for stochastic climate prediction. *Proceedings of the National Academy of Sciences* **96**, 14687–14691 (1999).
- [95] Franzke, C., Majda, A. J. & Vanden-Eijnden, E. Low-Order Stochastic Mode Reduction for a Realistic Barotropic Model Climate. *Journal of the Atmospheric Sciences* **62**, 1722–1745 (2005).
- [96] Pavliotis, G. A. & Stuart, A. M. *Multiscale Methods*, vol. 53 of *Texts Applied in Mathematics* (Springer, New York, NY, 2008).
- [97] Neelin, J. D. The Slow Sea Surface Temperature Mode and the Fast-Wave Limit: Analytic Theory for Tropical Interannual Oscillations and Experiments in a Hybrid Coupled Model. *Journal of the Atmospheric Sciences* **48**, 584–606 (1991).
- [98] Sardeshmukh, P. D. & Penland, C. Understanding the distinctively skewed and heavy tailed character of atmospheric and oceanic probability distributions. *Chaos: An Interdisciplinary Journal of Nonlinear Science* **25**, 036410 (2015).
- [99] Martinez-Villalobos, C., Vimont, D. J., Penland, C., Newman, M. & Neelin, J. D. Calculating State-Dependent Noise in a Linear Inverse Model Framework. *Journal of the Atmospheric Sciences* **75**, 479–496 (2018).
- [100] Jin, F.-F., Lin, L., Timmermann, A. & Zhao, J. Ensemble-mean dynamics of the ENSO recharge oscillator under state-dependent stochastic forcing. *Geophysical Research Letters* **34** (2007).
- [101] Thual, S., Majda, A. J., Chen, N. & Stechmann, S. N. Simple stochastic model for El Niño with westerly wind bursts. *Proceedings of the National Academy of Sciences* **113**, 10245–10250 (2016).
- [102] Levine, A. F. Z. & Jin, F. F. A simple approach to quantifying the noise–ENSO interaction. Part I: Deducing the state-dependency of the windstress forcing using monthly mean data. *Climate Dynamics* **48**, 1–18 (2017).
- [103] Bianucci, M. & Mannella, R. On the determination of the optimal parameters in the CAM model. *Chaos: An Interdisciplinary Journal of Nonlinear Science* **31**, 033113 (2021).

- [104] Sardeshmukh, P. D., Compo, G. P. & Penland, C. Need for Caution in Interpreting Extreme Weather Statistics. *Journal of Climate* **28**, 9166–9187 (2015).
- [105] Bianucci, M. Analytical probability density function for the statistics of the ENSO phenomenon: Asymmetry and power law tail. *Geophysical Research Letters* **43**, 386–394 (2016).
- [106] Smith, P. J. A Recursive Formulation of the Old Problem of Obtaining Moments from Cumulants and Vice Versa. *The American Statistician* **49**, 217–218 (1995).
- [107] Matis, T. I. & Guardiola, I. G. Achieving moment closure through cumulant neglect. *The Mathematica Journal* **12** (2010).
- [108] Gottwald, G. A., Tziperman, E. & Fedorov, A. An improved noise model for representing westerly wind bursts in the recharge oscillator model of ENSO. *Nonlinear Processes in Geophysics* **33**, 373–383 (2026).
- [109] Geng, L. & Jin, F.-F. Insights into ENSO Diversity from an Intermediate Coupled Model. Part II: Role of Nonlinear Dynamics and Stochastic Forcing. *Journal of Climate* **36**, 7527–7547 (2023).
- [110] Papanicolaou, G. C. & Kohler, W. Asymptotic theory of mixing stochastic ordinary differential equations. *Communications on Pure and Applied Mathematics* **27**, 641–668 (1974).
- [111] Bellenger, H., Guilyardi, E., Leloup, J., Lengaigne, M. & Vialard, J. ENSO representation in climate models: From CMIP3 to CMIP5. *Climate Dynamics* **42**, 1999–2018 (2014).
- [112] Planton, Y. Y. *et al.* Evaluating Climate Models with the CLIVAR 2020 ENSO Metrics Package. *Bulletin of the American Meteorological Society* **102**, E193–E217 (2021).
- [113] Weeks, E. & Tziperman, E. Challenges in Determining Whether ENSO Is a Damped or a Self-Sustained Oscillation. *Geophysical Research Letters* **52**, e2025GL116328 (2025).
- [114] Larson, S. M. & Kirtman, B. P. The Pacific Meridional Mode as an ENSO Precursor and Predictor in the North American Multimodel Ensemble. *Journal of Climate* **27**, 7018–7032 (2014).
- [115] Martinez-Villalobos, C. & Vimont, D. J. The Role of the Mean State in Meridional Mode Structure and Growth. *Journal of Climate* **29**, 3907–3921 (2016).

[116] Thomas, E. E. & Vimont, D. J. Modeling the Mechanisms of Linear and Nonlinear ENSO Responses to the Pacific Meridional Mode. *Journal of Climate* **29**, 8745–8761 (2016).

Acknowledgments. This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Biological and Environmental Research, under Award Number DE-SC0025595. This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof. This is IPRC publication X and SOEST contribution Y.

Declarations

Funding. CM acknowledges support from Proyecto ANID Fondecyt Iniciación código 11250471. BD acknowledges support from ANID through Concurso de Fortalecimiento al Desarrollo Científico de Centros Regionales 2020-R20F0008-CEAZA and Centro de Investigación Oceanográfica en el Pacífico Sur-Oriental COPAS Coastal FB210021. MFS acknowledges support from the U.S. Department of Energy, Office of Science, Office of Biological and Environmental Research, under Award Number DE-SC0025595.

Competing interests. The authors have no relevant financial or non-financial interests to disclose.

Author contributions. Cristian Martinez-Villalobos conceived the study, developed the methodology, performed the analyses, and wrote the original draft. Boris Dewitte, Soong-Ki Kim, and Malte F. Stuecker contributed to the conceptual development of the study, interpretation of the results, and revision of the manuscript. All authors discussed the results and approved the final manuscript.

Data availability. All datasets analyzed in this study are publicly available. Daily NOAA Optimum Interpolation Sea Surface Temperature (OISST) data are available from the NOAA Physical Sciences Laboratory at <https://www.psl.noaa.gov/data/gridded/data.noaa.oisst.v2.highres.html>. NCEP/NCAR Reanalysis 1 data are available from the NOAA Physical Sciences Laboratory at <https://psl.noaa.gov/data/reanalysis/reanalysis.shtml>. The GLORYS2V4 ocean reanalysis is available through the Copernicus

Marine Service at <https://doi.org/10.48670/moi-00024>. CMIP6 model output is available through the Earth System Grid Federation at <https://aims2.llnl.gov/search/cmip6/>. No new observational or climate-model datasets were generated in this study.

Use of AI. During manuscript preparation, ChatGPT (OpenAI) was used as an assistive tool to help organize, draft, and refine manuscript text based on author-provided scientific content; to support reference and internal-consistency checks; to verify algebra and assist with the completion and presentation of selected derivational steps in the Supplementary Information, based on author-developed equations, analytical approaches, and pre-existing calculations; to assist with coding and debugging for selected analysis and visualization tasks; and to generate the schematic shown in Fig. 1 based on an author-developed conceptual design, which was subsequently edited by the authors. All AI-assisted outputs were reviewed, verified, and edited by the authors before inclusion. The scientific questions, model design, mathematical framework, underlying equations and assumptions, analysis choices, interpretation of the results, and conclusions remained under the authors' responsibility.

Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- [20260918SI.pdf](#)