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Welding in the World

Prediction of plastic deformation zone width in SAW-welded structural steel butt joints using numerical simulations and analytical models

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Abstract

Results of numerical simulations based on temperature field distributions, which were used to determine the width of the plastic deformation zone in structural steel butt joints, are presented in this paper. The simulations were performed according to the principles of the design of experiments (DOE) for two plate thickness ranges corresponding to those used in marine ship hulls and offshore structures, employing process parameters representative of the SAW process. A Gaussian disc heat source model was adopted in the analyses. The obtained predictive relationships showed very good agreement with the numerical simulation results. The proposed approach accounts for the technological and design parameters affecting the width of the plastic deformation zone. Furthermore, the method developed for evaluating the simulation results enables the use of approximation relationships to predict the width of the plastic deformation zone in butt joints with different plate thickness ranges. The derived predictive relationships were compared with existing analytical models in order to identify the formula providing the best approximation.

Keywords: plastic zone, butt weld, design of experiment, prediction, regression analysis, shipbuilding

Nomenclature

b_{is} – width of inherent strain zone, (i.e., half-width) (mm)

b_{pl} – plastic deformation zone (i.e., half-width) (mm)

b_{pl-DOE} – width of the plastic zone (i.e., half-width) measurements in numerical simulations (mm)

b_{pl-tp} – width of the plastic zone (i.e., half-width) determined from numerical simulations, adequately for thin plates (i.e. structural steels of ship hull) (mm)

b_{pl-thp} – width of the plastic zone (i.e., half-width) determined based on numerical simulations, adequately for thick plates (i.e. structural steels for offshore construction) (mm)

B - breadth of analysed plate (mm)

c - specific heat capacity (J/kg°C)

DOE – design of experiments

E – Young's modulus (N/mm²)

F – Snedecor F-distribution

F_{cr} – Critical value of Snedecor F-distribution

FEM - Finite Element Method
 h – plate thickness (mm)
 HAZ - Heat Affected Zone
 I - welding current (A)
 k_e - type of current coefficient
 L - length of analysed plate (mm)
 q_l - linear heat input (kJ/mm)
 Q – arc energy per unit length (J/m)
 r_0 – heat source radius (mm)
 R - multiple correlation coefficient
 R^2 – squared multiple correlation coefficient
 R^2_{adj} – adjusted squared correlation coefficient
 SAW – submerged arc welding with wire electrode
 t - time (s)
 T – temperature (°C)
 T_0 – starting/ambient temperature (°C)
 T_{av} – average temperature rise (°C)
 T_{Yl} - first yield temperature (°C)
 U - welding voltage (V)
 v_s - welding speed (mm/min)
 x - longitudinal coordinate (in the direction of motion of the heat source) (m)
 x_1, x_2, x_3, x_4 – independent variables selected for the experiment
 y - transverse coordinate (perpendicular to the direction of motion of the heat source) (m)
 y_i – dependent variable (width of the plastic zone size)
 α - coefficient of expansion (1/°C)
 τ – delay time (s)
 η - heating efficiency coefficient
 λ – thermal conductivity (W/m°C)
 ν – Poisson's ratio
 ζ – normalized width of inherent strain zone with respect to b_0 which is the width for an infinitely wide plate,
 ρ - density (kg/m³)
 σ_{YB} - yield stress of base metal (N/mm²)

1. Introduction

The prediction of welding-induced distortions is a critical aspect of the design and fabrication of large steel structures, including marine and offshore constructions. Welding processes inherently generate complex thermomechanical cycles involving localized heating, melting, and solidification, followed by rapid cooling. These transient phenomena give rise to residual stresses and distortions that may persist throughout the service life of a structure. Accurate prediction of such distortions is therefore essential for ensuring dimensional accuracy, minimizing corrective operations, and maintaining structural integrity. As reported in numerous studies, welding-induced deformations remain a significant source of assembly inaccuracies and increased manufacturing costs in industrial practice [1-3].

In the Submerged Arc Welding (SAW) process, which is widely used for joining thick structural steels in large-scale applications, the combination of high heat input and deep penetration intensifies thermal gradients and plastic deformation within the heat-affected zone (HAZ) and the weld metal. Traditional thermo-elasto-plastic finite element analyses (TEP-FEM) can

predict welding-induced distortions with high accuracy; however, their computational cost becomes prohibitive when large or geometrically complex structures are considered. Recent advances in welding simulation have shifted towards hybrid and simplified modelling approaches in which inherent strain—the irreversible component of strain induced by welding thermal cycles—is employed as an equivalent load to predict the final deformation. The pioneering work of Ueda et al. [4, 5] established inherent strain as the primary source of welding residual stresses and demonstrated its effectiveness in predicting welding distortions.

The estimation of the extent of the inherent strain region is closely linked to the prediction of welding-induced plastic deformation zones. Consequently, accurate determination of the plastic deformation zone width constitutes a key element of simplified welding distortion analyses based on equivalent load and inherent strain methodologies.

Accurate prediction of the plastic deformation zone requires realistic thermal modelling of the welding process. The classical moving heat source theory developed by Rosenthal [6] provided the first analytical description of temperature fields in welded plates and established a fundamental framework for understanding heat transfer during welding. Despite the availability of more sophisticated heat source models in contemporary numerical simulations, Rosenthal's solutions remain essential for interpreting the influence of welding heat input, plate thickness, and material thermal properties on temperature fields, thermal gradients, and the resulting deformation mechanisms.

Accurate estimation of the width of the plastic deformation zone (b_{pl}), often referred to as the inherent strain zone, is a prerequisite for the successful application of inherent strain-based modelling approaches. This parameter governs the spatial distribution of equivalent loads and therefore has a direct influence on the accuracy of distortion predictions. Previous studies have demonstrated that b_{pl} is affected by welding heat input, joint geometry, thermal gradients, and restraint conditions, and should therefore be determined using experimental measurements, numerical simulations, or analytical formulations [3, 7].

Another important aspect of welding distortion prediction is the thermal modelling approach employed. Gaussian heat source models remain among the most widely used representations of welding heat input in numerical simulations. They enable physically realistic control of heat flux distribution and penetration characteristics, both of which have a direct influence on local temperature fields and, consequently, on the development and extent of the plastic deformation zone. Recent investigations have shown that variations in Gaussian heat source parameters can significantly affect the size of the heat-affected zone (HAZ) as well as the magnitude of welding-induced distortions [1].

Furthermore, statistical methodologies based on Design of Experiments (DOE) have become increasingly important in numerical welding research. DOE enables the systematic identification of influential process parameters and the development of efficient surrogate models, thereby reducing the computational burden associated with extensive thermo-elasto-plastic simulations.

Analytical approaches for estimating the extent of the plastic deformation zone have also received considerable attention. These methods, generally based on simplified heat transfer models and elastic–plastic theory, seek to provide computationally efficient estimates of plastic zone boundaries and residual stress distributions. Several studies have demonstrated that analytical formulations incorporating welding heat input can predict plastic deformation zone dimensions with good agreement to results obtained from finite element simulations [8, 9].

The available approaches relevant to the prediction of the plastic deformation zone width in SAW-welded structural steel butt joints are critically reviewed in this study. Based on this background, the work aims to develop a robust predictive methodology integrating planned numerical simulations, analytical models, and validated heat source representations. The

proposed framework is intended to provide reliable estimates of the plastic deformation zone width (b_{pl}) in large-scale welded structures used in marine and offshore applications.

The plastic deformation zone surrounding a weld is of fundamental importance in the analysis of welding-induced distortions, as it defines the region in which irreversible strains are generated by thermomechanical loading. The concept of inherent strain, commonly defined as the non-recoverable strain remaining after unloading, has served as a cornerstone of welding distortion prediction methodologies for several decades. Ueda and Yuan demonstrated that inherent strain consists of contributions arising from weld metal shrinkage, plastic deformation of the adjacent material, and thermal expansion mismatch. They further showed that inherent strain constitutes the fundamental source of welding residual stresses and the resulting deformation of welded structures [4].

The width of this zone, commonly denoted as b_{pl} , defines the spatial extent of inherent strains applied in simplified welding distortion models. As such, it has a direct and significant influence on the magnitude and distribution of the predicted distortions. Experimental observations and numerical simulations consistently indicate that an increase in the width of the plastic deformation zone is accompanied by greater transverse shrinkage and more pronounced angular distortion. For example, recent studies on thin-plate laser-welded butt joints demonstrated that larger weld gaps promote the development of more extensive plastic strain regions, leading to increased welding-induced deformation [3].

Advanced thermo-elasto-plastic finite element models (TEP-FEM) provide detailed information on the mechanisms governing the formation and evolution of plastic deformation zones during welding. For instance, Wang et al. demonstrated, using coupled thermal-mechanical simulations, that peak temperatures and thermal gradients play a decisive role in determining the distribution of plastic strains in welded thin-walled beams. Their study also confirmed the validity of inherent strain-based predictions through comparison with experimental results [7]. Furthermore, other investigations have shown that geometric characteristics of welded components, including plate dimensions, curvature, and restraint conditions, exert a significant influence on plastic deformation patterns and the magnitude of the resulting welding distortions [2].

Understanding the morphology and spatial extent of b_{pl} is essential for simplified welding distortion prediction methods based on equivalent loads and inherent strain formulations. Previous studies have shown that accurate representation of the plastic deformation zone significantly improves the accuracy and computational efficiency of distortion prediction methods based on inherent strain theory [10,11].

In summary, accurate determination of the width of the plastic deformation zone remains a fundamental requirement for modern welding distortion prediction methodologies. Whether derived from experimental measurements, numerical simulations, or analytical formulations, this parameter governs the representation of inherent strains and equivalent loads, thereby directly affecting the reliability and accuracy of distortion predictions. Consequently, reliable estimation of b_{pl} is of particular importance in the design and fabrication of large-scale welded structures used in marine and offshore applications.

The geometrical interpretation of both the plastic deformation zone and the inherent strain zone has its origins in the pioneering studies of Ueda and co-workers. Their work demonstrated that the width of the plastic deformation zone and the effective width of the inherent strain zone are closely related, although they should not be regarded as identical quantities. This distinction is of particular importance in inherent strain-based distortion prediction methods, where inherent strain distributions are converted into equivalent loads applied to simplified numerical models. Accurate representation of both zones is therefore essential for achieving reliable distortion predictions [4, 5].

Gaussian heat source models remain among the most widely used approaches for representing welding heat input in numerical simulations. Their popularity stems from the ability to control penetration characteristics, surface heat flux distribution, and overall heat input in a physically meaningful manner. As a result, they are particularly well suited to arc welding processes such as Submerged Arc Welding (SAW). Numerous studies have demonstrated that variations in Gaussian heat source parameters significantly influence temperature fields, heat flux concentration, and the extent of the heat-affected zone (HAZ), thereby affecting the formation of plastic deformation zones and the magnitude of welding-induced distortions [12].

Accurate modelling of the thermal field is a prerequisite for reliable estimation of b_{pl} , since the boundary of the plastic deformation zone is closely associated with regions in which peak temperatures exceed material-dependent thresholds for the onset of plastic yielding. Gaussian heat source models have been widely and successfully employed in welding simulations of plate structures, providing realistic representations of heat transfer and enabling accurate reproduction of experimentally observed deformation behaviour [13].

Moreover, recent numerical investigations have demonstrated that accurate calibration of Gaussian heat source parameters significantly improves the extraction of inherent strain distributions from thermo-elasto-plastic finite element analyses (TEP-FEM). As a result, the predictive capability and computational efficiency of simplified inherent strain-based distortion prediction methods can be substantially enhanced [7].

Design of Experiments (DOE) methodologies have become increasingly important in the development of predictive models for welding-induced deformation. By enabling the systematic identification of significant process parameters, DOE supports the efficient characterization of factors governing plastic deformation zone width and welding distortion. DOE-based approaches have been successfully applied to develop predictive regression models and response-surface approximations capable of replacing computationally expensive thermo-elasto-plastic analyses while maintaining satisfactory engineering accuracy [12,14-16].

Analytical approaches have also been developed to provide simplified and computationally efficient estimates of the extent of plastic deformation zones. These methods generally seek to establish quantitative relationships among welding heat input, temperature fields, and the extent of plastic deformation. Bhardwaj and Ratnayake proposed an analytical formulation based on welding heat input for estimating welding-induced plastic zone dimensions and demonstrated good agreement with finite element predictions [8]. In a related study, Pei, Song, and Dong derived an analytical expression relating plastic zone boundaries to maximum temperature distributions. Their formulation was validated through comparison with residual stress distributions obtained from thermo-elasto-plastic simulations, confirming its applicability to welding distortion analyses [9].

More recently, Grams [17] developed analytical predictive formulations that relate material properties, welding heat input, and structural geometry to both the extent of the plastic deformation zone and the evolution of residual stresses. A key feature of these formulations is the introduction of the concept of the first yield temperature, which enables the width of the plastic deformation zone to be estimated directly from fundamental thermal and mechanical material properties. By combining physically based analytical relationships with parameters commonly used in welding simulations, these formulations provide an important link between classical analytical approaches and modern numerical methodologies for welding distortion prediction.

Collectively, these studies demonstrate that analytical formulations can serve as accurate and computationally efficient alternatives to numerical simulations for estimating the width of the plastic deformation zone. Such approaches are particularly valuable during the preliminary design stage and in welding process optimization, where rapid engineering assessments are often required.

2. Heat source model

2.1. Governing Heat Conduction Equation

Transient heat transfer during welding is governed by the Fourier–Kirchhoff equation:

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (\lambda \nabla T) + Q(x, y, z, t) \quad (1)$$

For isotropic materials with temperature-independent thermal conductivity, Eq. (1) can be written as:

$$\frac{\partial T}{\partial t} = a \nabla^2 T + \frac{Q}{\rho c} \quad (2)$$

where

$$a = \frac{\lambda}{\rho c} \quad (3)$$

is the thermal diffusivity.

The analytical foundation for describing temperature fields generated by a moving welding heat source was established by Rosenthal, who derived closed-form solutions of Eq. (1) for a moving point heat source and demonstrated the influence of welding speed, heat input, and plate thickness on the resulting temperature distribution [6].

Rosenthal's solutions further revealed the transition from quasi-two-dimensional to three-dimensional heat flow conditions with increasing plate thickness, providing the basis for the classification of welded plates according to their thermal behaviour.

2.2. Rosenthal Heat Source Model

For a moving heat source travelling at a constant velocity v , Rosenthal proposed the following quasi-steady-state solution:

$$T - T_0 = \frac{\eta Q}{2\pi\lambda r} \exp\left(\frac{v(x - r)}{2a}\right) \quad (4)$$

Although Rosenthal's model remains fundamental for interpreting welding thermal cycles, its point heat source assumption cannot accurately reproduce the finite dimensions of a real welding arc. Consequently, modern finite element analyses typically employ distributed heat source models.

2.3. Surface Gaussian Heat Source Model

In numerical simulations of welding processes, particularly for structural steel butt joints welded by the Submerged Arc Welding (SAW) process, the selection of an appropriate heat source model is crucial for obtaining realistic predictions of temperature fields, thermal gradients, and the resulting plastic deformation. Among the available heat input representations, the surface Gaussian heat source model remains one of the most widely used due to its mathematical simplicity, numerical stability, and ability to provide a physically realistic description of arc energy distribution. These characteristics make it particularly suitable for the present study focused on predicting the width of the plastic deformation zone.

The Gaussian heat source model is based on the assumption that the heat flux delivered to the workpiece reaches its maximum at the centre of the heat source and decreases radially according to a normal (Gaussian) distribution. This assumption is consistent with experimental observations of welding arc characteristics, where both temperature and current density exhibit approximately Gaussian radial distributions.

In its surface formulation, the heat source is represented as a two-dimensional heat flux distribution applied to the surface of the workpiece. Consequently, it is particularly suitable for welding processes in which heat input is predominantly concentrated near the surface, including SAW applications characterized by stable arc behaviour and uniform energy transfer.

In transient thermal–mechanical analyses, the Gaussian heat source is typically modelled as a moving surface heat flux travelling along the weld line at a constant welding speed, v_s :

$$q(x, y, t) = \frac{3Q\eta}{\pi r_0^2} \exp\left(-3\frac{y^2}{r_0^2}\right) \exp\left(-3\frac{[x + v_s(t - \tau)]^2}{r_0^2}\right) \quad (5)$$

The theoretical temperature distributions and the geometrical relationships defining the moving Gaussian heat source are illustrated in Fig. 1.

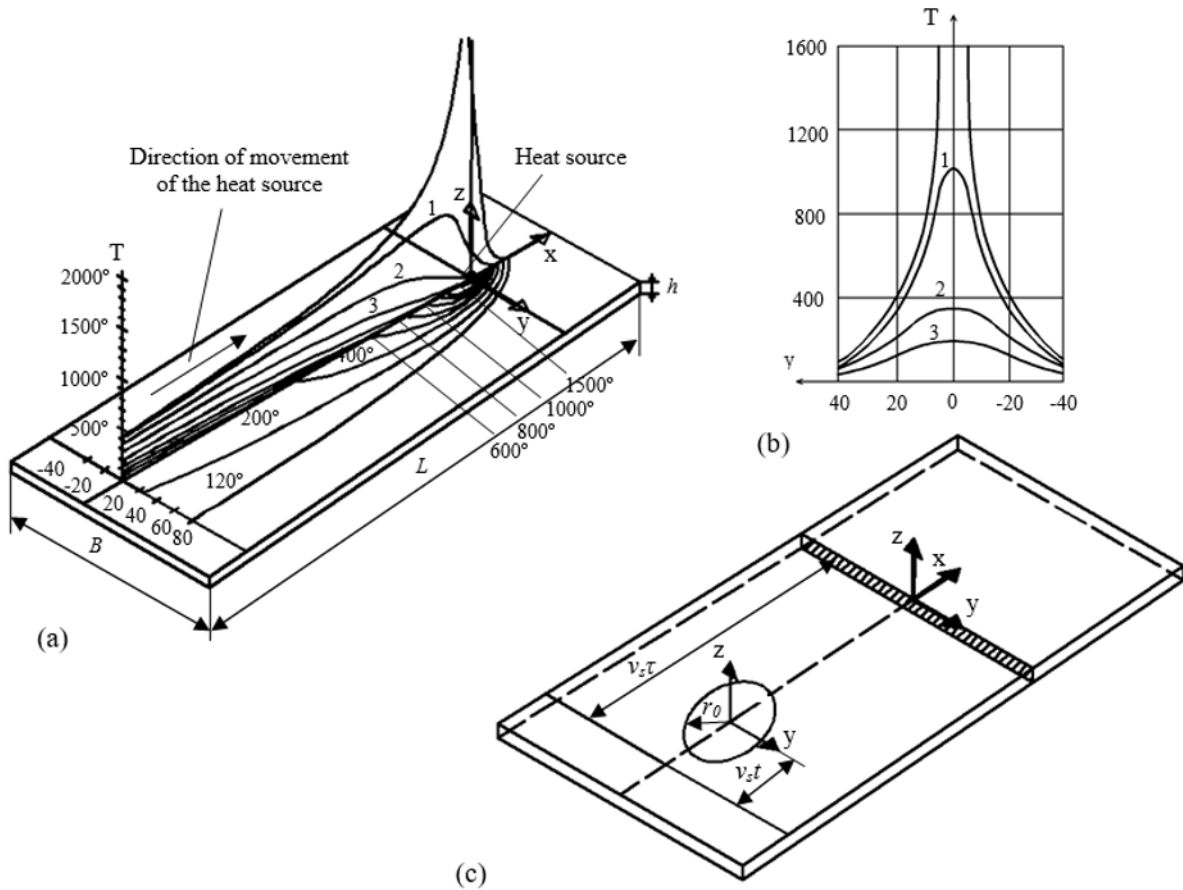


Fig. 1 Theoretical temperature distributions and heat source model:
 (a) temperature distribution along the direction of heat source movement,
 (b) temperature distribution transverse to the direction of heat source movement,
 (c) geometric relationships of the heat source model

The coefficient 3 appearing in both the exponential term and the normalization factor ensures that the integral of the heat flux distribution over the entire surface is equal to the effective heat

input applied to the workpiece. Although alternative normalization coefficients may be encountered in different implementations of the Gaussian heat source model, the underlying concept of an exponentially decaying radial heat flux distribution remains the same.

Accurate determination of the thermal efficiency factor is essential for reliable thermal modelling, since overestimation may result in unrealistically large weld pools and excessive heat penetration, whereas underestimation may lead to insufficient melting and inaccurate prediction of temperature fields and thermal gradients.

The parameter r_0 governs the spatial extent of the heat flux distribution and therefore plays a critical role in determining thermal gradients, peak temperatures, and the resulting deformation patterns. Accurate determination of this parameter is essential for reliable thermal modelling and may be achieved using several approaches:

- (a) Empirical estimation, based on measured weld bead dimensions or the effective arc footprint.
- (b) Inverse modelling and calibration, in which experimentally recorded thermal cycles, obtained for example from thermocouple measurements or infrared thermography, are reproduced numerically through iterative adjustment of r_0 .
- (c) Physics-based estimation, where the characteristic radius is derived from welding arc behaviour, process parameters, and flux layer characteristics.

For SAW butt-welded structural steels, reported values of r_0 typically range from approximately 4 to 10 mm, depending on welding current, electrode diameter, heat input, and flux composition. Since r_0 directly influences heat concentration and penetration characteristics, its calibration is often a key step in ensuring agreement between simulated and experimentally measured temperature fields.

2.4. Relevance to Predicting Plastic Deformation Zones

Since the width of the plastic deformation zone is strongly dependent on the temperature distribution generated during welding, the accuracy of the heat source model directly affects the predicted value of b_{pl} . For SAW applications, the surface Gaussian heat source provides an adequate representation of the thermal field and was therefore adopted in the present study.

3. Experimental study – finite element simulation

3.1. Numerical Experiment Design

Numerical simulations were conducted according to the principles of experimental design theory. The numerical model was treated as a so-called black box into which purposefully selected technological and design parameters were introduced. These parameters constituted independent variables affecting the width of the plastic deformation zone in the welded joint.

The factors determining the size of this zone were assumed to be:

- (a) For the first group of plates, referred to as thin plates (i.e., plates with thicknesses typical of marine hull structures): the welding heat input, the yield strength of the base material, and the component thickness.
- (b) For the second group of plates, referred to as thick plates (i.e., plates used in offshore structures): the welding heat input, the preheating temperature (i.e., the initial temperature), and the component thickness.

The width of the plastic deformation zone, recorded at the output of the black-box model, was adopted as the dependent variable. A graphical representation of the structure of the planned numerical experiment is shown in Fig. 2.

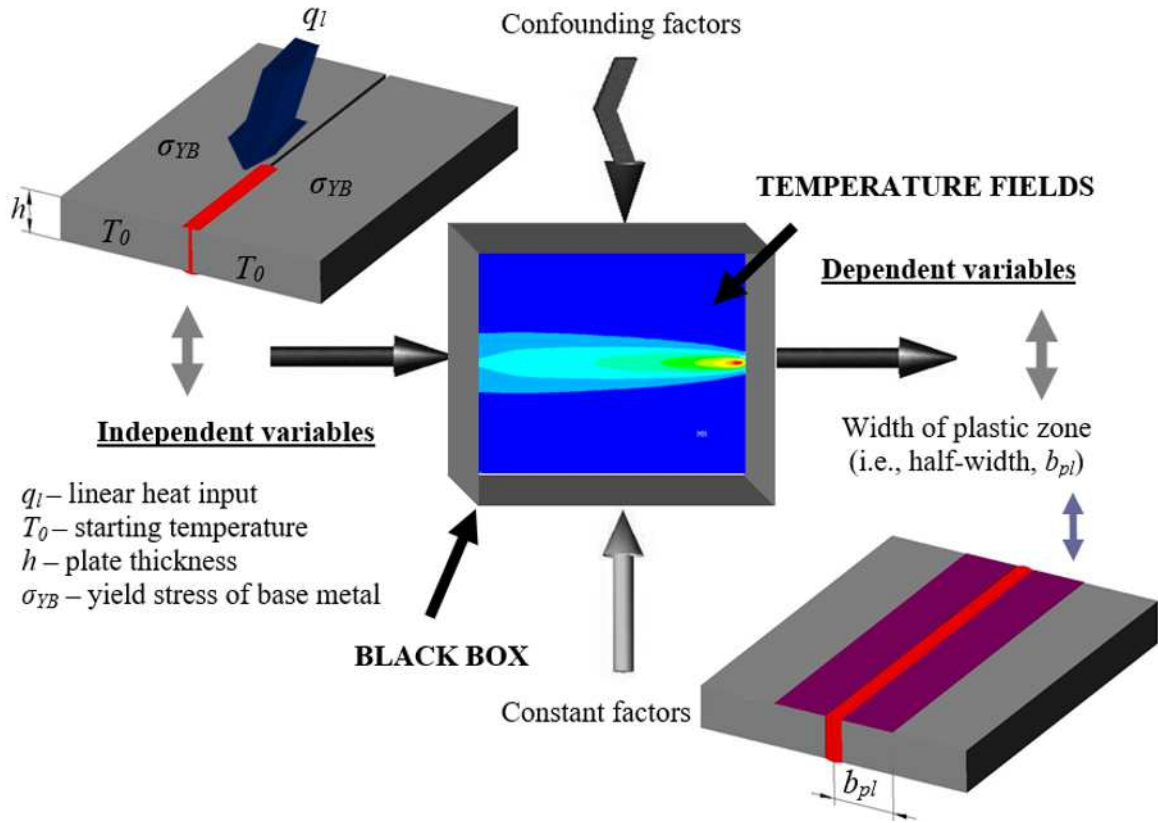


Fig. 2 Black-box model of the planned numerical experiment used for predicting the width of the plastic deformation zone

The model also incorporated two categories of factors associated with the computational process:

- Noise factors, encompassing numerical uncertainties arising, for example, from the discretization of the model.
- Fixed factors, including the adopted modelling assumptions and parameters that were not varied during the study.

Since these factors are neither controllable nor directly measurable, they were not considered experimental variables. However, their effects are implicitly embedded in the simulation results, which is characteristic of experiments based on the adopted methodology.

The selection of the independent variables considered critical to the width of the plastic deformation zone in the analysed welded joints was based on the thermomechanical mechanisms occurring during the welding thermal cycle as well as on the physical properties of the welded components. In particular:

- The welding heat input is the primary parameter describing the amount of heat introduced into the material during the welding process. It directly affects the temperature distribution within the material and, consequently, the thermal gradients, the extent of overheating, and the size of the heat-affected zone (HAZ). Since the HAZ is subjected to the highest stresses and strains, the welding heat input deterministically governs the width of the plastic deformation zone. Higher heat input results in longer cooling times and a larger extent of plasticization, leading to an increase in the width of the investigated zone. This variable was included in both plate groups because, in each case, it determines the intensity of heat input to the welded joint, regardless of plate thickness or material grade [18-21]. The welding heat input was calculated using Eq. (6).

$$q_l = \frac{\eta \cdot U \cdot I}{v_s} \cdot k_e \quad (6)$$

We note that for SAW $\eta = 0.91\text{--}0.99$.

- (b) The yield strength of the base material determines the stress threshold at which permanent plastic deformation occurs. Lower yield strength promotes the development of a larger plastically deformed region around the weld pool due to the action of welding-induced residual stresses, thereby increasing the width of the plastic deformation zone. For plates of moderate thickness, commonly used in ship hull structures, the mechanical response of the material to thermal loading is a key factor governing the extent of plastic deformation. In contrast, for thicker plates used in offshore structures, the influence of this variable becomes less pronounced, as the overall response is governed primarily by large thermal gradients and the necessity to control the initial (preheating) temperature [22, 23].
- (c) The thickness of the welded component is a key factor governing heat flow and the thermal boundary conditions associated with the welding process. An increase in thickness enhances the material's ability to conduct heat into the bulk of the section, thereby affecting the cooling characteristics and, consequently, the extent of plastic deformation. While this effect is relatively moderate for thin plates, it becomes dominant for thick plates, where the thermal mass of the material strongly influences the temperature field and cooling behaviour [23-25]. For the purposes of this study, a qualitative classification of plates into thin and thick categories, based on thermal phenomena associated with welding, is presented in Fig. 3.
- (d) For plates employed in offshore structures, preheating is a standard technological measure used to reduce cooling rates and mitigate the risk of cold cracking [26]. The initial temperature of the material affects both microstructural evolution and the extent of plastic deformation. An increase in the preheating temperature reduces thermal gradients and welding-induced shrinkage stresses, thereby influencing the width of the plastic deformation zone. Consequently, this variable was included in the analysis of thick plates, for which thermal effects are more pronounced and generally outweigh the influence of the material's mechanical properties [27-29].

The selection of the experimental design was guided by three criteria [30, 31]: informativeness, feasibility, and efficiency. Based on these criteria and taking into account the specific characteristics of the analysed process, a three-level Box–Behnken design (BBD) was selected for the numerical experiment. The design consisted of fifteen experimental runs for three independent variables, each varied at three levels (i.e., minimum, medium, and maximum), as shown in Table 1. The Box–Behnken design has previously been applied, among others, in [32] to develop a regression model for optimizing the parameters of the Gas Metal Arc Welding (GMAW) process.

The ranges assigned to the independent variables were selected to reflect the technological and design parameters encountered under real operating conditions, including industrial practice, as summarized in Table 2. The technological parameters corresponding to the three levels of welding heat input adopted for the analysed welding process are provided in Table 3.

The ranges cover the full range of welding conditions encountered in industrial practice, including combinations located near the technological limits of the investigated design space. This approach was adopted intentionally in order to ensure that the resulting predictive relationships remained applicable over the entire range of practical welding conditions considered in the study.

Table 1 Box–Behnken design matrix for three independent variables with normalized factor levels (-1, 0, +1)

Numerical Experiment No.	x_1 – Welding Heat Input, q_l (tp, hp)	x_2 – Yield Strength, σ_{YB} (tp) / Initial Temperature, T_0 (hp)	x_3 – Plate Thickness, $\underline{h}$ (tp, hp)
1	-1	-1	0
2	+1	-1	0
3	-1	+1	0
4	+1	+1	0
5	-1	0	-1
6	+1	0	-1
7	-1	0	+1
8	+1	0	+1
9	0	-1	-1
10	0	+1	-1
11	0	-1	+1
12	0	+1	+1
13	0	0	0
14	0	0	0
15	0	0	0

Table 2 Actual levels of the independent variables adopted in the Box–Behnken experimental design

Independent variable (unit)	Plastic deformation zone width model					
	Prediction of b_{pl} for Thin Plates (b_{pl} -tp)			Prediction of b_{pl} for Thick Plates (b_{pl} -hp)		
	min.	centre	max.	min.	centre	max.
q_l (kJ/mm)	1.1	1.7	2.3	1.1	1.7	2.3
σ_{YB}^a (N/mm ²)	230	330	430	-	-	-
T_0 (°C)	-	-	-	100	150	200
h (mm)	6	16	26	30	40	50

^aThe yield strength levels adopted in the experimental design were represented by selected structural steel grades: ASTM A36 corresponding to $\sigma_{YB} = 230$ MPa, ASTM A572 Grade 50 corresponding to $\sigma_{YB} = 330$ MPa, and CMn steel corresponding to $\sigma_{YB} = 430$ MPa. For thick plates used in offshore structures, S355G10+M steel was adopted as the representative material.

Table 3 Technological parameters of the SAW process corresponding to the three levels of welding heat input adopted in the experimental design (based on [33])

No.	Level of ranges of changes (-)	Heating efficiency coefficient (-)	Current (A)	Voltage (V)	Welding speed (mm/min)
1	min.	0.99	350	27	500
2	centre		450	29	450
3	max.		500	31	400

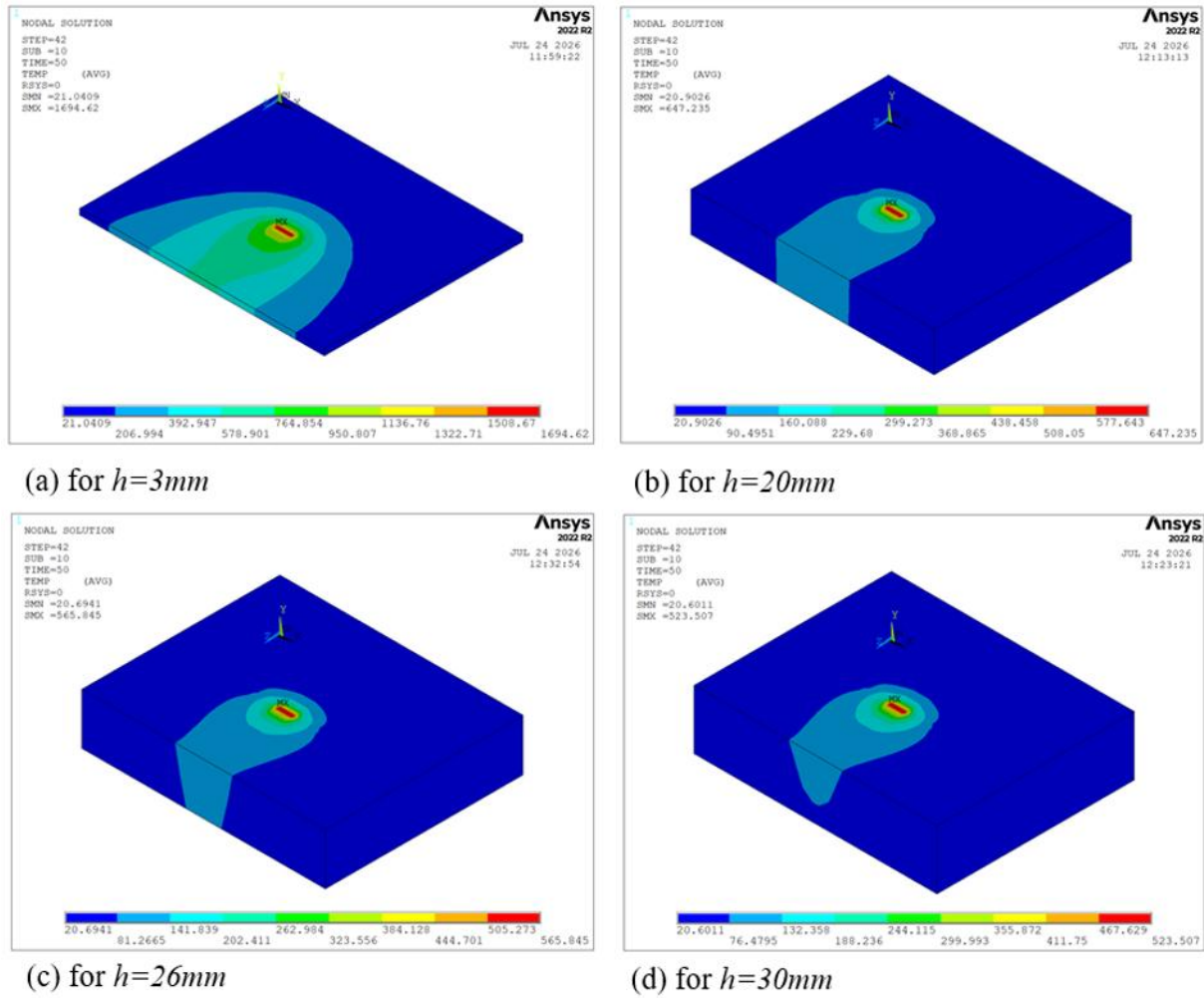


Fig. 3 Comparison of temperature distributions in steel plates of different thicknesses: (a) 3 mm, (b) 20 mm, (c) 26 mm, and (d) 30 mm

Figure 3 provides qualitative evidence supporting the adopted division of plates into thin-plate and thick-plate categories. It should be noted that this classification is based primarily on the physical interpretation of the obtained temperature distributions. The numerical results confirm the validity of the adopted ranges, namely 6–26 mm for thin plates and 30–50 mm for thick plates.

A noticeable change in the character of the temperature field is observed between the cases of $h = 26\text{ mm}$ and $h = 30\text{ mm}$, indicating a transition to a different heat conduction regime characteristic of massive structural elements. The thickness of 26 mm may therefore be regarded as the upper boundary of the transitional region, whereas a thickness of 30 mm already exhibits the thermal behaviour typical of thick plates.

The temperature distribution patterns obtained from the numerical simulations and presented in Fig. 3 are in good agreement with classical heat-transfer theories for welding processes. In particular, they reflect Rosenthal's analytical solutions for a moving heat source [6] and the widely reported transition from quasi-two-dimensional heat flow in thin plates to three-dimensional heat flow in thick sections [23, 24], [34].

With increasing plate thickness, the temperature gradient through the thickness becomes more pronounced, while the temperature field progressively localizes within the near-surface region. This change in thermal behaviour is characteristic of thick structural elements and is consistent with experimental observations and numerical investigations concerning the influence of plate

thickness on welding thermal cycles, heat flow mechanisms, and the development of the heat affected zone (HAZ).

The observed transition between the cases of $h = 26$ mm and $h = 30$ mm therefore provides additional physical justification for the adopted classification of plates into thin (6–26 mm) and thick (30–50 mm) categories.

3.2. Thermal modeling

The temperature field in the analysed structural steel butt joints was simulated using thermo-elasto-plastic finite element analyses performed in ANSYS and implemented through APDL (ANSYS Parametric Design Language) scripts.

A surface Gaussian heat source model was adopted, as it is commonly used in numerical simulations of welding processes, particularly when the heat input may be represented as a distributed surface heat flux (see Fig. 3 and Section 2.3, Eq. (5)). The selected model provides an accurate representation of the temperature field and the resulting plastic deformation zone width while maintaining reasonable computational efficiency.

The numerical models consisted of butt-welded plates with constant in-plane dimensions ($L \times B = 600 \times 250$ mm). The plate thickness (h) was treated as a variable parameter and selected according to the corresponding experimental design configuration (see Fig. 1, Table 1, and Table 2).

Transient thermal analyses were performed using a moving Gaussian heat source and PLANE55 four-node, two-dimensional thermal solid elements.

A constant value of the effective heat source radius ($r_0 = 5$ mm) was adopted throughout the study. Although the characteristic radius of a Gaussian heat source may vary with welding current, electrode diameter, arc characteristics, and process configuration, a fixed value was intentionally selected in order to isolate the influence of the primary variables considered in the Design of Experiments, namely welding heat input, plate thickness, yield strength, and preheating temperature. The adopted value lies within the range commonly reported for SAW processes and was considered representative of the investigated welding conditions. Consequently, variations in heat source radius were not included as an additional experimental factor.

A non-uniform finite element discretization was adopted in all numerical models (Fig. 4). To ensure accurate representation of the Gaussian heat source with an effective radius (r_0) of 5 mm, element widths of 2.5 mm and 1.25 mm were used within the weld region for thin and thick plates, respectively.

In regions remote from the weld, a mesh size of 5 mm was applied. Additional local refinement was introduced in the area used for determining the width of the plastic deformation zone (b_{pl}), where the minimum element width was reduced to 0.25 mm. To avoid abrupt changes in discretization density, a smooth mesh gradation was employed using element widths of 5 mm, 2 mm, 1 mm, 0.5 mm, and 0.25 mm. The adopted mesh refinement strategy is consistent with recent studies demonstrating that local refinement in the vicinity of the weld is sufficient for accurate prediction of temperature fields and welding distortions [35,36].

Outside the refined zone, towards the plate boundaries, element widths of 5 mm, 10 mm, and 20 mm were applied. The element length was maintained at 5 mm throughout the models. The resulting meshes consisted of 9559 nodes and 9360 elements for the thin and thick-plate models. Transient thermal analyses were conducted using a time increment of $\Delta t = 0.2$ s. Depending on the welding speed (v_s) corresponding to the selected levels of welding heat input (see Table 3), the delay factor (τ) adopted in the moving heat source model was equal to 0.60 s, 0.67 s, and 0.75 s, respectively.

The initial temperature of the numerical model was assumed to be equal to the ambient temperature of 20°C. Heat losses to the surrounding environment were represented by convection boundary conditions applied to all exposed surfaces. A constant convective heat transfer coefficient of 15 W/(m²·°C) was assumed throughout the analysis.

The width of the plastic deformation zone was determined on the basis of the maximum temperature distribution by adopting, as its boundary, the isotherm corresponding to the onset of yielding of the material.

Grams [17] introduced a concept of the first yield temperature as the temperature required to initiate plastic deformation. This temperature corresponds to the intersection of the elastic and plastic material response curves and represents the threshold at which irreversible deformation begins to develop. According to [17], the first yield temperature may be expressed as:

$$T_{Y1} = \Delta T_{Y1} + T_0 = \frac{\sigma_{YB}}{\alpha \cdot E} \Big|_{T_{Y1}} + T_0 \quad (7)$$

Consequently, the width of the plastic deformation zone was taken as the maximum width of the first yield isotherm, corresponding to the temperature field defined by $T(x, y) = T_{Y1}$.

The measurement of the plastic deformation zone width was performed at the mid-length section of the plate, where the influence of the welding start and end regions was minimized.

The thermophysical properties of the analysed steel grades were adopted from literature sources and are shown in Fig. 5. Temperature-dependent thermal conductivity and specific heat capacity were incorporated into the model, whereas latent heat effects and phase transformation phenomena were neglected. Since the objective of the analysis was limited to determining temperature distributions and the resulting plastic deformation zone width, the influence of chemical composition on metallurgical transformations was not considered.

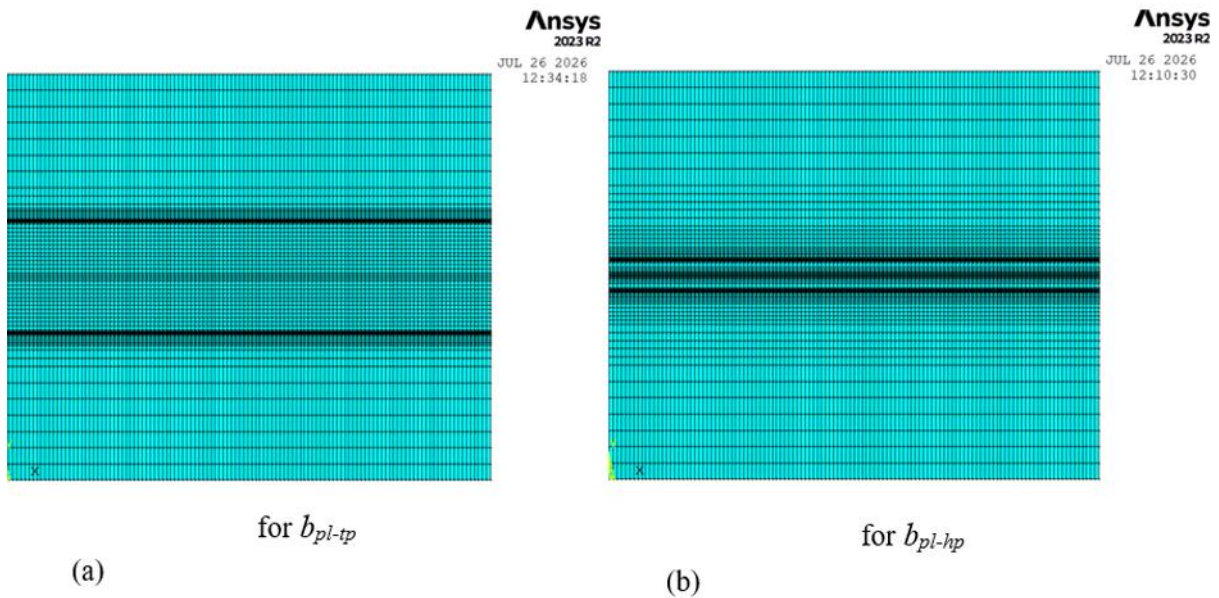


Fig. 4 Non-uniform finite element discretization used in the thermal analysis: (a) for b_{pl-tp} , (b) for b_{pl-hp}

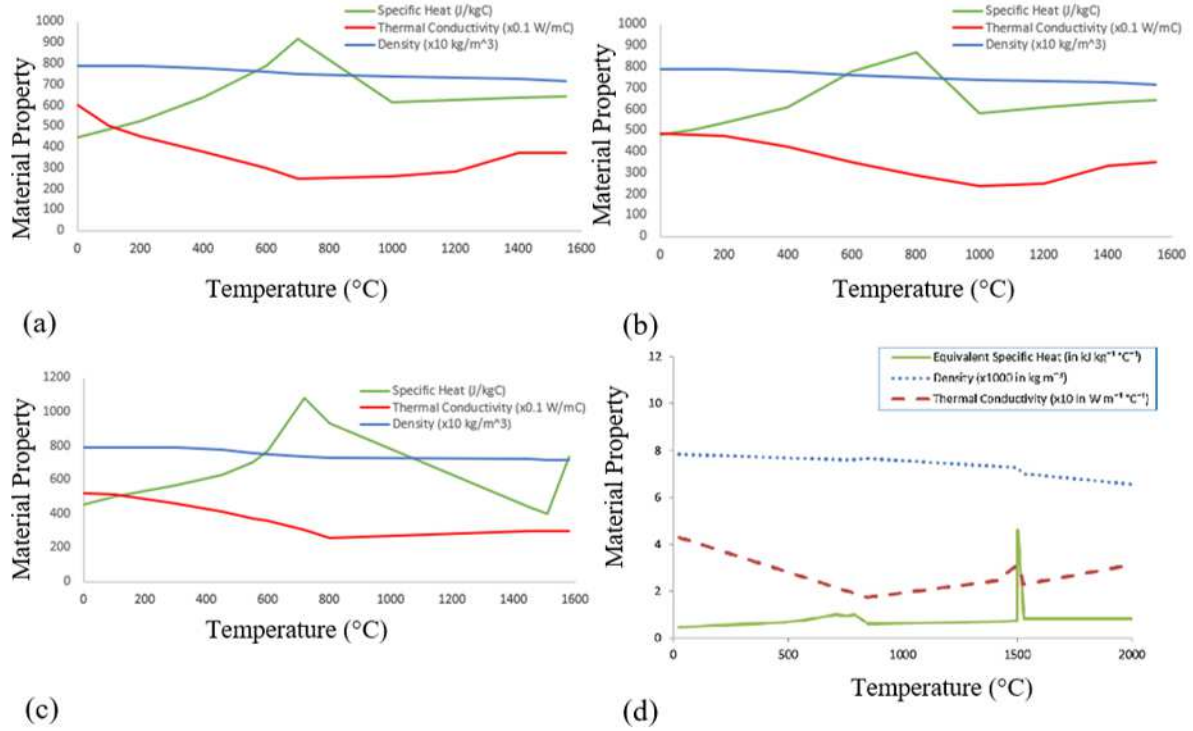


Fig. 5 Temperature-dependent thermophysical properties of the structural steels used in the numerical simulations: (a) ASTM A36, (b) ASTM A572 Grade 50, (c) CMn steel, and (d) S355G10+M (based on [17], [13], [37, 38])

3.3. Verification of the numerical model

The developed numerical model was verified through comparison with benchmark results reported in the literature. As a reference, the numerical model developed by Grams [17] for ASTM A36 structural steel and AISI 316L stainless steel was selected. Verification against two materials exhibiting substantially different thermal properties was considered a more stringent test of the model than verification performed for a single material only.

The thermophysical properties adopted for ASTM A36 were those shown in Fig. 5(a), whereas the properties of AISI 316L stainless steel were taken directly from [17].

At this point, it should be emphasized that, although the analytical formulations proposed by Bhardwaj and Ratnayake [8] and by Pei et al. [9] constitute valuable contributions to the prediction of plastic deformation zone dimensions, they could not be adopted for either purpose pursued in the present study, for two distinct reasons.

First, with respect to the comparative analysis presented in Section 4, both formulations define the plastic zone boundary using a criterion ($\Delta T_{max} \geq 2\Delta T_{Y1}$, corresponding to the onset of residual yield-magnitude strain after the complete heating–cooling cycle) that is not equivalent to the first-yield-temperature criterion ($T = T_{Y1}$) adopted in the present study following Grams [17]. Consequently, a direct comparison between the plastic zone widths obtained in the present study and those predicted by [8] and [9] would not be physically meaningful, as the two quantities are not defined on a consistent basis.

Second, with respect to the verification of the numerical model described in Section 3.2, neither [8] nor [9] provides a sufficiently detailed and reproducible numerical framework — i.e., complete material properties, heat source parameters, and modelling procedure — that could be reconstructed under identical welding and material conditions. In contrast, the work of Grams [17] reports the numerical model, material properties, heat source parameters, and

methodology in sufficient detail to enable a direct and reproducible comparison. Moreover, the concept of the first yield temperature introduced by Grams [17] forms an integral part of the methodology adopted in the present work, which further supports its selection as the reference case for verification.

Verification was carried out using the same welding process parameters as those employed by Grams [17]. A summary of these parameters, together with the remaining components of the Gaussian heat source model, is given in Table 4. The reference numerical model consisted of a plate measuring $2400 \times 2000 \times 3$ mm and employed a cylindrical heat source representation. Local mesh refinement to an element width of 2.5 mm was applied in the vicinity of the heat source, and the model was discretized using eight-node isoparametric brick elements.

The resulting comparison between the numerical results obtained in the present study and the reference data reported by Grams [17] is provided in Table 5.

Table 4 Summary of welding parameters, heat source radius, and delay factor adopted from Grams [17]

I (A) [17]	U (V) [17]	v_s (mm/min) [17]	η (-) [17]	q_l (J/mm) [17]	r_0 (mm) [17]	τ (s)
98	16	300	0.77	240	5	1.00

Table 5 Comparison of plastic deformation zone widths obtained from the developed numerical model and the reference data reported by Grams [17]

Mat.	σ_{YB} (MPa) [17]	E (GPa) [17]	α (1/°C) [17]	T_0 (°C) [17]	T_{YI} (°C) [17]	b_{pl} Sim. (mm) [17]	b_{pl} Sim. (mm)	b_{pl} Sim. (diff.) (%)
A36	201	197	12.6×10^{-6}	20	101	63	56.75	9.92
316L	195	192	15.1×10^{-6}	20	87	71	65.25	8.10

The results summarized in Table 5 demonstrate that the developed numerical model is capable of predicting the width of the plastic deformation zone (b_{pl}) with satisfactory accuracy for both analysed materials. The differences between the predicted values and the reference results reported by Grams [17] did not exceed approximately 8% for ASTM A36 steel and 10% for AISI 316L stainless steel. Considering the substantial differences in the thermophysical properties of the two materials, the obtained agreement confirms the reliability of the adopted modelling approach and supports its application in subsequent numerical analyses.

From an engineering perspective, the observed level of agreement was considered sufficient for the purposes of deriving predictive relationships for the plastic deformation zone width and analysing the influence of welding process parameters.

The verification study revealed discrepancies of approximately 8–10% between the predicted and reference values of the plastic deformation zone width. This level of agreement is considered acceptable for engineering applications involving welding distortion assessment, particularly in view of the simplifications adopted in the thermal model and the differences between the numerical implementations used for comparison.

The observed discrepancies may be attributed not only to differences in element type and time-step size, but also to variations in mesh topology, heat source representation, numerical integration procedures, and the treatment of temperature-dependent material properties.

Importantly, the practical implications of these differences are limited, since the plastic deformation zone width is not the final design quantity but rather an intermediate parameter employed in simplified distortion prediction methodologies. In equivalent-load and inherent-

strain approaches, the resulting deformation is additionally affected by structural stiffness, boundary conditions, restraint conditions, and joint geometry.

Consequently, an uncertainty of approximately 8–10% in the prediction of b_{pl} is expected to result in a comparable or smaller uncertainty in the predicted distortion response. Such accuracy remains within the range generally accepted for preliminary engineering assessments and the analysis of large-scale welded structures.

It should be noted that the verification procedure was limited by the availability of suitable reference data in the literature. At present, only a small number of studies provide sufficient information to enable direct comparison of plastic deformation zone widths under fully defined welding conditions. Consequently, the verification was performed using the benchmark numerical results reported by Grams [17] for two materials exhibiting substantially different thermophysical properties, namely ASTM A36 and AISI 316L. Although additional validation against experimental measurements or independent numerical studies would further strengthen the proposed methodology, such data were not available for the analysed welding configurations. It should also be emphasized that the primary objective of the present study was not the development of a new thermo-elasto-plastic finite element model, but the derivation of predictive relationships for estimating the width of the plastic deformation zone. The verified FEM model therefore served as a physically based data source for the Design of Experiments framework and the subsequent development of regression models.

3.4. Development of predictive equations for the plastic deformation zone width

The simulation results were analysed in accordance with the statistical procedures recommended in [30, 31], [12, 15-16], using the STATISTICA software package. Regression coefficients were estimated at a significance level of $\alpha = 0.05$, corresponding to a 95% confidence level.

The statistical analysis revealed that all independent variables selected for the study significantly affected the plastic deformation zone width in the thin-plate model (b_{pl-tp}). Two variables were significant as main effects, while all interaction terms were also found to be statistically significant. For the thick-plate model (b_{pl-hp}), two variables were identified as significant, comprising one main effect and two interaction effects. The values of the adjusted coefficient of determination (R_{adj}^2) and the Snedecor F-statistic for the developed models are presented in Table 6. Detailed regression summaries and the corresponding sets of significant variables included in the predictive models are given in Table 7 and Table 8.

Table 6 Comparison of adjusted squared multiple correlation coefficients and Snedecor's F-distribution

Independent variable	R_{adj}^2	Snedecor's F distribution based on regression analysis	Critical value F_{cr} , from statistical tables (for $\alpha = 0.05$)
b_{pl-tp}	0.96951387	$F(6,8)=75.204$ $p<1.343E-07$	$F_{cr}(6,8) = 3.58$
b_{pl-hp}	0.97708965	$F(3,11)=200.03$ $p<7.102E-10$	$F_{cr}(3,11) = 3.59$

Table 7 Summary of significant variables when performing stepwise regression methods for b_{pl-tp}

Summary of the regression of the dependent variable b_{pl-tp} for a stepwise regression with backward elimination: $R = 0.99125141$; $R^2 = 0.98257935$; $R_{adj}^2 = 0.96951387$; $F(6,8)=75.204$ $p<1.343E-07$ Standard error of estimation: 5.3605				
Variable	Coef.	SE Coef.	t-s	p
Constant	37.9753	15.14217	2.50792	0.036489
q_l	102.0544	11.08775	9.20425	0.000016
h	-8.5903	1.42915	-6.01074	0.000320
$q_l \cdot \sigma_{YB}$	-0.1502	0.02341	-6.41742	0.000205
$q_l \cdot h$	-1.6400	0.45469	-3.60681	0.006914
$\sigma_{YB} \cdot h$	0.0079	0.00236	3.34416	0.010168
h^2	0.1790	0.02774	6.45203	0.000198

Table 8 Summary of significant variables when performing stepwise regression methods for b_{pl-hp}

Summary of the regression of the dependent variable b_{pl-hp} for a stepwise regression with forward elimination: $R = 0.99095863$; $R^2 = 0.98199901$; $R_{adj}^2 = 0.97708965$; $F(3,11)=200.03$ $p<7.102E-10$ Standard error of estimation: 0.61513				
Variable	Coef.	SE Coef.	t-s	p
Constant	-5.89081	2.592417	-2.2723	0.044127
q_l	18.04394	3.193902	5.6495	0.000149
$q_l \cdot h$	-0.14512	0.012309	-11.7901	0.000000
q_l^2	-1.26749	0.50300	-2.5199	0.0285

The resulting polynomial predictive relationships describing the width of the plastic deformation zone as a function of the selected technological and design parameters are expressed by Eq. (8) for b_{pl-tp} and Eq. (9) for b_{pl-hp} .

$$b_{pl-tp} = 37.9753 + 102.0544 \cdot q_l - 8.5903 \cdot h - 0.1502 \cdot q_l \cdot \sigma_{YB} - 1.6400 \cdot q_l \cdot h + 0.0079 \cdot \sigma_{YB} \cdot h + 0.1790 \cdot h^2 \quad (8)$$

$$b_{pl-hp} = -5.89081 + 18.04394 \cdot q_l - 0.14512 \cdot q_l \cdot h - 1.26749 \cdot q_l^2 \quad (9)$$

The regression analysis resulted in the selection of six significant terms for the b_{pl-tp} model and three significant terms for the b_{pl-hp} model.

As shown in Table 6, the calculated Snedecor F-statistics for both regression models (Eqs. (8) and (9)) are substantially greater than the corresponding critical values (F_{cr}) for the relevant degrees of freedom. The critical values were determined from the statistical tables reported in [30]. Moreover, the associated p -values are considerably lower than the adopted significance level ($\alpha = 0.05$), indicating a high level of statistical significance.

Therefore, the predictive relationships expressed by Eqs. (8) and (9) can be regarded as statistically valid and suitable for estimating the width of the plastic deformation zone (b_{pl}) as a function of the selected technological and design parameters. For the thin-plate model (b_{pl-tp}), these parameters include welding heat input (q_l), yield strength (σ_{YB}), and plate thickness (h), whereas for the thick-plate model (b_{pl-hp}) they comprise welding heat input (q_l), initial temperature (T_0), and plate thickness (h).

The adjusted coefficients of determination reported in Tables 7 and 8 demonstrate a very high level of agreement between the regression models and the numerical simulation results. The values of R_{adj}^2 were approximately 0.97 for the b_{pl-tp} model and 0.98 for the b_{pl-hp} model, indicating that the developed regression surfaces explain nearly all of the variability observed in the simulation data.

The analysis of the obtained regression models indicates that welding heat input (q_l) is the most influential factor affecting the width of the plastic deformation zone in butt-welded joints. As a parameter directly associated with the technological conditions of the welding process, it was identified as the dominant variable in both developed models. In the case of the b_{pl-tp} model, plate thickness (h) was also found to be highly significant.

The most important interaction effects were observed between welding heat input (q_l) and plate thickness (h) for both b_{pl-tp} and b_{pl-hp} , as well as between welding heat input (q_l) and the yield strength of the base material (σ_{YB}) in the b_{pl-tp} model.

The initial (preheating) temperature (T_0) was not identified as a statistically significant predictor within the investigated range of 100–200°C and was therefore excluded from the final regression model describing the plastic deformation zone width for thick plates (b_{pl-hp}). It should be emphasized, however, that the statistical insignificance of T_0 does not imply that preheating has no effect on the welding process. Rather, it indicates that, within the investigated range, its influence on the width of the plastic deformation zone was not statistically significant. Preheating remains an important technological measure affecting cooling rates, microstructural evolution, residual stresses, and susceptibility to cold cracking.

The relationships among the technological and design parameters can be effectively visualized using three-dimensional response surface plots. Owing to their graphical form, such plots allow the simultaneous presentation of only two variables, either as individual factors or as interaction terms. Response surfaces for the most influential variables are shown in Fig. 6. The plots further confirm the conclusions drawn from the statistical analysis. In particular, welding heat input (q_l), represented by both its linear and quadratic terms in the b_{pl-tp} model (Fig. 6a), and the interaction between welding heat input (q_l) and plate thickness (h) in the b_{pl-hp} model (Fig. 6b), exert a substantially greater influence on the width of the plastic deformation zone than the remaining parameters. An increase in these variables results in a significant increase in the predicted width of the plastic deformation zone. This behaviour is consistent with the physical mechanisms governing heat transfer during welding, whereby higher heat input and greater thermal penetration promote the development of a wider region subjected to plastic deformation.

Analysis of the residual normal probability plots presented in Fig. 7a and Fig. 7b indicates satisfactory agreement between the residual distributions and the theoretical normal distribution. Most of the data points are located close to the reference line, while the minor deviations observed at the tails of the distributions do not suggest any substantial violation of the normality assumption. Furthermore, no pronounced outliers indicating potential model inadequacy were identified. The obtained results therefore confirm that the developed regression models satisfy the normality assumption to an acceptable degree and may be considered statistically adequate for predicting the width of the plastic deformation zone within the range of variables investigated in the present study.

The high quality of the developed regression models is further confirmed by the prediction plots presented in Fig. 8. The values of the dependent variables obtained from the numerical simulations are plotted on the horizontal axes, whereas the corresponding values calculated from the regression equations (8) and (9) are plotted on the vertical axes.

The distribution of data points in Fig. 8a and Fig. 8b shows a clear concentration around the 45° reference line, which represents perfect agreement between the predicted and reference

values. This observation indicates that the developed regression models accurately reproduce the analysed relationships.

It should be noted that a necessary condition for obtaining reliable predictions using the developed approximation polynomials is that the technological and design parameters employed remain within the experimental domain defined by the adopted design of experiments. Extrapolation beyond the investigated ranges of the independent variables may lead to a reduction in prediction accuracy and should therefore be avoided.

It should also be noted that the regression model for the thin-plate group was developed from a relatively limited experimental design consisting of fifteen simulation cases. Although the developed model exhibited excellent statistical indicators and all retained terms were found to be statistically significant, the available number of residual degrees of freedom remained limited. Consequently, the predictive relationships should primarily be regarded as engineering approximation models applicable within the investigated experimental domain.

A comparison between the plastic deformation zone widths obtained from the planned numerical simulations and the corresponding values predicted by Eqs. (8) and (9) is presented in Fig. 9.

The close agreement between the predicted and simulated values confirms the validity of the developed regression models and demonstrates their suitability for estimating the width of the plastic deformation zone within the investigated range of technological and design parameters.

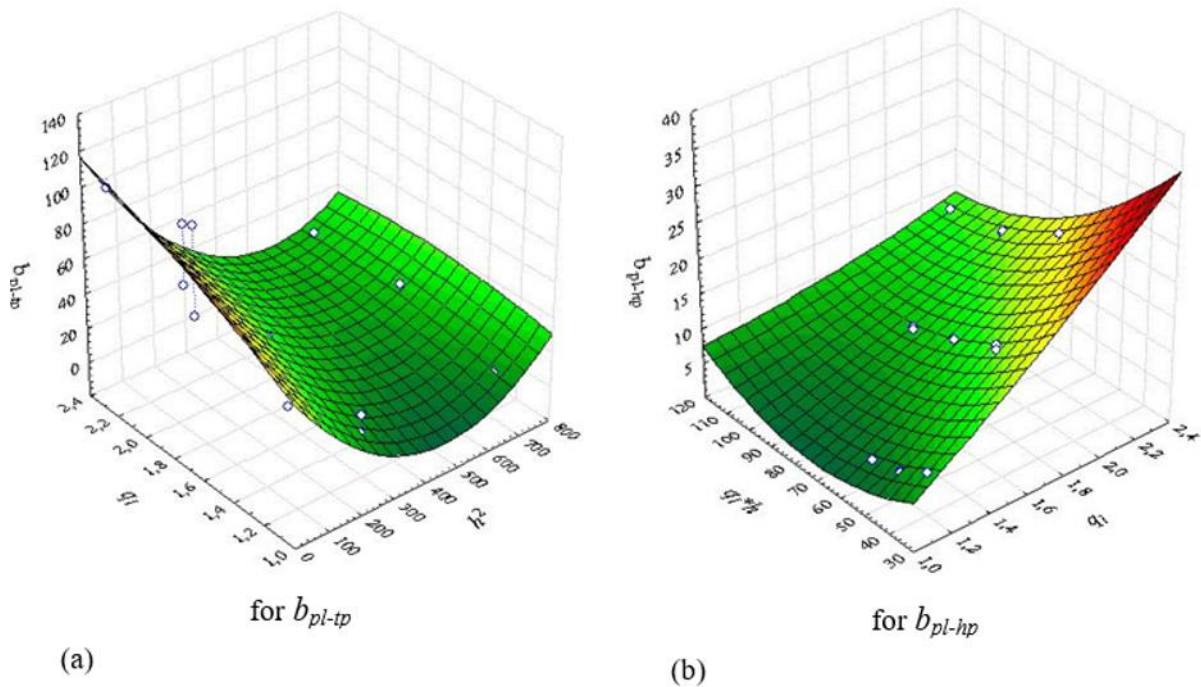


Fig. 6 Influence of technological and design parameters on the width of the plastic deformation zone: (a) effect of welding heat input (q_l) and plate thickness (h) on b_{pl-tp} ; (b) effect of welding heat input (q_l) and the interaction between welding heat input and plate thickness ($q_l \times h$) on b_{pl-hp}

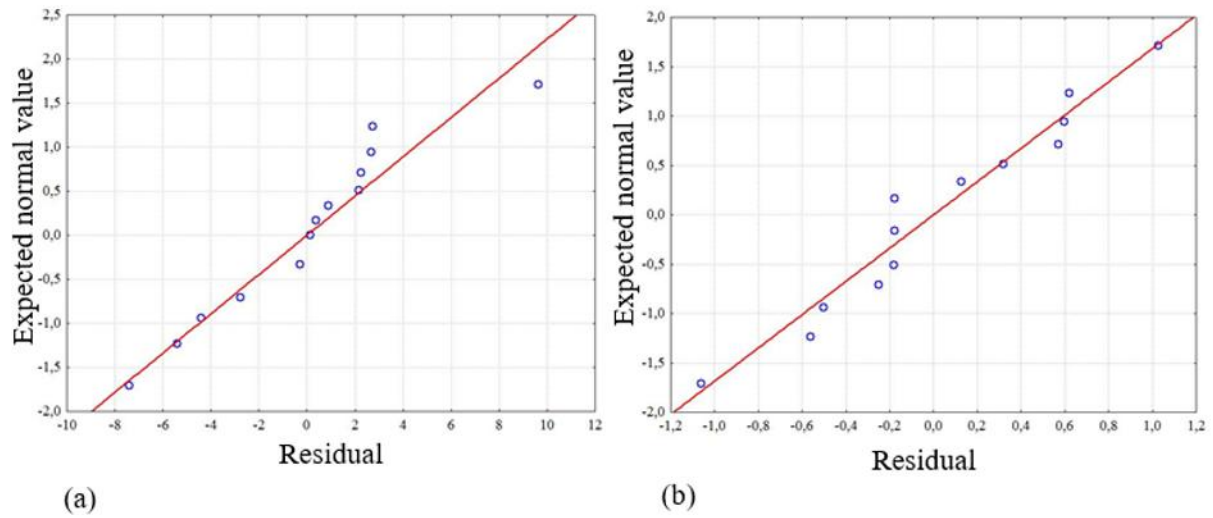


Fig. 7 Assessment of the predictive performance of the developed regression models: normal probability plots of residuals for (a) b_{pl-tp} and (b) b_{pl-hp}

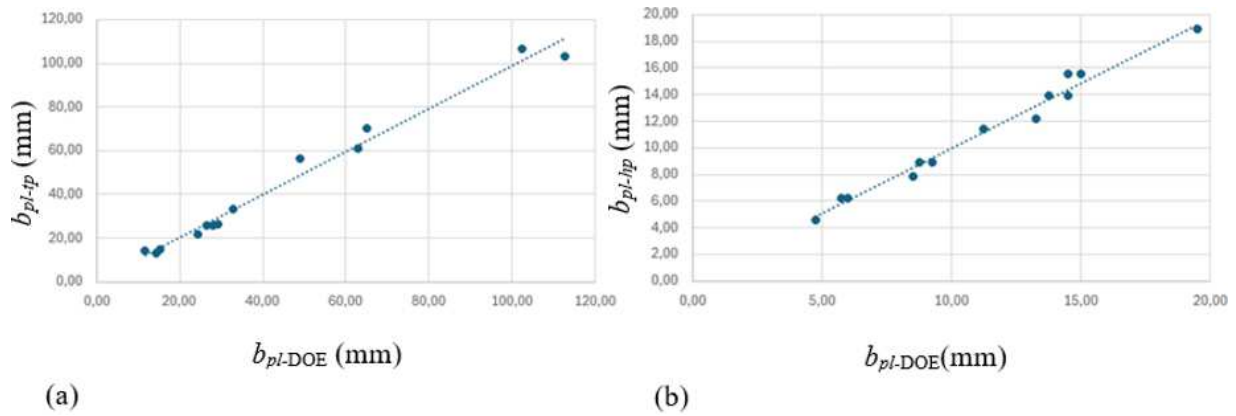


Fig. 8 Predicted versus simulated values of the plastic deformation zone width: (a) b_{pl-tp} and (b) b_{pl-hp}

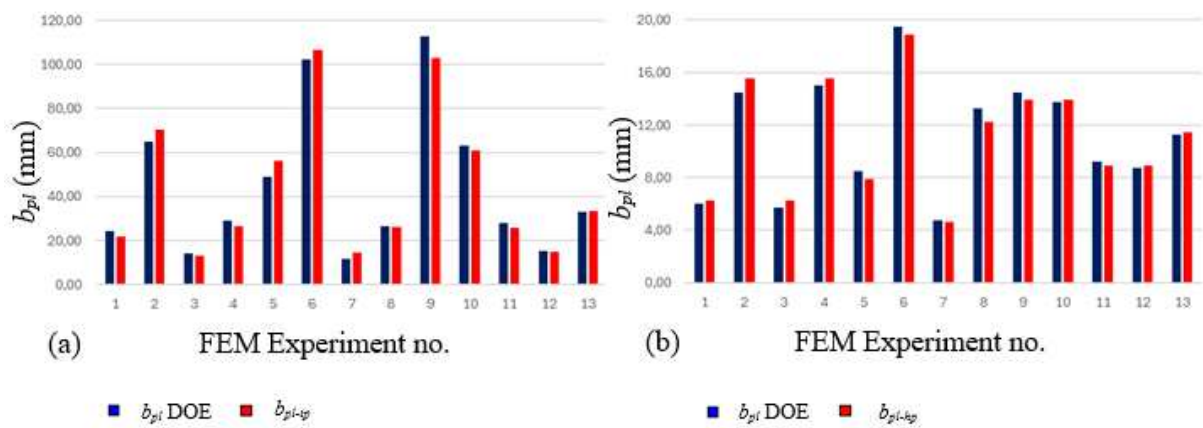


Fig. 9 Comparison of plastic deformation zone widths obtained from numerical simulations and predicted by the developed regression relationships: (a) b_{pl-tp} and (b) b_{pl-hp} (numbering of FEM experiments corresponds to the experiment numbers given in the first column of Table 1)

Based on the analysis of the plots presented in Fig. 9(a) and Fig. 9(b), the following observations can be made:

- (a) A substantially greater variation in the predicted plastic deformation zone widths was observed for the b_{pl-tp} group. The obtained values ranged from 11.75 mm to 112.75 mm, corresponding to a total variation of 101.00 mm. In contrast, the b_{pl-hp} group exhibited a considerably narrower range, from 4.75 mm to 19.50 mm, corresponding to a variation of 14.75 mm. This difference was observed despite both groups covering the same plate thickness range of 20 mm.
- (b) The differences between the plastic deformation zone widths obtained from numerical simulations and those predicted by the regression relationships ranged from 0.20 mm to 9.65 mm for b_{pl-tp} and from 0.13 mm to 1.06 mm for b_{pl-hp} . These discrepancies correspond to relative differences of 0.83–18.78% and 1.28–7.99%, respectively.
- (c) The observed differences may be attributed to the fact that identical ranges of welding heat input were adopted for both groups, in accordance with industrial welding practice (see Section 3.1, Table 2 and Table 3), despite significant differences in plate thickness. The first group consists of plates that may be classified as thin and medium-thickness plates, whereas the second group comprises plate thicknesses characteristic of thick-section structures.
- (d) For the structural steels used in marine ship hull structures (Fig. 9a), the smallest discrepancies between simulation results and regression predictions were obtained for experimental runs corresponding to the intermediate level of welding heat input, namely Runs 12 and 13, for which the differences were 0.20 mm and 0.27 mm, respectively. The largest discrepancies were observed for runs corresponding to the minimum and maximum heat input levels, namely Run 5 (7.39 mm) and Run 9 (9.65 mm).
- (e) For the structural steels used in offshore structures (Fig. 9b), the smallest discrepancies were obtained for simulation runs corresponding to the minimum heat input level, namely Runs 1 and 7, with differences of 0.25 mm and 0.13 mm, respectively, and for the intermediate heat input level, namely Runs 10, 12, and 13, for which the differences were 0.18 mm. The largest discrepancies were observed for runs corresponding to the maximum heat input level, namely Run 2 (1.06 mm) and Run 8 (1.03 mm).
- (f) It should be emphasized that, although welding heat input was identified as the most influential factor affecting the width of the plastic deformation zone, other variables also contributed significantly to the developed predictive models. In particular, plate thickness played an important role in both models, while the yield strength of the base material was additionally significant in the b_{pl-tp} model. These findings are consistent with the statistical analyses presented in Tables 7 and 8 and are reflected in the predictive relationships given by Eqs. (8) and (9).

4. Comparative Analysis of Predictive Relationships

4.1. Predictive relationships reported in the literature

The objective of this analysis is to verify the values of the plastic deformation zone width obtained from the developed predictive relationships (Eqs. (8) and (9)) through comparison with other analytical formulations available in the literature, as well as with the results obtained from numerical simulations. Among the analytical relationships proposed by researchers investigating welding-induced stresses and distortions in butt-welded joints, two formulations published by Ueda et al. [4, 5] were selected for comparative evaluation and are expressed by

Eqs. (10) and (11). The comparison was intended to assess the predictive capability of the developed models and to identify the analytical formulation that most accurately reproduces the plastic deformation zone widths obtained from the numerical simulations.

It should be noted that the analytical formulation proposed by Grams [17] was already employed in the verification stage of the numerical model (Section 3.3) and additionally forms the basis for the definition of the first yield temperature adopted in the present study. Consequently, the comparative analysis in this section focuses on the classical relationships of Ueda and Yuan [4, 5], which explicitly relate the widths of the plastic deformation zone and the inherent strain zone to welding process parameters and therefore constitute a suitable benchmark for the developed predictive equations.

$$b_{pl} = \frac{0.242 \cdot \alpha \cdot E \cdot q_l}{c \cdot \rho \cdot h \cdot \sigma_{YB}} \quad (10)$$

$$b_{is} = \xi \cdot b_{pl} \quad (11)$$

where:

$$\xi = 1 - \frac{0.27 \cdot \alpha \cdot E \cdot T_{av}}{\sigma_{YB}} \quad (12)$$

$$T_{av} = \frac{q_l}{2 \cdot c \cdot \rho \cdot h \cdot B} \quad (13)$$

4.2. Comparative Analysis

In the analytical relationships, material properties are assumed to be temperature-independent for the sake of simplicity. The material properties adopted for the analysed cases are summarized in Table 9.

The results of the comparative analysis between the plastic deformation zone widths obtained from the planned numerical simulations and those predicted by the developed regression relationships (Eqs. (8) and (9)), as well as by the analytical formulations reported in the literature, are presented in Tables 10 and 11 and illustrated in Fig. 10. The comparison was performed to evaluate the predictive capability of the developed regression models and to assess their accuracy relative to both finite element simulation results and established analytical approaches available in the literature. Table 10 summarizes the minimum, maximum, and average absolute prediction errors obtained for the analysed predictive relationships. The results indicate that the regression equations developed in the present study provide substantially lower average prediction errors than the analytical relationships reported in the literature. Table 11 provides an engineering-oriented classification of prediction errors by reporting the number of cases falling within predefined absolute error intervals. This representation facilitates practical assessment of prediction accuracy and allows direct comparison of the frequency of low- and high-error predictions among the investigated relationships.

Table 9 Thermal material properties for structural steels

Material properties	for b_{pl-tp}	for b_{pl-hp}
c (J/g°C)	0.670	0.665
E (N/mm ²)	210000	210000
α (1/°C)	0.000012	0.000012
ρ (g/mm ³)	0.00785	0.00785
q_l (kJ/mm)	According to Table 2	According to Table 2
σ_{YB} (N/mm ²)	According to Table 2	355

Table 10 Statistical summary of prediction errors between FEM results and analytical/predictive relationships

Predictive formula	Predictive error					
	$ b_{pl-DOE} - b_{pl-tp} $			$ b_{pl-DOE} - b_{pl-hp} $		
	min. (mm)	max. (mm)	min. (mm)	max. (mm)	min. (mm)	max. (mm)
b_{pl-tp}	0.20	9.65	3.18	-	-	-
b_{pl-hp}	-	-	-	0.13	1.06	0.47
b_{pl} Ueda	2.54	32.00	11.25	1.89	5.74	3.56
b_{is} Ueda	0.84	14.96	4.96	1.38	4.35	3.02

Table 11 Number of prediction cases falling within specified absolute error ranges

Predictive formula	Number of cases					
	$ b_{pl-DOE} - b_{pl-tp} $			$ b_{pl-DOE} - b_{pl-hp} $:		
	0÷5 mm	5÷10 (mm)	10÷15(mm)	0÷5 (mm)	5÷10 (mm)	10÷15(mm)
b_{pl-tp}	12	3	0	-	-	-
b_{pl-hp}	-	-	-	15	0	0
b_{pl} Ueda	7	3	2	13	2	0
b_{is} Ueda	10	4	1	15	0	0

Note: If the sum of the values in a given category is less than 15, the remaining results correspond to plastic deformation zone widths falling outside the range of 10–15 mm.

Based on the analysis of the results presented in Fig. 10(a), Fig. 10(b), Table 10, and Table 11, the following conclusions can be drawn:

- Application of the analytical relationships described by Eqs. (10) and (11) confirmed the previously observed trend of considerably greater variation in the plastic deformation zone widths for the b_{pl-tp} group than for the b_{pl-hp} group.
- The highest predictive accuracy, both qualitatively and quantitatively, was obtained using the developed approximation equations (8) and (9). For the structural steels used in marine ship hull structures (Fig. 10a), the differences between the numerical simulation results and the predicted values were within the range of 0–5 mm for 12 out of 15 cases (Table 11), while in 11 of these cases the differences did not exceed 3 mm. Only three cases (Runs 2, 5, and 9) exhibited discrepancies in the range of 5–10 mm (see also Fig. 9a). Even better agreement was observed for the offshore structural steels (Fig. 10b), where all differences were below 5 mm and only two cases (Runs 2 and 8) exceeded 1 mm (see also Fig. 9b).
- The inherent strain zone relationship, Eq. (11), also provided good agreement for both groups of steels. For the offshore steel group, all 15 differences between the predicted and simulated results fell within the range of 0–5 mm. However, for more than half of the cases (eight observations), the discrepancies exceeded 3 mm. For the shipbuilding steel group, the differences relative to the FEM results were within the range of 0–5 mm for ten cases,

5–10 mm for four cases, and 10–15 mm for one case (Run 9). The average discrepancy for this relationship was 4.96 mm.

- (d) The Ueda plastic deformation zone relationship, Eq. (10), provided generally good agreement for the offshore steel group. In 13 cases, the differences relative to the FEM results were below 5 mm, while only two cases exceeded this value, namely Run 6 (5.74 mm) and Run 10 (5.14 mm). In contrast, the agreement obtained for the shipbuilding steel group was substantially lower. Seven cases exhibited discrepancies below 5 mm (average value: 4.14 mm), three cases fell within the range of 5–10 mm (average value: 7.30 mm), and two cases were within the range of 10–15 mm. In the remaining three cases, the differences exceeded 15 mm, reaching 16.48 mm for Run 5, 32.00 mm for Run 6, and 31.41 mm for Run 9.

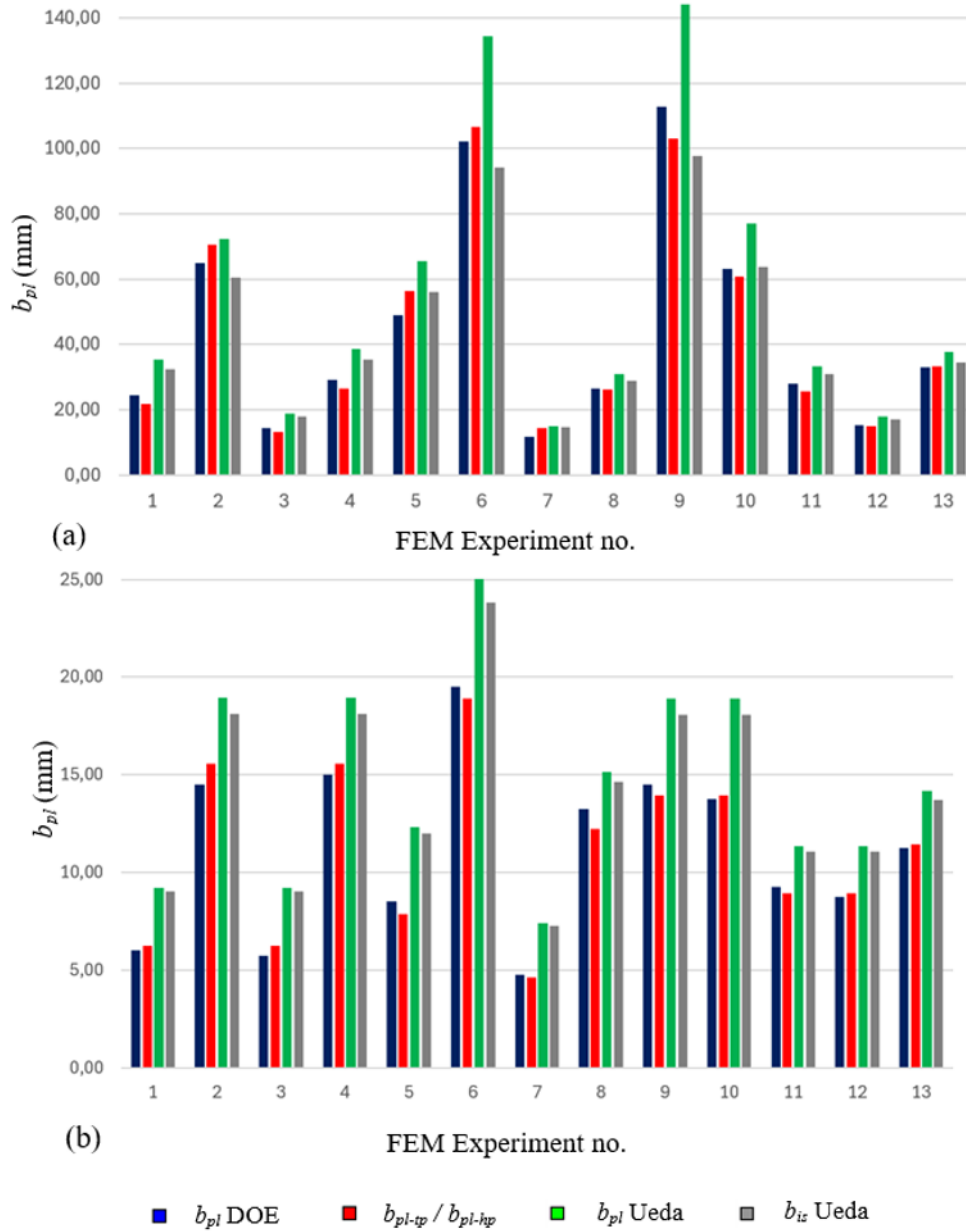


Fig. 10 Comparison of plastic deformation zone widths, including the inherent strain zone, obtained from numerical simulations and predicted by all analytical and predictive relationships considered in the comparative analysis: (a) b_{pl-tp} and (b) b_{pl-hp} . The FEM Experiment No. corresponding to the experimental design matrix presented in Table 1

- (e) Experimental Runs 6 and 9 repeatedly appeared among the cases exhibiting the least favourable agreement, particularly for the shipbuilding steel group. Both runs correspond to the minimum plate thickness level ($h = 6$ mm) combined with medium and maximum welding heat input levels of 1.7 kJ/mm and 2.3 kJ/mm, respectively (see Table 1 and Table 2). It should be noted that the welding heat input levels adopted in the study were selected on the basis of industrial welding practice and correspond to combinations of current, voltage, and welding speed specified in Company Standard T081-03 [33] (Table 3). Consequently, all points included in the experimental design represent technologically feasible SAW conditions. Nevertheless, the cases involving 6 mm plates combined with medium and high heat input levels should be regarded as near-limit welding conditions, for which the sensitivity of the plastic deformation zone width to thermal loading becomes particularly pronounced. This observation provides a plausible explanation for the increased discrepancies observed in Runs 6 and 9.
- (f) This combination resulted in plastic deformation zone widths that significantly differed from the values observed in the remaining cases, which is also evident in Fig. 10(a).
- (g) Furthermore, it was observed that the predictive relationships developed in the present study (Eqs. (8) and (9)) generally yielded results that were closer to the inherent strain zone relationship, Eq. (11), than to the Ueda plastic deformation zone relationship, Eq. (10). This characteristic makes the proposed models particularly attractive for estimating the width of the inherent strain zone, which can subsequently be employed in simplified approaches for predicting welding-induced distortions.

5. Conclusions

The investigations presented in this study allow the following conclusions and observations to be drawn:

- (a) It was demonstrated that the width of the plastic deformation zone in butt-welded joints made of structural steels can be effectively predicted using regression models developed on the basis of a planned set of numerical simulations. The adopted Gaussian surface heat source model proved to be adequate for this purpose, as confirmed by the verification and comparative analyses.
- (b) The Design of Experiments (DOE) analyses performed for two groups of steels commonly used in shipbuilding and offshore structures enabled the clear identification of the key parameters governing the width of the plastic deformation zone. The results demonstrated that welding heat input and plate thickness are the dominant factors influencing the extent of the plastic deformation zone.
- (c) The developed predictive models exhibited a very high level of agreement with the numerical simulation results, confirming their suitability for engineering analyses and design applications. The adjusted coefficients of determination exceeded 0.97 for both models, indicating excellent agreement between the regression surfaces and the simulation data. Consequently, the proposed relationships can be considered reliable tools for predicting the width of the plastic deformation zone within the investigated range of technological and design parameters.
- (d) The obtained results demonstrated that two distinct predictive models are required for accurate estimation of plastic zone width. Separate regression equations were developed for thin plates (6–26 mm) and thick plates (30–50 mm), reflecting different thermal behaviour associated with plate thickness.
- (e) Heat input was identified as the most influential parameter controlling the plastic deformation zone width in both plate groups, while plate thickness constituted the second most important factor.

- (f) Contrary to expectations, the preheating temperature did not appear as a statistically significant variable in the predictive model developed for thick plates. Within the investigated range of 100–200°C, its influence on the plastic zone width was negligible compared with the effects of heat input and plate thickness. This result suggests that moderate variations in preheating temperature primarily affect thermal cycles and cooling rates rather than the maximum spatial extent of the plastic deformation zone.
- (g) The comparison of the developed regression models with analytical relationships available in the literature further confirmed the validity and predictive capability of the proposed formulations. The results demonstrated that the developed models provide predictions that are consistent with established analytical approaches while exhibiting substantially better agreement with the numerical simulation results. Consequently, the proposed relationships may be regarded as reliable tools for estimating the width of the plastic deformation zone in butt-welded structural steel joints.
- (h) Furthermore, it was demonstrated that the developed numerical model can be used to estimate the width of the inherent strain zone. The obtained results indicate that the proposed modelling approach provides a suitable basis for determining both the plastic deformation zone width and the extent of the inherent strain zone, which constitutes a key input parameter in simplified methods for predicting welding-induced distortions based on equivalent loads and inherent strain formulations.

CRediT authorship contribution statement

Tomasz Urbański: Conceptualization, Methodology, Writing - original draft, Investigation, Visualization.

Maciej Taczala: Formal analysis, Writing - review & editing, Supervision.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Figure and Table Captions

Fig. 1 Theoretical temperature distributions and heat source model: (a) temperature distribution along the direction of heat source movement, (b) temperature distribution transverse to the direction of heat source movement, (c) geometric relationships of the heat source model

Fig. 2 Black-box model of the planned numerical experiment used for predicting the width of the plastic deformation zone

Fig. 3 Comparison of temperature distributions in steel plates of different thicknesses: (a) 3 mm, (b) 20 mm, (c) 26 mm, and (d) 30 mm

Fig. 4 Non-uniform finite element discretization used in the thermal analysis: (a) for b_{pl-tp} , (b) for b_{pl-hp}

Fig. 5 Temperature-dependent thermophysical properties of the structural steels used in the numerical simulations: (a) ASTM A36, (b) ASTM A572 Grade 50, (c) CMn steel, and (d) S355G10+M (based on [17], [13], [37, 38])

Fig. 6 Influence of technological and design parameters on the width of the plastic deformation zone: (a) effect of welding heat input (q_l) and plate thickness (h) on b_{pl-tp} ; (b) effect of welding heat input (q_l) and the interaction between welding heat input and plate thickness ($q_l \times h$) on b_{pl-hp}

Fig. 7 Assessment of the predictive performance of the developed regression models: normal probability plots of residuals for (a) b_{pl-tp} and (b) b_{pl-hp}

Fig. 8 Predicted versus simulated values of the plastic deformation zone width: (a) b_{pl-tp} and (b) b_{pl-hp}

Fig. 9 Comparison of plastic deformation zone widths obtained from numerical simulations and predicted by the developed regression relationships: (a) b_{pl-tp} and (b) b_{pl-hp} (numbering of FEM experiments corresponds to the experiment numbers given in the first column of Table 1)

Fig. 10 Comparison of plastic deformation zone widths, including the inherent strain zone, obtained from numerical simulations and predicted by all analytical and predictive relationships considered in the comparative analysis: (a) b_{pl-tp} and (b) b_{pl-hp} . The FEM Experiment No. corresponding to the experimental design matrix presented in Table 1

Table 1 Box–Behnken design matrix for three independent variables with normalized factor levels (−1, 0, +1)

Table 2 Actual levels of the independent variables adopted in the Box–Behnken experimental design

Table 3 Technological parameters of the SAW process corresponding to the three levels of welding heat input adopted in the experimental design (based on [33])

Table 4 Summary of welding parameters, heat source radius, and delay factor adopted from Grams [17]

Table 5 Comparison of plastic deformation zone widths obtained from the developed numerical model and the reference data reported by Grams [17]

Table 6 Comparison of adjusted squared multiple correlation coefficients and Snedecor's F-distribution

Table 7 Summary of significant variables when performing stepwise regression methods for b_{pl-tp}

Table 8 Summary of significant variables when performing stepwise regression methods for b_{pl-hp}

Table 9 Thermal material properties for structural steels

Table 10 Statistical summary of prediction errors between FEM results and analytical/predictive relationships

Table 11 Number of prediction cases falling within specified absolute error ranges