

Supplementary Information

Risk-aware and trackability-constrained trajectory planning for autonomous vehicles in unstructured road environments

Xuan Yang, Enjun Bai*, Jingsong Liu, Maochen Ge, Yongwang Chai, Shuang Chen, Wenyu Ma
Shenyang Ligong University, Shenyang, Liaoning, China

Supplementary Table S1. Effect of trackability-constrained B-spline optimization.

Metric	Before optimization	After optimization
Maximum curvature (m^{-1})	0.279587	0.207353
Minimum clearance (m)	1.153256	1.292732
Path length (m)	158.556	161.040
Tracking curvature constraint satisfied	No	Yes

Supplementary Table S2. Dynamic-obstacle handling strategies.

Obstacle	Behaviour	Conflict type	Selected strategy	Outcome
O1	Moving crossing obstacle	Transient	S-T deceleration	Safe
O2	Faster crossing obstacle	Transient	S-T acceleration	Safe
O3	Moves and then remains stationary	Persistent spatial blockage	Hybrid A* + S-T replanning	Safe

Supplementary Table S3. Closed-loop tracking performance.

Metric	Result
Maximum lateral error, normal tracking	0.016 m
Maximum heading error, normal tracking	1.29°
Maximum lateral error, full trajectory	0.0678 m
Maximum heading error, full trajectory	2.470°
Terminal position error	0.1117 m
Steering saturation	No

The main tracking interval is approximately 0-125 s; full-trajectory metrics include the low-speed terminal motion.

Supplementary Table S4. Start-goal configurations used in the multi-scenario evaluation.

Scenario	Start grid [r,c]	Goal grid [r,c]	Start (x,y) (m)	Goal (x,y) (m)
S1	[24,16]	[82,4]	(8.0, 12.0)	(2.0, 41.0)
S2	[25,104]	[99,162]	(52.0, 12.5)	(81.0, 49.5)
S3	[40,83]	[68,16]	(41.5, 20.0)	(8.0, 34.0)
S4	[23,178]	[88,160]	(89.0, 11.5)	(80.0, 44.0)

Supplementary Table S5. Numerical configurations of the dynamic obstacles.

Obstacle	Initial [y,x] (m)	Size [y,x] (m)	Velocity [y,x] (m/s)	Time schedule (s)	Behaviour
O1	[18.0, 48.0]	[1.8, 1.2]	[0.35, 0.00]	appear 22; move 22–38; disappear 48	transient / slowdown
O2	[18.5, 74.5]	[1.4, 0.8]	[0.43, 0.00]	appear/move 54–78; disappear 78	transient / speed-up
O3	[45.0, 12.0]	[4.0, 2.8]	[-0.50, 0.00]	appear/move 90–98; persistent after stop	persistent / Hybrid A*

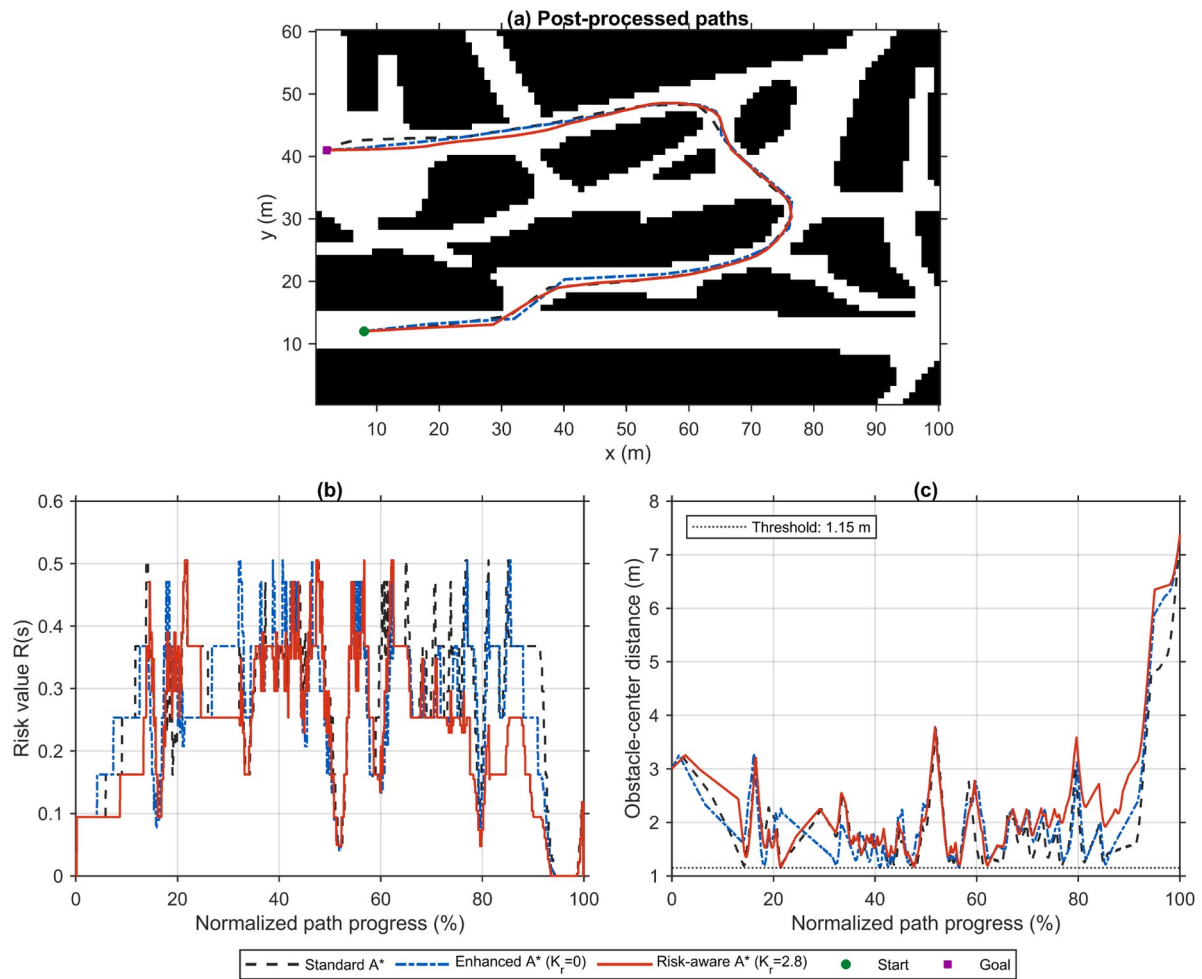
Dynamic-obstacle positions and sizes use the [y,x] convention, consistent with the [row,column] directions of the planning grid.

Supplementary Table S6. Main vehicle-model and MPC parameters for closed-loop tracking.

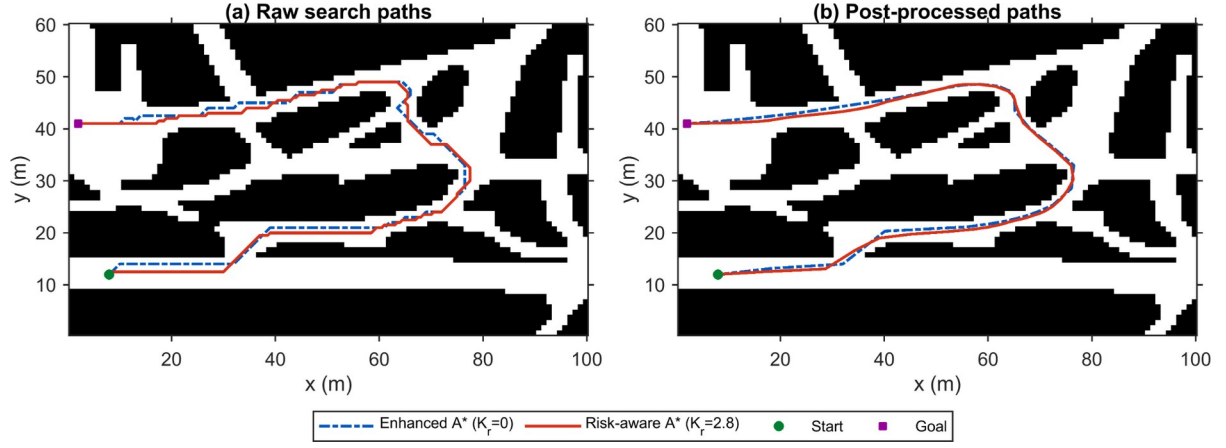
Parameter	Symbol	Value
Sampling interval	T_s	0.05 s
Prediction horizon	N_p	30
Vehicle mass	m	1412 kg
Yaw moment of inertia	I_z	1536.7 kg·m ²
CG-to-front-axle distance	l_f	0.9 m
CG-to-rear-axle distance	l_r	1.1 m
Front cornering stiffness	C_f	125400 N/rad
Rear cornering stiffness	C_r	133800 N/rad
Lateral-error weight	Q_y	120
Heading-error weight	Q_ψ	220
Lateral-velocity weight	Q_{vy}	0.05
Yaw-rate weight	Q_r	8.0
Steering-correction penalty	R_{corr}	3.5
Steering-increment penalty	$R_{\Delta\delta}$	2.5
Steering-increment limit	$\Delta\delta_{max}$	3.5°/sample
Steering-correction limit	$\delta_{corr,max}$	7.0°

Supplementary Figures

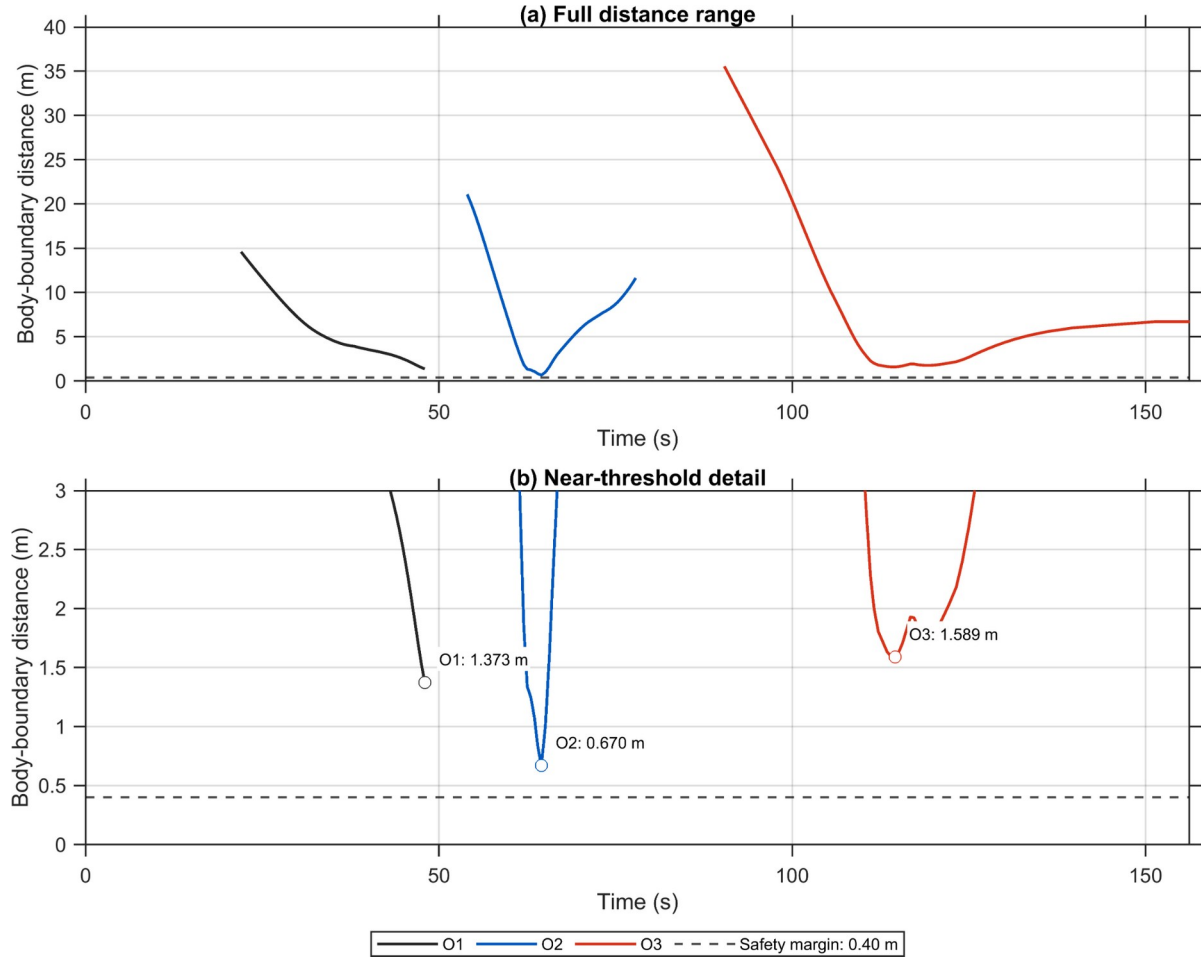
Supplementary Figure S1. Representative S1 path and profile comparison. a, Post-processed paths for standard A*, enhanced A* without risk ($K_r = 0$), and risk-aware A* ($K_r = 2.8$). b, Pathwise risk value $R(s)$ as a function of normalized path progress. c, Obstacle-centre distance along the path; the dotted line marks the 1.15 m hard threshold.



Supplementary Figure S2. Risk-term ablation under identical search and post-processing settings. a, Raw search paths. b, Post-processed paths. Enhanced A* without risk ($K_r = 0$) and risk-aware A* ($K_r = 2.8$) share the same turning penalty, adaptive heuristic and post-processing procedure, so the comparison isolates the environmental-risk term.



Supplementary Figure S3. Sampled dynamic-obstacle safety check. a, Body-boundary distance to O1-O3 over the sampled trajectory intervals. b, Enlarged near-threshold view. The sampled minima are 1.373 m, 0.670 m and 1.589 m for O1, O2 and O3, respectively, all above the 0.40 m body-boundary safety margin. The figure represents a discrete sampling check rather than a continuous-time proof.



Supplementary Figure S4. Control-reference lateral acceleration along the final planned trajectory. The dashed horizontal lines denote the nominal $\pm 0.50 \text{ m/s}^2$ lateral-acceleration limits used for the reference-level feasibility check. The planning speed envelope uses the more conservative value described in Methods.

