

Equilibrium Transmission and Dynamics in Price Networks

HyeongWoo Kim

harry17@hanyang.ac.kr

Hanyang University

Research Article

Keywords:

Posted Date: September 29th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-11069697/v1>

License:   This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Additional Declarations: No competing interests reported.

Equilibrium Transmission and Dynamics in Price Networks

Anonymous Author(s) Affiliation Omitted for Double-Blind Review

Abstract

This paper develops a network model of heterogeneous price judgments in which the same network structure governs both equilibrium transmission and subsequent dynamic adjustment. The key object is the mode-specific equilibrium transmission coefficient

$$T_j = \frac{1 - r}{1 - r + kg_j},$$

which measures how fundamental heterogeneity is transmitted into price judgments in network mode j . We show that, conditional on the common behavioral parameters (r, λ) , this equilibrium object provides an alternative representation of the characteristic polynomial governing the subsequent dynamics of the same mode. The resulting transmission–dynamics restriction implies that equilibrium transmission and dynamic adjustment must be jointly consistent across network modes.

The model also shows that aggregate prices need not reveal cross-sectional disagreement. Under uniform aggregation, only the consensus component is visible in the aggregate price, while disagreement modes can remain heterogeneous and dynamically distinct. More generally, non-uniform aggregation can make disagreement modes observable in the aggregate. Equilibrium disagreement is characterized by

$$D^* = n^{-1} \sum_{j \in D} T_j^2 \theta_j^2,$$

providing a mode-level decomposition of cross-sectional dispersion. The framework therefore yields testable restrictions that combine cross-sectional equilibrium information with time-series adjustment and cannot be assessed from aggregate price behavior alone.

1 Introduction

Aggregate prices are often interpreted as evidence that heterogeneous economic judgments have converged. In a strategic environment, however, this inference need not be valid. Individual agents may continue to hold systematically different price judgments even when the aggregate price is stable or converges to a stationary value. The reason is that the aggregate price is a projection of the cross-sectional state: variation orthogonal to the aggregation weights can remain economically relevant while being invisible in the aggregate. This raises a more specific question for models of heterogeneous expectations and strategic interaction: *can the cross-sectional transmission of heterogeneity observed in equilibrium place restrictions on how that heterogeneity subsequently adjusts over time?*

This paper develops a network model of strategic price formation in which agents form forward-looking expectations about the judgments of others. The interaction structure is represented by a fixed matrix G , while heterogeneity in fundamentals is represented by a vector θ . The resulting equilibrium and dynamic system can be decomposed into an aggregate component and a set of cross-sectional disagreement modes. This decomposition provides a useful distinction between two objects that are often conflated: aggregate convergence and individual consensus. In particular, stability of the aggregate component does not imply that disagreement modes have vanished. More importantly, the modal decomposition allows the same network mode to be tracked from its equilibrium transmission into subsequent dynamic adjustment. In particular, stability of the aggregate component does not imply that disagreement modes have vanished.

The main contribution is to show that the equilibrium transmission of fundamental heterogeneity into a particular network mode also constrains the dynamic adjustment properties of that same mode. Let g_j

denote the eigenvalue of G associated with mode j . In the stationary equilibrium, the transmission of fundamental heterogeneity into the corresponding price mode is

$$T_j \equiv \frac{c_j^*}{\theta_j} = \frac{1-r}{1-r+kg_j}, \quad (1)$$

where r measures persistence in individual judgment and k measures the strength of strategic interaction. Thus, T_j has a direct economic interpretation as a mode-specific equilibrium transmission coefficient: it measures how fundamental heterogeneity in mode j is transmitted into equilibrium price heterogeneity. Depending on the network mode and parameter values, this transmission may represent attenuation, amplification, or a sign reversal rather than a literal fraction of the fundamental heterogeneity.

The same network mode follows a second-order dynamic equation,

$$z_{j,t+1} = [r - k(1+\lambda)g_j] z_{j,t} + k\lambda g_j z_{j,t-1}, \quad (2)$$

where λ captures the strength of forward-looking adjustment. The equilibrium transmission coefficient and the dynamic law are linked because both are generated by the same underlying network mode g_j . Using (1) to eliminate g_j from (2) yields the paper's central structural restriction:

$$q_j(\mu | T_j) = \mu^2 - \left[r - (1+\lambda)(1-r)\frac{1-T_j}{T_j} \right] \mu - \lambda(1-r)\frac{1-T_j}{T_j}. \quad (3)$$

The algebraic elimination itself is straightforward. The economic content of the restriction is that T_j is an equilibrium cross-sectional object, whereas $q_j(\mu | T_j)$ governs subsequent adjustment of the same network mode. Consequently, conditional on (r, λ) , equilibrium transmission T_j restricts the admissible dynamic roots of that mode. Equilibrium disagreement and dynamic persistence are therefore not independent objects in the model: the cross-sectional transmission observed in equilibrium contains structural information about how the corresponding disagreement mode adjusts over time.

The restriction becomes more informative when considered jointly across network modes. Because r and λ are common behavioral parameters, the model implies that there must exist a common pair (r, λ) such that the dynamic law of every mode is consistent with its equilibrium transmission coefficient:

$$\exists (r, \lambda) \text{ such that } q_j(\mu) = q_j(\mu | T_j; r, \lambda) \quad \text{for all relevant modes } j. \quad (4)$$

Thus, the model imposes a cross-mode restriction rather than merely a mode-by-mode relationship. Equilibrium transmission coefficients observed across modes jointly constrain the admissible collection of dynamic adjustment processes. This provides a basis for studying equilibrium cross-sectional heterogeneity and subsequent time-series adjustment within a common structural framework.

A further implication concerns the information contained in aggregate prices. The aggregate component depends only on the projection of individual judgments onto the aggregation direction, whereas disagreement modes are orthogonal to that projection under the benchmark network assumptions. Hence distinct cross-sectional configurations can generate the same aggregate price while implying different equilibrium disagreement and different mode-specific adjustment. In the stationary benchmark, disagreement dispersion can be written as

$$D^* = \frac{1}{n} \sum_{j \in \mathcal{D}} T_j^2 \theta_j^2, \quad (5)$$

where $\mathcal{D}$ denotes the disagreement modes. Aggregate stability therefore does not by itself identify either the magnitude or the structure of underlying disagreement, nor does it recover the mode-specific dynamic adjustment implied by the equilibrium transmission coefficients.

The paper makes three main contributions. First, it derives an *equilibrium-to-dynamics restriction* linking the mode-specific equilibrium transmission coefficient T_j to the characteristic polynomial governing subsequent adjustment of the same network mode, as given in (3). Second, because the behavioral parameters (r, λ) are common across modes, the model generates a cross-mode restriction: the collection of equilibrium transmission coefficients must be jointly consistent with a common dynamic law. This turns the equilibrium-to-dynamics mapping into a potentially testable restriction rather than a mode-specific algebraic relationship.

Third, the model shows that aggregate prices alone need not reveal this information: economies with observationally equivalent aggregate equilibrium prices can differ in their underlying disagreement structure and in the dynamic adjustment of their disagreement modes.

The contribution is therefore not simply that networks affect price dynamics or that heterogeneous beliefs may persist. These mechanisms are already central to the literatures on heterogeneous expectations, disagreement, strategic interaction, and networked economic behavior Brock and Hommes (1998); Allen et al. (2006); Hong and Stein (2007); Galeotti et al. (2010); Ozsoylev and Walden (2011); Panchenko et al. (2013). The distinctive object here is the *equilibrium-to-dynamics mapping*: a mode-specific equilibrium statistic generated by the network structure places a restriction on the subsequent dynamic law of that same mode. Moreover, the common-parameter structure makes this restriction jointly testable across modes. The resulting framework connects cross-sectional equilibrium information to time-series adjustment without requiring the aggregate price alone to reveal the underlying disagreement structure.

The remainder of the paper proceeds as follows. Section 2 positions the framework relative to the literatures on heterogeneous beliefs, disagreement, strategic interaction, and network-based asset pricing. Section 3 develops the economic environment and specifies the individual behavioral adjustment rule. Section 4 characterizes stationary equilibrium and the mode-specific transmission coefficient. Section 5 derives the corresponding dynamic system and its stability properties. Section 6 establishes the equilibrium-to-dynamics restriction and the associated cross-mode common-parameter restriction. Section 7 derives the implications for aggregate prices and disagreement dispersion, while Section 8 discusses observational equivalence, identification, and testable restrictions. Additional proofs, extensions, and numerical illustrations are reported in the Appendix.

2 Related Literature

This paper is related to four strands of literature: heterogeneous beliefs and expectation-driven asset pricing, disagreement and its persistence, strategic interaction in economic networks, and the identification of heterogeneous states from aggregate outcomes. Across these literatures, the effects of cross-sectional heterogeneity, network interaction, and dynamic adjustment have been studied extensively. The narrower question addressed here is whether an equilibrium measure of how heterogeneity is transmitted into prices can itself impose restrictions on the subsequent adjustment of that heterogeneity.

A first strand studies asset pricing and macroeconomic dynamics under heterogeneous beliefs. In these models, agents may assign different probabilities to future states or use different forecasting rules, so that belief heterogeneity affects equilibrium prices and their dynamics Brock and Hommes (1998); Allen et al. (2006). Chang Chang (2007), for example, incorporates social interactions and heterogeneous beliefs into a simple asset-pricing model and studies their implications for equilibrium and dynamic stability. Our framework shares this emphasis on cross-sectional heterogeneity but differs in the object used to connect equilibrium and dynamics. Rather than introducing heterogeneous expectations solely through exogenous forecasting types, the framework generates individual price judgments through strategic interaction and represents the resulting heterogeneity through network modes. The key object is then the mode-specific equilibrium transmission coefficient, which can be related directly to the subsequent dynamic adjustment of the same mode.

A second literature studies disagreement, including its magnitude, persistence, and implications for economic outcomes. This literature shows that disagreement and the cross-sectional distribution of beliefs can matter for asset prices and other market outcomes Chen et al. (2002); Hong and Stein (2007); Xiouros and Zapatero (2024). Our analysis focuses instead on how the underlying heterogeneity is transmitted into equilibrium prices and how that transmission relates to subsequent adjustment. For network mode j , we define

$$T_j \equiv \frac{c_j^*}{\theta_j} = \frac{1 - r}{1 - r + kg_j}, \quad (6)$$

which measures the equilibrium transmission of fundamental heterogeneity into price heterogeneity. The relevant question is therefore not only whether disagreement persists, but whether its equilibrium transmission places a structural restriction on the subsequent adjustment of the same cross-sectional mode.

A third and particularly close strand incorporates social or strategic interaction into models of beliefs and asset prices. Panchenko et al. (2013) study heterogeneous beliefs with local network interactions and show that network topology affects stability and price dynamics. More recently, Hatcher and Hellmann (2025) study how network structure and performance-based belief updating jointly affect belief and asset-price dynamics, including convergence and stability. These studies show that network structure can affect equilibrium outcomes, information transmission, stability, and convergence (Galeotti et al. (2010); Ozsoylev and Walden (2011); Hatcher and Hellmann (2024)). The distinction here is not whether networks affect equilibrium or dynamics, but whether an equilibrium statistic generated by the network can constrain the dynamics of the same mode. In the present framework, the network generates a mode-specific transmission coefficient, and the corresponding equilibrium transmission can be used to eliminate the latent network eigenvalue from the dynamic law.

This restriction is obtained by eliminating the network eigenvalue from the dynamic system. For a mode with transmission T_j , the characteristic polynomial is

$$q_j(\mu | T_j) = \mu^2 - \left[r - (1 + \lambda)(1 - r) \frac{1 - T_j}{T_j} \right] \mu - \lambda(1 - r) \frac{1 - T_j}{T_j}. \quad (7)$$

Thus, conditional on (r, λ) , equilibrium transmission contains information about the subsequent adjustment of the same cross-sectional mode. The economic content of the mapping is not the algebraic elimination itself, but the fact that an equilibrium cross-sectional object is informative about a subsequent time-series process generated by the same structural interaction mechanism. This equilibrium-to-dynamics restriction is the central distinction of the present framework.

A fourth strand concerns aggregation and identification. In models of social learning and information aggregation, aggregate outcomes need not fully reveal the underlying cross-sectional distribution of beliefs or opinions (DeGroot (1974); DeMarzo et al. (2003); Golub and Jackson (2010); Acemoglu and Ozdaglar (2011)). Our framework yields a transparent version of this problem: under the benchmark network assumptions, the aggregate price is a projection onto the consensus direction, while disagreement modes are orthogonal to that projection. Different cross-sectional configurations can therefore generate the same aggregate price while implying different disagreement and mode-specific dynamics. This aggregation problem is relevant here because the cross-sectional information needed to recover equilibrium transmission is precisely the information that aggregate prices may discard.

The paper is therefore positioned at the intersection of these literatures, but its main purpose is not to add another model in which networks, disagreement, or heterogeneous expectations affect prices. The relevant connection is between the equilibrium and dynamic objects generated by the same network mode. The heterogeneous-beliefs literature motivates cross-sectional expectations, the disagreement literature motivates dispersion, the network literature provides the interaction structure, and the aggregation literature highlights the information lost in aggregate outcomes. The present framework uses these elements to derive a more restrictive cross-sectional-to-dynamic implication: the equilibrium transmission coefficient of a network mode constrains the dynamic law of that same mode.

This distinction also determines the empirical content of the model. Aggregate price data alone need not reveal the magnitude or composition of disagreement. Cross-sectional information, such as individual forecasts, price judgments, or other measures of heterogeneous expectations, can be used to recover mode-specific transmission. The model then implies that these transmission measures should be systematically related to the corresponding dynamic adjustment according to (7). Because the behavioral parameters (r, λ) are common across modes, the restriction can also be evaluated jointly: the observed collection of equilibrium transmission coefficients and mode-specific dynamic coefficients should be consistent with a common pair (r, λ) . This cross-mode restriction provides the basis for the identification and empirical design developed later in the paper.

Table 1: Positioning Relative to Closest Literature

Literature	Main object	Main finding	Distinction from the present framework
Muth (1961)	Rational expectations	Expectations and price formation	Does not derive a mode-specific transmission object linking cross-sectional heterogeneity to subsequent dynamic adjustment.
Morris and Shin (2002); Hellwig (2005); Angeletos and Pavan (2007)	Information aggregation and strategic interaction	Information structure and strategic interaction affect equilibrium outcomes.	The present framework links the equilibrium transmission of heterogeneity to the dynamics of the same cross-sectional mode.
Brock and Hommes (1998); Allen, Morris, and Shin (2006)	Heterogeneous beliefs and asset-price dynamics	Heterogeneous expectations generate persistence and nontrivial price dynamics.	Dynamics here arise from strategic interaction and are indexed by equilibrium transmission rather than heterogeneous forecasting types.
Hong and Stein (2007); Chen, Hong, and Stein (2002); Scheinkman and Xiong (2003)	Disagreement and asset prices	Disagreement and heterogeneous beliefs affect prices and market outcomes.	The contribution is the restriction linking the equilibrium transmission of a disagreement mode to its subsequent dynamic adjustment.
DeGroot (1974); Vayanos, and Zwiebel (2003); Golub and Jackson (2010)	Social learning and opinion aggregation	Network interaction affects convergence, disagreement, and aggregation.	Network modes are used to derive an observable transmission object, after which the latent network eigenvalue is eliminated from the dynamic law.
Galeotti et al. (2010); Ozsoylev and Walden (2011); Panchenko et al. (2013); Hatcher and Hellmann (2024, 2025)	Networks and economic dynamics	Network structure affects information transmission, equilibrium, and dynamic adjustment.	The central object is not network topology itself but the mode-specific transmission coefficient generated by the network.
Present framework	Equilibrium transmission and dynamic adjustment	T_j measures equilibrium transmission of fundamental heterogeneity.	The same network mode links equilibrium transmission to subsequent dynamic adjustment: $T_j \rightarrow q_j(\mu) \rightarrow \rho_j$. The common behavioral parameters (r, λ) impose a cross-mode restriction, so the collection of equilibrium transmission coefficients must be jointly consistent with the corresponding dynamic laws.

3 Economic Environment

Consider an economy with n agents indexed by $i = 1, \dots, n$. Each agent forms a price judgment about the same underlying economic object. Let $C_{i,t}$ denote agent i 's judgment at date t , and let $C_t = (C_{1,t}, \dots, C_{n,t})^\top$ collect individual judgments. The corresponding fundamental benchmark is represented by

$$\theta = (\theta_1, \dots, \theta_n)^\top,$$

where θ_i denotes the benchmark price level toward which agent i 's judgment is partially adjusted. The benchmark may be interpreted as the agent-specific fundamental component of the judgment, such as a private valuation, signal-based benchmark, or other persistent fundamental assessment. The model does not require these benchmarks to arise from a particular information-acquisition or utility-maximization process. Heterogeneity in θ_i therefore represents differences in the fundamental component of agents' price judgments, while the interaction mechanism itself is held common across agents.

The aggregate price is a weighted projection of individual judgments,

$$P_t = w^\top C_t, \quad w_i \geq 0, \quad \sum_{i=1}^n w_i = 1.$$

The benchmark analysis uses uniform weights, $w_i = 1/n$, so that the aggregate direction is proportional to the vector $\mathbf{1}$. The distinction between P_t and the full vector C_t is central: the aggregate price contains only the component of individual judgments that is visible through the aggregation operator, whereas cross-sectional components can remain hidden from aggregate observations.

Agents interact through a fixed network represented by the matrix G . The element G_{ij} is the weight that agent i assigns to agent j 's judgment when forming the strategically relevant network-weighted judgment of others. Thus, conditional on the current vector of judgments, $(GC_t)_i$ summarizes the judgment of the agents to whom agent i is connected, with larger G_{ij} indicating a stronger interaction weight. The matrix G therefore describes the structure of strategic interaction rather than the source of fundamental heterogeneity. We impose the benchmark conditions

$$G = G^\top, \quad G_{ij} \geq 0, \quad G_{ii} = 0, \quad G\mathbf{1} = \mathbf{1}. \quad (8)$$

Thus G is symmetric and row-stochastic, with zero diagonal. The nonnegativity, row-stochasticity, and zero-diagonal conditions ensure that $(GC_t)_i$ is a weighted average of the judgments of other agents. We additionally assume that the associated interaction network is connected. This makes the consensus eigenvalue $g_1 = 1$ unique, while the remaining eigenmodes span the disagreement subspace. Symmetry and stochasticity imply that G admits an orthonormal eigenbasis; more general nonsymmetric interaction matrices are left outside the benchmark analysis.

The behavioral adjustment process contains two distinct channels. First, agents have intrinsic persistence in their judgments, captured by $0 \leq r < 1$. Second, they respond to the judgments of others with interaction strength $k > 0$. The parameter r therefore governs the persistence of an agent's own judgment, while k determines the strength of strategic substitution against the strategically relevant judgments of other agents. The fundamental vector θ and the interaction matrix G consequently play different economic roles: θ determines the source of cross-sectional heterogeneity, whereas G determines how that heterogeneity is transmitted across agents.

The interaction is forward-looking. Agent i does not condition only on the contemporaneous judgment of others, but forms an expectation about their next-period judgment. Define the relevant network-weighted judgment of agents other than i by

$$C_{-i,t} \equiv \sum_{j \neq i} G_{ij} C_{j,t} = (GC_t)_i, \quad (9)$$

where the equality follows from the zero-diagonal condition $G_{ii} = 0$. We use the anticipatory rule

$$\hat{C}_{-i,t+1} = C_{-i,t} + \lambda(C_{-i,t} - C_{-i,t-1}), \quad \lambda \geq 0. \quad (10)$$

The parameter λ measures the extent to which agents extrapolate recent changes in the judgments of others. When $\lambda = 0$, the expected next-period judgment coincides with the current judgment; larger values place greater weight on the recent direction of movement.

To specify individual behavior, suppose that agent i follows a behavioral partial-adjustment rule combining adjustment toward its fundamental benchmark with strategic substitution against the strategically relevant expected judgment of others:

$$C_{i,t+1} - C_{i,t} = (1 - r)(\theta_i - C_{i,t}) - k\hat{C}_{-i,t+1}. \quad (11)$$

The first term captures partial adjustment toward the fundamental, while the second captures strategic substitution against the expected judgment of others. The parameter r governs individual persistence, and k measures the strength of strategic interaction. This specification is a behavioral adjustment law rather than a static individual optimization problem.

For the benchmark parameterization, the resulting vector system can be written in reduced form as

$$C_{t+1} = (1 - r)\theta + [rI - k(1 + \lambda)G]C_t + k\lambda GC_{t-1}. \quad (12)$$

Equation (12) is the central dynamic representation of the model. It shows that the same interaction matrix G governs both the stationary equilibrium and the dynamic adjustment of individual judgments, so that the network structure can be tracked consistently across the equilibrium and dynamic systems.

A stationary equilibrium C^* therefore satisfies

$$C^* = (1 - r)\theta + [rI - kG]C^*, \quad (13)$$

and hence

$$C^* = (1 - r)[(1 - r)I + kG]^{-1}\theta, \quad (14)$$

whenever the inverse exists. The next section uses the structure of G to decompose (14) into the aggregate and disagreement modes and to derive the mode-specific equilibrium transmission coefficient.

4 Equilibrium Transmission

We now characterize how fundamental heterogeneity is transmitted into equilibrium price judgments. The key advantage of the benchmark network assumptions in (8) is that they separate the consensus direction from the cross-sectional disagreement directions. This separation allows equilibrium heterogeneity to be indexed by the eigenmodes of the interaction matrix.

Let (g_j, v_j) denote an eigenvalue–eigenvector pair of G ,

$$Gv_j = g_j v_j. \quad (15)$$

Under the benchmark assumptions, the consensus vector $\mathbf{1}$ is the unique eigenvector direction associated with the consensus eigenvalue $g_1 = 1$. Connectedness ensures that this eigenvalue is simple, while the remaining eigenvectors span the non-consensus, or disagreement, subspace. We denote the set of these non-consensus modes by $\mathcal{D}$.

Write the fundamental and equilibrium judgment vectors in the corresponding mode representation,

$$\theta = \sum_j \theta_j v_j, \quad C^* = \sum_j c_j^* v_j. \quad (16)$$

Substituting this representation into the stationary equilibrium (14) gives, for each mode,

$$c_j^* = \frac{1 - r}{1 - r + kg_j} \theta_j. \quad (17)$$

Thus each network mode acts as a distinct transmission channel for fundamental heterogeneity.

We define the equilibrium transmission coefficient of mode j by

$$T_j \equiv \frac{1 - r}{1 - r + kg_j}. \quad (18)$$

Then

$$c_j^* = T_j \theta_j. \quad (19)$$

When $\theta_j \neq 0$, this coefficient can equivalently be recovered as

$$T_j = \frac{c_j^*}{\theta_j}. \quad (20)$$

The coefficient T_j is the central equilibrium object of the analysis. It measures the mode-specific transmission of fundamental heterogeneity into equilibrium price heterogeneity. A value of T_j close to one means that the corresponding cross-sectional component of fundamentals is transmitted strongly into individual price judgments, whereas a smaller value indicates stronger compression of that component through strategic interaction. Outside the stable region, T_j need not lie in $[0, 1]$; within the stable region the Jury conditions imply $T_j > 0$, although T_j may still exceed one for modes associated with negative network eigenvalues.

Equation (18) shows that transmission is mode-specific. The same interaction mechanism can therefore compress different components of fundamental heterogeneity by different amounts. In particular, the consensus mode has $g_1 = 1$ and hence

$$T_1 = \frac{1 - r}{1 - r + k}. \quad (21)$$

For a disagreement mode $j \in \mathcal{D}$, transmission instead depends on the corresponding eigenvalue g_j . Thus the equilibrium transmission profile contains information about how the network transforms different components of fundamental heterogeneity.

The aggregate equilibrium follows immediately from the consensus component. Under uniform aggregation,

$$P^* = \frac{1 - r}{1 - r + k} \bar{\theta}, \quad \bar{\theta} \equiv \frac{1}{n} \mathbf{1}^\top \theta. \quad (22)$$

Thus the aggregate stationary price depends on the fundamental only through its projection onto the consensus direction. The remaining components of θ do not enter (22) directly. This is the source of the aggregation problem emphasized in this paper: aggregate equilibrium can be fully determined even when individual judgments remain heterogeneous.

To make this distinction explicit, define the disagreement component of the equilibrium judgment by

$$C_D^* = \sum_{j \in \mathcal{D}} c_j^* v_j. \quad (23)$$

Using (18), this becomes

$$C_D^* = \sum_{j \in \mathcal{D}} T_j \theta_j v_j. \quad (24)$$

Hence disagreement is not an independent residual introduced after solving for the aggregate price. It is the component of fundamental heterogeneity that is transmitted through the non-consensus network modes.

In the symmetric benchmark, where the eigenvectors can be chosen orthonormally, the cross-sectional disagreement dispersion is

$$D^* \equiv \frac{1}{n} \|C_D^*\|^2 = \frac{1}{n} \sum_{j \in \mathcal{D}} T_j^2 \theta_j^2. \quad (25)$$

Equation (25) provides a direct quantitative link between the source of heterogeneity, network structure, and equilibrium disagreement. It also shows why aggregate stability alone cannot determine the extent of individual disagreement.

The mode-specific transmission profile

$$\{T_j : j \in \mathcal{D}\}$$

therefore summarizes how the network maps fundamental heterogeneity into cross-sectional equilibrium heterogeneity. Different networks can generate different transmission profiles even when they generate the same aggregate stationary price. Conversely, a given network can transmit different components of fundamental heterogeneity at different intensities.

For the remainder of the paper, however, the importance of the transmission profile is not purely static. Because each T_j is generated by the same network eigenvalue g_j that enters the dynamic system, the equilibrium transmission coefficient can potentially impose restrictions on the subsequent adjustment process of the corresponding mode. The next sections formalize this equilibrium-to-dynamics connection.

The result can be summarized as follows.

Proposition 4.1 (Mode-specific equilibrium transmission). . *Suppose $0 \leq r < 1$, $k > 0$, and G satisfies (8). For every eigenmode j for which $1 - r + kg_j \neq 0$, the stationary equilibrium satisfies*

$$c_j^* = T_j \theta_j, \quad T_j = \frac{1 - r}{1 - r + kg_j}. \quad (26)$$

Hence equilibrium transmission is jointly determined by the persistence parameter r , the interaction strength k , and the eigenvalue g_j of the corresponding network mode.

The proposition establishes the static mapping from fundamental heterogeneity to equilibrium transmission. The next section derives the dynamic law governing these same modes. Section 6 then establishes the central equilibrium-to-dynamics connection: the same network eigenvalue that determines T_j also enters the characteristic polynomial governing the subsequent adjustment of mode j . Consequently, the equilibrium transmission coefficient is not merely a static description of cross-sectional heterogeneity; conditional on the common behavioral parameters, it places a structural restriction on the mode-specific dynamic law.

5 Dynamic Adjustment

We next characterize the dynamic adjustment of individual price judgments. The vector system in (12) has a particularly useful structure: the same interaction matrix G that determines equilibrium transmission also determines the propagation of each cross-sectional mode over time. Consequently, after decomposing the system into eigenmodes, the multidimensional dynamic problem reduces to a collection of scalar second-order difference equations. Under the symmetry assumption on G , the eigenvectors can be chosen to form an orthonormal basis, so the modal decomposition is exact and the modes evolve independently.

Let v_j be an eigenvector of G associated with eigenvalue g_j as in (15), and write the deviation of individual judgments from their stationary equilibrium as

$$C_t - C^* = \sum_j z_{j,t} v_j. \quad (27)$$

Because C^* satisfies the stationary equilibrium condition, subtracting that condition from (12) gives

$$C_{t+1} - C^* = [rI - k(1 + \lambda)G](C_t - C^*) + k\lambda G(C_{t-1} - C^*).$$

Projecting this vector equation onto eigenmode j yields

$$z_{j,t+1} = a_j z_{j,t} + b_j z_{j,t-1}. \quad (28)$$

where

$$a_j \equiv r - k(1 + \lambda)g_j, \quad b_j \equiv k\lambda g_j. \quad (29)$$

Thus the same network eigenvalue that indexes equilibrium transmission also determines the contemporaneous and lagged responses of the corresponding mode.

The characteristic polynomial associated with (28) is

$$q_j(\mu) = \mu^2 - a_j \mu - b_j. \quad (30)$$

Let $\mu_{j,+}$ and $\mu_{j,-}$ denote its two roots. The evolution of mode j is therefore a linear combination of $\mu_{j,+}^t$ and $\mu_{j,-}^t$, with the precise path determined by the initial conditions. The modulus of the dominant root determines the asymptotic persistence of the mode, while the signs and relative magnitudes of the two roots determine whether adjustment is monotone or alternating.

For the benchmark model, local asymptotic stability of mode j requires both characteristic roots to lie inside the unit circle. Since (30) can be written as

$$\mu^2 - a_j\mu - b_j = 0,$$

the standard second-order stability conditions are

$$1 - a_j - b_j > 0, \quad 1 + a_j - b_j > 0, \quad 1 + b_j > 0. \quad (31)$$

Substituting (29) gives

$$1 - r + k(1 + \lambda)g_j - k\lambda g_j > 0 \quad (32)$$

$$1 + r - k(1 + \lambda)g_j - k\lambda g_j > 0, \quad (33)$$

$$1 + k\lambda g_j > 0. \quad (34)$$

Equivalently, the first two conditions can be written as

$$1 - r + kg_j > 0, \quad (35)$$

and

$$1 + r - k(1 + 2\lambda)g_j > 0. \quad (36)$$

These conditions make clear that stability is mode-specific. The same structural parameters can therefore generate stable adjustment in one network mode and instability in another when the associated eigenvalues differ sufficiently.

The consensus mode deserves separate treatment. Since $g_1 = 1$, its dynamic coefficients are

$$a_1 = r - k(1 + \lambda), \quad b_1 = k\lambda. \quad (37)$$

Under the benchmark uniform aggregation rule, the aggregate price inherits the dynamics of the consensus component. Accordingly, aggregate convergence is governed by the roots of $q_1(\mu)$, whereas cross-sectional disagreement is governed by the roots $q_j(\mu)$ for $j \in D$. With non-uniform aggregation weights, disagreement modes may also load into the aggregate price; this extension is developed in Section 7.6. This distinction is important because aggregate stability does not imply that all disagreement modes converge at the same rate, or even that they share the same dynamic adjustment pattern.

For a disagreement mode $j \in \mathcal{D}$, define its asymptotic persistence by

$$\rho_j \equiv \max \{|\mu_{j,+}|, |\mu_{j,-}|\}. \quad (38)$$

When $\rho_j < 1$, the mode converges to its stationary equilibrium; values of ρ_j closer to one correspond to more persistent deviations. If the dominant root is negative, the mode exhibits alternating adjustment. If the roots are complex, the mode can instead display damped oscillatory adjustment whenever their common modulus is below one. These are dynamic implications of the same network mode that determines equilibrium transmission.

The connection to equilibrium transmission can be made explicit by eliminating the network eigenvalue. From (18),

$$T_j = \frac{1 - r}{1 - r + kg_j}, \quad (39)$$

and therefore

$$kg_j = (1 - r) \frac{1 - T_j}{T_j}. \quad (40)$$

Substitution into (29) yields

$$a_j = r - (1 + \lambda)(1 - r) \frac{1 - T_j}{T_j}, \quad (41)$$

$$b_j = \lambda(1 - r) \frac{1 - T_j}{T_j}. \quad (42)$$

Hence the characteristic polynomial can be expressed without the network eigenvalue:

$$q_j(\mu | T_j) = \mu^2 - \left[r - (1 + \lambda)(1 - r) \frac{1 - T_j}{T_j} \right] \mu - \lambda(1 - r) \frac{1 - T_j}{T_j}. \quad (43)$$

Equation (43) provides the formal representation linking the equilibrium and dynamic analyses. Section 4 established T_j as the mode-specific measure of equilibrium transmission, while the present section shows how the dynamic law of the same mode can be re-expressed in terms of that equilibrium object. The next section uses this representation to state the paper's main equilibrium-to-dynamics restriction and its cross-mode implications.

Proposition 5.1 (Mode-specific dynamic representation). *Suppose that the assumptions of Proposition 4.1 hold and that the dynamic system is given by (12). For every network eigenmode j , the deviation from stationary equilibrium satisfies (28), with coefficients given by (29). Moreover, whenever $T_j \neq 0$, the characteristic polynomial can be written entirely as a function of (T_j, r, λ) according to (43).*

The proposition establishes the representation needed to connect an equilibrium cross-sectional object to the dynamic adjustment of the same mode. Section 6 develops this connection as the paper's main structural restriction.

6 Main Result: Equilibrium Transmission and Dynamic Adjustment

The preceding sections establish two representations of the same network mode. First, the equilibrium transmission coefficient T_j summarizes how fundamental heterogeneity is transmitted into the stationary price judgment of mode j . Second, the same network mode follows a second-order dynamic system whose coefficients determine its subsequent adjustment.

At the level of a single mode, the connection between these two representations is algebraic: T_j is a one-to-one transformation of the network eigenvalue g_j , under the maintained regularity conditions. Thus, the substitution of T_j for g_j should not be interpreted as a new source of within-mode variation. The economic content of the result comes instead from the fact that T_j is an equilibrium cross-sectional object, whereas (a_j, b_j) describe subsequent time-series adjustment, and the behavioral parameters (r, λ) are common across network modes.

The main structural implication is therefore a cross-mode restriction. Equilibrium transmission coefficients observed across modes cannot be combined with arbitrary mode-specific dynamic laws: all modes must be jointly consistent with the same values of (r, λ) . This converts the within-mode algebraic representation into a testable restriction linking cross-sectional equilibrium information to subsequent dynamic adjustment.

6.1 The transmission–persistence restriction

From (40), the network eigenvalue associated with mode j can be recovered from its equilibrium transmission:

$$kg_j = (1 - r) \frac{1 - T_j}{T_j}. \quad (44)$$

Substituting this expression into the dynamic coefficients gives

$$a_j(T_j) = r - (1 + \lambda)(1 - r) \frac{1 - T_j}{T_j}, \quad (45)$$

and

$$b_j(T_j) = \lambda(1 - r) \frac{1 - T_j}{T_j}. \quad (46)$$

The characteristic polynomial of mode j is therefore

$$q_j(\mu | T_j) = \mu^2 - a_j(T_j)\mu - b_j(T_j). \quad (47)$$

This representation removes the network eigenvalue from the dynamic equation. The resulting restriction is important because T_j is an equilibrium object, whereas the roots of $q_j(\mu \mid T_j)$ describe subsequent adjustment over time. The model therefore links a cross-sectional equilibrium object to the time-series behavior of the same network mode.

This yields the central theorem.

Theorem 6.1 (Equilibrium transmission restricts mode dynamics). *Suppose that $0 \leq r < 1$, $\lambda \geq 0$, and $k > 0$, and let the interaction matrix G satisfy (8). For any network eigenmode j for which*

$$1 - r + kg_j \neq 0,$$

define

$$T_j = \frac{1 - r}{1 - r + kg_j}.$$

Then

$$kg_j = (1 - r) \frac{1 - T_j}{T_j}, \quad (48)$$

and, conditional on the structural parameters (r, λ) , the characteristic polynomial of the dynamic adjustment of mode j is uniquely determined by its equilibrium transmission coefficient:

$$q_j(\mu \mid T_j) = \mu^2 - \left[r - (1 + \lambda)(1 - r) \frac{1 - T_j}{T_j} \right] \mu - \lambda(1 - r) \frac{1 - T_j}{T_j}. \quad (49)$$

Thus, conditional on (r, λ) , the equilibrium transmission coefficient places a structural restriction on the dynamic law of the same network mode. If the roots of $q_j(\mu \mid T_j)$ lie inside the unit circle, the corresponding mode is asymptotically stable.

Interpretation of the theorem. The theorem is not a claim that the equilibrium transmission coefficient contains information about dynamics independently of the network eigenvalue. Within a single mode, T_j and g_j are algebraically equivalent conditional on the structural parameters. The substantive restriction arises when the relationship is imposed jointly across modes: the same behavioral parameters (r, λ) must map every observed equilibrium transmission coefficient into the dynamic coefficients of the corresponding mode.

The theorem is stronger than the statement that network structure affects price persistence. Network structure enters both equilibrium and dynamics through g_j , but the theorem eliminates this latent network object and expresses the dynamic law directly in terms of the equilibrium transmission coefficient. Conditional on (r, λ) , an observed equilibrium transmission coefficient therefore restricts the set of dynamic adjustment processes that the model can generate for the corresponding mode.

This restriction also has an identification interpretation. If equilibrium transmission can be estimated for a cross-sectional mode, then the model does not permit its subsequent dynamic behavior to vary freely while holding (r, λ) fixed. Thus, cross-sectional equilibrium information places restrictions on the dynamic adjustment process that cannot be recovered from the aggregate equilibrium price alone.

Proposition 6.2 (Cross-mode common-parameter restriction). *Suppose that the assumptions of Theorem 6.1 hold and that (r, λ) are common across network modes. Then the collection of equilibrium transmission coefficients and dynamic coefficients must satisfy*

$$a_j = r - (1 + \lambda)(1 - r) \frac{1 - T_j}{T_j}, \quad b_j = \lambda(1 - r) \frac{1 - T_j}{T_j}, \quad \forall j \in \mathcal{D}. \quad (50)$$

Equivalently, there must exist a common pair (r, λ) such that

$$q_j(\mu) = q(\mu \mid T_j; r, \lambda), \quad \forall j \in \mathcal{D}. \quad (51)$$

Thus, while the mapping between T_j and g_j is algebraic within a single mode, the joint consistency of (T_j, a_j, b_j) across modes is a cross-mode structural restriction of the model.

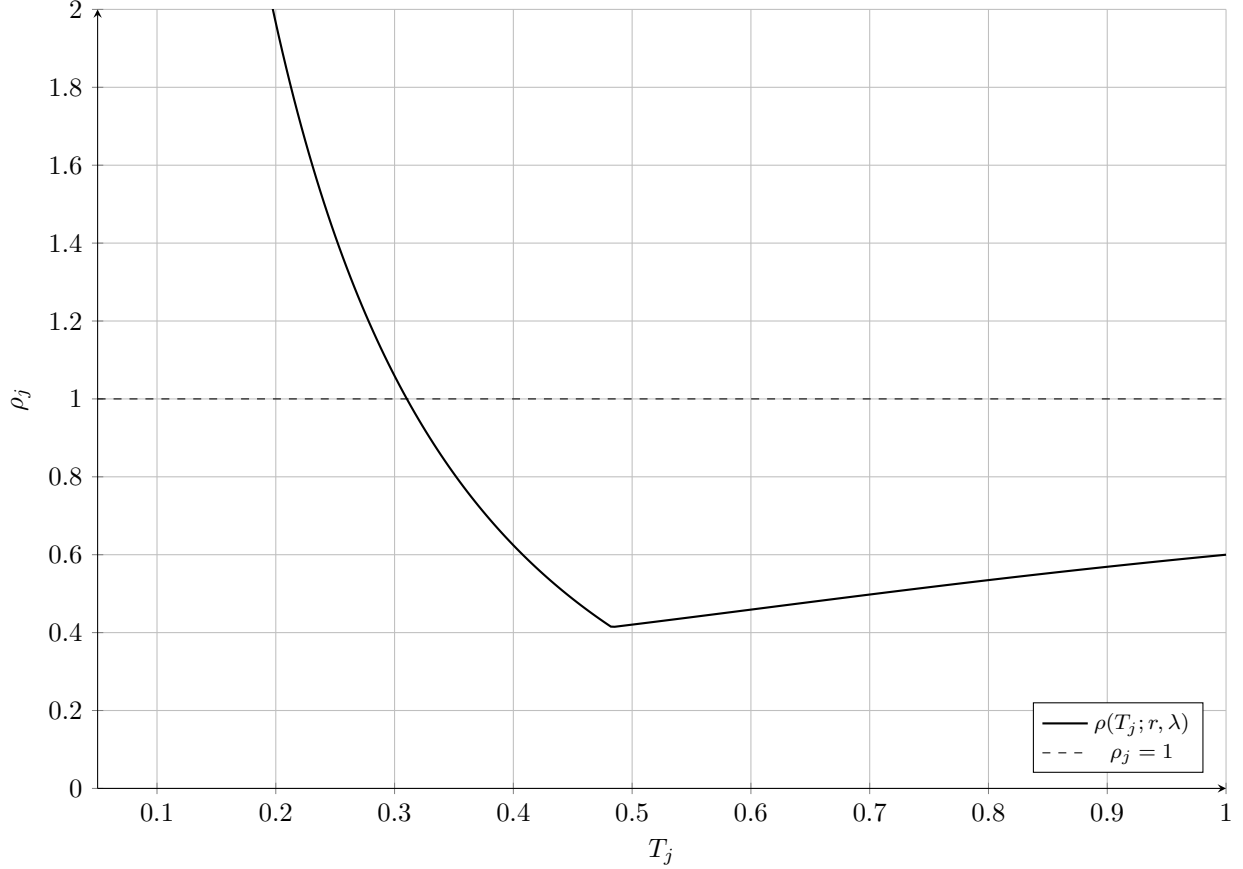


Figure 1: Equilibrium transmission and dynamic persistence. The figure illustrates the structural mapping $T_j \mapsto \rho(T_j; r, \lambda)$ implied by (53) for a representative parameterization $(r, \lambda) = (0.6, 0.4)$. The horizontal line $\rho_j = 1$ marks the boundary between the stable and unstable regions of the modal dynamics. The figure is a visualization of the analytical restriction and does not constitute an additional calibration or empirical assumption.

The proposition shifts the empirical content of the model from a mode-specific reparameterization to a joint cross-mode restriction. Within a single mode, replacing g_j with T_j does not introduce an independent source of variation. Across modes, however, the same behavioral parameters (r, λ) must rationalize the observed relationship between equilibrium transmission and dynamic adjustment. Hence the model can be rejected even when each mode is individually consistent with the dynamic representation.

6.2 Persistence implications

Let

$$\rho(T_j; r, \lambda) \equiv \max \{ |\mu_+(T_j; r, \lambda)|, |\mu_-(T_j; r, \lambda)| \}. \quad (52)$$

denote the asymptotic persistence of mode j . The theorem implies that

$$\rho_j = \rho(T_j; r, \lambda). \quad (53)$$

This mapping is the principal quantitative signature of the model.

The economic content of the mapping follows from (45)–(46). The equilibrium transmission coefficient determines both the contemporaneous and lagged coefficients governing the subsequent adjustment of the same mode. Consequently, T_j is not merely a static measure of the amount of disagreement that survives in equilibrium. Conditional on (r, λ) , it restricts how that disagreement can evolve over time.

The model also generates a useful cross-sectional prediction. Consider two disagreement modes j and ℓ with the same structural parameters (r, λ) . For distinct transmission coefficients, the corresponding characteristic polynomials differ generically:

$$T_j \neq T_\ell \implies q_j(\mu | T_j) \neq q_\ell(\mu | T_\ell) \text{ generically.}$$

Hence heterogeneity in equilibrium transmission maps into heterogeneity in the characteristic polynomials and, generically, into heterogeneity in dynamic persistence. This implication is stronger than predicting that disagreement is persistent on average: it predicts systematic *dispersion in the dynamic adjustment properties across modes*.

Corollary 6.3 (Transmission-based prediction of persistence). *Under the assumptions of Theorem 6.1, conditional on (r, λ) , the dynamic persistence of each network mode is pinned down by its equilibrium transmission coefficient through (53). Consequently, cross-sectional differences in T_j imply corresponding differences in the characteristic polynomials and, generically, in the persistence of disagreement modes.*

The corollary provides the most direct empirical interpretation of the model. Suppose that individual-level observations allow the researcher to estimate equilibrium transmission for several cross-sectional components. The model does not require those components to have identical adjustment paths. Instead, it predicts that their dynamic behavior should satisfy the structural mapping in (53). A rejection of this mapping would therefore reject the joint equilibrium and dynamic structure of the model, rather than merely indicating that one particular network specification is misspecified.

6.3 Stability and the convergent region

The transmission–persistence relationship is meaningful only in the region in which the corresponding equilibrium and dynamic system are well-defined. From (18), a finite transmission coefficient requires

$$1 - r + kg_j \neq 0. \tag{54}$$

For asymptotic convergence, the roots of $q_j(\mu | T_j)$ must additionally satisfy

$$|\mu_{j,+}| < 1, \quad |\mu_{j,-}| < 1.$$

Equivalently, the second-order stability conditions in (31) must hold. These restrictions define the economically relevant region in which equilibrium transmission can be interpreted jointly with dynamic persistence.

Importantly, the theorem itself does not require every disagreement mode to be stable. The mapping in (49) continues to characterize the linear dynamics whenever the characteristic polynomial is well-defined. Stability determines whether those dynamics converge to equilibrium; it is a separate restriction on the admissible parameter and network region. This distinction prevents the main result from being reduced to a stability assumption.

6.4 Why the result matters economically

The economic significance of the result is that it connects a cross-sectional equilibrium object and a time-series object that are usually analyzed separately. Equilibrium transmission measures how strongly fundamental heterogeneity survives strategic interaction at a point in time. Dynamic persistence measures how the corresponding mode adjusts after a disturbance. The model predicts that, because both objects are generated by the same network mode, they must satisfy the structural restriction in Theorem 6.1.

This provides a sharper prediction than aggregate convergence. Two economies may have identical aggregate prices and identical aggregate convergence rates while differing in their cross-sectional transmission profiles. The model predicts that these differences are not innocuous: conditional on common (r, λ) , they imply different admissible dynamic laws for the underlying disagreement modes. Aggregate observations can therefore conceal economically meaningful differences in the adjustment process.

The result also clarifies the precise novelty of the framework relative to models in which networks, heterogeneous beliefs, or disagreement affect prices independently. The contribution is not the introduction

of any one of these ingredients in isolation. It is the structural restriction that the *equilibrium transmission of heterogeneity through a network mode constrains the dynamic adjustment of that same mode*. The model thus turns an equilibrium cross-sectional object into a restriction on subsequent dynamic behavior.

The next section uses this restriction to derive observable implications for aggregate prices, disagreement dispersion, and observational equivalence. It also develops the cross-sectional dispersion prediction that provides a natural empirical counterpart to the transmission–persistence mapping.

Cross-mode restriction. The restriction becomes testable across modes because the behavioral parameters (r, λ) are common to all network modes. Thus, for any two modes j and ℓ , their equilibrium transmission coefficients and dynamic coefficients cannot vary independently. In particular, the model requires the existence of common parameters (r, λ) such that

$$q_j(\mu) = q(\mu \mid T_j; r, \lambda) \quad \text{for all } j. \quad (55)$$

This cross-mode restriction is stronger than the mode-by-mode representation alone: it implies that the entire collection of equilibrium transmission coefficients and dynamic laws must be jointly consistent with a common pair of behavioral parameters.

7 Aggregate Prices, Disagreement, and Quantitative Predictions

The transmission–persistence restriction in Section 6 has direct implications for observables. In particular, the model implies that aggregate prices and cross-sectional disagreement contain different information about the underlying state of the economy. Aggregate prices depend only on the consensus component of the equilibrium state, whereas the cross-sectional distribution of individual judgments contains information about the transmission of non-consensus fundamental components.

7.1 Aggregate prices and hidden disagreement

Under the benchmark uniform aggregation rule, the stationary aggregate price is

$$P^* = T_1 \bar{\theta}, \quad T_1 = \frac{1-r}{1-r+k}, \quad (56)$$

where $\bar{\theta} = n^{-1} \mathbf{1}^\top \theta$. Equation (56) implies that the aggregate price depends on the fundamental only through its consensus projection. The disagreement modes do not enter the aggregate directly.

This generates a simple stationary aggregate-price equivalence result.

Theorem 7.1 (Aggregate stationary-price equivalence). *Suppose two economies have the same aggregate fundamental component $\bar{\theta}$ and the same consensus transmission coefficient T_1 . Then they generate the same stationary aggregate price P^* , even if their disagreement-mode fundamentals $\{\theta_j : j \in \mathcal{D}\}$ or their disagreement transmission profile $\{T_j : j \in \mathcal{D}\}$ differ.*

The theorem establishes that equality of stationary aggregate prices does not imply equality of the underlying cross-sectional state. In particular, two economies can be indistinguishable from aggregate stationary price data while having different levels and compositions of individual disagreement. The distinction follows directly from the orthogonal decomposition of the equilibrium state.

The corresponding equilibrium disagreement dispersion is

$$D^* = \frac{1}{n} \sum_{j \in \mathcal{D}} T_j^2 \theta_j^2. \quad (57)$$

Thus, holding the aggregate fundamental component fixed, changes in the network structure can alter disagreement without changing the aggregate stationary price. Aggregate price stability can therefore coexist with substantial cross-sectional dispersion.

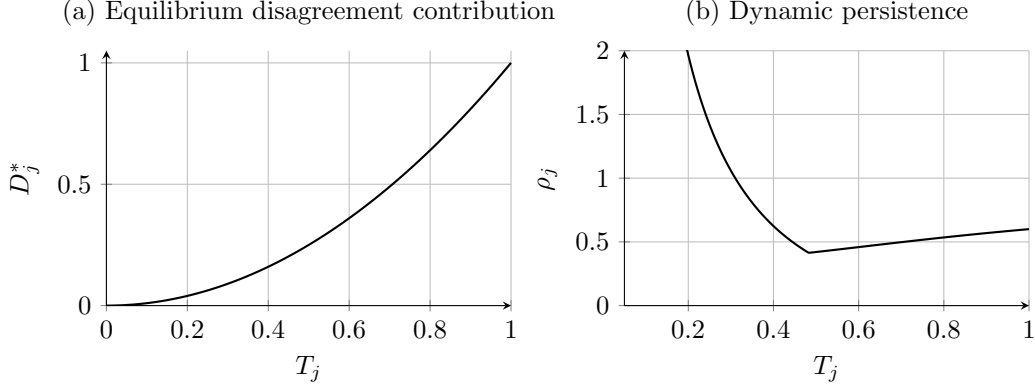


Figure 2: The joint static–dynamic signature of equilibrium transmission. Panel (a) illustrates the mode-level equilibrium contribution to cross-sectional disagreement, $D_j^* = n^{-1}T_j^2\theta_j^2$, under the normalization $n^{-1}\theta_j^2 = 1$. Panel (b) illustrates the corresponding dynamic persistence $\rho_j = \rho(T_j; r, \lambda)$ for the representative parameterization $(r, \lambda) = (0.6, 0.4)$. The two panels illustrate how the same equilibrium transmission coefficient enters both the cross-sectional contribution of a disagreement mode and its subsequent dynamic adjustment. The figure is illustrative and does not impose an additional behavioral relationship between the two objects.

7.2 A quantitative cross-sectional signature

Rather than treating D^* only as an aggregate dispersion statistic, consider the contribution of an individual disagreement mode,

$$D_j^* \equiv \frac{1}{n}T_j^2\theta_j^2. \quad (58)$$

The object D_j^* measures the amount of equilibrium cross-sectional disagreement attributable to fundamental component θ_j after that component has been transmitted through the interaction structure. It therefore combines two distinct forces: the magnitude of the underlying heterogeneous fundamental, θ_j , and the transmission of that heterogeneity into equilibrium judgments, T_j . The square arises because the object is a contribution to cross-sectional variance rather than a signed disagreement measure.

The total equilibrium disagreement can consequently be written as

$$D^* = \sum_{j \in \mathcal{D}} D_j^*. \quad (59)$$

This decomposition makes clear why aggregate prices and cross-sectional dispersion contain different information. The aggregate price depends only on the consensus projection, whereas D_j^* records how much of a specific non-consensus fundamental component remains visible across individual judgments.

The measurement interpretation also gives a quantitative prediction. Holding θ_j fixed, a mode with larger equilibrium transmission contributes more to cross-sectional disagreement:

$$D_j^* \propto T_j^2.$$

Thus the model predicts not only that disagreement may survive aggregation, but also how the equilibrium contribution of a given fundamental disagreement mode scales with its transmission coefficient.

Combining this measurement object with the dynamic result in Section 6 gives the joint restriction

$$D_j^* = \frac{1}{n}T_j^2\theta_j^2, \quad \rho_j = \rho(T_j; r, \lambda). \quad (60)$$

Hence the same transmission coefficient enters two empirically distinct dimensions of a disagreement mode: its equilibrium contribution to cross-sectional dispersion and its subsequent dynamic adjustment. The economic content of the model is therefore not exhausted by the level of disagreement. It links the measured cross-sectional contribution of a mode to the dynamic law governing that same mode.

7.3 The cross-mode transmission–dynamics signature

The strongest empirical implication of the model is not simply that disagreement is persistent, or that persistence differs across network modes. It is that the equilibrium transmission of a mode restricts the dynamic law of that same mode, conditional on common structural parameters (r, λ) . Section 6 establishes this restriction theoretically; here we formulate it as a cross-mode empirical test.

For each disagreement mode $j \in \mathcal{D}$, let T_j denote its equilibrium transmission coefficient and let $q_j(\mu)$ denote the characteristic polynomial governing its observed dynamic adjustment. The model imposes the cross-mode restriction

$$H_0 : \exists (r, \lambda) \text{ such that } q_j(\mu) = q(\mu | T_j; r, \lambda) \quad \forall j \in \mathcal{D}, \quad (61)$$

where

$$q(\mu | T; r, \lambda) = \mu^2 - \left[r - (1 + \lambda)(1 - r) \frac{1 - T}{T} \right] \mu - \lambda(1 - r) \frac{1 - T}{T}. \quad (62)$$

Thus the null hypothesis is a joint restriction across modes rather than a collection of independent mode-specific regressions. The same pair (r, λ) must rationalize the relationship between equilibrium transmission and dynamic adjustment for every observed disagreement mode. The network eigenvalues g_j do not enter the empirical restriction directly: they are eliminated through the equilibrium transmission coefficient.

This formulation gives the model a sharper empirical signature than a generic association between disagreement and persistence. The distinction from a purely algebraic reparameterization is important for empirical interpretation. If the model were considered mode by mode, the substitution of T_j for g_j would add no independent restriction: the two quantities are algebraically equivalent conditional on the structural parameters. The additional content arises because the same (r, λ) must rationalize all modes simultaneously. Consequently, the model can be rejected even when each mode can be fit individually: the mode-specific estimates of (r, λ) may fail to coincide. The cross-mode restriction therefore uses the joint variation in equilibrium transmission and dynamic adjustment rather than the algebraic identity linking T_j and g_j within a single mode. A reduced-form relationship between T_j and a persistence statistic could arise from many mechanisms. By contrast, (61) requires the observed dynamic coefficients themselves to satisfy the two-parameter structural restriction generated by the common values of (r, λ) .

In particular, writing

$$q_j(\mu) = \mu^2 - a_j \mu - b_j,$$

the model requires

$$a_j = r - (1 + \lambda)(1 - r) \frac{1 - T_j}{T_j}, \quad b_j = \lambda(1 - r) \frac{1 - T_j}{T_j}, \quad \forall j \in \mathcal{D}. \quad (63)$$

Hence variation in T_j generates simultaneous restrictions on both the contemporaneous coefficient a_j and the lag coefficient b_j . The prediction is therefore about the structure of the dynamic response, not merely its persistence.

A useful scalar implication is obtained by defining

$$\rho_j = \max \{ |\mu_{j,+}|, |\mu_{j,-}| \}. \quad (64)$$

Under H_0 ,

$$\rho_j = \rho(T_j; r, \lambda), \quad \forall j \in \mathcal{D}. \quad (65)$$

The persistence relation is therefore a lower-dimensional implication of the stronger polynomial restriction in (61). Empirically, the polynomial restriction should be treated as the primary test and the persistence mapping as its most transparent summary.

This distinction matters for empirical interpretation. If two modes have different transmission coefficients, the model predicts different dynamic laws even when their aggregate contributions are not separately visible. Thus the null does not impose homogeneous persistence across modes. Instead, it predicts systematic heterogeneity in dynamic behavior that is quantitatively disciplined by equilibrium transmission.

A direct empirical implementation is to estimate (T_j, a_j, b_j) for a set of disagreement modes and ask whether there exists a common pair (r, λ) satisfying (63) for all modes. The test therefore combines cross-sectional and time-series information: equilibrium observations identify the transmission profile $\{T_j\}$, while

the dynamic path of the same modes identifies (a_j, b_j) . Rejection can be interpreted as a rejection of the joint equilibrium-to-dynamics mechanism, rather than simply as evidence against a particular network specification.

7.4 What aggregate data cannot reveal

Aggregate stationary-price equivalence implies that identical aggregate prices need not correspond to identical disagreement structures or dynamic adjustment paths. In particular, two economies with identical $\bar{\theta}$ and T_1 can generate the same P^* while differing in their disagreement components and their subsequent mode-specific dynamics.

This implies that aggregate price convergence should not be interpreted as evidence that individual judgments have converged unless additional cross-sectional information rules out persistent disagreement modes.

7.5 Testable implications

The model produces three principal quantitative implications.

First, equilibrium disagreement is mode-specific and satisfies

$$D_j^* = \frac{1}{n} T_j^2 \theta_j^2.$$

Second, conditional on (r, λ) , the dynamic coefficients satisfy the cross-mode restriction

$$a_j = r - (1 + \lambda)(1 - r) \frac{1 - T_j}{T_j}, \quad b_j = \lambda(1 - r) \frac{1 - T_j}{T_j}.$$

Third, aggregate stationary prices cannot in general recover the disagreement profile because only the consensus projection enters P^* .

These implications suggest an empirical strategy that combines cross-sectional and time-series information. One can estimate the equilibrium transmission profile $\{T_j\}$ from individual-level judgments, estimate the corresponding dynamic coefficients $\{a_j, b_j\}$, and test whether a common pair (r, λ) satisfies the structural restriction across modes. The resulting test directly evaluates whether equilibrium transmission and subsequent dynamic adjustment are jointly consistent with the model.

7.6 Non-uniform aggregation and aggregate price dynamics

The benchmark aggregation rule assigns equal weight to all individual judgments. This assumption is useful for isolating the consensus mode, but it is not necessary for the equilibrium and dynamic decomposition itself. Consider instead an aggregate price defined by

$$P_t = w^\top C_t, \quad w^\top \mathbf{1} = 1, \tag{66}$$

where w is a vector of aggregation weights satisfying $w_i \geq 0$ and $w^\top \mathbf{1} = 1$. The benchmark case corresponds to $w = n^{-1} \mathbf{1}$.

Using the orthonormal modal decomposition, write the deviation from stationary equilibrium as

$$C_t - C^* = \sum_{j=1}^n z_{j,t} v_j. \tag{67}$$

Then the aggregate price deviation satisfies

$$P_t - P^* = w^\top (C_t - C^*) = \sum_{j=1}^n \omega_j z_{j,t}, \quad \omega_j \equiv w^\top v_j. \tag{68}$$

Under uniform aggregation, $w = n^{-1}\mathbf{1}$ and $w^\top v_j = 0$ for every disagreement mode $j \in D$. The benchmark result that aggregate prices load only on the consensus mode therefore follows as a special case. Under non-uniform aggregation, however, ω_j need not vanish for $j \in D$. Hence disagreement modes can contribute directly to aggregate price movements.

Proposition 7.2 (Non-uniform aggregation transmits disagreement dynamics). *Suppose that the benchmark network and behavioral assumptions hold and that aggregate prices are defined by (66). For any disagreement mode $j \in D$ with $\omega_j = w^\top v_j \neq 0$, a disturbance to mode j affects the aggregate price through (68). In particular, the aggregate component generated by that mode has the same characteristic roots $\mu_{j,+}$ and $\mu_{j,-}$ as the underlying mode.*

Consequently, whenever the mode is dynamically stable, its contribution to aggregate price adjustment has asymptotic persistence ρ_j . Conditional on the common structural parameters (r, λ) , this persistence is therefore linked to the mode's equilibrium transmission coefficient through

$$\rho_j = \rho(T_j; r, \lambda).$$

Proof. For a disturbance confined to mode j , the modal decomposition gives $C_t - C^* = z_{j,t}v_j$. Equation (68) then implies

$$P_t - P^* = \omega_j z_{j,t}.$$

Since $z_{j,t}$ satisfies the second-order recurrence with characteristic polynomial $q_j(\mu)$, multiplication by the constant loading ω_j does not alter its characteristic roots. The persistence of the aggregate component generated by mode j is therefore ρ_j . The final statement follows from the transmission–persistence restriction in Section 6. $\square$

The proposition provides a direct channel through which cross-sectional heterogeneity can become observable in aggregate price dynamics. Under uniform aggregation, disagreement is orthogonal to the aggregate and is therefore hidden from aggregate price movements. Under non-uniform aggregation, the same disagreement modes can become observable in the aggregate whenever their aggregation loadings ω_j are nonzero. The model thus separates two distinct mechanisms: the network determines the equilibrium transmission and dynamic adjustment of each mode, whereas the aggregation weights determine how strongly each mode is represented in the observed aggregate price.

For multiple disagreement modes, aggregate price dynamics are generally a weighted sum of their respective dynamic responses. The aggregate series therefore need not inherit the persistence of any single mode. Instead, its behavior depends jointly on the modal loadings $\{\omega_j\}$, the mode-specific characteristic roots, and the disturbances exciting each mode. The single-mode result above provides the cleanest interpretation of the mechanism.

8 Identification Conditions and Empirical Testability

The preceding results imply that aggregate and cross-sectional data contain different information about the equilibrium. The aggregate price depends only on the consensus component and therefore does not, in general, identify the disagreement modes or their transmission coefficients. The empirical content of the model comes from combining aggregate, cross-sectional, and dynamic information, with cross-sectional and dynamic observations providing the information that aggregate prices cannot reveal.

8.1 What Is and Is Not Identified from Aggregate Data

Aggregate stationary prices identify the consensus product $T_1\bar{\theta}$, but not the disagreement transmission profile. This is a structural feature of the benchmark model: the aggregate projection eliminates the disagreement modes under the symmetric interaction structure. Consequently, different cross-sectional states can generate the same stationary aggregate price.

The identification failure is structural rather than statistical. Under the benchmark symmetric interaction structure, the aggregate projection eliminates the disagreement modes because the consensus vector is orthogonal to every disagreement eigenmode. Hence different cross-sectional states can be observationally equivalent in aggregate stationary price data.

Within the present framework, cross-sectional observations provide the additional information required to distinguish such economies. For mode j , the equilibrium relationship is

$$c_j^* = T_j \theta_j, \quad T_j = \frac{1 - r}{1 - r + kg_j}, \quad (69)$$

so that, whenever $\theta_j \neq 0$ and T_j is well-defined,

$$T_j = \frac{c_j^*}{\theta_j}.$$

Thus the transmission profile is not recoverable from aggregate prices alone, but can be recovered from cross-sectional observations when the corresponding fundamental components are observed or consistently estimated.

This recovery is relative to the modal decomposition induced by the interaction structure. Under the benchmark assumption that G is observed or consistently estimated and symmetric, the empirical construction maps the interaction structure into its eigenbasis and decomposes the fundamental and equilibrium judgment vectors into the corresponding components:

$$G \longrightarrow \{v_j\} \longrightarrow \{\theta_j, c_j^*\} \longrightarrow \{T_j\}.$$

The dynamic observations must then be expressed in the same modal basis so that estimated transmission and dynamic coefficients refer to the same network mode.

The distinction is central to the empirical content of the model. Aggregate data identify the consensus product $T_1 \bar{\theta}$, whereas cross-sectional information identifies the mode-specific transmission coefficients that govern disagreement. Dynamic observations provide the additional information needed to test whether these equilibrium transmission coefficients impose the restrictions derived in Section 6.

8.2 Identification of the Behavioral Parameters

The modal representation also provides a direct route to identifying the behavioral parameters. Define

$$s_j \equiv \frac{1 - T_j}{T_j}. \quad (70)$$

The dynamic coefficients then satisfy

$$a_j = r - (1 + \lambda)(1 - r)s_j, \quad b_j = \lambda(1 - r)s_j. \quad (71)$$

Adding the two coefficients yields

$$a_j + b_j = r - (1 - r)s_j, \quad (72)$$

and therefore

$$r = \frac{s_j + a_j + b_j}{1 + s_j}. \quad (73)$$

Whenever $s_j \neq 0$, the extrapolation parameter is subsequently recovered from

$$\lambda = \frac{b_j}{(1 - r)s_j}. \quad (74)$$

These expressions provide mode-level recovery formulas rather than an unconditional claim of statistical identification. Their empirical use requires that T_j , a_j , and b_j be identified from the available cross-sectional and time-series data, that the relevant denominators be nonzero, and that the benchmark modal decomposition be valid.

8.3 Cross-Mode Overidentifying Restrictions

Because (r, λ) are common behavioral parameters, the recovery formulas above must yield the same parameter values across all informative modes. For any two modes j and ℓ with nonzero s_j and s_ℓ , the model requires

$$\frac{s_j + a_j + b_j}{1 + s_j} = \frac{s_\ell + a_\ell + b_\ell}{1 + s_\ell} = r. \quad (75)$$

Conditional on the recovered value of r , it also requires

$$\frac{b_j}{(1 - r)s_j} = \frac{b_\ell}{(1 - r)s_\ell} = \lambda. \quad (76)$$

Thus multiple disagreement modes generate overidentifying restrictions: the same behavioral parameters must be recovered from each informative mode.

8.4 Cross-sectional dispersion as an observable implication

The equilibrium dispersion result provides a complementary observable object. Under the benchmark orthogonal decomposition,

$$D^* = \frac{1}{n} \sum_{j \in \mathcal{D}} T_j^2 \theta_j^2. \quad (77)$$

Thus observed cross-sectional dispersion provides information about the transmission of non-consensus fundamental components that is absent from the aggregate stationary price.

When the modal components can be tracked over time, the same decomposition also provides the link between cross-sectional measurement and dynamic identification. Dispersion itself is not the structural test; rather, it helps recover and measure the mode-specific equilibrium objects that enter the equilibrium-to-dynamics restriction.

8.5 A feasible empirical design

A direct implementation can proceed in four steps. First, estimate the interaction structure G and construct its modal decomposition. Second, estimate the equilibrium transmission coefficients from

$$T_j = \frac{c_j^*}{\theta_j},$$

using observed or consistently estimated fundamental components. Third, estimate the mode-specific dynamic coefficients (a_j, b_j) from the time-series behavior of the same modal components. Fourth, recover the behavioral parameters (r, λ) and test whether the same parameter values satisfy the cross-mode restrictions in (75) and (76).

The identifying variation is therefore cross-sectional and dynamic rather than purely aggregate. A rejection may arise because equilibrium transmission is inconsistent with the observed dynamics, because a common set of structural parameters cannot explain the cross-mode relationship, or because the benchmark network assumptions do not adequately describe the underlying interaction structure. These possibilities can be distinguished by examining which component of the structural restriction fails.

The empirical strategy therefore provides a direct test of the paper's central mechanism: whether cross-sectional equilibrium transmission contains information about subsequent mode-specific adjustment. An aggregate specification may reproduce the same stationary price while remaining silent about the disagreement structure and its dynamics. The additional cross-sectional and time-series information allows the model's equilibrium-to-dynamics restriction to be tested directly.

9 Conclusion

This paper develops a framework in which strategic interaction determines both the equilibrium transmission of heterogeneous fundamentals and the subsequent dynamic adjustment of the same cross-sectional components. The central equilibrium object is the mode-specific transmission coefficient

$$T_j = \frac{c_j^*}{\theta_j} = \frac{1-r}{1-r+kg_j}. \quad (78)$$

It measures how strongly fundamental heterogeneity in network mode j is transmitted into equilibrium price judgments.

The main result is that T_j is not merely a static measure of disagreement. Eliminating the network eigenvalue from the dynamic system gives

$$q_j(\mu | T_j) = \mu^2 - \left[r - (1+\lambda)(1-r)\frac{1-T_j}{T_j} \right] \mu - \lambda(1-r)\frac{1-T_j}{T_j}, \quad (79)$$

so that, conditional on (r, λ) , equilibrium transmission restricts the characteristic polynomial governing the corresponding mode. The model therefore links a cross-sectional equilibrium object to a time-series property through a common structural mechanism.

Two further implications follow. First, aggregate prices generally identify only the consensus component of the underlying state. Economies with the same aggregate fundamental and consensus transmission can therefore be observationally equivalent in aggregate stationary prices while differing in their disagreement structure. Second, equilibrium disagreement is mode-specific and depends on both the magnitude of the underlying heterogeneous fundamental and its network transmission. Thus the framework connects aggregate prices, cross-sectional disagreement, and mode-specific dynamic adjustment without requiring the aggregate price to reveal the full underlying state.

The contribution is not the introduction of heterogeneous beliefs, disagreement, strategic interaction, or network structure in isolation. Each of these ingredients has been studied extensively. The distinctive result is the joint equilibrium–dynamic restriction: conditional on common adjustment parameters, the amount of heterogeneity transmitted through a network mode restricts the dynamic law of that same mode. This provides an empirically testable signature that requires both cross-sectional and time-series information and is generally not visible in aggregate price data alone.

The analysis is deliberately developed under a benchmark interaction environment that permits an analytical mode decomposition. Its main limitations are the common adjustment parameters and the benchmark network structure. Extending the framework to more general network structures, heterogeneous adjustment parameters, or endogenous network formation is therefore a natural direction for future work. Such extensions would clarify whether, and under what conditions, the equilibrium–dynamic restriction survives beyond the benchmark environment.

The central message is that equilibrium transmission of heterogeneous fundamentals can contain information about subsequent dynamic adjustment when both are generated by the same strategic interaction structure. The framework makes this connection explicit as a quantitative restriction linking cross-sectional transmission, disagreement, and mode-specific dynamic adjustment. The resulting equilibrium-to-dynamics restriction provides a basis for testing whether cross-sectional equilibrium patterns are consistent with the subsequent evolution of disagreement.

A Behavioral Adjustment Representation

This appendix specifies the individual adjustment rule and derives the dynamic system used in the main text. The purpose is to make explicit the behavioral assumptions underlying (12) without introducing additional structure beyond the benchmark environment.

For agent i , let the relevant network-weighted judgment of other agents be denoted by $\hat{C}_{-i,t+1}$. The individual adjustment rule is specified as

$$C_{i,t+1} - C_{i,t} = (1-r)(\theta_i - C_{i,t}) - k\hat{C}_{-i,t+1}. \quad (80)$$

The first term captures partial adjustment of the agent's judgment toward its fundamental benchmark. The second captures strategic substitution against the strategically relevant judgment of other agents. The parameter r governs individual persistence, while k measures the strength of strategic interaction.

The adjustment rule is a reduced-form behavioral specification rather than a first-order condition derived from a static optimization problem. Its role is to represent two economically distinct forces in the adjustment process: correction toward the agent's own fundamental benchmark and strategic response to the network-weighted judgment of other agents. This keeps the behavioral structure transparent while allowing the equilibrium and dynamic implications to be derived from the same adjustment mechanism.

The extrapolative expectation component is governed by the rule introduced in the main text,

$$\hat{C}_{-i,t+1} = C_{-i,t} + \lambda(C_{-i,t} - C_{-i,t-1}), \quad \lambda \geq 0. \quad (81)$$

This rule takes the current network-weighted judgment of others as the baseline forecast and extrapolates a fraction λ of its most recent change. Thus $\lambda = 0$ corresponds to a non-extrapolative expectation, while $\lambda > 0$ introduces forward-looking adjustment based on the recent direction of the interaction term. The parameter λ therefore controls the extent to which current strategic adjustment responds to the recent movement of the network-weighted judgment.

Collecting the individual adjustment rules and expressing the network-weighted judgments through G yields

$$C_{t+1} = (1 - r)\theta + [rI - k(1 + \lambda)G]C_t + k\lambda GC_{t-1}. \quad (82)$$

This is the dynamic system stated in (12). In particular, the interaction matrix enters contemporaneously with coefficient $-k(1 + \lambda)$ and with one lag through $k\lambda G$. No additional behavioral parameter is required to obtain the second-order network dynamics.

The stationary system follows by imposing $C_{t+1} = C_t = C_{t-1} = C^*$ in (82). The resulting condition is

$$C^* = (1 - r)\theta + (rI - kG)C^*, \quad (83)$$

which is the stationary equilibrium equation used in Section 4.

The specification therefore generates the model's equilibrium and dynamic coefficients from a common behavioral mechanism: adjustment toward the fundamental benchmark, strategic interaction through the network, and extrapolation of recent network movements. The subsequent appendices use this common system to derive the equilibrium transmission coefficient, the equilibrium-to-dynamics restriction, and the associated stability and observability results.

B Equilibrium and Mode Decomposition

This appendix derives the stationary equilibrium and its decomposition into network modes. The derivation establishes formally the equilibrium transmission coefficient used in Sections 4 and 6.

Starting from the dynamic system in (82), impose stationarity,

$$C_{t+1} = C_t = C_{t-1} = C^*.$$

The stationary equilibrium therefore satisfies

$$C^* = (1 - r)\theta + [rI - kG]C^*, \quad (84)$$

or, equivalently,

$$[(1 - r)I + kG]C^* = (1 - r)\theta. \quad (85)$$

Whenever

$$\det[(1 - r)I + kG] \neq 0, \quad (86)$$

the stationary equilibrium is uniquely given by

$$C^* = (1 - r)[(1 - r)I + kG]^{-1}\theta. \quad (87)$$

We now decompose the equilibrium according to the eigenmodes of the interaction matrix. Let (g_j, v_j) satisfy

$$Gv_j = g_j v_j. \quad (88)$$

For the benchmark network, the consensus vector $\mathbf{1}$ is an eigenvector with eigenvalue $g_1 = 1$. The remaining eigenvectors span the non-consensus, or disagreement, subspace. Under the symmetry assumption on G , the eigenvectors can be chosen to form an orthonormal basis. If an eigenvalue has multiplicity greater than one, the transmission coefficient is common to the corresponding eigenspace, so the economically relevant object is the projection onto that eigenspace rather than a particular basis vector within it. Write

$$\theta = \sum_j \theta_j v_j, \quad C^* = \sum_j c_j^* v_j. \quad (89)$$

Substituting these expansions into (85) and using (88) gives

$$\sum_j (1 - r + kg_j) c_j^* v_j = (1 - r) \sum_j \theta_j v_j. \quad (90)$$

Linear independence of the eigenvectors implies, mode by mode,

$$(1 - r + kg_j) c_j^* = (1 - r) \theta_j. \quad (91)$$

Hence

$$c_j^* = \frac{1 - r}{1 - r + kg_j} \theta_j. \quad (92)$$

Define the equilibrium transmission coefficient of mode j by

$$T_j \equiv \frac{1 - r}{1 - r + kg_j}. \quad (93)$$

Then

$$c_j^* = T_j \theta_j. \quad (94)$$

When $\theta_j \neq 0$, this coefficient can equivalently be recovered as

$$T_j = \frac{c_j^*}{\theta_j}. \quad (95)$$

The coefficient is well-defined whenever

$$1 - r + kg_j \neq 0. \quad (96)$$

Thus the transmission coefficient is not an additional primitive of the model. It is determined by the individual persistence parameter r , the interaction strength k , and the eigenvalue of the network mode.

The consensus mode follows by setting $g_1 = 1$:

$$T_1 = \frac{1 - r}{1 - r + k}. \quad (97)$$

Under uniform aggregation, the consensus coefficient is the component that enters the aggregate stationary price. In contrast, for $j \in \mathcal{D}$, the coefficients T_j determine how the corresponding non-consensus fundamental components survive in individual equilibrium judgments.

Finally, (95) can be inverted whenever $T_j \neq 0$:

$$kg_j = (1 - r) \frac{1 - T_j}{T_j}. \quad (98)$$

This identity is the key algebraic step used in Appendix C to eliminate the latent network eigenvalue from the dynamic characteristic polynomial.

Proposition B.1 (Equilibrium mode decomposition). *Under (86), the stationary equilibrium admits the mode representation*

$$c_j^* = T_j \theta_j, \quad T_j = \frac{1 - r}{1 - r + kg_j}.$$

Consequently, conditional on (r, k) , the equilibrium transmission profile is given by the eigenvalues of the interaction matrix.

C Main Theorem and Stability

This appendix proves the transmission–persistence result stated in Section 6 and characterizes the corresponding stability region. The proof consists of two steps. First, the network eigenvalue is eliminated using the equilibrium transmission coefficient. Second, the resulting characteristic polynomial is related to the stability conditions of the second-order difference equation.

From (82), subtract the stationary equilibrium condition (84). For a deviation

$$C_t - C^* = \sum_j z_{j,t} v_j,$$

the coefficient of eigenmode j satisfies

$$z_{j,t+1} = [r - k(1 + \lambda)g_j] z_{j,t} + k\lambda g_j z_{j,t-1}. \quad (99)$$

Define

$$a_j \equiv r - k(1 + \lambda)g_j, \quad b_j \equiv k\lambda g_j. \quad (100)$$

The characteristic equation associated with (99) is therefore

$$q_j(\mu) = \mu^2 - a_j \mu - b_j = 0. \quad (101)$$

The equilibrium transmission coefficient derived in (95) satisfies

$$T_j = \frac{1 - r}{1 - r + kg_j}. \quad (102)$$

For $T_j \neq 0$, this relationship can be inverted to obtain

$$kg_j = (1 - r) \frac{1 - T_j}{T_j}. \quad (103)$$

Substituting (103) into (100) gives

$$a_j = r - (1 + \lambda)(1 - r) \frac{1 - T_j}{T_j}, \quad (104)$$

and

$$b_j = \lambda(1 - r) \frac{1 - T_j}{T_j}. \quad (105)$$

Consequently, the characteristic polynomial can be written entirely in terms of the equilibrium transmission coefficient and the adjustment parameters:

$$\boxed{q_j(\mu | T_j) = \mu^2 - \left[r - (1 + \lambda)(1 - r) \frac{1 - T_j}{T_j} \right] \mu - \lambda(1 - r) \frac{1 - T_j}{T_j}}. \quad (106)$$

Theorem C.1 (Equilibrium transmission restricts mode dynamics). *Suppose $0 \leq r < 1$, $\lambda \geq 0$, $k > 0$, and G satisfies (8). For every eigenmode j for which*

$$1 - r + kg_j \neq 0,$$

define

$$T_j = \frac{1 - r}{1 - r + kg_j}.$$

Then, whenever $T_j \neq 0$,

$$kg_j = (1 - r) \frac{1 - T_j}{T_j}, \quad (107)$$

and, conditional on the structural parameters (r, λ) , the characteristic polynomial governing deviations of mode j from stationary equilibrium is

$$q_j(\mu \mid T_j; r, \lambda) = \mu^2 - \left[r - (1 + \lambda)(1 - r) \frac{1 - T_j}{T_j} \right] \mu - \lambda(1 - r) \frac{1 - T_j}{T_j}. \quad (108)$$

Thus, conditional on (r, λ) , the equilibrium transmission coefficient places a structural restriction on the dynamic law of the same network mode.

Proof. Equation (99) follows by projecting the linear deviation system onto an eigenvector of G . Its characteristic polynomial is therefore (101). From (102),

$$kg_j = (1 - r) \frac{1 - T_j}{T_j}.$$

Substituting this expression into both coefficients in (100) gives (104) and (105), and hence (108). Therefore, conditional on (r, λ) , the characteristic polynomial of mode j can be expressed entirely in terms of its equilibrium transmission coefficient T_j . $\square$

Proposition C.2 (Common-parameter cross-mode restriction). *Suppose that the behavioral parameters (r, λ) are common across network modes. Then the collection of equilibrium transmission coefficients and dynamic characteristic polynomials must satisfy*

$$q_j(\mu) = q(\mu \mid T_j; r, \lambda) \quad \text{for all } j, \quad (109)$$

where

$$q(\mu \mid T; r, \lambda) = \mu^2 - \left[r - (1 + \lambda)(1 - r) \frac{1 - T}{T} \right] \mu - \lambda(1 - r) \frac{1 - T}{T}. \quad (110)$$

Thus, multiple network modes generate cross-mode restrictions because the same behavioral parameters must account jointly for their equilibrium transmission and dynamic adjustment.

Proof. The result follows by applying Theorem C.1 to each network mode while holding the behavioral parameters (r, λ) fixed across modes. $\square$

C.1 Stability and the Convergent Region

The theorem establishes the structural transmission–persistence restriction without imposing stability. Stability is a separate property of the roots of the characteristic polynomial. For the second-order difference equation

$$z_{j,t+1} = a_j z_{j,t} + b_j z_{j,t-1},$$

the roots of (101) lie strictly inside the unit circle if and only if

$$1 - a_j - b_j > 0, \quad 1 + a_j - b_j > 0, \quad 1 + b_j > 0. \quad (111)$$

These are the standard Jury conditions for the polynomial $\mu^2 - a_j \mu - b_j$.

Using (104) and (105), the first condition becomes

$$1 - r + (1 - r) \frac{1 - T_j}{T_j} > 0. \quad (112)$$

Since $1 - r > 0$, this is equivalent to

$$\frac{1 - r}{T_j} > 0, \quad (113)$$

and hence to $T_j > 0$ under the maintained assumption $0 \leq r < 1$.

The second condition becomes

$$1 + r - (1 + 2\lambda)(1 - r) \frac{1 - T_j}{T_j} > 0. \quad (114)$$

The third condition is

$$1 + \lambda(1 - r) \frac{1 - T_j}{T_j} > 0. \quad (115)$$

Equivalently, in terms of the primitive network eigenvalue, the same conditions are

$$1 - r + k g_j > 0, \quad (116)$$

$$1 + r - k(1 + 2\lambda) g_j > 0, \quad (117)$$

and

$$1 + k\lambda g_j > 0. \quad (118)$$

Thus stability is mode-specific and depends on the eigenvalue associated with the relevant disagreement component. Importantly, these conditions are not part of the transmission–persistence identity itself; they restrict the region in which the resulting dynamic system converges.

Corollary C.3 (Persistence mapping). *Under the assumptions of Theorem C.1, let $\mu_{j,+}(T_j; r, \lambda)$ and $\mu_{j,-}(T_j; r, \lambda)$ denote the two roots of (108). Then the asymptotic persistence of mode j is*

$$\rho_j = \max \{ |\mu_{j,+}(T_j; r, \lambda)|, |\mu_{j,-}(T_j; r, \lambda)| \}. \quad (119)$$

Hence, conditional on (r, λ) , ρ_j is a function of the equilibrium transmission coefficient T_j .

Proof. The result follows directly from the definition of asymptotic persistence and Theorem C.1, which uniquely determines the characteristic polynomial from (T_j, r, λ) . $\square$

No monotonicity of ρ_j in T_j is imposed by the theorem. Accordingly, the benchmark model does not imply that greater equilibrium transmission necessarily generates either greater or lower persistence without additional restrictions on the parameter region.

D Aggregate Observational Equivalence under Uniform Aggregation

This appendix formalizes the aggregate observational-equivalence result stated in Section 7. The result follows from the orthogonal decomposition of the equilibrium state into the consensus component and the disagreement subspace.

Let the consensus eigenvector be

$$v_1 = \frac{1}{\sqrt{n}} \mathbf{1},$$

and let $\{v_j\}_{j \in \mathcal{D}}$ denote an orthonormal basis for the disagreement subspace. Thus,

$$v_1^\top v_j = 0, \quad j \in \mathcal{D}. \quad (120)$$

The stationary equilibrium can be written as

$$C^* = c_1^* v_1 + \sum_{j \in \mathcal{D}} c_j^* v_j. \quad (121)$$

Multiplying by $n^{-1} \mathbf{1}^\top$ gives

$$\begin{aligned} P^* &\equiv \frac{1}{n} \mathbf{1}^\top C^* \\ &= \frac{1}{n} \mathbf{1}^\top c_1^* v_1 + \frac{1}{n} \sum_{j \in \mathcal{D}} c_j^* \mathbf{1}^\top v_j. \end{aligned} \quad (122)$$

By (120),

$$\mathbf{1}^\top v_j = 0, \quad j \in \mathcal{D},$$

so that

$$P^* = \frac{1}{n} \mathbf{1}^\top c_1^* v_1. \quad (123)$$

Since $v_1 = n^{-1/2} \mathbf{1}$,

$$P^* = \frac{c_1^*}{\sqrt{n}}. \quad (124)$$

The aggregate stationary price therefore depends only on the consensus mode.

For the fundamental vector, write analogously

$$\theta = \theta_1 v_1 + \sum_{j \in \mathcal{D}} \theta_j v_j. \quad (125)$$

From (92),

$$c_1^* = T_1 \theta_1.$$

Hence

$$P^* = T_1 \frac{\theta_1}{\sqrt{n}} = T_1 \bar{\theta}, \quad \bar{\theta} \equiv \frac{1}{n} \mathbf{1}^\top \theta. \quad (126)$$

This reproduces the aggregate equilibrium relationship in (56).

Theorem D.1 (Aggregate observational equivalence). *Consider two economies A and B satisfying the benchmark assumptions. Suppose that*

$$\bar{\theta}_A = \bar{\theta}_B \quad \text{and} \quad T_{1,A} = T_{1,B}. \quad (127)$$

Then their stationary aggregate prices are identical:

$$P_A^* = P_B^*. \quad (128)$$

This conclusion holds regardless of differences in their disagreement components $\{\theta_{j,A}\}_{j \in \mathcal{D}}$ and $\{\theta_{j,B}\}_{j \in \mathcal{D}}$ or in their corresponding transmission coefficients $\{T_{j,A}\}_{j \in \mathcal{D}}$ and $\{T_{j,B}\}_{j \in \mathcal{D}}$.

Proof. By (126),

$$P_A^* = T_{1,A} \bar{\theta}_A, \quad P_B^* = T_{1,B} \bar{\theta}_B.$$

The conditions in (127) therefore imply

$$P_A^* = T_{1,A} \bar{\theta}_A = T_{1,B} \bar{\theta}_B = P_B^*.$$

The disagreement components do not enter the aggregate projection because they are orthogonal to the consensus vector. Hence differences in those components do not affect stationary aggregate prices. $\square$

The theorem does not imply that the two economies are economically identical. Their cross-sectional equilibrium states can differ substantially. In particular, under the orthonormal decomposition, the equilibrium disagreement component is

$$C_{\mathcal{D}}^* = \sum_{j \in \mathcal{D}} T_j \theta_j v_j. \quad (129)$$

Its squared Euclidean magnitude is

$$\|C_{\mathcal{D}}^*\|^2 = \sum_{j \in \mathcal{D}} T_j^2 \theta_j^2, \quad (130)$$

where the cross-products vanish by orthonormality. Consequently, the cross-sectional dispersion measure used in the main text is

$$D^* = \frac{1}{n} \sum_{j \in \mathcal{D}} T_j^2 \theta_j^2. \quad (131)$$

Thus, aggregate observational equivalence is compatible with

$$D_A^* \neq D_B^*$$

and with different mode-specific dynamic laws. Aggregate price data alone therefore cannot generally recover the disagreement transmission profile. Additional cross-sectional information is required to distinguish economies that are observationally equivalent at the aggregate level.

E Disagreement Dispersion and Empirical Objects

This appendix derives the mode-specific disagreement measure used in Section 7. The result follows directly from the orthogonal mode decomposition of the stationary equilibrium.

Let the disagreement component of the stationary equilibrium be

$$C_{\mathcal{D}}^* = \sum_{j \in \mathcal{D}} c_j^* v_j = \sum_{j \in \mathcal{D}} T_j \theta_j v_j, \quad (132)$$

where the second equality follows from (92). Under the orthonormality of the disagreement eigenvectors,

$$v_j^\top v_\ell = \mathbf{1}_{\{j=\ell\}}, \quad j, \ell \in \mathcal{D},$$

the squared magnitude of the disagreement component is

$$\begin{aligned} \|C_{\mathcal{D}}^*\|^2 &= \left(\sum_{j \in \mathcal{D}} T_j \theta_j v_j \right)^\top \left(\sum_{\ell \in \mathcal{D}} T_\ell \theta_\ell v_\ell \right) \\ &= \sum_{j \in \mathcal{D}} T_j^2 \theta_j^2. \end{aligned} \quad (133)$$

Define cross-sectional disagreement dispersion by the average squared deviation from the consensus component. Then

$$D^* = \frac{1}{n} \sum_{j \in \mathcal{D}} T_j^2 \theta_j^2. \quad (134)$$

Thus the contribution of disagreement mode j is

$$D_j^* \equiv \frac{1}{n} T_j^2 \theta_j^2, \quad (135)$$

so that

$$D^* = \sum_{j \in \mathcal{D}} D_j^*.$$

Equation (135) provides the mode-level cross-sectional object associated with equilibrium disagreement. The same transmission coefficient T_j also enters the dynamic characteristic polynomial through (108). Thus the static and dynamic objects share a common equilibrium transmission component, while remaining distinct observables.

This relationship clarifies the empirical role of cross-sectional dispersion. Aggregate prices identify the consensus projection, whereas D_j^* contains information about the non-consensus components that is lost under aggregation. Observing both the cross-sectional distribution and its time-series evolution therefore permits a direct test of the joint static–dynamic restriction.

When the corresponding modal components can be observed over time, the equilibrium dispersion identity and the dynamic transmission restriction can be evaluated jointly using estimates of the same mode-specific transmission coefficient. Sampling and measurement error then affect the empirical implementation, but do not alter the underlying theoretical identities.

References

- J. F. Muth, “Rational Expectations and the Theory of Price Movements,” *Econometrica* **29**, 315–335 (1961).
 S. Morris and H. S. Shin, “Social Value of Public Information,” *American Economic Review* **92**, 1521–1534 (2002), doi:10.1257/000282802762024610.
 C. Hellwig, “Heterogeneous Information and the Welfare Effects of Public Information Disclosures,” *American Economic Review* **95**, 473–479 (2005).

- G.-M. Angeletos and A. Pavan, “Efficient Use of Information and Social Value of Information,” *Econometrica* **75**, 1103–1142 (2007), doi:10.1111/j.1468-0262.2007.00783.x.
- W. A. Brock and C. H. Hommes, “Heterogeneous Beliefs and Routes to Chaos in a Simple Asset Pricing Model,” *Journal of Economic Dynamics and Control* **22**, 1235–1274 (1998), doi:10.1016/S0165-1889(98)00011-6.
- F. Allen, S. Morris, and H. S. Shin, “Beauty Contests and Iterated Expectations in Asset Markets,” *Review of Financial Studies* **19**, 719–752 (2006), doi:10.1093/rfs/hhj036.
- S.-K. Chang, “A Simple Asset Pricing Model with Social Interactions and Heterogeneous Beliefs,” *Journal of Economic Dynamics and Control* **31**, 1300–1325 (2007).
- H. Hong and J. C. Stein, “Disagreement and the Stock Market,” *Journal of Economic Perspectives* **21**, 109–128 (2007), doi:10.1257/jep.21.2.109.
- J. Chen, H. Hong, and J. C. Stein, “Breadth of Ownership and Stock Returns,” *Journal of Financial Economics* **66**, 171–205 (2002), doi:10.1016/S0304-405X(02)00223-4.
- J. A. Scheinkman and W. Xiong, “Overconfidence and Speculative Bubbles,” *Journal of Political Economy* **111**, 1183–1219 (2003), doi:10.1086/378531.
- C. Xiouros and F. Zapatero, “Disagreement, Information Quality and Asset Prices,” *Journal of Financial Economics* **153**, 103774 (2024), doi:10.1016/j.jfineco.2023.103774.
- M. H. DeGroot, “Reaching a Consensus,” *Journal of the American Statistical Association* **69**, 118–121 (1974), doi:10.1080/01621459.1974.10480137.
- P. M. DeMarzo, D. Vayanos, and J. Zwiebel, “Persuasion Bias, Social Influence, and Unidimensional Opinions,” *Quarterly Journal of Economics* **118**, 909–968 (2003), doi:10.1162/00335530360698469.
- B. Golub and M. O. Jackson, “Naive Learning in Social Networks and the Wisdom of Crowds,” *American Economic Journal: Microeconomics* **2**, 112–149 (2010), doi:10.1257/mic.2.1.112.
- A. Galeotti, S. Goyal, M. O. Jackson, F. Vega-Redondo, and L. Yariv, “Network Games,” *Review of Economic Studies* **77**, 218–244 (2010), doi:10.1111/j.1467-937X.2009.00570.x.
- D. Acemoglu and A. Ozdaglar, “Opinion Dynamics and Learning in Social Networks,” *Dynamic Games and Applications* **1**, 3–49 (2011).
- H. N. Ozsoylev and J. Walden, “Asset Pricing in Large Information Networks,” *Journal of Economic Theory* **146**, 2252–2280 (2011), doi:10.1016/j.jet.2011.10.003.
- V. Panchenko, S. Gerasymchuk, and O. V. Pavlov, “Asset Price Dynamics with Heterogeneous Beliefs and Local Network Interactions,” *Journal of Economic Dynamics and Control* **37**, 2623–2642 (2013).
- M. Hatcher and T. Hellmann, “Communication, Networks and Asset Price Dynamics: A Survey,” *Journal of Economic Interaction and Coordination* **19**, 1–58 (2024), doi:10.1007/s11403-023-00395-8.
- M. Hatcher and T. Hellmann, “Networks, Beliefs, and Asset Prices,” *Journal of Economic Dynamics and Control* **173**, 105059 (2025).