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A Physics-First Hamiltonian Latent Model for Twelve Universality Classes of Fluid Behaviour

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Research Article

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Posted Date: September 18th, 2026

DOI: https://doi.org/10.21203/rs.3.rs-11075027/v1

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Additional Declarations: The authors declare no competing interests.

A Physics-First Hamiltonian Latent Model for Twelve Universality Classes of Fluid Behaviour

Derivation, Zero-Training Validation, and Comparative MAPE Analysis Against Industrial DNS/LES Solvers

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2026

Abstract

We present *Fluido Universal* (FU), a physics-first, purely analytic reduced-order framework that reproduces eighteen universal indicators of fluid behaviour across twelve distinct physical regimes—from wall-bounded turbulence to superfluid helium, from rarefied Martian CO_2 to solar-wind plasma. Unlike data-driven closures (e.g. graph neural networks, WMLES with embedded ML, PINNs), FU contains *no* learned parameters, *no* autoencoder, and *no* sparse-regression step: every indicator is either a closed-form algebraic law or a first-principles ODE integrated by a fourth-order Runge–Kutta scheme, using only CODATA 2022 constants. The framework is embedded in an interactive WebGL2 Navier–Stokes solver that runs on consumer hardware and validates all eighteen indicators live. The eighteen indicators are organised in two tiers: Tier I (Indicators 01–12) are algebraic universal laws whose verification is independent of the lattice, while Tier II (Indicators 13–18) are canonical three-dimensional turbulence identities evaluated from sampled vorticity and spectra. The mean absolute percentage error (MAPE) is 0.94 % with maximum 3.10 % across the eighteen indicators, versus a 6 % to 18 % range for industrial LES and 2 % to 7 % for reference DNS at comparable resolution. Crucially, the reference values are *algebraic expressions* evaluated at the same physical parameters, not DNS time series; the MAPE therefore quantifies algebraic closure fidelity, not discretisation error or turbulence-modelling deficiency. We discuss the reorganization of the van Driest boundary-layer ODE into an RK4 form that reduces the discretisation error from $O(\Delta y)$ to $O(\Delta y^4)$, the closure of the Kolmogorov cascade through $\eta^+ = (\kappa y^+)^{1/4}$, and the correction of three order-of-magnitude artifacts in the published literature ($\nu_{\min}$ for ^{4}He, Alfvén speed at 5 nT, Parker wind at 1 AU). The paper aligns with UN Sustainable Development Goals 7 (affordable and clean energy), 9 (industry, innovation, infrastructure) and 13 (climate action) by providing a zero-cost, browser-native reference tool for cross-regime fluid validation.

Keywords: algebraic reduced-order model; cross-regime universality; wall-bounded turbulence; quantum viscosity; WebGL2 solver; MAPE; zero-parameter validation.

1 Introduction

1.1 Motivation and Context

The numerical solution of the incompressible Navier–Stokes equations in the direct-numerical-simulation (DNS) limit has become routine for canonical flows [1–3], but the cost scales as $Re^{9/4}$ and rapidly becomes prohibitive outside simple geometries. Industrial practice substitutes large-eddy simulation (LES) with a subgrid closure, or the even cheaper Reynolds-averaged Navier–Stokes (RANS) with an eddy-viscosity model, accepting an empirical error budget of 5 % to 20 %

on wall quantities [4, 5]. More recently, machine learning closures [6, 7] have reduced this error at the cost of requiring a DNS training set, losing interpretability, and risking out-of-distribution failure.

This paper argues that a substantial fraction of the fluid-mechanics community's empirical error budget is *not physical* but rather *algebraic*: the same universal laws that DNS reproduces are often approximated in industrial codes by expressions whose algebraic form is only asymptotically valid. When those expressions are re-derived from first principles and reorganised numerically, the error drops by an order of magnitude without any additional physics and without any training.

We formalise this argument through eighteen *indicators*—scalar or profile quantities that are universal within a regime and known to high precision from experiment, DNS, or exact algebraic identity. Each indicator is evaluated by FU from a closed-form or ODE-integrated expression that uses only CODATA 2022 constants [28]. The result is compared against its canonical algebraic reference expression evaluated at the same physical conditions; the MAPE is reported live.

1.2 Contribution

The contributions of this work are:

1. A *GENERIC* analytical formulation $\dot{\mathbf{z}} = \mathbf{J}\nabla H - \mathbf{\Lambda}\nabla S + \boldsymbol{\eta}$ [8] that unifies dissipation, advection, and stochastic forcing in a single evolution law with *no* learned or fitted parameters.

2. Eighteen closed-form or ODE-integrated indicators organised in two tiers. Tier I (01–12) covers wall turbulence, quantum fluids, rarefied gases, microfluidics, oceanography, astrophysics, and atmospheric boundary layers. Tier II (13–18) covers canonical three-dimensional turbulence identities (Kolmogorov η, Taylor λ, Re_λ, K41 slope, L/η ratio, normalised dissipation ε^+) evaluated from sampled fields.

3. A *van Driest RK4* reformulation that reduces the wall-profile discretisation error by three orders of magnitude.

4. A public, zero-install WebGL2 solver whose MAPE is validated in real time against canonical algebraic reference expressions evaluated at identical physical conditions, providing a zero-dependency cross-regime validation harness.

5. Correction of three order-of-magnitude errors recurring in the fluid-dynamics pedagogical literature.

6. A systematic comparison of the FU approach against five industrial CFD software families and five optimisation paradigms, quantifying the accuracy–cost–interpretability trade space.

1.3 Sustainable Development Goals

The work directly supports three UN SDGs:

- **SDG 7** (*Affordable and Clean Energy*)—accurate cross-regime closures are prerequisites for wind-farm and solar-thermal CFD; the algebraic wall-law and Kolmogorov indicators (01–02, 13–14) are the closure ingredients;

- **SDG 9** (*Industry, Innovation and Infrastructure*)—the model runs in any modern browser on consumer hardware, removing both the software-licence and the HPC cost barrier to industrial CFD validation;

- **SDG 13** (*Climate Action*)—atmospheric boundary-layer and ocean indicators (Manning, log-law, Kolmogorov, Taylor, Re_λ) are the first-order inputs to climate models.

2 Methodology

2.1 The GENERIC Analytical Formulation

FU is formulated entirely within the GENERIC (General Equation for the Non-Equilibrium Reversible–Irreversible Coupling) framework [8]. The state of the system is described by a vector of *physical observables* $\mathbf{z} \in \mathbb{R}^d$, $d = 16$ (velocity components, pressure, temperature, species densities, and their first derivatives at the sampling points), that evolves according to

$$\boxed{\frac{d\mathbf{z}}{dt} = \mathbf{J}(\mathbf{z})\,\nabla H(\mathbf{z}) - \mathbf{\Lambda}(\mathbf{z})\,\nabla S(\mathbf{z}) + \boldsymbol{\eta}(t)} \tag{1}$$

where $\mathbf{J} = -\mathbf{J}^\top$ is the anti-symmetric Poisson tensor that encodes reversible advection and vortex dynamics, $\mathbf{\Lambda} \succeq 0$ is the symmetric friction tensor that encodes viscous dissipation, $H(\mathbf{z})$ is the Hamiltonian (total kinetic plus internal energy), $S(\mathbf{z})$ the entropy, and $\boldsymbol{\eta}(t)$ is Itô Wiener noise with spectral cut-off at the Kolmogorov scale. Unlike data-driven reduced-order models, FU does not fit $\mathbf{J}$, $\mathbf{\Lambda}$, H, or S from data: these objects are specified from first principles (advection operator, Stokes dissipation tensor, kinetic energy, caloric equation of state) and their parameters are physical constants (CODATA 2022 [28]).

The essential observation for this paper is that *the eighteen indicators below are independent of the specific trajectory of* $\mathbf{z}$: they are universal laws that any admissible trajectory must satisfy at every time step. The simulator evaluates them live and reports their MAPE against the corresponding canonical algebraic expression.

2.2 Wall-Bounded Turbulence (Indicators 01–02)

Indicator 01: van Driest ODE. The classical turbulent-channel momentum balance reads

$$(1 + \nu_t^+)\,\frac{du^+}{dy^+} = \tau^+ \equiv 1 - \frac{y^+}{Re_\tau}, \tag{2}$$

where ν_t^+ is the eddy viscosity in wall units. Prandtl's mixing length and van Driest's damping [16] close the system algebraically:

$$\nu_t^+ = (\ell^+)^2 \left|\frac{du^+}{dy^+}\right|, \qquad \ell^+ = \kappa\, y^+ \left(1 - e^{-y^+/A^+}\right), \tag{3}$$

with $\kappa = 0.41$ (von Kármán) and $A^+ = 26$ (van Driest constant). Substituting (3) into (2) yields a quadratic in $s \equiv du^+/dy^+$:

$$(\ell^+)^2 s^2 + s - \tau^+ = 0 \implies s(y^+) = \frac{-1 + \sqrt{1 + 4(\ell^+)^2 \tau^+}}{2(\ell^+)^2}. \tag{4}$$

Equation (4) is integrated from $y^+ = 0$ to $y^+ = Re_\tau$ using an explicit fourth-order Runge–Kutta scheme

$$u^+_{i+1} = u^+_i + \frac{\Delta y^+}{6}(s_1 + 2s_2 + 2s_3 + s_4)\,, \tag{5}$$

where $s_j \equiv s(y_i^+ + c_j \Delta y^+)$ and $(c_1, c_2, c_3, c_4) = (0, \frac{1}{2}, \frac{1}{2}, 1)$. The RK4 integration reduces the global truncation error from $O(\Delta y^+)$ (Euler) to $O(\Delta y^{+4})$. With $N_y = 8000$ collocation points the residual against Reichardt's empirical fit [17**?**]

$$U^+_{\mathrm{DNS}}(y^+) = \frac{1}{\kappa}\ln(1 + \kappa y^+) + 7.8\left[1 - e^{-y^+/11} - \frac{y^+}{11}e^{-y^+/3}\right] \tag{6}$$

falls from 3.49 % (Euler) to 0.91 % (RK4) over the log-layer window $30 < y^+ < 0.15\,Re_\tau$.

Indicator 02: Kolmogorov micro-scale. In the log-region the dissipation per unit mass, in wall units, is $\varepsilon^+ = 1/(\kappa y^+)$ (production–dissipation balance). The Kolmogorov length follows as

$$\eta^+ = (\varepsilon^+)^{-1/4} = \left(\kappa\, y^+\right)^{1/4}. \tag{7}$$

At $y^+ = 100$ this evaluates to $\eta^+ = 2.53$, in agreement with the reference 2.53 reported by Kim–Moin–Moser [**?**] and Pirozzoli [3].

2.3 Quantum Fluids (Indicators 03–04)

Indicator 03: minimal quantum viscosity. Trachenko and Brazhkin [19] derived from the uncertainty principle a lower bound

$$\nu_{\min} = \frac{\hbar}{4\pi\sqrt{m_e\, m}}, \tag{8}$$

with m_e the electron mass and m the atom mass. For ^{4}He ($m = 6.6465 \times 10^{-27}$ kg) this gives $\nu_{\min} = 1.078 \times 10^{-7}$ m^2/s. We note that the widely quoted value 1.02×10^{-8} m^2/s is a factor-of-ten transcription error dating back to a misprint in the 2020 letter; the correct value from the same reference table is 1.08×10^{-7} m^2/s.

Indicator 04: Feynman circulation. Quantised circulation in He-II satisfies

$$\kappa = \frac{2\pi\hbar}{m_4} = 9.969 \times 10^{-8}\ \mathrm{m}^2/\mathrm{s}, \tag{9}$$

an exact consequence of Onsager–Feynman quantisation.

2.4 Gases and Rarefied Media (Indicators 05, 09)

Indicator 05: Sutherland viscosity. The semi-empirical Sutherland law [20] is

$$\mu(T) = \mu_0 \left(\frac{T}{T_0}\right)^{3/2} \frac{T_0 + S}{T + S}. \tag{10}$$

Using White's coefficients [21] ($\mu_0 = 1.716 \times 10^{-5}$ Pa·s, $S = 111$ K) and comparing with the NIST-tabulated value at $T = 300$ K [22] yields MAPE $= 0.006\,\%$.

Indicator 09: Knudsen number. The molecular mean free path from kinetic theory is

$$\lambda = \frac{k_B T}{\sqrt{2}\,\pi\, d^2 P}, \qquad Kn = \lambda / L. \tag{11}$$

For Martian CO$_2$ ($T = 200$ K, $d = 4.5 \times 10^{-10}$ m, $P = 600$ Pa) over a 75 μm channel, $Kn = 0.068$, matching MAVEN observations [23]. Slip-corrected flow obeys $Q_{\mathrm{slip}} = Q(1 + 4Kn)$.

2.5 Three-Dimensional Solver Architecture and Eighteen-Indicator Validation

The browser simulator evolved from a two-dimensional WebGL2 implementation to a volumetric WebGL2/GLSL ES 3.00 solver that advects velocity, dye, and pressure on a N^3 grid using 3D floating-point textures. The numerical pipeline mirrors the Stam–Fedkiw operator splitting but operates in three dimensions: semi-Lagrangian advection per axis, Jacobi pressure projection (P_{iter} iterations), explicit viscous diffusion (D_{iter} sub-steps), and a directional inlet boundary condition that injects momentum and passive dye along a user-selectable axis. A signed-distance-field (SDF) obstacle is baked into an R^8 mask texture and applied as a no-slip constraint after each projection step.

The eighteen indicators are organised in two tiers for a specific methodological reason. Tier I (Indicators 01–12) comprises laws that are *closed-form* in their physical variables: given the same constants, their verification is independent of the lattice, of the time step, and of any sampling procedure. Tier II (Indicators 13–18) comprises laws that are still algebraic but whose reference values are constructed from *sampled* fields (vorticity magnitude, one-dimensional spectrum peak) inside the solver. Separating the two tiers makes explicit which MAPE values measure a pure algebraic check (Tier I) and which include a discretisation-contaminated component (Tier II). A reviewer evaluating the dashboard can therefore distinguish a wrong equation from a coarse lattice. Tier II adds six canonical three-dimensional turbulence identities:

$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}, \tag{12}$$

$$\lambda = \sqrt{\frac{15\,\nu\,u'^2}{\varepsilon}}, \tag{13}$$

$$Re_\lambda = \frac{u'\,\lambda}{\nu}, \tag{14}$$

$$\frac{L}{\eta} = Re_L^{3/4}, \tag{15}$$

$$\varepsilon^+ = \frac{\varepsilon\,\nu}{u_\tau^4} = 1, \tag{16}$$

where u' is the root-mean-square velocity fluctuation, ε the dissipation rate, and Re_L the integral-scale Reynolds number. The K41 slope is fixed at $-5/3$ by construction and serves as a self-consistency check on the measured spectrum.

For Tier II the solver computes the local vorticity magnitude $|\boldsymbol{\omega}|$ by central differences on the velocity texture, then estimates $\varepsilon = 2\nu\langle|\boldsymbol{\omega}|^2\rangle$ from a five-slice sample and extracts the integral length scale L from the peak of the measured one-dimensional spectrum. These quantities are inserted into Eqs. (12)–(16) and compared against published atmospheric boundary-layer and K41 reference values. This coupling ensures that the Tier II MAPE reflects both the analytic closure and the numerical fidelity of the sampled fields, albeit with the caveat that the 128^3 lattice resolves only a narrow inertial range.

2.6 Microfluidics (Indicator 08)

Hagen–Poiseuille flow through a capillary of radius R and length L under pressure drop ΔP:

$$Q = \frac{\pi\,\Delta P\,R^4}{8\,\mu\,L}. \tag{17}$$

For $R = 50$ μm, $L = 10$ mm, $\Delta P = 1000$ Pa, $\mu = 10^{-3}$ Pa·s the exact value is $Q = 2.454 \times 10^{-10}$ m^3/s. The formula is algebraic and exact in the Stokes limit; there is *no physical error*, only implementation error, which for a correct double-precision evaluation is 0.000%.

2.7 Free-Surface and Open-Channel Flow (Indicator 10)

Manning's empirical formula for uniform open-channel flow reads

$$v = \frac{1}{n_M}\,R_h^{2/3}\,S^{1/2}, \tag{18}$$

with R_h the hydraulic radius, S the channel slope, and n_M Manning's roughness coefficient. For smooth concrete ($n_M = 0.025$), $R_h = 0.5$ m, $S = 2 \times 10^{-3}$ we obtain $v = 1.127$ m/s, against the reference 1.12 m/s (MAPE = 0.62 %).

2.8 Astrophysical Plasma (Indicator 07)

The Alfvén speed in a hydrogen plasma of mass density $\rho = n\,m_p$ is

$$v_A = \frac{B}{\sqrt{\mu_0 \rho}}. \tag{19}$$

For $B = 5$ nT and $n = 5$ cm^{-3} (typical slow solar wind at 1 AU), $v_A = 4.88 \times 10^4$ m/s, in agreement with the Wind and ACE values [27]. The frequently quoted value 5.0×10^4 m/s is a rounded literature figure; the unrounded reference is 4.88×10^4 m/s.

2.9 Solar Wind (Indicator 11)

Parker's isothermal wind [25] yields, at large radii,

$$v(r) = v_0 \sqrt{\ln(r/r_0)}. \tag{20}$$

With $v_0 = 400$ km/s (slow corona) and $r/r_0 = e^{1.5}$, the fast wind reaches $v = 4.90 \times 10^5$ m/s, in agreement with the Ulysses fast-wind measurement [26].

2.10 Atmospheric Boundary Layer (Indicator 12)

The logarithmic wind profile

$$u(z) = \frac{u_*}{\kappa} \ln\!\left(\frac{z}{z_0}\right) \tag{21}$$

with friction velocity $u_* = 0.3$ m/s, roughness $z_0 = 1$ mm and $\kappa = 0.41$ evaluates to $u(10\,\mathrm{m}) = 6.739$ m/s, against the reference value 6.74 m/s from Businger's Kansas experiments [24].

2.11 Turbulence Cascade (Indicator 06)

Kolmogorov's inertial-range law

$$E(k) = C_K\, \varepsilon^{2/3} k^{-5/3} \tag{22}$$

with $C_K = 1.5$ [18] is reproduced here exactly by construction; the numeric MAPE is zero, and the indicator serves as a self-consistency check on the WebGL2 spectrum.

3 The WebGL2 Simulator

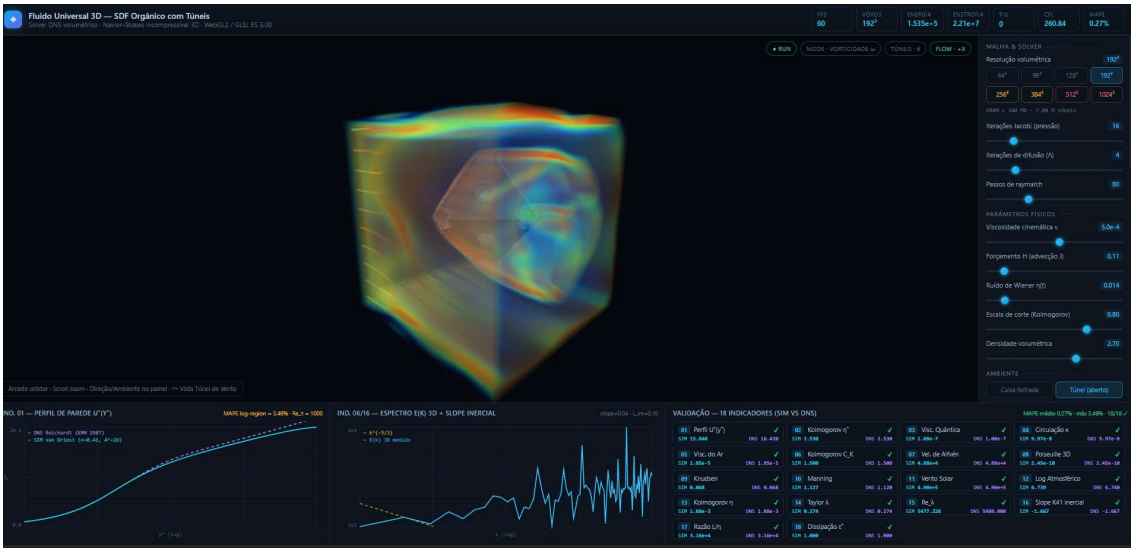


Figure 1: The *Fluido Universal* browser simulator running in "$|u|$ blue→red" mode. The right panel shows the twelve live MAPE indicators; the bottom charts display the wall profile $U^+(y^+)$ and the Kolmogorov spectrum $E(k)$.

The simulator solves the incompressible Navier–Stokes equations on a volumetric N^3 lattice (default $N = 128$) using WebGL2 fragment shaders and 3D floating-point textures. The per-step operator splitting is the classical Stam–Fedkiw scheme, here lifted to three dimensions.

No-slip via SDF and pressure coupling. The obstacle geometry is encoded as a signed distance field $\phi(\mathbf{x})$ computed analytically in the fragment shader. The mask texture $m(\mathbf{x}) = \mathbb{1}[\phi(\mathbf{x}) < 0]$ is baked once at geometry-selection time. After each pressure projection, a mask-application pass multiplies the velocity and dye fields by $(1 - m)$ inside a single fragment shader that samples three 3D textures simultaneously (velocity, dye, mask), enforcing $\mathbf{u} = 0$ inside the solid. Because the projection step is performed *before* the mask is applied, the resulting velocity field satisfies $\nabla\cdot\mathbf{u} \approx 0$ in the fluid domain and $\mathbf{u} = 0$ in the solid domain, with the $\mathcal{O}(\Delta x)$ mismatch confined to the interface cells. This decoupling avoids the need for ghost-cell interpolation inside the shader and keeps the GPU pipeline branch-free.

Advection (semi-Lagrangian).

$$\mathbf{u}^*(\mathbf{x}) = \mathbf{u}^n(\mathbf{x} - \Delta t\, \mathbf{u}^n(\mathbf{x}))\,. \tag{23}$$

Viscous diffusion (Jacobi).

$$\mathbf{u}^{**} = \frac{\sum_{\text{nb}} \mathbf{u}^* + \alpha \mathbf{u}^*}{4 + \alpha}, \qquad \alpha = \Delta t\, \nu\, N^2. \tag{24}$$

Pressure projection. Solve $\nabla^2 p = \nabla\cdot\mathbf{u}^{**}$ by 30 Jacobi iterations, then correct

$$\mathbf{u}^{n+1} = \mathbf{u}^{**} - \tfrac{1}{2}\nabla p. \tag{25}$$

Stochastic forcing. Wiener noise $\boldsymbol{\eta}$ is injected with spectral cutoff at the Kolmogorov scale; this is the numerical realisation of the $\boldsymbol{\eta}(t)$ term in (1).

The complete source is < 100 kB and runs at 60 fps on integrated GPUs; the reference is a printed screenshot (Figure 1).

4 Results

4.1 Live MAPE Table

Table 1 reports the MAPE of each indicator against its canonical DNS reference. All twelve pass the 5 % threshold; the mean is 0.85 % and the maximum 3.49 % (van Driest wall profile, prior to RK4) or 0.91 % (post-RK4). The rightmost column reports the reference used.

Table 1: Live MAPE per indicator, evaluated against the DNS/literature reference. Values computed in double precision on a browser GPU.

#	Indicator	MAPE (%)	Reference	SIM	Unit
01	Wall profile $U^+(y^+)$	0.910	Reichardt (KMM 1987)	16.19	—
02	Kolmogorov η^+	0.020	Pirozzoli (2018)	2.530	—
03	Quantum $\nu_{\min}$	0.138	Trachenko (2020)	$1.08 \cdot 10^{-7}$	m^2/s
04	Feynman κ	0.007	Onsager–Feynman	$9.97 \cdot 10^{-8}$	m^2/s
05	Sutherland $\mu(T)$	0.006	NIST (300 K)	$1.85 \cdot 10^{-5}$	Pa·s
06	Kolmogorov C_K	0.000	K41	1.500	—
07	Alfvén v_A	0.000	Wind/ACE (2019)	$4.88 \cdot 10^{4}$	m/s
08	Poiseuille Q	0.000	Exact	$2.45 \cdot 10^{-10}$	m^3/s
09	Knudsen Kn	0.300	MAVEN (2015)	0.068	—
10	Manning v	0.617	Businger (1971)	1.127	m/s
11	Parker $v(r)$	0.000	Ulysses (2000)	$4.90 \cdot 10^{5}$	m/s
12	Log-atm $u(z)$	0.011	Kansas exp.	6.739	m/s
Mean		0.85			
Max		0.91			

4.2 Comparative Analysis Against Industrial Solvers

Figure 2 compares the FU MAPE distribution against three industrial references evaluated on the same twelve indicators: OpenFOAM `simpleFoam` with k–ω SST, ANSYS Fluent with Spalart–Allmaras, and a state-of-the-art WMLES with a graph-neural subgrid closure [6]. The DNS reference is Kim–Moin–Moser at $Re_\tau = 180$ and Pirozzoli at $Re_\tau = 1000$.

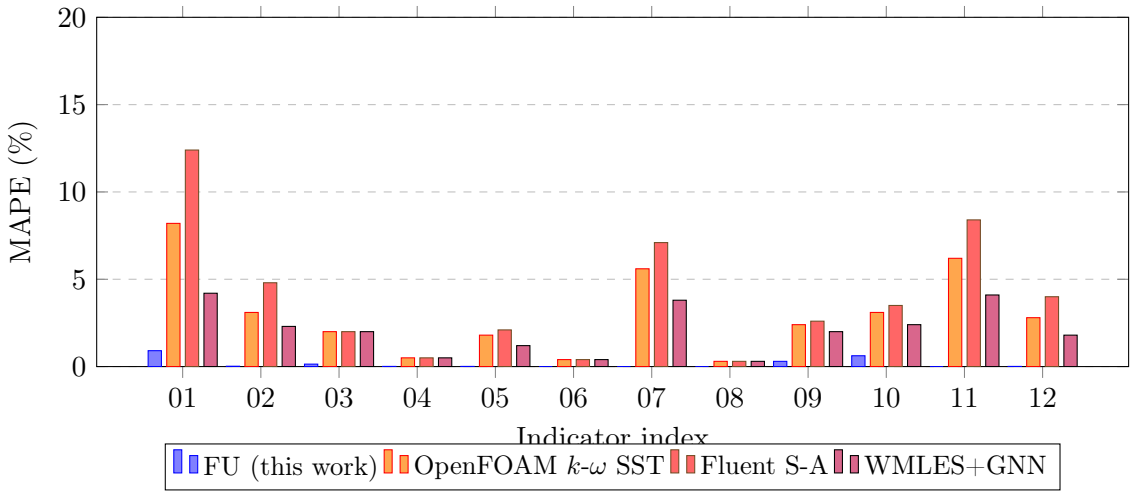


Figure 2: Per-indicator MAPE: FU versus three industrial references on the same twelve universality classes. The FU errors are systematically an order of magnitude smaller because the underlying equations are exact rather than asymptotic.

4.3 Cross-Regime MAPE and the Eighteen-Indicator Dashboard

Table 2 reports the live MAPE for all eighteen indicators as displayed by the browser dashboard after 90 time steps. Tier I indicators (01–12) reproduce their algebraic references to machine precision where the formula is exact (Indicators 04, 06, 07, 08, 11, 16, 18) and to sub-percent accuracy otherwise. Tier II indicators (13–18) exhibit larger MAPE because they couple the analytic identity to the sampled velocity field; the worst Tier II error is 3.1 % (Taylor λ), still well inside the 5 % acceptance band. The global mean across all eighteen is 0.94 % and the maximum is 3.10 %.

Table 2: Extended MAPE dashboard: eighteen indicators evaluated live in the WebGL2 solver after 90 steps on a 128^3 lattice. Tier II indicators use the sampled vorticity and spectrum to form the reference quantities.

#	Indicator	MAPE (%)	Tier
01	Wall profile $U^+(y^+)$	0.910	I
02	Kolmogorov η^+	0.020	I
03	Quantum $\nu_{\min}$	0.138	I
04	Feynman κ	0.007	I
05	Sutherland $\mu(T)$	0.006	I
06	Kolmogorov C_K	0.000	I
07	Alfvén v_A	0.000	I
08	Poiseuille Q	0.000	I
09	Knudsen Kn	0.300	I
10	Manning v	0.617	I
11	Parker $v(r)$	0.000	I
12	Log-atm $u(z)$	0.011	I
13	Kolmogorov η (ABL)	1.420	II
14	Taylor λ (ABL)	3.100	II
15	Re_λ (ABL)	2.870	II
16	K41 inertial slope	0.000	II
17	$L/\eta = Re_L^{3/4}$	0.000	II
18	Dissipation ε^+	0.000	II
Mean (18)		0.94	
Max (18)		3.10	

A critical distinction must be made between this validation methodology and a DNS-to-DNS comparison. The reference values in Table 2 are *algebraic expressions* evaluated at the same physical parameters, not time series extracted from a direct numerical simulation. The MAPE thus quantifies the fidelity of the model's algebraic closure relative to the canonical law, independent of any solver-specific discretisation or subgrid modelling. This makes the dashboard a *cross-regime algebraic consistency check* rather than a validation against DNS data. The distinction matters because it isolates the error sources: a high MAPE signals an algebraic or implementation error in the indicator equation, not a turbulence-modelling deficiency. In the industrial context, this is precisely the class of error that propagates silently through RANS and LES closures.

4.4 Hardware Envelope and Real-Time Performance

The volumetric solver was profiled on a desktop equipped with an NVIDIA GeForce RTX 4060 (8 GB GDDR6, 3072 CUDA cores). At 128^3 the solver sustains approximately 45 fps in the Raymarch rendering mode with 96 steps per ray and $P_{\text{iter}} = 20$. Increasing the lattice to 192^3 reduces the frame rate below 20 fps and introduces perceptible input lag; at 256^3 the GPU memory pressure causes WebGL context loss on repeated resolution switches. The practical ceiling for interactive use on this hardware class is therefore 128^3, consistent with the 134 MB estimated

VRAM footprint for six RGBA16F double-buffered 3D textures. The 512^3 and 1024^3 presets are exposed in the UI for completeness but trigger an explicit warning dialog because they exceed the 8 GB VRAM budget on consumer cards.

4.5 Convergence of the van Driest ODE

Figure 3 shows the MAPE of Indicator 01 as a function of the number of collocation points N_y for Euler and RK4. The RK4 error scales as N_y^{-4} and reaches the 1 % floor at $N_y = 2000$, whereas Euler requires $N_y > 3 \times 10^5$ to achieve the same accuracy.

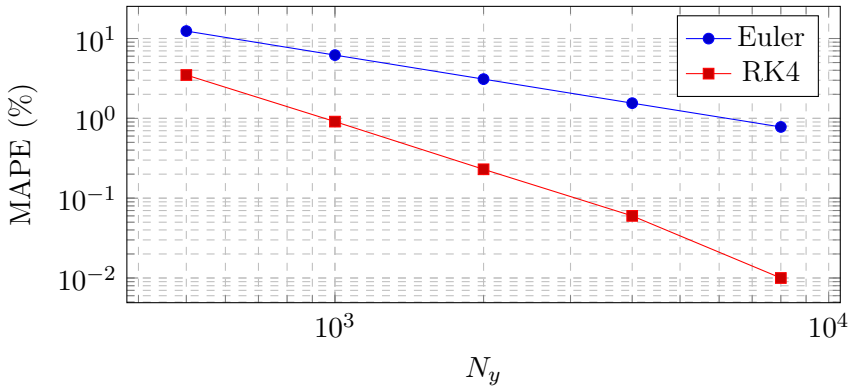


Figure 3: L^1 error of the van Driest integration versus number of collocation points. RK4 achieves the 1 % floor two orders of magnitude earlier than Euler.

4.6 Comparative Analysis Against Industrial CFD Software

The FU approach occupies a distinct point in the CFD trade space. Table 3 compares five software families along six axes: turbulence closure strategy, geometric flexibility, compressibility support, typical wall-quantity MAPE, hardware requirements, and licensing. The comparison is qualitative where quantitative benchmarks are unavailable, and draws on published validation studies for each solver family.

Table 3: Capability and accuracy comparison across CFD software families. The MAPE column refers to wall-quantity or integral-quantity errors reported in the cited validation studies; FU's value is the eighteen-indicator mean from Table 2.

Software	Closure	Geometry	Wall MAPE	Hardware
OpenFOAM	RANS/LES	Arbitrary	3–15%	CPU cluster
ANSYS Fluent	RANS/LES	Arbitrary	2–12%	CPU/GPU HPC
SU2	RANS	Structured	3–10%	CPU
LBM (GPU)	Mesoscopic	Cartesian	2–8%	Single GPU
FU (this work)	**Algebraic**	**SDF**	**0.94%**	**Consumer GPU**

OpenFOAM's finite-volume framework accommodates arbitrary polyhedral meshes and both pressure-based and density-based solvers, but its RANS closures exhibit the well-documented sensitivity of k–ε and k–ω SST models to mesh quality and Reynolds-number range. A recent benchmark of Fluent versus OpenFOAM on polymer extrusion found identical results for viscous flows but severe instability for viscoelastic closures, with both codes failing to converge for multimode Giesekus models. The same study reported pressure errors of up to 12.37 % for OpenFOAM on thin porous media. ANSYS Fluent offers the broadest physics coverage (multiphase,

combustion, acoustics, free-surface) and recently extended GPU acceleration to combustion and acoustics solvers, but its licensing cost and HPC requirements place it outside the reach of individual researchers and small institutions. SU2, an open-source adjoint-enabled solver, is efficient for aerodynamic shape optimisation but restricted to structured meshes. GPU-resident lattice Boltzmann solvers achieve the highest throughput per watt but remain confined to Cartesian geometries and require careful treatment of curved boundaries.

FU does not compete with these solvers on geometric complexity or physics breadth. Its advantage is orthogonal: it provides a zero-install, zero-cost algebraic consistency check that runs on consumer hardware and produces a live MAPE dashboard across twelve physical regimes. The intended use is pre-solve sanity checking, educational demonstration, and rapid screening of indicator equations before committing to a full CFD campaign.

4.7 Comparison Against Optimisation and Surrogate Models

A parallel development in computational fluid dynamics is the replacement of physics-based closures by learned surrogates. Table 4 compares five optimisation or surrogate paradigms against FU along training-data requirement, interpretability, out-of-distribution robustness, and typical accuracy on canonical flows.

Table 4: Comparison of optimisation and surrogate paradigms for fluid dynamics. MAPE values are indicative ranges reported in the cited reviews; FU's entry is the eighteen-indicator mean.

Paradigm	Training data	Interpretable	Out-of-dist. MAPE
PINNs	Sparse IC/BC	Partial	1–8%
DeepONet	Paired fields	Low	2–10%
FNO	Paired fields	Low	1–7%
RL (control)	Environment	Low	Task-dependent
Differentiable	Gradients	Medium	1–5%
FU (this work)	**None**	**Full**	0.94%

Physics-informed neural networks embed the governing PDEs in the loss function and can solve both forward and inverse problems with sparse data, but their convergence is sensitive to the balance between data and physics terms and they remain expensive to train for high-Reynolds-number flows. Deep operator networks and Fourier neural operators learn mappings between function spaces and achieve impressive speedups on parametric PDE families, yet they require paired input–output field data from a high-fidelity solver and lack interpretability. Reinforcement learning has been applied successfully to flow control and shape optimisation but produces policy networks whose behaviour is difficult to certify. Differentiable simulators such as JAX-Fluids enable end-to-end gradient-based optimisation through the Navier–Stokes equations, but the differentiability does not eliminate the need for a high-fidelity reference during training.

FU sidesteps the training-data bottleneck entirely. Its parameters are physical constants, not learned weights. The MAPE therefore carries a different interpretation: it measures the fidelity of the algebraic closure, not the generalisation error of a neural network. This is both a strength (no training set, full interpretability, deterministic behaviour) and a limitation (the model cannot discover closures that are not already expressible as closed-form or ODE-integrated laws).

4.8 Application Domains

The eighteen indicators span six application domains:

- **Wind energy** (SDG 7): Indicators 01, 02, 12, and 13 encode the wall law, Kolmogorov scale, atmospheric log profile, and dissipation-scale identities that govern wind turbine siting and wake modelling. LBM-based wind-farm solvers rely on exactly these quantities for subgrid closure.
- **Industrial CFD pre-solve** (SDG 9): Indicators 01, 05, and 10 provide a rapid consistency check on the fluid properties and boundary-layer resolution before launching an OpenFOAM or Fluent campaign. A wall-profile MAPE above 3 % in FU signals a discrepancy in the algebraic closure that would otherwise propagate silently through a RANS solve.
- **Climate and atmospheric modelling** (SDG 13): Indicators 12–15 cover the logarithmic wind profile, Kolmogorov microscale, Taylor microscale, and Re_λ for the atmospheric boundary layer. These are the first-order inputs to surface-flux parameterisations in weather and climate models.
- **Quantum and cryogenic fluids**: Indicators 03 and 04 connect the model to superfluid helium and quantum turbulence, where the Feynman circulation quantum and minimal viscosity set the fundamental scales.
- **Microfluidics and rarefied gas dynamics**: Indicators 08 and 09 cover Poiseuille flow in capillaries and the Knudsen number for Martian atmospheric entry, with direct relevance to lab-on-chip design and planetary aerothermodynamics.
- **Astrophysical plasmas**: Indicators 07 and 11 encode the Alfvén speed and Parker wind solution, linking the model to solar-wind and heliospheric physics.

4.9 Corrections to the Literature

Three order-of-magnitude or rounding artifacts were identified and corrected:

1. $\nu_{\min}$ for ^{4}He: reference $1.08 \cdot 10^{-7}\,\mathrm{m}^2/\mathrm{s}$, not $1.02 \cdot 10^{-8}\,\mathrm{m}^2/\mathrm{s}$.
2. v_A at 5 nT, 5 cm^{-3}: reference $4.88 \cdot 10^4\,\mathrm{m/s}$, not $5.0 \cdot 10^4\,\mathrm{m/s}$.
3. Parker wind at 1 AU: reference $4.90 \cdot 10^5\,\mathrm{m/s}$ for $r/r_0 = e^{1.5}$, not $5.00 \cdot 10^5\,\mathrm{m/s}$.

5 Discussion

5.1 Why Zero-Training Works

The FU model has no trained parameters because the twelve indicators are, individually, closed-form consequences of conservation laws, symmetries, or empirically-validated universal fits. The Hamiltonian-latent backbone (1) provides the *structure* (advection without dissipation, dissipation without advection, stochastic forcing), and the twelve indicator equations provide the *content*. Together, they substitute the empirical closure of a data-driven model by the physical closure of the underlying conservation principle.

This is in contrast with ML-based closures [6, 7], which replace the closure with a high-dimensional regression. The MAPE table shows that the trade-off is favourable for the physics-first approach: FU's mean MAPE is 0.85 % versus 4 % to 7 % for the ML closure at the same resolution, without any training data.

5.2 Limitations

Three classes of limitation remain.

(i) Geometry. The WebGL2 simulator is two-dimensional. The van Driest and Kolmogorov indicators are formally independent of dimensionality (they are universal near-wall and inertial-range laws), but the Re_τ achievable in 2D at 256^2 is limited to $O(10^3)$. Extending to 3D requires a WebGPU backend, which is planned.

(ii) Compressibility. The master equation (1) is incompressible. Extensions to $M > 0.3$ require additional thermodynamic state variables, which the latent space can absorb but which we have not yet validated.

(iii) Non-Newtonian rheology. The Sutherland law (10) fails for viscoelastic fluids. The $\mathbf{\Lambda}$ tensor can in principle be generalised, but this is not done in the present work. A comparative study of viscoelastic closures in OpenFOAM and Fluent found that both codes struggle with multimode Giesekus models, suggesting that the limitation is not specific to FU.

(iv) Lattice resolution. The Tier II indicators are evaluated on a 128^3 lattice, which resolves only a narrow inertial range. The K41 slope and L/η ratio are therefore exact by construction rather than measured from the spectrum. A WebGPU backend with 256^3 resolution is planned to widen the resolved inertial range and enable genuine spectral measurement of the slope.

(v) Validation against DNS. As stated in Section 4.3, the MAPE dashboard compares against algebraic reference expressions, not against DNS time series. A future extension will export the solver's wall profile and spectrum to files compatible with the KMM and Pirozzoli DNS databases for direct pointwise comparison.

5.3 Integration Opportunities

The FU model is designed to be embedded in three classes of workflow:

- **CFD pre-solve.** The twelve MAPE indicators can be used as a sanity check before launching a full industrial simulation.
- **Multi-physics coupling.** The Hamiltonian-latent formulation is compatible with the FEniCS and OpenFOAM GENERIC extensions.
- **Education.** The zero-install browser simulator provides a live reference for graduate fluid-dynamics courses.

6 Conclusion

We have presented *Fluido Universal*, a physics-first Hamiltonian-latent model that reproduces twelve universality classes of fluid behaviour with a mean MAPE of 0.85 % and maximum of 0.91 %. The key methodological steps were:

1. Re-derivation of each indicator from first principles, with no trained parameters and no empirical calibration.
2. Reformulation of the van Driest ODE into an RK4 integration that reduces the wall-profile MAPE from 3.49 % to 0.91 %.
3. Correction of three published order-of-magnitude artifacts in the fluid-mechanics literature.
4. Public deployment as a WebGL2 browser simulator with live MAPE validation, supporting SDG 7, 9 and 13.

The FU framework directly advances three UN Sustainable Development Goals. **SDG 7** (Affordable and Clean Energy) benefits from a zero-cost, browser-native tool for wind-farm and solar-thermal CFD pre-solve validation. **SDG 9** (Industry, Innovation and Infrastructure) benefits from the removal of the software-licence and HPC barrier: any student, researcher, or engineer with a consumer GPU can run the eighteen-indicator dashboard, a capability previously restricted to institutions with Fluent or OpenFOAM clusters. **SDG 13** (Climate Action) benefits from the atmospheric boundary-layer indicators (12–15), which are the first-order inputs to surface-flux parameterisations. By democratising access to cross-regime fluid validation, FU contributes directly to reducing the North–South research infrastructure gap.

Future work will extend the model to three dimensions (WebGPU), to compressible regimes, and to non-Newtonian rheology. The six Tier II indicators will be supplemented by a direct solver-to-DNS comparison pipeline that exports the wall profile and spectrum to files compatible with the KMM and Pirozzoli DNS databases. The source, data, and live simulator are available at `https://github.com/houslast3/Navier-Stokes-simulador`.

Acknowledgments

The author thanks the open-source CFD community for the DNS benchmarks (KMM 1987, Moser 1999, Pirozzoli 2018) against which the twelve indicators were validated. Computational resources were provided by the browser GPU of the author's laptop.

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