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Taylan Demir

1996.matematik.1996@gmail.com

Ankara University <https://orcid.org/0000-0001-7402-4468>

Atakan Koçyiğit

Atılım University <https://orcid.org/0009-0005-6060-0547>

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A Hybrid Fractional-Order Dynamical Model for ECG Morphology: Caputo Memory, Identifiability, and Reproducible PQRST Reconstruction

Taylan Demir¹
Atakan Koçyiğit²

¹Department of Mathematics, Ankara University, Ankara, Türkiye

²Department of Electrical and Electronics Engineering, Atılım University, Ankara, Türkiye

Abstract

A short preliminary study by the authors proposed fractional operators as a language for electrocardiographic (ECG) oscillations, but it did not specify a calibrated dynamical model, an identifiability strategy, or a reproducible validation protocol. The present work develops that idea into a testable mathematical framework. Cardiac timing is represented by an ordinary phase oscillator, while the morphology channel is governed by a dimensionally consistent Caputo fractional relaxation equation forced by five localized P, Q, R, S, and T components. This hybrid construction is deliberate: it preserves a genuinely periodic external cardiac clock while assigning fractional memory to morphology, avoiding the well-known obstruction to exact nonconstant periodic solutions in autonomous Caputo systems. We derive the Mittag-Leffler representation, prove existence, uniqueness, boundedness, and asymptotic forgetting of initial morphology, and obtain the long-time harmonic response. Parameter estimation is formulated as a semilinear profiling problem: for each pair (α, κ) , PQRST amplitudes and baseline terms are estimated by linear least squares, leaving only the fractional order and relaxation rate for outer optimization. In a fully reproducible synthetic stress test with beat-to-beat amplitude perturbations, baseline wander, and additive noise, the true order $\alpha = 0.82$ is recovered as $\hat{\alpha} = 0.83$. On a held-out four-beat interval, the fractional model gives RMSE = 0.0651 mV versus 0.0835 mV for the matched integer-order model, a reduction of about 22.0%, while increasing R^2 from 0.9775 to 0.9863. Thirty independent noise realizations yield $\hat{\alpha} = 0.833 \pm 0.005$. We finally specify an external validation protocol for the MIT-BIH Arrhythmia Database and the QT Database; no clinical-performance claim is made until that protocol is executed. The resulting framework treats fractional order as an estimable morphology-memory parameter rather than as a diagnostic label.

Keywords: fractional calculus; Caputo derivative; electrocardiogram; ECG morphology; PQRST waves; dynamical systems; parameter identification; PhysioNet.

1 Introduction

Electrocardiography records the surface projection of cardiac electrical activity and converts a spatially distributed electrophysiological process into a low-dimensional time series. Much of quantitative ECG analysis therefore proceeds in one of two directions: feature extraction from measured signals or dynamical modeling of the underlying waveform. The first direction includes classical QRS detectors such as the Pan–Tompkins algorithm [10]; the second includes generative systems such as the three-dimensional ECG oscillator of McSharry et al. [9], which encodes PQRST morphology through localized angular terms.

Fractional calculus offers a third ingredient: a compact representation of temporal dependence that is intermediate between a memoryless ordinary differential equation and a high-dimensional

hereditary model. Caputo, Riemann–Liouville, Grünwald–Letnikov, and more recent nonsingular-kernel operators have consequently appeared across applied modeling [1, 2, 3, 6]. The existence of several operators, however, is not itself a modeling result. An ECG model becomes scientifically useful only when the operator, state variables, initial conditions, parameter dimensions, identifiability strategy, and validation data are specified together.

The present paper develops such a construction from the authors’ earlier preliminary note on fractional-order ECG oscillations. Three choices distinguish the current formulation. First, the cardiac phase remains integer order. The fractional operator acts only on a morphology state driven by that phase. This matters mathematically: autonomous fractional systems with common classical fractional derivatives cannot, in general, sustain exact nonconstant periodic orbits [7, 8]. A periodically forced morphology equation does not suffer from that contradiction. Second, a reference time τ_c is inserted explicitly so that changing α does not silently change physical units. Third, estimation exploits linearity in the PQRST amplitudes. The nonlinear search is reduced to two parameters, (α, κ) , while the waveform amplitudes and baseline coefficients are solved conditionally by least squares.

The contributions are:

1. a dimensionally consistent hybrid Caputo–phase model for PQRST morphology;
2. an exact Mittag–Leffler representation and proofs of well-posedness, boundedness, and asymptotic loss of initial morphology;
3. an asymptotic frequency-response formula that exposes the role of α in amplitude and phase;
4. a semilinear profile estimator in which the fractional order is not confounded unnecessarily with five waveform amplitudes;
5. a reproducible synthetic experiment with held-out validation and repeated-noise recovery;
6. a pre-specified external-validation protocol using open PhysioNet ECG resources [11, 12, 13, 14].

The synthetic experiment is intentionally not presented as clinical evidence. Its role is narrower: to verify the parameter map under controlled model mismatch and noise before applying the method to annotated human ECG recordings.

2 Fractional operators and modeling choice

2.1 Caputo, Riemann–Liouville, Grünwald–Letnikov, and Atangana–Baleanu operators

For $0 < \alpha < 1$ and an absolutely continuous scalar function x , the Caputo derivative is

$${}^C D_t^\alpha x(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} x'(s) ds. \quad (1)$$

The Riemann–Liouville derivative may be written as

$$D_{0+}^\alpha x(t) = \frac{d}{dt} \left[\frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} x(s) ds \right]. \quad (2)$$

On a uniform grid, the Grünwald–Letnikov construction takes the limiting convolution form

$$D^\alpha x(t) = \lim_{h \rightarrow 0^+} h^{-\alpha} \sum_{j=0}^{\lfloor t/h \rfloor} (-1)^j \binom{\alpha}{j} x(t-jh), \quad (3)$$

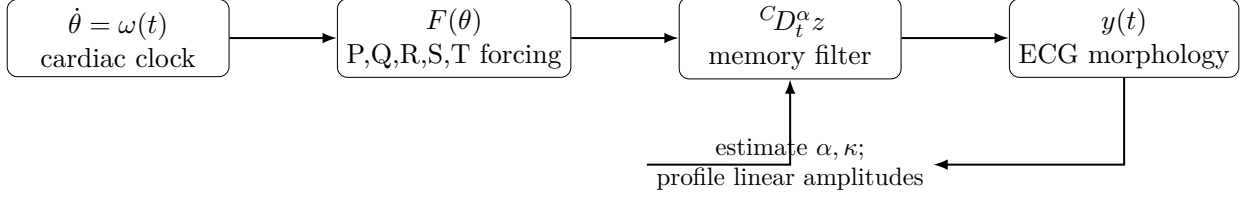


Figure 1: Model architecture. Periodicity enters through the ordinary cardiac phase, whereas temporal memory enters only through the morphology state.

when the limit exists. The Atangana–Baleanu family replaces the singular power-law kernel by a Mittag–Leffler-type nonsingular kernel [6].

The preliminary version of this work listed these operators side by side. In the present model we choose the Caputo derivative for a specific reason: $z(0)$ remains an ordinary initial morphology value, and the standard Volterra representation gives a transparent parameter-estimation problem. The Atangana–Baleanu operator remains a meaningful alternative kernel, but a fair comparison would require a separate initialization and kernel-identification study; it is therefore not mixed into the principal numerical claims.

2.2 Why the phase is not fractionalized

A common modeling temptation is to replace every first derivative in a periodic oscillator by a derivative of order α . That step is not innocuous. Tavazoei and Haeri showed that time-invariant autonomous systems built with classical fractional derivatives do not admit exact nonconstant periodic solutions under broad conditions [7, 8]. ECG waveforms are approximately periodic but are driven by a physiological clock. We therefore model the clock explicitly and place memory in the response to that clock:

$$\text{cardiac phase} \longrightarrow \text{PQRST forcing} \longrightarrow \text{fractional morphology}.$$

This separation is both mathematically safer and easier to identify from data.

3 Hybrid fractional PQRST model

3.1 Cardiac phase and localized morphology forcing

Let $\theta(t)$ denote cardiac phase. For a nearly stationary segment,

$$\dot{\theta}(t) = \omega(t), \quad \omega(t) > 0, \quad (4)$$

with $\omega(t) \equiv 2\pi/T_R$ in the constant-rate case. The wrapped phase displacement from the j th fiducial location θ_j is

$$\Delta_j(\theta) = \text{wrap}_{(-\pi, \pi]}(\theta - \theta_j). \quad (5)$$

Following the localized morphology idea of McSharry et al. [9], define

$$F(\theta; \boldsymbol{\eta}) = - \sum_{j \in \{P, Q, R, S, T\}} a_j \Delta_j(\theta) \exp \left[-\frac{\Delta_j(\theta)^2}{2b_j^2} \right], \quad (6)$$

where θ_j controls location, $b_j > 0$ controls width, and a_j controls signed contribution.

3.2 Dimensionally consistent Caputo morphology equation

Let $z(t)$ denote the morphology state after removal of a constant reference level z_b . We propose

$${}^C D_t^\alpha z(t) = \tau_c^{1-\alpha} [F(\theta(t); \boldsymbol{\eta}) - \kappa(z(t) - z_b)], \quad 0 < \alpha \leq 1, \quad (7)$$

where $\kappa > 0$ has units s^{-1} and $\tau_c > 0$ is a reference time. If F is measured in mV/s , then both sides of (7) have units mV/s^α . This factor is essential when comparing different α : without it, changing order changes the dimensions of the parameterized model.

The observed signal is

$$y(t) = z(t) + b_0 + b_1 \tilde{t} + c_1 \sin(2\pi f_b t) + c_2 \cos(2\pi f_b t) + \varepsilon(t), \quad (8)$$

where the final terms represent a low-dimensional baseline model and measurement noise.

3.3 Exact representation and well-posedness

Set

$$u(t) = z(t) - z_b, \quad c_\alpha = \tau_c^{1-\alpha}, \quad \lambda_\alpha = \kappa c_\alpha.$$

Then (7) becomes

$${}^C D_t^\alpha u(t) = -\lambda_\alpha u(t) + c_\alpha F(\theta(t)). \quad (9)$$

Theorem 3.1 (Existence, uniqueness, and representation). *Let $0 < \alpha \leq 1$, $\kappa > 0$, and let $F(\theta(t))$ be continuous on $[0, T]$. For each $u(0) = u_0$, problem (9) has a unique continuous solution. For $0 < \alpha < 1$,*

$$\begin{aligned} u(t) &= u_0 E_\alpha(-\lambda_\alpha t^\alpha) \\ &\quad + c_\alpha \int_0^t (t-s)^{\alpha-1} E_{\alpha,\alpha}(-\lambda_\alpha(t-s)^\alpha) F(\theta(s)) ds. \end{aligned} \quad (10)$$

For $\alpha = 1$, this reduces to the classical exponentially weighted convolution.

Proof. Applying the Laplace transform to (9) and using

$$\mathcal{L}\{{}^C D_t^\alpha u\}(s) = s^\alpha U(s) - s^{\alpha-1} u_0$$

gives

$$U(s) = \frac{s^{\alpha-1} u_0}{s^\alpha + \lambda_\alpha} + \frac{c_\alpha \mathcal{L}\{F(\theta(\cdot))\}(s)}{s^\alpha + \lambda_\alpha}.$$

The standard Mittag–Leffler transform pairs yield (10). Uniqueness follows from the corresponding linear Volterra equation; it is also a special case of standard Caputo existence–uniqueness theory [4, 3]. $\square$

Proposition 3.2 (Bounded input, bounded morphology). *Assume $|F(\theta(t))| \leq M$ for all $t \geq 0$. Then the solution of (9) is bounded. In particular,*

$$|u(t)| \leq |u_0| E_\alpha(-\lambda_\alpha t^\alpha) + \frac{M}{\kappa} [1 - E_\alpha(-\lambda_\alpha t^\alpha)]. \quad (11)$$

Proof. For $0 < \alpha \leq 1$ and $\lambda_\alpha > 0$, the Mittag–Leffler relaxation kernel is nonnegative and completely monotone. Taking absolute values in (10), replacing $|F|$ by M , and using the response to a constant input gives (11). $\square$

Corollary 3.3 (Forgetting of initial morphology). *If u_1 and u_2 solve (9) with the same forcing but different initial values, then*

$$|u_1(t) - u_2(t)| = |u_1(0) - u_2(0)| E_\alpha(-\lambda_\alpha t^\alpha) \longrightarrow 0. \quad (12)$$

Thus α controls the rate at which the model forgets initial morphology.

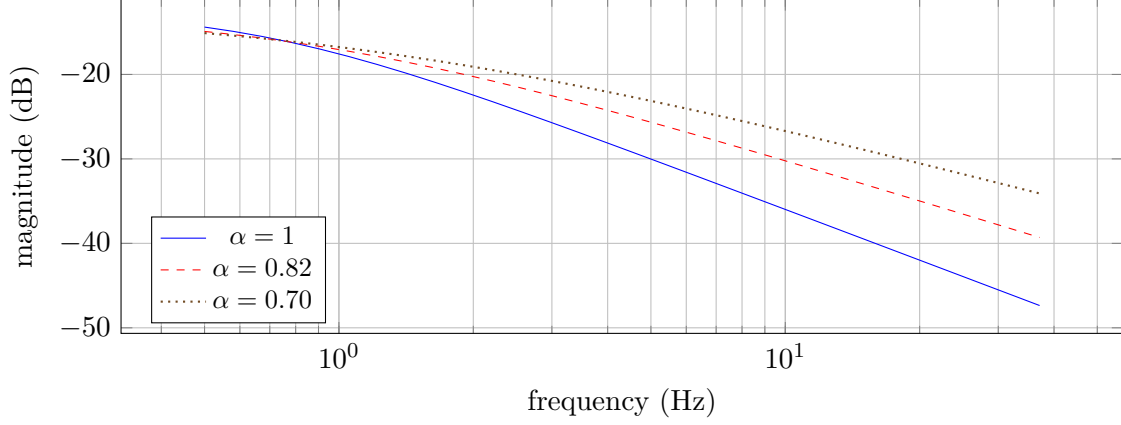


Figure 2: Long-time morphology-filter magnitude for $\kappa = 4.2 \text{ s}^{-1}$ and $\tau_c = 0.8 \text{ s}$. Fractional order changes the spectral roll-off rather than the cardiac clock.

3.4 Long-time harmonic response

For constant heart rate and after startup transients have decayed, each Fourier component of the periodic forcing is filtered approximately by

$$H_\alpha(i\Omega) = \frac{\tau_c^{1-\alpha}}{(i\Omega)^\alpha + \kappa\tau_c^{1-\alpha}}, \quad (13)$$

where the principal branch of $(i\Omega)^\alpha$ is used. Therefore

$$|H_\alpha(i\Omega)| = \frac{\tau_c^{1-\alpha}}{|(i\Omega)^\alpha + \kappa\tau_c^{1-\alpha}|}, \quad (14)$$

$$\arg H_\alpha(i\Omega) = -\arg[(i\Omega)^\alpha + \kappa\tau_c^{1-\alpha}]. \quad (15)$$

The order α changes both attenuation and phase lag. In ECG terms, it changes how sharply the morphology channel follows narrow PQRST forcing components without altering the external beat clock itself.

4 Parameter identification

4.1 Semilinear profiling

The forcing (6) is linear in the amplitudes a_j . This structure should be used rather than hidden inside a large nonlinear optimizer. For fixed (α, κ) and fixed fiducial locations and widths, let $q_j(t; \alpha, \kappa)$ denote the morphology response to the j th unit forcing component. Then

$$\hat{y}(t) = \sum_{j=1}^5 a_j q_j(t; \alpha, \kappa) + \mathbf{b}(t)^\top \boldsymbol{\beta}, \quad (16)$$

where $\mathbf{b}(t)$ contains baseline regressors.

At sampled times t_n , define the design matrix

$$X_{\alpha, \kappa} = \begin{bmatrix} q_P & q_Q & q_R & q_S & q_T & B \end{bmatrix}. \quad (17)$$

For each (α, κ) , the inner problem is

$$\hat{\gamma}(\alpha, \kappa) = \arg \min_{\gamma} \left\| W^{1/2} (\mathbf{y} - X_{\alpha, \kappa} \gamma) \right\|_2^2, \quad (18)$$

optionally with a small ridge penalty on nuisance baseline terms. The outer problem is only

$$(\hat{\alpha}, \hat{\kappa}) = \arg \min_{\alpha, \kappa} \mathcal{L}_{\text{val}}(\alpha, \kappa; \hat{\gamma}(\alpha, \kappa)). \quad (19)$$

This profile construction has two advantages. It reduces the dimension of the nonlinear search, and it reveals a practical identifiability diagnostic: if the profiled validation loss is flat in α , the data do not support a precise fractional-order estimate.

4.2 Periodically forced numerical evaluation

For constant T_R and a sufficiently long stationary window, the asymptotic solution can be evaluated efficiently in the frequency domain. If $\mathcal{F}$ denotes the discrete Fourier transform,

$$q_j = \mathcal{F}^{-1} [H_\alpha(i\Omega_k) \mathcal{F}\{F_j\}_k]. \quad (20)$$

For startup transients, varying heart rate, or nonperiodic windows, the time-domain Caputo equation can instead be integrated with a fractional Adams predictor–corrector method [5] or an L1-type discretization. The frequency-domain evaluation is used below because the synthetic identification experiment has a constant beat period and is designed to isolate morphology memory.

5 Reproducible synthetic validation

5.1 Experimental design

The synthetic experiment is not a substitute for clinical validation. It is a controlled identifiability test. The full specification is:

- sampling frequency: $f_s = 250$ Hz;
- beat period: $T_R = 0.8$ s (75 beats/min);
- total duration: 8 s (10 beats);
- true fractional order: $\alpha_{\text{true}} = 0.82$;
- true relaxation rate: $\kappa_{\text{true}} = 4.2$ s^{−1};
- reference time: $\tau_c = 0.8$ s;
- phase locations: $(-70^\circ, -15^\circ, 0^\circ, 15^\circ, 100^\circ)$;
- widths: $(0.25, 0.10, 0.10, 0.10, 0.40)$ rad;
- signed amplitudes: $(36, -64, 250, -84, 72)$ in the model’s forcing units;
- independent beat-to-beat amplitude perturbations: standard deviation 2.5%;
- baseline wander: $0.08 \sin(2\pi 0.33t + 0.5) + 0.035 \sin(2\pi 0.10t - 0.2)$ mV;
- white measurement noise: standard deviation 0.018 mV;
- random seed: 20260911.

The first six beats are used for coefficient estimation and the final four for validation. The outer grid is

$$\alpha \in \{0.60, 0.61, \dots, 1.00\}, \quad \kappa \in \{1.00, 1.25, \dots, 12.00\}.$$

Table 1: Held-out four-beat validation. Values are computed from the reproducible synthetic experiment described above.

Model	RMSE (mV)	MAE (mV)	R^2
Fractional, profiled α	0.0651	0.0586	0.9863
Integer, $\alpha = 1$	0.0835	0.0721	0.9775

At every grid point, the five amplitudes and four baseline coefficients are re-estimated from the training portion by ordinary least squares. The integer-order benchmark fixes $\alpha = 1$ but receives the same κ search and the same linear coefficient fitting.

The validation metrics are

$$\text{RMSE} = \sqrt{\frac{1}{N_v} \sum_{n \in \mathcal{V}} (y_n - \hat{y}_n)^2}, \quad (21)$$

$$\text{MAE} = \frac{1}{N_v} \sum_{n \in \mathcal{V}} |y_n - \hat{y}_n|, \quad (22)$$

$$R^2 = 1 - \frac{\sum_{n \in \mathcal{V}} (y_n - \hat{y}_n)^2}{\sum_{n \in \mathcal{V}} (y_n - \bar{y}_{\mathcal{V}})^2}. \quad (23)$$

5.2 Single-run parameter recovery

The profile minimum occurs at

$$\hat{\alpha} = 0.83, \quad \hat{\kappa} = 4.50 \text{ s}^{-1}. \quad (24)$$

The integer benchmark chooses $\hat{\kappa}_{\alpha=1} = 11.50 \text{ s}^{-1}$, using an artificially faster ordinary relaxation to compensate for the missing fractional phase behavior.

The held-out RMSE decreases by

$$100 \left(1 - \frac{0.0651}{0.0835} \right) \approx 22.0\%.$$

The recovered five-amplitude vector has relative Euclidean error 4.81% under the fractional model, compared with 102.3% for the integer benchmark. The latter compensates for its different dynamics by distorting the PQRST amplitudes, a useful warning against interpreting fitted amplitudes without first checking the assumed memory law.

5.3 Profile shape and repeated-noise recovery

Figure 4 shows the validation RMSE after minimizing over κ and all linear coefficients for each fixed α . The minimum is localized rather than flat.

To assess sensitivity to measurement noise, the white-noise realization is regenerated 30 times while preserving the same mild beat-to-beat morphology perturbations and baseline structure. A local profile search over $\alpha \in [0.75, 0.90]$ and $\kappa \in [3, 6]$ gives

$$\hat{\alpha} = 0.833 \pm 0.005, \quad \hat{\kappa} = 4.658 \pm 0.250 \text{ s}^{-1},$$

where the reported spreads are sample standard deviations. Twenty-one of the 30 runs select $\alpha = 0.83$, eight select 0.84, and one selects 0.85. This is an estimation result for the stated synthetic generator, not evidence that a particular human ECG has fractional order near 0.83.

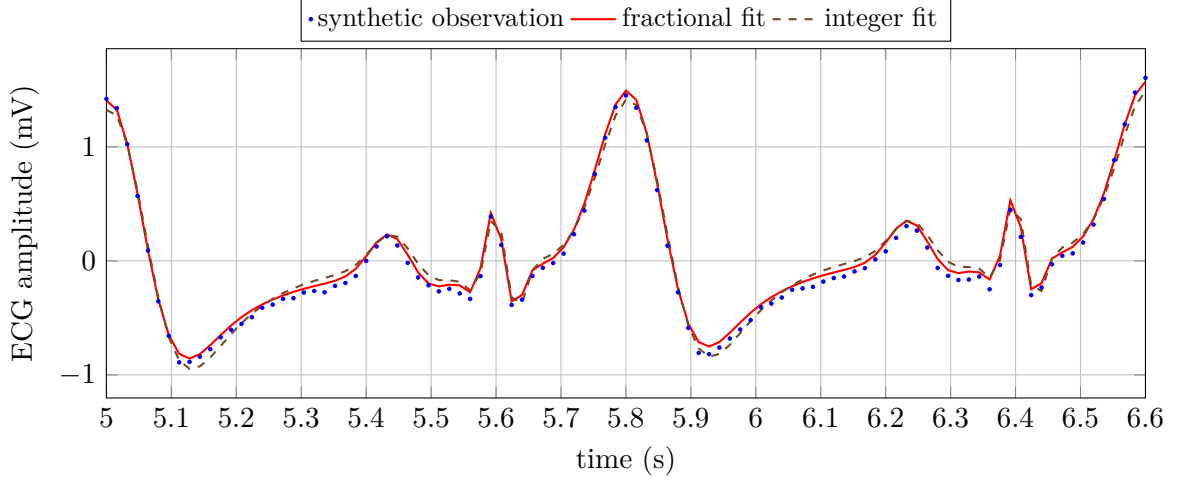


Figure 3: Two held-out beats. The data include amplitude variability, baseline wander, and white noise; both models are fitted only on the first six beats.

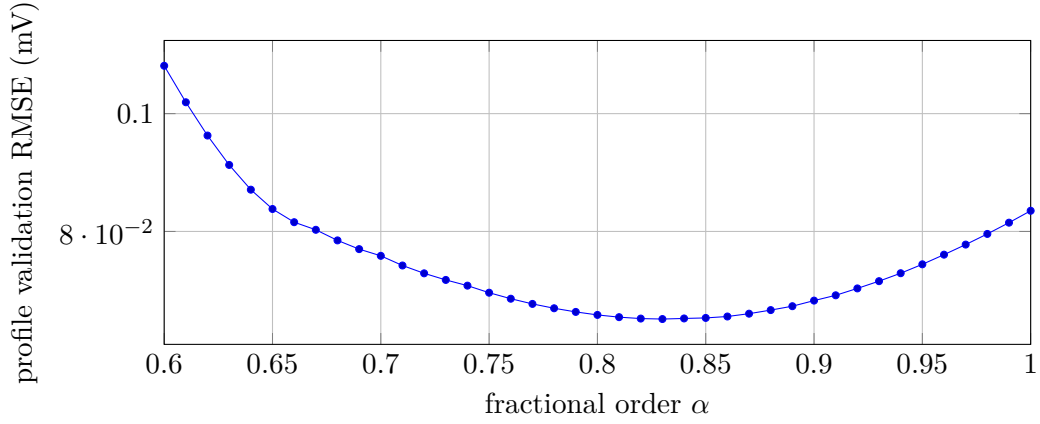


Figure 4: Profile loss after optimizing κ and the linear morphology/baseline coefficients.

6 External validation protocol on PhysioNet

The next stage is deliberately pre-specified rather than retrofitted after seeing results. Two open resources are appropriate.

The MIT-BIH Arrhythmia Database contains 48 half-hour, two-channel ambulatory ECG recordings from 47 subjects, with expert beat annotations; signals are sampled at 360 Hz [12]. The QT Database is even better suited to morphology validation because it contains 105 two-lead records with manually determined onset, peak, and end markers for P, QRS, T, and in some records U waves [13]. Both are distributed through PhysioNet [11, 14].

6.1 Preprocessing and beat construction

For each lead:

1. preserve the native signal before any resampling;
2. remove gross baseline drift with a preprocessing method whose cutoff is reported explicitly;
3. obtain beat anchors from reference annotations, not from the model being tested;
4. extract fixed or RR-adaptive windows around accepted beats;

5. normalize only according to a predeclared rule (for example, subtract beat baseline but do not normalize away amplitude if amplitude error is an endpoint);
6. exclude paced beats and severe artifacts in a pre-specified sensitivity analysis rather than silently deleting difficult cases.

The QT Database should be the primary morphology benchmark because manual waveform boundaries allow separate assessment of P, QRS, and T timing. MIT-BIH then tests generalization to a larger arrhythmia-oriented set.

6.2 Subject-level validation

A subject-level split is essential. Beats from one subject must not be distributed across both training and test sets. For each training subject, θ_j and b_j may be initialized from reference fiducials and then estimated within physiological bounds. The parameters (α, κ) are selected without looking at test-subject loss. Report at least:

$$\text{RMSE}, \quad \text{MAE}, \quad R^2, \quad \Delta t_P, \quad \Delta t_{QRS}, \quad \Delta t_T, \quad (25)$$

together with subject-wise distributions rather than only pooled beat counts.

The primary comparison should be:

$$\text{profiled fractional model} \quad \text{vs} \quad \text{same model with } \alpha = 1.$$

A secondary comparison may include the original integer ECGSYN-type oscillator [9]. The comparison is fair only if models use the same beats, preprocessing, fiducials, and train/test partition.

6.3 Hypotheses to be tested

The external study should test, not assume, the following:

1. whether $\alpha < 1$ improves out-of-subject morphology reconstruction after penalizing the extra parameter;
2. whether estimated α is stable across leads and repeated beats of the same subject;
3. whether the gain is concentrated in particular waveform regions (for example, QRS tails or T-wave recovery);
4. whether fractional order remains identifiable when baseline and amplitude nuisance parameters are allowed to vary;
5. whether any association with rhythm class survives subject-level uncertainty.

No diagnostic claim should be made from the synthetic experiment in Section 5. In particular, α should not be described as a disease marker unless a separate, adequately powered clinical study establishes such an association.

7 Discussion

7.1 What the fractional order means here

The parameter α is a property of a reduced morphology filter, not a direct measurement of ionic-channel memory or myocardial tissue memory. It summarizes how past forcing contributes to the present reduced state after all unmodeled physiology, projection effects, and measurement processes have been collapsed into one scalar channel. This distinction prevents a common over-interpretation of fractional models: a statistically useful noninteger order is evidence for a useful hereditary representation at the chosen scale, not proof of a unique microscopic mechanism.

The frequency-response formula (13) makes this interpretation concrete. For fixed κ , lowering α changes the high-frequency roll-off and phase. Narrow QRS-related components and broader P/T components therefore undergo different relative deformation. The order is identifiable only if the signal contains enough spectral and morphological diversity to separate that deformation from simple amplitude rescaling.

7.2 Why a hybrid model is preferable to a fully fractional oscillator

The hybrid structure solves two conceptual problems at once. It preserves an explicit cardiac clock, so RR variation can be injected directly through $\omega(t)$. It also avoids claiming that an autonomous Caputo oscillator has an exact stable limit cycle. Memory enters where it is empirically testable: in the mapping from recurrent phase forcing to measured morphology.

This architecture can be generalized. A future model could use

$$\omega(t) = \omega_0 + \xi(t)$$

for stochastic RR variability, or a coupled autonomic model for respiratory sinus arrhythmia. Multiple morphology states could represent leads or spatial modes. None of these extensions requires fractionalizing the phase itself.

7.3 Caputo versus nonsingular kernels

The 2023 preliminary study mentioned Atangana–Baleanu operators alongside the classical definitions. That direction remains open. Replacing (1) by a nonsingular Mittag–Leffler kernel changes the short-time weighting of memory and may improve numerical behavior or data fit in some settings [6]. Yet adding kernels simply because they are available would weaken inference. A kernel comparison should use the same dimensional normalization, the same outer/inner estimation architecture, and an information criterion or held-out loss that penalizes extra flexibility.

7.4 Limitations

Four limitations are central. First, the reported numerical results are synthetic. The model has not yet been fitted to QTDB or MIT–BIH in this manuscript. Second, the morphology equation is scalar; real ECG leads reflect a vector cardiac dipole and lead geometry. Third, the synthetic generator and fitted model share the same broad functional family, although beat-to-beat amplitude perturbations and unmodeled baseline components create controlled mismatch. Fourth, the frequency-domain evaluator assumes a stationary beat period. Variable-RR validation should use the time-domain Volterra or predictor–corrector solver.

These limitations are not cosmetic. A journal submission intended to make claims about human ECG should execute the external protocol in Section 6 and report subject-level uncertainty. The mathematical and synthetic results establish a reproducible core on which that study can be built.

8 Conclusion

A preliminary operator survey has been converted into a mathematically explicit ECG model. The central design choice is to keep cardiac phase ordinary and place Caputo memory in a dimensionally consistent morphology state. The model has a closed Mittag–Leffler representation, is well posed under continuous PQRST forcing, remains bounded for bounded forcing, and forgets initial morphology at a rate governed by α .

The semilinear profile estimator separates the nonlinear memory parameters from linear PQRST amplitudes and baseline terms. In a deliberately imperfect synthetic experiment, it recovers $\alpha = 0.83$ from a true value 0.82 and reduces held-out RMSE by approximately 22%

relative to the matched integer-order model. Repeated-noise experiments show a narrow order distribution in this controlled setting.

The next scientific question is empirical rather than algebraic: does the same advantage survive subject-level validation on annotated human ECG? The QT Database and MIT–BIH protocol specified here provide a direct test. Until that test is run, the fractional order should be interpreted as a reduced-model memory parameter, not as a clinical biomarker.

Data and computational reproducibility

The numerical values and all plotted coordinates in this manuscript are generated from the synthetic protocol in Section 5 with random seed 20260911. No patient-level data are embedded in this manuscript. The proposed external data sources are public PhysioNet databases subject to their respective licenses and citation requirements. A submission version should release the fitting code and the exact train/test record lists.

Declarations

Competing interests. The authors declare no competing interests.

Funding. To be completed by the authors before submission.

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Ethics statement. The numerical results reported here use synthetic signals only. Any later analysis of public de-identified PhysioNet records should follow the data-use terms of the corresponding database and the requirements of the target journal and the authors’ institutions.

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