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PoincAÍré: Classification Learning of Spin-Orbit Dynamical Structures via Poincaré Surface of Section

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Abstract

Poincaré surfaces of section (PSS) are a well-established method for visualizing the structure and behavior of trajectories in dynamical systems. By recording the intersections of trajectories with two-dimensional cross sections in phase space, they enable the identification of periodic, quasi-periodic, and chaotic motion. However, their construction and analysis generally require the numerical integration of a large number of trajectories over sufficiently long timescales, making this approach computationally demanding and potentially time-consuming, particularly when exploring extensive parameter spaces. PoincAÍré is an ensemble of open-source machine learning models developed to accelerate the analysis of the spin-orbit motion of an elongated satellite under the gravitational influence of a spherical central body using the PSS approach. The first model employs Convolutional Neural Network (CNN) and Vision Transformer (ViT) architectures trained to automatically classify three distinct dynamical regimes from PSS images: resonance, circulation, and chaos, achieving an accuracy of $\sim 99\%$. The second model is based on the Extreme Gradient Boosting (XGBoost) algorithm and uses the system's initial conditions as input to predict the probability of the trajectory belonging to each dynamical class. This approach significantly speeds up parameter-space exploration, enabling high-resolution dynamical maps to be

generated without explicitly integrating every trajectory. This novel methodology can classify 10^5 different initial conditions in a matter of seconds, with an accuracy of $\sim 97\%$.

Keywords: Spin-orbit resonance, Poincaré surface of section, Machine learning, Dynamical maps

1 Introduction

Poincaré surfaces of section (or just Poincaré maps) are usually adopted for studies of two-dimension systems, where they produce plots that make it easier to visualize dynamical characteristics of the trajectories, such as periodic, quasi-periodic or chaotic behavior. This technique is commonly used in problems of celestial mechanics and many other physical and mathematical areas. Though effective, the identification and classification of the trajectories are made visually, one by one, and are computationally expensive when requiring exhaustive parameter space exploration to establish classification thresholds. Therefore, in recent years, approaches that use different types of artificial intelligence have been proposed to quickly and automatically generate Poincaré maps, classify trajectories, and/or extract system features from them.

In the current work, we present a numerical package based on machine learning techniques to facilitate studies of Poincaré maps of a spin-orbit dynamical system, which describes the rotational dynamics of a satellite moving around a central body and whose center of mass orbits on a Keplerian ellipse. Following we present brief descriptions on a list of papers that are relevant to our work. One of the first interesting works on this line was made by Yip in the early 1990s [1]. In order to classify trajectories from Poincaré maps, he used rules based on features obtained from a study where an orbit was associated to a minimal spanning tree, divided into subgraphs.

The planar circular restricted three-body problem (PCR3BP) is a classical example, where each Poincaré section corresponds to one energy level (Jacobi constant). Even atlases of large sets of these plots for the Earth-Moon [2] and Jupiter-Sun [3, 4] mass ratios have been made in the past. Higher dimensional systems introduce difficulties in the analysis of the dynamical characteristics of the trajectories. Borders-Motta and Winter [5] studied Poincaré maps around a 3D irregular asteroid. They were able to map stable and chaotic regions finding correlations between those regions and the contribution of the third dimension of the system to the trajectory dynamics. Bonasera and Bosanac [6] used data mining techniques to autonomously group higher-dimensional crossings on a Poincaré periapsis map that captures spatial trajectories at a single energy level in the Sun-Earth circular restricted three-body problem. Once made the classification of the trajectories, according to their characteristics, for each initial condition of a set of Poincaré maps, it can be produced a dynamical stability map. The dynamical map is a plot showing the characteristics of the trajectories in a range of initial conditions. In the case of orbital dynamics around irregular bodies, it is usually made as a plot of initial semi-major axis (a) versus initial eccentricity

(e), indicating for which values the trajectories are regular/stable (periodic or quasi-periodic orbit) and which are chaotic and/or unstable (occurs collision or escape from the nearby region) [[7],[8],[9]].

Celletti and collaborators [10] studied capabilities of deep learning in the classification of regular and chaotic motions in Hamiltonian systems. Taking into account time series data, they used convolutional neural networks to classify types of motion. The spin-orbit problem was considered as a case study. They also showed the ability to predict types of motion from observations, adopting InceptionTime deep convolutional neural networks to make cartographic study of analogous albeit not identical dynamical systems. In order to identify chaotic behavior on the dynamics of small bodies in the Solar system, Carruba and collaborators [11] proposed the use of an approach based on the autocorrelation function of time series. This allowed them to infer unpredictable time series. They tested the approach for Poincaré maps of the Hénon-Heiles system, but did not get very sharp results. Jiménez-López and Garcia-Garrido [12] investigated the use of support vector machines to classify regular and chaotic trajectories in symplectic maps and Hamiltonian systems through Lagrangian descriptors. Their models achieved very good performance in the classification of trajectories for several dynamical systems, including the Hénon-Heiles system ($> 90\%$).

In devices built to contain hot plasmas it is important to understand the confinement properties. A useful tool for such studies is the Poincaré map obtained through numerical integration of a magnetic field line. Burby, Tang and Maulik [13] presented a neural network architecture that learned from actual Poincaré maps, conserving the magnetic flux (symplectic condition) and generating accurate approximate evaluations of those. Also, having the goal of analyzing the magnetic field topology derived from numerical tokamak simulations, Kamath [14] utilized a machine learning approach. His research detailed addressed two primary challenges: (i) the generation of a high-quality training dataset containing a minimal number of mislabeled orbits, and (ii) the transformation of the Poincaré section coordinates into features that are discriminating, despite the similarity among orbits belonging to distinct classes. Ruth and Bindel [15] proposed a kernel-based method to learn a single labeling function that is approximately invariant under the symplectic map.

The present study introduce a novel machine learning methodology for generating high-resolution dynamical maps directly from the system's initial conditions, without the need to explicitly integrate each trajectory. This method enables the classification of 10^5 distinct initial conditions within a few seconds, achieving an accuracy of approximately 97%.

This paper is structured as follows. Section 2 introduces the dynamical system investigated in this work. Section 3 presents the key concepts underlying the Machine learning (ML) algorithms employed. Section 4 describes the methodology and evaluates the performance of the ML algorithms. Section 5 show an analysis of the structures observed in the Poincaré maps obtained from our simulations. Finally, Section 6 presents the concluding remarks.

2 Dynamical System

In this section, we present the mathematical formulation used to understand the dynamics of an elongated satellite orbiting a massive central body. We present the equations describing the angular motion of an elongated body around a central body, as well as the Poincaré Surface of Section method. The methodology used in this section is the same as that adopted in [16].

The physical problem of interest in this article concerns the orbital motion of an elongated artificial satellite with planar rotation under the gravitational attraction of a central massive body. The satellite's orbit has an eccentricity e , which will vary over a range of values. We adopt dimensionless units in which the semi-major axis and mean motion are set to $a = 1$ and $n = 1$, respectively; consequently, the orbital period is $T = 2\pi$. In Figure 1, the variable f is the true anomaly, r is the distance between the central massive body, at point P and the satellite, and γ is the angle between r and the longest main axes of moment of inertia of satellite, which connects the satellite to the point A on the reference line. The variable θ is the one that gives the angular orientation of the satellite in the orbit, and is given by $\theta = f - \gamma$.

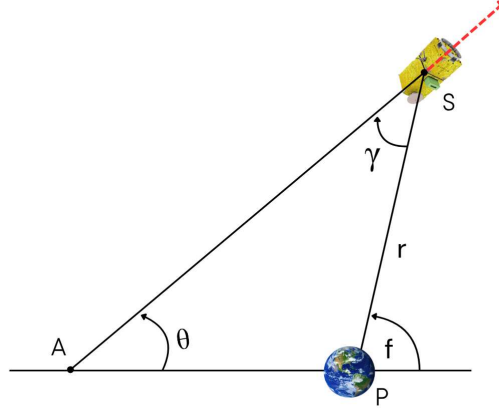


Fig. 1: Representation of an artificial satellite (S) orbiting around a central massive body (P).

Danby [17], and Goldreich and Peale [18] showed a differential equation that models the behavior of the angle θ in cases where there is no influence of tidal torques, given by

$$\ddot{\theta} + \frac{\omega^2}{2r^3} \sin[2(\theta - f)] = 0, \quad (1)$$

where the parameter ω represents the asphericity of the body, which depends on the geometry of the body of interest, that is, the principal moments of inertia. The value of ω is given by

$$\omega^2 = 3 \frac{(B - A)}{C}, \quad (2)$$

in which A , B , and C are the principal moments of inertial and obey $A < B < C$. To complete the mathematical analysis and simulations, we use the following equations to obtain the values of r and f

$$r = \frac{1 - e^2}{1 + e \cos(f)}, \quad (3)$$

and

$$\dot{f} = \frac{(1 + e \cos(f))^2}{(1 - e^2)^{3/2}}. \quad (4)$$

2.1 Poincaré Surface of Section

The Poincaré Surface of Section is a fundamental tool in the qualitative analysis of dynamical systems, particularly in problems involving chaotic behavior. In this method, the continuous evolution of a trajectory in phase space is replaced by a set of points obtained from intersections of the orbit with a surface transverse to the flow [19]. With this method, it is possible to convert a continuous flux in phase space into a discrete map. Consider the flow that models the physical problem as follows

$$\dot{\theta} = f(\mathbf{x}), \quad (5)$$

in which $\mathbf{x} = \mathbf{x}(t)$ represents the coordinates of the n -dimensional system. Figure 2, shows a scheme of Poincaré surface of section. The surface Σ is the plane that intersects the flux f in a determined region of phase space. The points x_k , x_{k+1} , and x_{k+2} represent the $k + 2$ intersections of the flow f in the surface Σ . The collection of points generated by this intersection shows the behavior of the dynamical system.

Figure 3 presents the results of the Poincaré method for a system with $\omega = 0.3$, while the satellite eccentricity e varies from 0 to 0.2. The initial condition for θ is fixed at 0° , whereas $\dot{\theta}$ ranges from -1 to 4 with a step size of $\Delta\dot{\theta} \sim 0.05$. The phase-space section in the $(\theta, \dot{\theta})$ plane is constructed by recording the state of the system at each perigee passage of the satellite. There are three easily identifiable regions in the section. The first is in green, consisting of points spaced in the way that does not form any type of pattern; this region is known as sea of chaos, and becomes denser as the perturbations in the system increases. The second region, represented in blue, shows the resonance islands, and the crossings that form these islands represents quasi-periodic trajectories. The third region, shown in red, represents circulation.

The Poincaré surface of section can pose a significant challenge for curve classification task. As illustrated in Figure 4, the separatrix region is particularly difficult to classify because it often exhibits characteristics of multiple dynamical regimes. In addition, some resonant and circulation curves may be misclassified because small

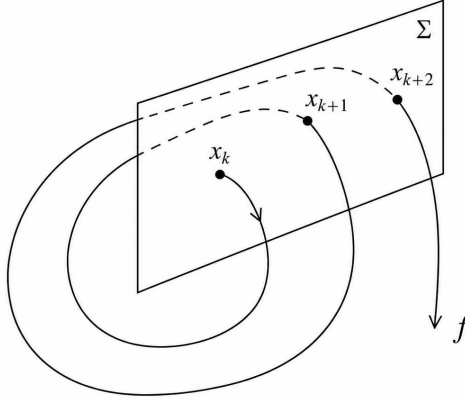


Fig. 2: A schematic representation of Poincaré Surface of Section.

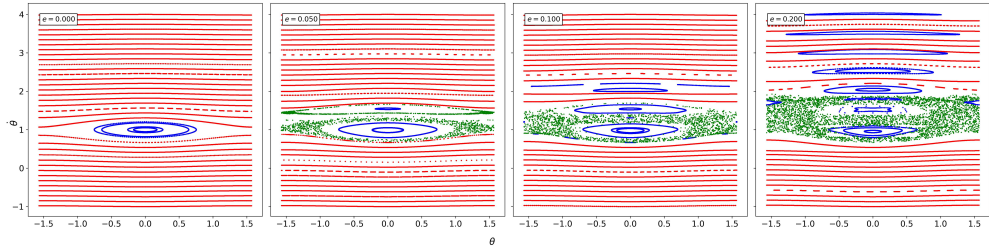


Fig. 3: Poincaré surface of section for four different orbital eccentricities, between 0 to 0.20, and with $\omega = 0.3$. The colors indicate the three dynamical classes as resonance (blue), circulation (red), and chaos (green).

resonant islands can resemble circulation patterns when they are represented by an insufficient number of points, as shown in the second and fourth examples of Figure 4.

Manually classifying the Poincaré surfaces of section becomes an extremely time-consuming and challenging task. To address this, one solution is to create a training set to extract features from each curve and enable automatic classification via machine learning, thereby reducing manual effort and the time spent on this task. However, it is important to note the potential challenges arising from the non-obvious classification of some curves, as previously discussed. In the following section, we describe the machine learning methods applied to classify the Poincaré surfaces of sections.

3 Machine Learning

This section provides an overview of the ML methods employed in this study, introducing the fundamental concepts required to understand their application and performance. These concepts will be revisited throughout the following sections, particularly in the analysis and discussion of the results. In the last decade, ML techniques

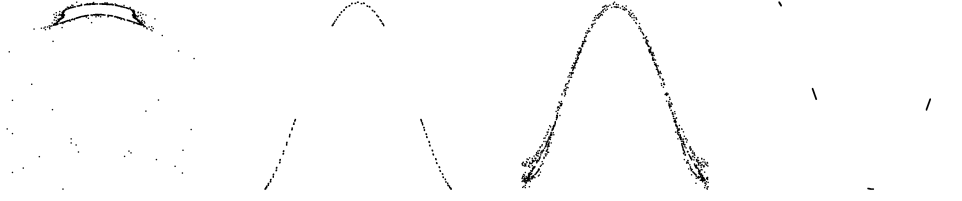


Fig. 4: Examples of possible misclassifications of Poincaré surface of sections.

have been increasingly applied to image classification in several areas of astronomy, including galaxy classification ([20],[21]) and the identification of asteroids in resonances [22]. Among the most prominent algorithms in computer vision are Convolutional Neural Networks (CNNs) and Vision Transformers (ViTs), both of which are described in the following sections. In addition, Extreme Gradient Boosting (XGBoost) is introduced as a complementary machine learning method for further analysis of the database generated from the image classification process, together with the evaluation metrics used to assess the performance of the different models.

3.1 Convolutional Neural Network

CNNs introduced by [23] are a type of neural networks. Neural networks are computational models inspired by the structure of the human brain. They consist of interconnected computational units, called neurons, which process and transmit information through weighted connections. CNNs are designed to recognize spatial patterns in images. A CNN is composed by three main layers: convolutional, pooling, and fully-connected layers [24]. The convolutional layer is responsible for automatically extracting relevant features from the input data by applying a set of learnable convolution kernels (or filters). Each kernel will determine the output of neurons of which are connected to local regions of the input through the calculation of the scalar product between their weights and the region connected to the input [25]. The result of a convolution is a feature map that highlights specific patterns such as edges, textures, or shapes. After the convolution operation, a nonlinear activation function, such as the Rectified Linear Unit (ReLU), is applied element-wise to introduce nonlinearity into the model. The pooling layer reduces the spatial dimensions of the input, decreasing the number of parameters while retaining the most relevant features. After several convolutional and pooling layers, one or more fully-connected layers integrate the extracted features by connecting all neurons in the previous layer to every single neuron of current layer to generate global semantic information. Finally, for classification tasks, the output layer uses these features to classify the input and produce the final prediction.

3.2 Vision Transformer

In the field of computer vision, the most common models are CNNs; recently, new architecture was developed by researchers from Google Research and the University of Toronto [26] with the Vision Transformer (ViT). The concept of a transformer

appears in Natural Language Processing (NLP) with self-attention-based architecture, a mechanism relating different positions of a single sequence to compute a representation of the sequence. The ViT applies the Transformer architecture in the image data, the basic idea is split the image in small parts with fixed sizes named patches. After the split, the data pass to the Transformer encoder block, where occurs the self-attention process and a feed-forward network is applied, which make transformations to the data independently for each position in the sequence. Finally the output passed through a fully connected layer with a softmax activation to obtain the predicted class probabilities.

3.3 XGBoost

XGBoost, introduced by [27], is an ensemble learning method based on the Gradient-Boosted Tree (GBT) proposed by [28]. The algorithm iteratively improves the model by computing residuals between observed values and predictions from previous trees. These residuals are then used to optimize subsequent trees, progressively reducing prediction error and enhancing the overall model performance. In contrast to traditional Gradient-Boosted Trees, which are constructed sequentially, XGBoost employs optimized parallelization strategies by partitioning the dataset into subsets during the tree-building process.

3.4 Performance Evaluation

To evaluate the performance of the machine learning models, metrics derived from the confusion matrix are employed. The confusion matrix indicates which classes were correctly classified and which were misclassified during the testing stage. As illustrated in Figure 5, correctly classified samples correspond to the True Positive (TP) and True Negative (TN) categories, while misclassified samples correspond to the False Positive (FP) and False Negative (FN) categories. Based on this, four main evaluation metrics can be defined: accuracy, recall, precision, and F1-score.

		TRUE CLASS	
		POSITIVE	NEGATIVE
PREDICTED CLASS	POSITIVE	TP	FP
	NEGATIVE	FN	TN

Fig. 5: Confusion matrix representation.

Accuracy measures the proportion of correctly classified instances relative to the total number of evaluated samples, while recall and precision represent the proportion of correctly identified positive instances with respect to the total number of actual positive instances and the total number of predicted positive instances, respectively. Finally, the F1-score combines precision and recall by calculating their harmonic mean:

$$\begin{aligned} \text{Accuracy} &= \frac{TP + TN}{TP + TN + FP + FN} & \text{Recall} &= \frac{TP}{TP + FN} \\ \text{Precision} &= \frac{TP}{TP + FP} & \text{F1-score} &= 2 \left(\frac{\text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}} \right) \end{aligned} \quad (6)$$

4 Implementation of the Classification Models

As described in Section 3, we implemented two machine learning architectures, CNN and ViT, to classify the Poincaré surface of section images. The XGBoost algorithm subsequently uses the dataset generated by the image classification models to train a machine learning model capable of predicting the dynamical class of each trajectory from the system’s initial conditions. In this section, we detail the training parameters adopted for each model and evaluate their predictive performance.

The training dataset for our machine learning models is generated by first step integrating Poincaré Surface of Section using the equations of motion presented in Section 2 for systems with $\omega = [0.3, 1.7]$, $\dot{\theta} = [-1, 3]$, $e = [0, 0.2]$ and $\theta = 0^\circ$.

In total, 4,200 different initial conditions are explored and classified into three classes: resonant, circulation, and chaotic as shown in the first step of the flowchart (Figure 6). The dataset is split into 80% of the images for training and validation, while the remaining 20% is reserved for testing. We employ two different machine learning approaches. The first model is based on a Convolutional Neural Network (CNN) using the MobileNetV2 architecture with transfer learning, while the second model use ViT-Tiny Patch architecture pre-trained on ImageNet and fine-tuned on the target dataset.

For the CNN models, the input images are resized to 128×128 pixels, and pixel values are normalized to the range $[0, 1]$. Training is performed using a batch size of 16 and 25 epochs, meaning that the model processes 16 samples at a time before updating its weights through backpropagation, and the entire dataset is passed through the network 25 times during training.

In the ViT approach, the images are resized to 224×224 pixels, the model was trained with a batch size of 32 and 40 epochs. In addition, a patch size of 16 was used, such that each input image was decomposed into 16×16 patches during the training process.

To improve generalization and reduce overfitting for both models, data augmentation techniques are applied during training. These include random horizontal flips, small rotations (up to 10°), and zoom variations of up to 10%.

Once the CNN and ViT models had been trained, and saved, they were the best-performing image classification model was selected and used to classify new sets of

randomly generated curves corresponding to different values of ω , e and $\dot{\theta}$ using approximately 300,000 initial conditions. The resulting classifications were stored in a new data frame containing the initial parameters, together with the probabilities associated with the chaotic, circulation, and resonant classes.

This dataset was then used to train an XGBoost model capable of predicting the probability of each condition belonging to one of the three classes. The trained model was subsequently used to generate the dynamic diagram presented as the final stage of the flowchart in Figure 6. In the following two sections, we present the performance of the CNN, ViT, and XGBoost models.

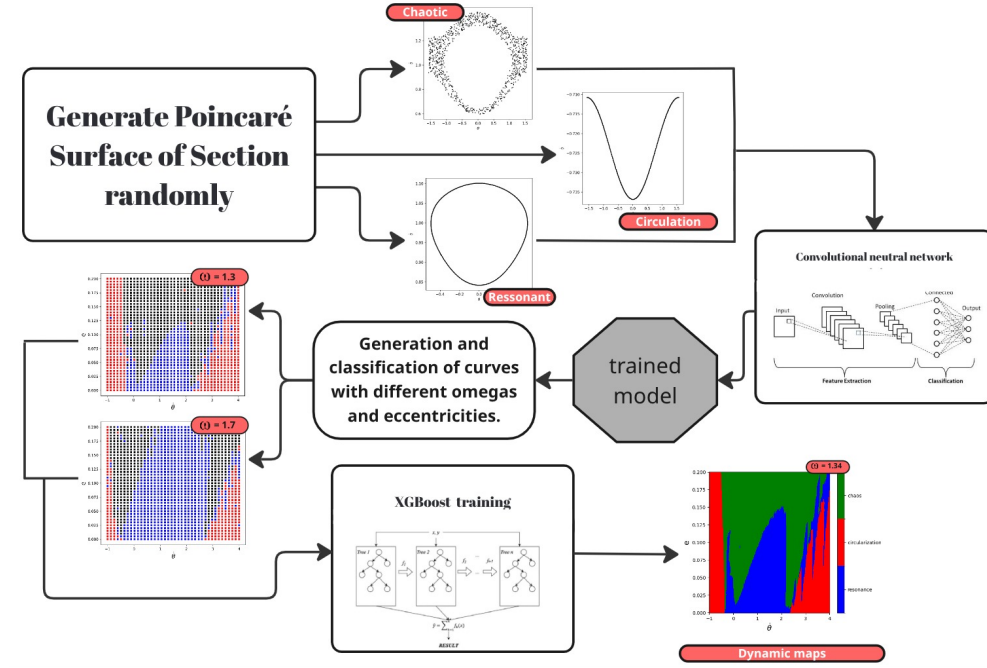


Fig. 6: Flowchart of the methodology stages.

4.1 Image Classifier

The models achieved high overall accuracies, reaching 0.986 for the CNN and 0.99 for the ViT model (Table 1). When the performance of each class is analyzed separately, both models continue to show excellent results, with the lowest metric value being a precision of 0.96 for the resonance class in the ViT model. Regarding precision, both models correctly classified 100% of the samples identified as chaotic conditions, while the ViT model also achieved 100% precision for the circulation class. For the resonant class, the models showed similar performances, with precision values of 0.98 for the CNN and 0.96 for the ViT model.

In terms of recall, the ViT model correctly identified 98% of the initial conditions belonging to the chaotic and circulation classes, and 100% of the samples associated with the resonant class. For the CNN model, the highest recall was obtained for the circulation class, correctly identifying all the samples in this class and 98% for the other two classes. The F1-score metric, both models achieved values between 0.98 and 0.99.

Metric	Chaos		Resonance		circulation	
	CNN	ViT	CNN	ViT	CNN	ViT
Precision	1.00	1.00	0.98	0.96	0.98	1.00
Recall	0.97	0.98	0.98	1.00	1.00	0.98
F1-score	0.98	0.99	0.98	0.98	0.99	0.99
Accuracy CNN = 0.986			Accuracy ViT = 0.990			

Table 1: Performance comparison between the CNN and ViT models.

Figure 7 shows the learning curves (loss versus epochs) for both models, where the loss represents the difference between the predicted class probabilities and the true class labels during training.

In each plot, the blue curve is the training loss, while the orange curve corresponds to the validation loss. As the number of epochs increases, the loss progressively decreases until reaching a stable value, indicating the convergence of the model. A large separation between the training and validation curves is generally associated with overfitting, a situation in which the model learns the training data too specifically and loses its ability to generalize well to unseen data.

A comparison of the model performances indicates a slight overfitting in the ViT model. This behavior can be observed from the larger gap between the training and validation loss curves when compared to the CNN model. For the CNN, the difference between the curves remains < 0.01 , whereas for the ViT model it reaches ~ 0.11 .

Despite both models showing very similar performance, the slight overfitting observed in the ViT model led us to select the CNN model as the best-performing approach. Hereafter, all subsequent analyses and results will be obtained using this ML model.

The mosaic in Figure 8 shows the performance metrics of the CNN model for PSS maps containing different numbers of plot points. Although all machine learning models were trained and tested using PSS maps with 1,000 plot points, we additionally evaluated the performance of the best model using maps with varying quantity of points.

For this analysis, we used the same test dataset and progressively reduced the number of plot points for each initial condition from 1,000 to 100, in steps of 100 points.

Overall, the performance metrics shown in Figure 8 indicate that a PSS curve sampled with approximately 500 data points provides sufficient resolution for the CNN model to achieve high classification performance, with all evaluation metrics

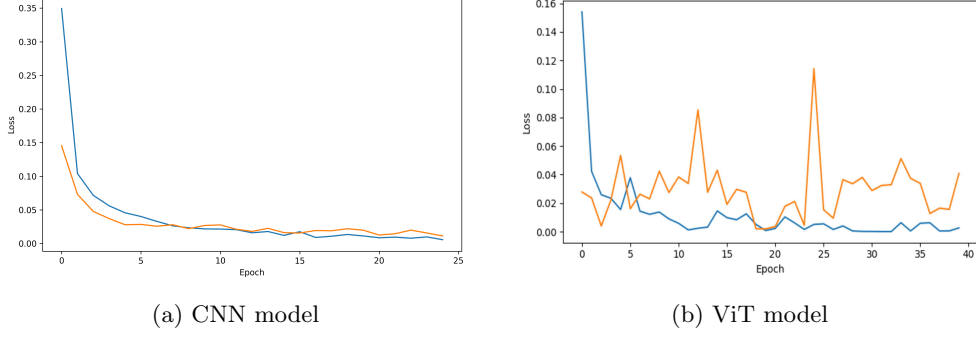


Fig. 7: Learning curves for both models, showing the training loss (blue) and validation loss (orange) during the training process. The CNN model achieved a better fit than the ViT model because it presented a smaller gap between the training and validation losses, indicating better generalization.

exceeding 95%. Beyond this resolution only marginal improvements are obtained by further increasing the number of sampled points, previous paper has shown that good classification performance can be achieved using PSS maps sampled with 1,000 data points [14]. Even when the PSS curves are sampled with only 100 data points, the CNN model still correctly classifies more than 85% of the cases, demonstrating that the model remains robust even at relatively low resolutions.

4.2 XGboost Classifier

The second step of this work is to develop a secondary classifier model, aiming to predicted the dynamical system regime: chaotic, circulation, and resonant. To generate the training dataset, we numerically simulated a sample of 292,900 different initial across the following grid: $\omega = [0.3, 1.7]$, with step size $\Delta\omega = 0.05$, $\dot{\theta} = [-1, 3]$ with $\Delta\dot{\theta} = 0.05$, $e = [0, 0.2]$ with $\Delta e = 0.002$ and $\theta = 0^\circ$.

All PSS curves were classified using the CNN model, producing a dataset subsequently used to train an XGBoost model. Table 2 shows the performance of our best-performing model after hyperparameter tuning. Prior to selecting XGBoost, we ran a Genetic Algorithm (an optimization technique inspired by natural selection that iteratively evolves candidate solutions to identify the fittest model) ten times to determine which machine learning algorithm would best address this classification problem. Similar applications of genetic algorithms for model selection can be found in references [29–31].

As shown in Table 2, the XGBoost model achieves a high classification performance, with an overall accuracy of $\sim 97\%$. The resonant and chaotic classes are correctly identified in 98% of the instances, while both classes also present a precision in prediction of 97%. The circulation regime exhibits a slightly lower performance, with a recall value of 93%.

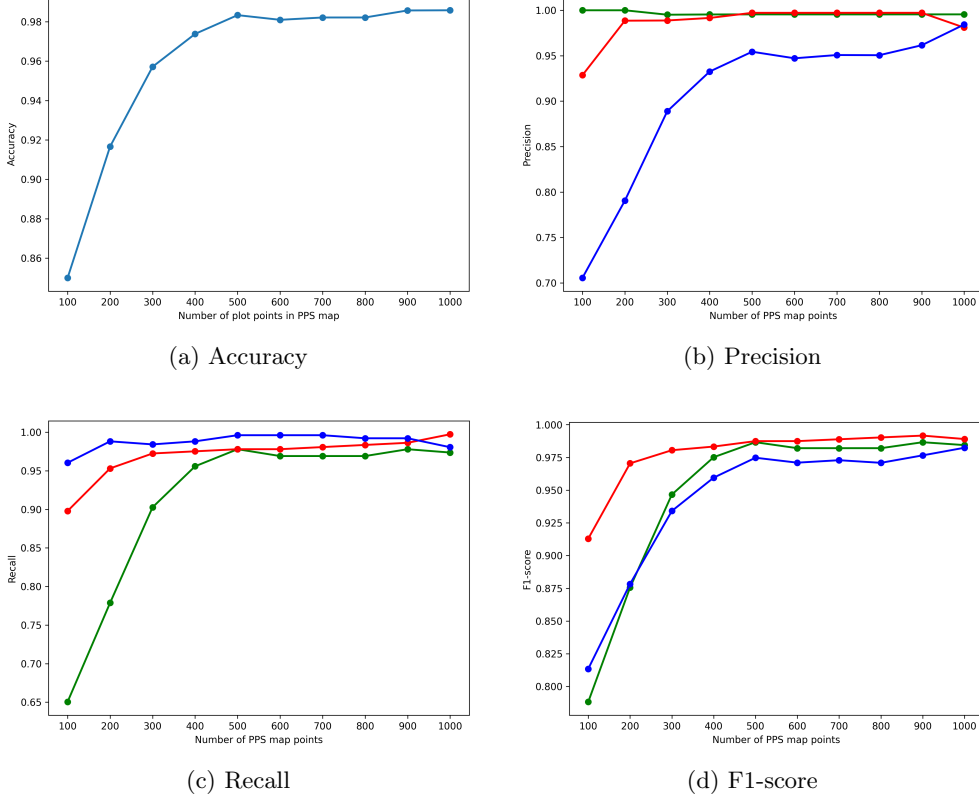


Fig. 8: Evaluation of the CNN model performance as a function of the number of plot points in the PSS maps. The metrics shown are accuracy, precision, recall, and F1-score for the three dynamical classes: resonance (blue), circulation (red), and chaos (green). The results indicate that using approximately 500 plot points is sufficient to achieve stable and high performance ($\gtrsim 0.95$ across all metrics), since beyond this value the improvements become marginal, indicating the convergence of the model performance with respect to map resolution.

The output of the XGBoost algorithm is the probability that a given instance belongs to a specific class, as shown in Figure 9. For each value of the satellite asphericity parameter, the probability that an initial condition belongs to a given class is represented by the color scale. These probability maps are presented for $\theta = 0^\circ$ and $\omega = 0.8$ and 1.34 , corresponding to the same cases analyzed in previous study [16].

The parameter $\omega = 0.8$ represents the lowest value identified for a parallelepiped shaped satellite (see Figure 2 of reference [16]). On the other hand, $\omega = 1.34$ is associated with the highest concentration of known satellites and also represents a triaxial ellipsoid with $a = 2b$, where a and b are the semi-major axes of the ellipsoid [16].

Metric	Chaos	Resonance	Circulation
Precision	0.97	0.97	0.96
Recall	0.98	0.99	0.93
F1-score	0.97	0.98	0.95
Accuracy XGBoost = 0.967			

Table 2: Performance of the XGBoost classifier.

By combining the probability maps shown in Figure 9 and assigning each instance to the class with the highest predicted probability, we obtain the dynamical maps ($e, \dot{\theta}$) of Figure 10. The panels a and c are the resulting dynamical maps for $\omega = 0.8$ and 1.34, respectively, while panels e-h are examples how the dynamical map is built from the information contained in the PSS maps.

Figure 10c shows the case $\omega = 1.34$, where the values of e and $\dot{\theta}$ were sampled on a grid with a very high resolution of 300×300 . Each black horizontal line corresponds to a PSS map, such as those shown in panels e-h, which were computed using only 20 different initial conditions in $\dot{\theta}$. The gray vertical lines in the PSS panels indicate the locations of these initial conditions, all computed with an initial angle of $\theta = 0^\circ$.

Regarding the blue regions in Figure 10, each island is associated with a distinct spin-orbit resonance. Comparing their locations in the $(e, \dot{\theta})$ with the resonance interval presented in Figures 5 and 6 of reference [16] allows the corresponding resonance of each island to be identified.

For both $\omega = 0.8$ and $\omega = 1.34$, each blue island represents a distinct spin-orbit resonance. We identified seven spin-orbit resonances discussed in reference [16], and their evolution across different values of ω will be examined in detail in Section 5. These resonances are the $1:1\alpha$, $1:1\beta$, $3:2$, $2:1$, $5:2$, $3:1$ and $7:2$ resonances and each resonance is represented by a well-defined blue island located in a distinct region of $\dot{\theta}$.

Figures 10b and d present the entropy maps associated with the performance of the predictive machine learning model. In thermodynamics, entropy is a measure of the disorder of a system. In a similar way, for classification problems, machine learning algorithms learn from a labeled dataset and attempt to identify patterns in the distribution of classes within the feature space.

Regions of the feature space dominated by a single class are generally easier to classify, since new instances are likely to belong to that same class. In contrast, regions where different classes are strongly mixed present a higher level of uncertainty, making it more difficult for the algorithm to determine the correct class of a new instance. Entropy provides a quantitative measure of this uncertainty and can therefore be used to assess the predictability of the class distribution in a given region of the feature space.

The normalized entropy is computed as

$$H_{nor} = - \frac{\sum_{i=1}^3 p_i \ln p_i}{\ln 3}, \quad (7)$$

The normalization factor $\ln 3$ ensures that the entropy ranges from 0 to 1, since three distinct classes are considered. Thus, $H_{nor} = 0$ corresponds to a completely

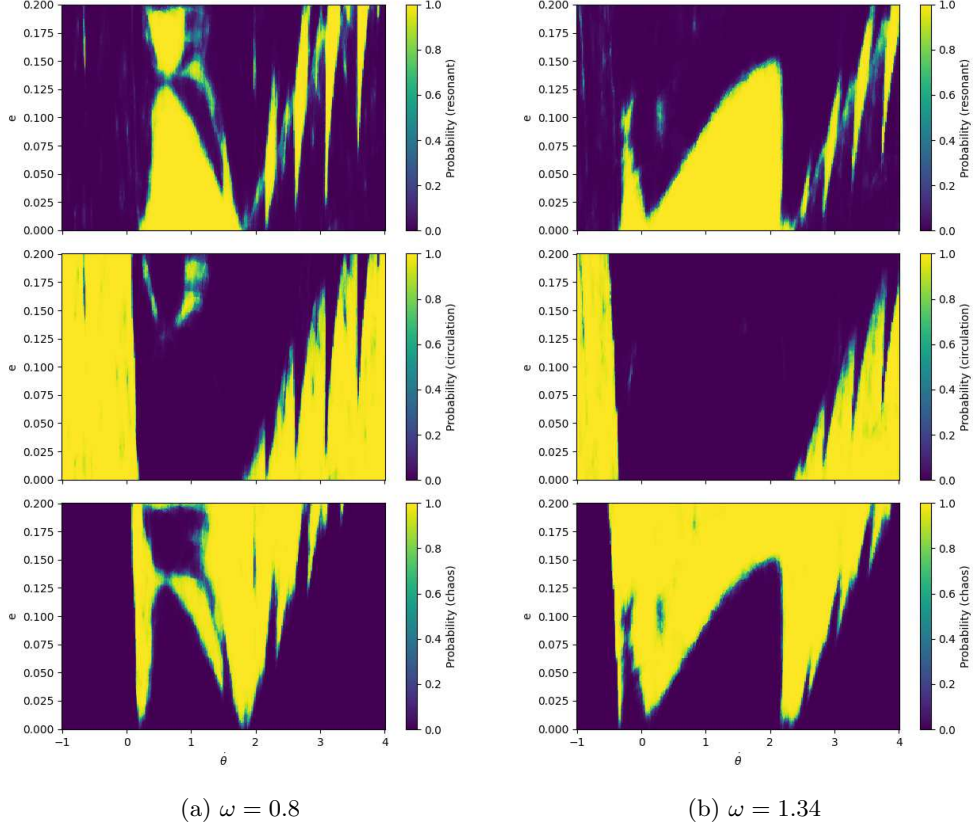


Fig. 9: Probability maps showing the likelihood of belonging to each dynamical class (resonance, circulation, or chaos), as predicted by the XGBoost model, for (a) $\omega = 0.8$ and (b) $\omega = 1.34$.

ordered region containing only a single class, whereas $H_{\text{nor}} = 1$ represents maximum disorder, with the three classes being equally represented.

In Figures 10b and d dark regions correspond to low entropy, indicating that the local parameter space is dominated by a single dynamical class and that the classification is therefore highly predictable. In contrast, reddish and yellowish regions correspond to $H_{\text{nor}} > 0.4$, where different dynamical classes coexist within a small region of the parameter space. These high-entropy regions typically coincide with the separatrices between dynamical classes, where small variations in the initial conditions can lead to qualitatively different dynamical outcomes.

For example, Figure 4 shows cases in which the PSSs simultaneously exhibit resonant and chaotic behavior, making these boundary regions intrinsically difficult to classify. The red boundaries generally delineate the transition between two dynamical classes, whereas the yellow boundaries indicate the intersection of three classes, representing the highest level of local uncertainty.

5 General Structure of the Phase Space

The evolution of the spin-orbit resonances and the overall structure of the phase space were investigated over the interval $0.3 \leq \omega \leq 1.7$ using the trained XGBoost model to generate the corresponding phase-space maps on a fine grid of initial conditions (resolution of 300×300). Figure 11 shows a subset of representative dynamical maps, while the complete sequence of 50 predicted maps is available as an online animation at the following website: <https://lealpinheiro.github.io/PoincAIre>.

In Figure 11, the upper-left panel shows the prediction for the system with $\omega = 0.3$, where the resonant islands were identified and the corresponding resonance types are indicated in the panels. We identified 10 distinct spin-orbit resonances, whose general starting coordinates are indicated in brackets (e, θ) and which extend beyond $e = 0.2$. The identified resonances are: 1:1 α at $(0, 1)$, 3:2 at $(0, 1.5)$, 5:3 at $(0.05819, 1.726)$, 2:1 at $(0.0087, 2.01)$, 4:9 at $(0.08562, 2.278)$, 5:2 at $(0.05284, 2.528)$, 4:11 at $(0.17191, 2.779)$, 3:1 at $(0.10836, 3.03)$, 7:2 at $(0.16455, 3.515)$, and 4:1 at $(0.18127, 4)$. For different values of the asymmetry parameter ω , most of the same resonances appear, but their regions generally shift toward higher θ values.

An analysis of these plots shows that, in general, the space always presents the three classes of trajectories: circulation (red), resonance (blue) and chaos (green). For very low values of $\omega = [0.03-0.04]$ the space is dominated by circulations. the resonances are located close to their expected θ value (for example, the 2:1 resonance appears for $\theta = 2$ and the 3:2 resonance appears for $\theta = 1.5$), independently of the eccentricity value (vertical blue stripes). The chaotic trajectories occupy a very small portion of the space, being located between strong resonances, such as the 1:1, 3:2 and 2:1 resonances.

As the value of ω increases, the space occupied by circulation reduces, while the chaotic regions increase significantly. Apart from the 1:1 resonance, all other resonances shift their location to higher values of θ , and these values also increase as a function of the eccentricity (inclined blue stripes).

The 1:1 resonance is always the largest one, and its evolution as a function of ω is not obvious, with many peculiarities. As shown in [16], this resonance bifurcates into 1:1 α and 1:1 β for values of $\omega > 1$ (the exact value depending on the eccentricity). Before the 1:1 β resonance appears, the 1:1 α shows a complex evolution. Its existence depends on the value of ω and the eccentricity. For example, in the case of $\omega = 0.5$ it exists only for $e < 0.1$. Its width in terms of θ is almost constant as a function of the eccentricity for very low values of ω , but, as ω increases it becomes much wider for near circular trajectories. For $\omega > 1.1$, the resonance 1:1 β shows its importance, presenting an impressive domain of the space, especially for low eccentricity values.

These predicted dynamical maps, composed of thin grades of conditions, show very fine details that would need huge time consuming to identify by generating PSS individually. For example, many small islands of weak resonances can be identified from the small blue islands immersed in the chaotic seas (green), as well as, the other small blue islands immersed in the wide regions of circulations (red).

6 Final Remarks

In the present work, we investigated the spin-orbit dynamics of an elongated satellite under the gravitational influence of a spherical central body through the analysis of Poincaré surfaces of section (PSS). To overcome the high computational cost associated with traditional numerical explorations of phase space, we incorporated machine learning techniques, leading to the development of PoincAÎré, an ensemble of open-source machine learning models for the efficient characterization of dynamical behavior. The proposed machine learning methodology is capable of generating high-resolution dynamical maps by replacing explicit trajectory integrations with machine learning predictions based only on the system’s initial conditions.

The CNN model identified three dynamical regimes: resonance, circulation, and chaos, with an accuracy of approximately 99%. Although our training and test sets contain 1,000 points per curve, an analysis of the evaluation metrics for different numbers of data points shows that the CNN model already achieves high performance with 500 points, with all evaluation metrics exceeding 95%. As the resolution decreases, the model performance also degrades. Additionally, the CNN has difficulty classifying points located near the separatrix regions.

The XGBoost model evaluated 10^5 initial conditions in only a few seconds, achieving an accuracy of about 97%. An important observation is that prediction uncertainty increases near the boundaries between classes, where small variations in the initial conditions can lead to transitions between the three dynamical regimes.

We explored the phase space within the ranges $\omega = [0.3, 1.7]$, $e = [0, 0.2]$ and $\theta = 0$, producing probability maps for the occurrence of the main spin-orbit resonances, the chaotic and circulation regions. We identified the seven spin-orbit resonances reported by [16]: $1:1\alpha$, $1:1\beta$, $3:2$, $2:1$, $5:2$, $3:1$, and $7:2$. In addition, a more detailed inspection of the islands discussed in Section 5 revealed four previously unidentified resonances: $5:3$, $4:9$, $4:11$, and $4:1$. These maps reveal the regions where each resonance exists and remains stable across different values of body elongation (ω) and orbital eccentricity (e), providing a highly detailed, comprehensive overview of the system dynamics. The model was trained within the specified parameter space; therefore, its predictions are limited to this domain.

The open-source machine-learning framework PoincAÎré, presented in this paper, is available for download at <https://lealpinheiro.github.io/PoincAÎre>.

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Conflict of interest

The authors declare no conflict of interest.

Author Contributions

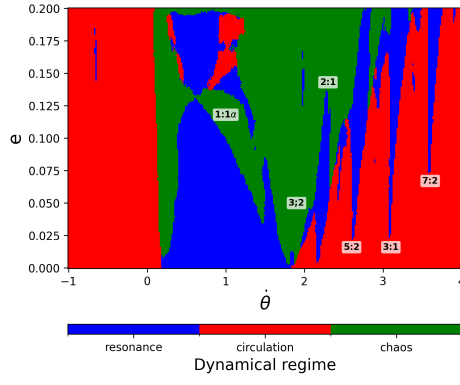
G.R. and T. F. L.L. P. developed the computational codes, performed the numerical analyses, and contributed to the interpretation of the results. O. C. W. and L. S. P. contributed to the discussion and interpretation of the results throughout the development of the study. All authors contributed to the writing and revision of the manuscript and approved the final version.

Data Availability

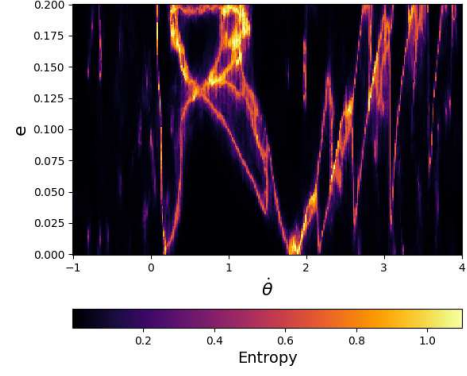
The open-source machine-learning framework PoincAIRE, presented in this paper, is available for download at <https://lealpinheiro.github.io/PoincAIRE>.

AI Declaration

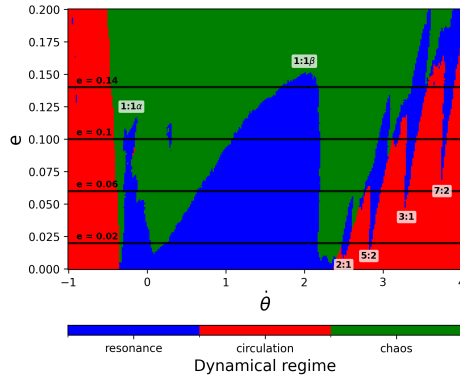
During the preparation of this work, the authors used ChatGPT and Gemini to improve the readability of the manuscript and to assist with the development of scripts for the GitHub Pages website. After using these tools, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.



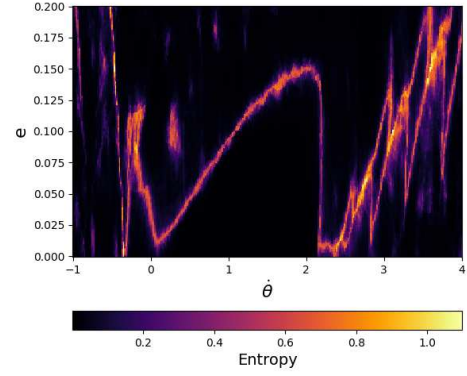
(a) Dynamical map for $\omega = 0.8$



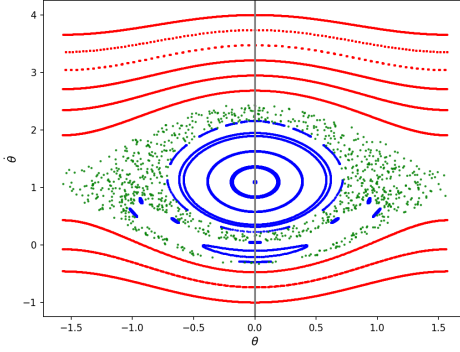
(b) Entropy map for $\omega = 0.8$



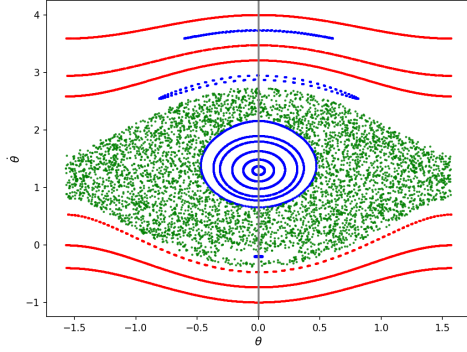
(c) Dynamical map for $\omega = 1.34$



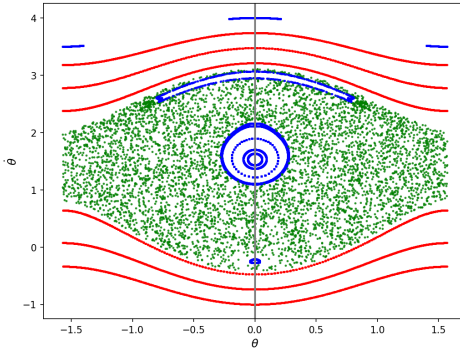
(d) Entropy map for $\omega = 1.34$



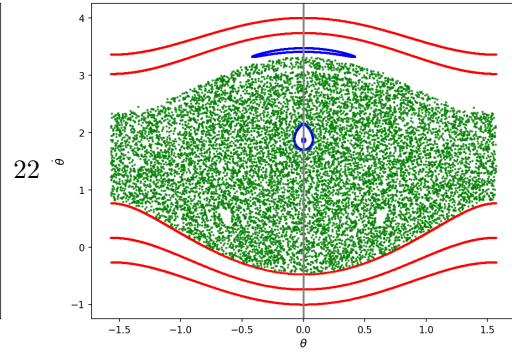
(e) PSS map for $\omega = 1.34$ and $e = 0.02$



(f) PSS map for $\omega = 1.34$ and $e = 0.06$



(g) PSS map for $\omega = 1.34$ and $e = 0.10$



(h) PSS map for $\omega = 1.34$ and $e = 0.14$

Fig. 10: Dynamical maps and corresponding entropy estimates for two values of ω , based on predictions from the XGBoost model. Panels a and b show the diagram of initial conditions (e, θ) and the entropy for $\omega = 0.8$, while panels c and d present the same quantities for $\omega = 1.34$. The three dynamical classes are resonance (blue), circulation (red), and chaos (green). Panels e–h show four examples of PSS maps for $\omega = 1.34$, corresponding to the eccentricities indicated by the black horizontal lines in panel c.

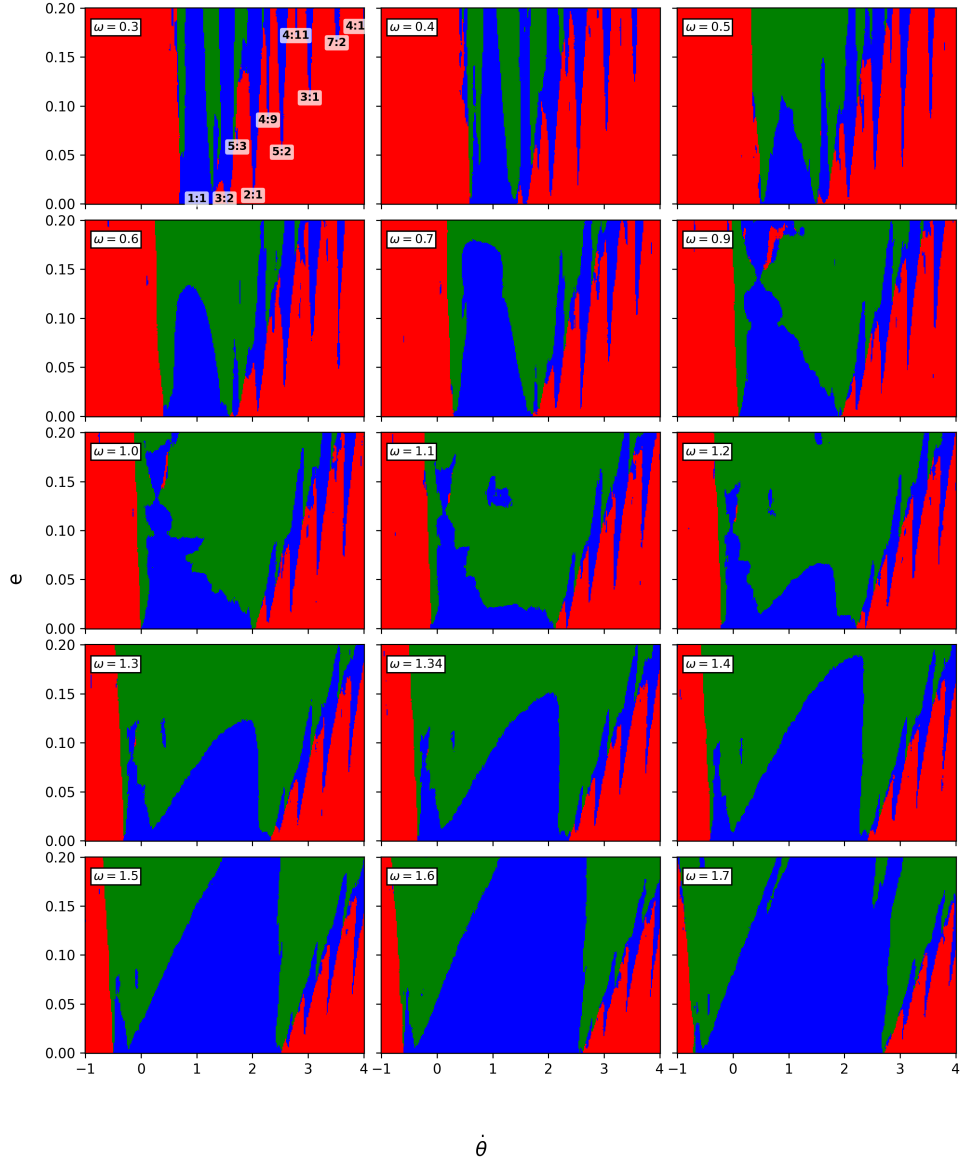


Fig. 11: Diagram of initial conditions ($e \times \dot{\theta}$) for $\omega \in [0.3, 1.7]$, classified by an XGBoost model into three dynamical states: resonance (blue), circulation (red), and chaos (green).