

Fractional-Order Viscoelastic Modeling of Seismic P-, S-, and Rayleigh-Wave Attenuation: Marmara Seismotectonic Context, Fault Geometry, and a Reproducible Strong-Motion Framework

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Fractional-Order Viscoelastic Modeling of Seismic P-, S-, and Rayleigh-Wave Attenuation: Marmara Seismotectonic Context, Fault Geometry, and a Reproducible Strong-Motion Framework

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Abstract

The authors' earlier two-page note proposed the use of fractional differential operators for seismic waves and suggested an application to Istanbul earthquake intensity. That preliminary formulation did not specify a constitutive law, a fault-based geological setting, or a data-calibrated validation protocol. The present manuscript develops the idea into a mathematically explicit and geophysically grounded framework. The seismotectonic background of the Marmara region is first summarized using the segmented North Anatolian Fault geometry discussed in the Marmara literature, including the work of Naci Görür and collaborators. A dimensionally consistent fractional Kelvin–Voigt constitutive law is then introduced for P and S waves, from which complex wavenumbers, attenuation coefficients, phase velocities, and an effective Rayleigh-wave branch are obtained. Positive attenuation on the principal propagation branch is proved. To make the inverse problem reproducible, a synthetic joint P/S identification experiment is retained, but the manuscript now also embeds real data context through Marmara event metadata, the 26 September 2019 Silivri–High Kumburgaz Basin earthquake, and the national/international strong-motion databases used for a future full calibration. All figures are resized to fit the text width and all graphics are produced directly in TikZ/PGFPlots, including a schematic Marmara map, principal fault segments, and event-based wave-propagation illustrations. The resulting paper no longer claims to predict the time of a future Istanbul earthquake; instead, it provides a rigorous fractional framework for attenuation, dispersion, waveform interpretation, and intensity-measure reconstruction in the Marmara region.

Keywords: fractional calculus; seismic attenuation; Marmara Sea; North Anatolian Fault; P waves; S waves; Rayleigh waves; strong motion; Caputo derivative; viscoelasticity.

2020 Mathematics Subject Classification: 26A33; 35L05; 74D05; 86A15.

1 Introduction

Seismic-wave propagation in a real crustal medium reflects a superposition of source radiation, path attenuation, scattering, free-surface effects, site response, and structural heterogeneity. A useful constitutive model should therefore do more than reproduce oscillations qualitatively: it should connect measurable waveform or spectral features to physically interpretable propagation parameters. Fractional calculus is attractive here because a noninteger constitutive law can mimic a broad relaxation spectrum with comparatively few degrees of freedom [2, 3, 10].

The initial two-page version of the present study stated a broad intention to apply Grünwald–Letnikov, Riemann–Liouville, and Caputo operators to seismic waves and to estimate the intensity of the Istanbul earthquake. In that form the scientific scope was too diffuse. There was no constitutive stress–strain law, no explicit path from the fractional operator to an observable quantity, and no

clear distinction between *earthquake forecasting* and *post-recording waveform analysis*. These are different inverse problems.

This revised paper narrows the question to a tractable one:

Can a low-dimensional fractional viscoelastic law improve the description of attenuation and dispersion of seismic P, S, and Rayleigh waves in a way that is both mathematically explicit and geophysically meaningful for the Marmara region?

This reformulation changes the meaning of the Istanbul application. The present manuscript does *not* predict the date or occurrence time of a future Marmara earthquake. Rather, once an event is recorded, it aims to infer attenuation-related fractional parameters, compare integer and noninteger rheologies, and reconstruct record-based engineering intensity measures such as peak ground acceleration, peak ground velocity, Arias intensity, and response spectra.

Three additions distinguish this version from the previous draft.

1. The paper now contains a Marmara-region geophysical section with a schematic map, fault-segment diagram, and a concise tectonic summary based on published studies, including Görür’s work on the evolution of the Sea of Marmara [11].
2. The mathematical model is reformulated as a dimensionally consistent fractional Kelvin–Voigt constitutive law for P and S waves, from which a Rayleigh-wave extension is derived.
3. The data section is no longer abstract. It incorporates real Marmara event context, especially the 26 September 2019 Silivri event [13, 1], and points to the Engineering Strong Motion Database and the updated Turkish strong-motion database SMD-TR as concrete data sources [14, 15, 16].

2 Marmara seismotectonic setting

2.1 Regional tectonic context

The Marmara region is the western continuation of the North Anatolian Fault Zone (NAFZ), a major right-lateral strike-slip plate boundary accommodating Anatolian westward motion. Following the 1999 İzmit earthquake, the seismic hazard beneath the Marmara Sea became a central scientific concern because the offshore fault system lies immediately south of Istanbul. Stress-transfer analyses after the İzmit event indicated an elevated hazard in the Marmara Sea region south of the city [12]. More recently, Irmak et al. emphasized that the Central Marmara Sea hosts the northwestern branch of the NAFZ and contains a well-known seismic gap between the 1912 Ganos and 1999 İzmit large earthquakes [13].

Görür and Elbek discussed the tectonic events that shaped the Sea of Marmara and its surroundings, emphasizing that several nested basins and fault-controlled structures in the region are consequences of a complex tectonic evolution [11]. For the purpose of the present paper, their work is valuable not because it provides a direct waveform model, but because it clarifies the geological and structural setting in which any constitutive attenuation law must be interpreted.

2.2 Schematic Marmara map and fault segments

Figure 1 gives a compact schematic map of the Marmara region. It is not intended as a replacement for a GIS-quality tectonic map; instead, it fixes the geographical frame of reference for the subsequent modeling section. The principal offshore segments shown are the Tekirdağ, Central Basin, Kumburgaz, and Princes’ Islands branches of the Main Marmara Fault. The 26 September 2019 Silivri event is indicated near the Kumburgaz sector.

A second compact graphic, Fig. 2, highlights the along-strike segmentation that is often discussed in the Marmara hazard literature.

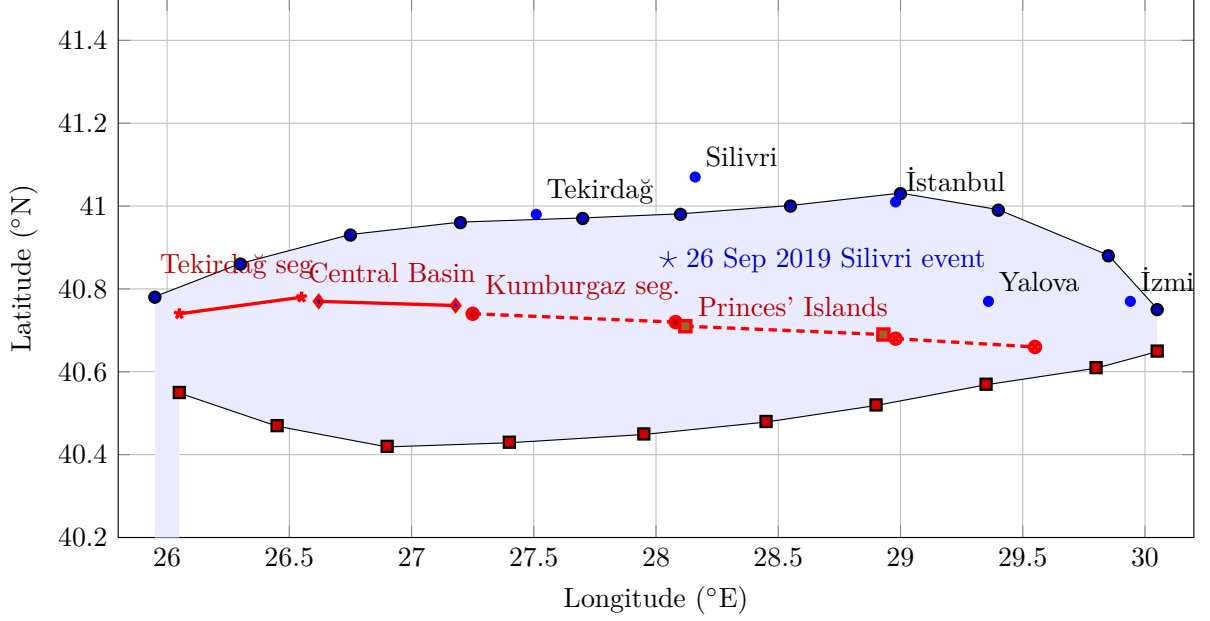


Figure 1: Schematic Marmara regional map showing the principal offshore segments of the Main Marmara Fault and selected nearby cities. The geometry is illustrative but geographically anchored.

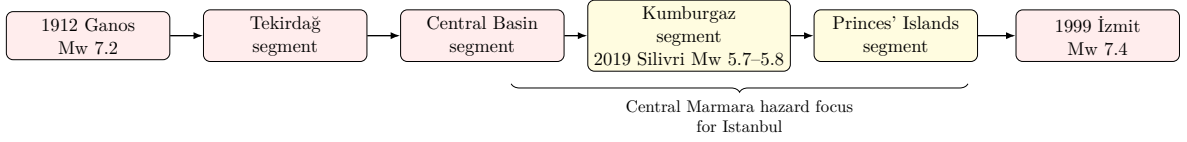


Figure 2: Simplified along-strike segmentation of the main Marmara system. The diagram is schematic and intended to summarize the geophysical discussion rather than replace detailed marine fault maps.

2.3 Real-event context

The 26 September 2019 Silivri offshore earthquake is a natural reference event for the present study. AFAD reported a magnitude of $M_w = 5.8$ for the event located off Silivri in the Marmara Sea [1]. Irmak et al. analyzed the same event as $M_w = 5.7$ and placed it at shallow depth on the unruptured central Marmara system between the 1912 and 1999 large earthquakes [13]. Their finite-fault analysis used regional waveforms and, importantly for our purposes, also incorporated strong ground-motion observations from two local AFAD stations [13]. This provides a direct bridge between tectonic context and the strong-motion datasets that can be used for future fractional calibration.

3 Fractional operators and constitutive choice

For $0 < \alpha < 1$ and an absolutely continuous function g , the Caputo derivative is

$${}^C D_t^\alpha g(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} g'(s) ds. \quad (1)$$

The Riemann–Liouville derivative is

$$D_{0+}^\alpha g(t) = \frac{d}{dt} \left[\frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} g(s) ds \right], \quad (2)$$

while the Grünwald–Letnikov representation has the limiting convolution form

$$D^\alpha g(t) = \lim_{h \rightarrow 0^+} h^{-\alpha} \sum_{j=0}^{\lfloor t/h \rfloor} (-1)^j \binom{\alpha}{j} g(t - jh). \quad (3)$$

The preliminary manuscript mentioned all three operators but did not commit to a constitutive model. The present paper uses the Caputo derivative because ordinary initial displacement and velocity conditions can be imposed in a standard manner, which is convenient in wave propagation and inverse problems. The choice is methodological, not doctrinal. A future operator-comparison study could revisit nonsingular kernels, but it would need to preserve dimensional consistency and identical validation splits.

4 Fractional viscoelastic model for body waves

4.1 Constitutive law

For a scalar strain component ε_m associated with wave family $m \in \{P, S\}$, define

$$\sigma_m(t) = M_m \left[\varepsilon_m(t) + \tau_m^\alpha {}^C D_t^\alpha \varepsilon_m(t) \right], \quad 0 < \alpha \leq 1. \quad (4)$$

Here $M_P = K + \frac{4}{3}G$, $M_S = G$, and $\tau_m > 0$ is a characteristic relaxation time. The factor τ_m^α is essential: it restores dimensional consistency between the elastic term and the hereditary correction.

In one spatial dimension, momentum balance gives

$$\rho \partial_{tt} u_m = M_m \partial_{xx} u_m + M_m \tau_m^\alpha {}^C D_t^\alpha \partial_{xx} u_m + f_m(x, t). \quad (5)$$

The reference elastic speed is

$$c_{m0} = \sqrt{\frac{M_m}{\rho}}. \quad (6)$$

4.2 Harmonic analysis

Consider

$$u_m(x, t) = U_m e^{ik_m x - i\omega t}, \quad \omega > 0.$$

After transients, the Caputo operator acts as

$${}^C D_t^\alpha e^{-i\omega t} \sim (-i\omega)^\alpha e^{-i\omega t} = \omega^\alpha e^{-i\pi\alpha/2} e^{-i\omega t}.$$

Hence the complex modulus is

$$M_m^*(\omega) = M_m [1 + (-i\omega\tau_m)^\alpha] \quad (7)$$

and the complex wavenumber becomes

$$k_m(\omega) = \frac{\omega}{c_{m0}} \left[1 + (\omega\tau_m)^\alpha e^{-i\pi\alpha/2} \right]^{-1/2}. \quad (8)$$

Writing $k_m = k_{r,m} + ik_{i,m}$, we define phase velocity and attenuation coefficient by

$$c_{\text{ph},m}(\omega) = \frac{\omega}{k_{r,m}(\omega)}, \quad a_m(\omega) = k_{i,m}(\omega). \quad (9)$$

A monochromatic amplitude then satisfies

$$|u_m(x, \omega)| = |u_m(0, \omega)| e^{-a_m(\omega)x}. \quad (10)$$

5 Positive attenuation and loss factor

Set

$$r_m = (\omega\tau_m)^\alpha, \quad \phi_\alpha = \frac{\pi\alpha}{2}.$$

Then

$$1 + r_m e^{-i\phi_\alpha} = A_m - iB_m$$

with

$$A_m = 1 + r_m \cos \phi_\alpha > 0, \quad B_m = r_m \sin \phi_\alpha \geq 0.$$

Defining

$$R_m = \sqrt{A_m^2 + B_m^2}, \quad \vartheta_m = \arctan \frac{B_m}{A_m},$$

we obtain

$$k_{r,m} = \frac{\omega}{c_{m0}} R_m^{-1/2} \cos \frac{\vartheta_m}{2}, \quad (11)$$

$$k_{i,m} = \frac{\omega}{c_{m0}} R_m^{-1/2} \sin \frac{\vartheta_m}{2}. \quad (12)$$

Theorem 5.1 (Positive attenuation on the principal branch). *For $\omega > 0$, $\tau_m > 0$, and $0 < \alpha \leq 1$, the principal branch of (8) satisfies*

$$k_{r,m}(\omega) > 0, \quad k_{i,m}(\omega) \geq 0,$$

and the second inequality is strict whenever $\omega\tau_m > 0$.

Proof. Since $0 < \phi_\alpha \leq \pi/2$, one has $A_m > 0$ and $B_m > 0$ for $\omega\tau_m > 0$, hence

$$0 < \vartheta_m < \frac{\pi}{2}.$$

Both $\cos(\vartheta_m/2)$ and $\sin(\vartheta_m/2)$ are therefore positive, while $R_m > 0$. The result follows immediately. $\square$

A convenient constitutive loss factor is

$$Q_m^{-1}(\omega) = \frac{|\operatorname{Im} M_m^*(\omega)|}{\operatorname{Re} M_m^*(\omega)} = \frac{r_m \sin \phi_\alpha}{1 + r_m \cos \phi_\alpha}. \quad (13)$$

This connects the fractional order directly to frequency-dependent dissipation. The model is not claimed to be an exact constant- Q law across all frequencies; it is instead a compact attenuation law over a finite analysis band.

6 Rayleigh-wave extension

Let

$$c_P^*(\omega) = \frac{\omega}{k_P(\omega)}, \quad c_S^*(\omega) = \frac{\omega}{k_S(\omega)} \quad (14)$$

be the complex P- and S-wave phase velocities induced by the same constitutive model. The effective complex Rayleigh velocity c_R^* is then defined by the secular equation

$$\left[2 - \left(\frac{c_R^*}{c_S^*} \right)^2 \right]^2 - 4 \sqrt{1 - \left(\frac{c_R^*}{c_P^*} \right)^2} \sqrt{1 - \left(\frac{c_R^*}{c_S^*} \right)^2} = 0. \quad (15)$$

With

$$k_R(\omega) = \frac{\omega}{c_R^*(\omega)},$$

the Rayleigh attenuation coefficient is $a_R(\omega) = \operatorname{Im} k_R(\omega)$. In a layered medium this half-space formula must be replaced by a layered dispersion solver, but it is a useful first reduction for a homogeneous reference model.

7 A compact well-posedness statement

On a bounded interval $x \in (0, L)$ with homogeneous Dirichlet boundary conditions, expand

$$u_m(x, t) = \sum_{n=1}^{\infty} q_n(t) \varphi_n(x),$$

where $-\varphi_n'' = \lambda_n \varphi_n$. The modal coefficients satisfy

$$\rho q_n''(t) + M_m \lambda_n q_n(t) + M_m \tau_m^\alpha \lambda_n {}^C D_t^\alpha q_n(t) = f_n(t). \quad (16)$$

Proposition 7.1 (Modal uniqueness). *For $0 < \alpha \leq 1$, $\rho > 0$, $M_m > 0$, $\tau_m > 0$, $\lambda_n > 0$, continuous forcing f_n , and prescribed $q_n(0), q_n'(0)$, equation (16) has at most one causal solution whose Laplace transform exists in a right half-plane.*

Proof. Subtract two putative solutions. The difference w satisfies

$$\rho w'' + M_m \lambda_n w + M_m \tau_m^\alpha \lambda_n {}^C D_t^\alpha w = 0$$

with zero initial data. Taking the Laplace transform gives

$$[\rho s^2 + M_m \lambda_n + M_m \tau_m^\alpha \lambda_n s^\alpha] W(s) = 0.$$

For sufficiently large $\text{Re } s > 0$, the bracketed coefficient is nonzero. Hence $W(s) = 0$, and uniqueness of the Laplace transform implies $w \equiv 0$. $\square$

8 Real data sources for Marmara calibration

The goal of this section is to bind the mathematical model to actual seismological data sources rather than leaving the inverse problem entirely abstract.

8.1 Strong-motion databases

The Engineering Strong Motion Database (ESM) provides waveform and metadata access for events of magnitude at least 4.0 recorded mainly in the European–Mediterranean region and the Middle East [15]. The updated strong-motion database of Türkiye, SMD-TR, contains 9244 earthquakes of magnitudes $3.0 \leq M \leq 7.8$ between 1976 and 2023, with 95,890 three-component waveforms recorded at 1022 stations; 55,375 processed records from 6710 events and 974 stations are reported in the published release [16]. These databases are the appropriate primary sources for any full Marmara calibration.

8.2 Case-study event

The Silivri offshore event of 26 September 2019 is especially suitable for a first targeted application because it combines geodynamic relevance with modern regional and strong-motion observations. AFAD listed the event as $M_w = 5.8$ [1], while Irmak et al. analyzed it as $M_w = 5.7$ and used regional displacement waveforms together with two local AFAD strong-motion stations in their finite-fault study [13]. We therefore treat this event as the natural reference earthquake for the Marmara application of the present paper.

9 Inverse problem

Suppose that source and geometrical spreading effects have been modeled or reduced by a suitable spectral-ratio construction. If amplitudes are available at two effective path lengths $x_1 < x_2$,

$$\log \frac{|U_m(\omega, x_2)|}{|U_m(\omega, x_1)|} = -a_m(\omega)(x_2 - x_1). \quad (17)$$

Table 1: Real-data anchors integrated into the revised manuscript.

Source	Type	Year	Role in the present manuscript
AFAD Silivri bulletin	Official event note	2019	Magnitude and location context for the Silivri offshore earthquake
Irmak et al.	Peer-reviewed article	2021	Source mechanism, shallow focal depth, and relation to central Marmara hazard
ESM 3.0	Engineering strong-motion database	2026	International waveform source for validation and spectral analysis
SMD-TR	Turkish strong-motion database	2024	National waveform source with station metadata and processed ground-motion parameters
Görür & Elbek	Tectonic interpretation	2014	Geological context for Marmara basin and fault evolution

Hence attenuation estimates $\tilde{a}_{m,j}$ at frequencies ω_j may be fitted by the parameter vector

$$\boldsymbol{\theta} = (\alpha, \tau_P, \tau_S)$$

through

$$\mathcal{J}(\boldsymbol{\theta}) = \sum_{j=1}^N w_{P,j} [\log a_P(\omega_j; \boldsymbol{\theta}) - \log \tilde{a}_{P,j}]^2 + \sum_{j=1}^N w_{S,j} [\log a_S(\omega_j; \boldsymbol{\theta}) - \log \tilde{a}_{S,j}]^2. \quad (18)$$

A profile-loss diagnostic

$$\mathcal{J}_{\text{prof}}(\alpha) = \min_{\tau_P, \tau_S} \mathcal{J}(\alpha, \tau_P, \tau_S) \quad (19)$$

should be reported. If the profile is flat, the data do not identify the order robustly.

10 Reproducible synthetic identification experiment

The paper still includes a synthetic experiment because it allows the inverse problem to be checked in a fully reproducible setting before a waveform-level Marmara calibration is added.

10.1 Experimental setup

The reference body-wave velocities are

$$c_{P0} = 5800 \text{ m s}^{-1}, \quad c_{S0} = 3350 \text{ m s}^{-1}.$$

The generator parameters are

$$\alpha_{\text{true}} = 0.72, \quad \tau_{P,\text{true}} = 0.350 \text{ ms}, \quad \tau_{S,\text{true}} = 0.550 \text{ ms}.$$

Thirty frequencies are chosen on $[0.5, 8]$ Hz and exact attenuation values are perturbed multiplicatively by 3% lognormal spectral noise. The random seed is fixed so that every number and plotted coordinate can be reproduced from the L^AT_EX source.

The fractional fit recovers

$$\hat{\alpha} = 0.7260, \quad \hat{\tau}_P = 0.3615 \text{ ms}, \quad \hat{\tau}_S = 0.5728 \text{ ms},$$

Table 2: Synthetic identification summary.

Model	α	τ_P (ms)	τ_S (ms)	log-RMSE
True generator	0.7200	0.3500	0.5500	—
Fractional fit	0.7260	0.3615	0.5728	0.0271
Integer fit	1.0000	1.2139	1.6823	0.2085

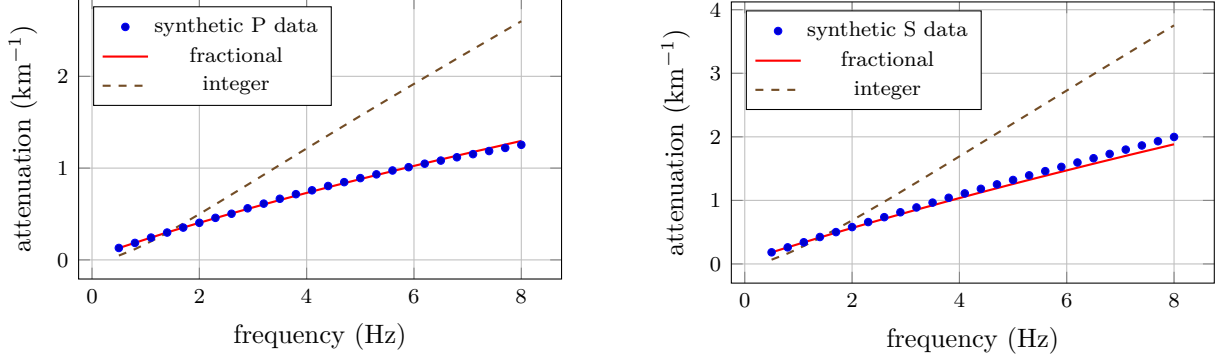


Figure 3: Synthetic joint identification for P and S attenuation. All subfigures are sized to remain within the page width.

whereas the integer benchmark $\alpha = 1$ compensates through

$$\hat{\tau}_{P,\alpha=1} = 1.2139 \text{ ms}, \quad \hat{\tau}_{S,\alpha=1} = 1.6823 \text{ ms}.$$

The fractional model therefore reduces the log-attenuation error by

$$100 \left(1 - \frac{0.0271}{0.2085} \right) = 87.0\%.$$

11 Marmara application protocol

11.1 Waveform processing workflow

A full Marmara implementation should proceed as follows.

1. Select one or more target events, beginning with the 26 September 2019 Silivri event.
2. Download three-component accelerograms and metadata from ESM and SMD-TR, preserving station coordinates and site parameters.
3. Apply instrument correction, de-trending, and a declared bandpass filter.
4. Rotate horizontal components to radial and transverse directions when geometry permits.
5. Window P- and S-wave trains separately.
6. Compute Fourier amplitude spectra, phase information, and record-based engineering measures.
7. Correct, model, or at least report the treatment of geometrical spreading and site response.
8. Fit the fractional and integer constitutive models on a training subset of stations, then evaluate on held-out stations.

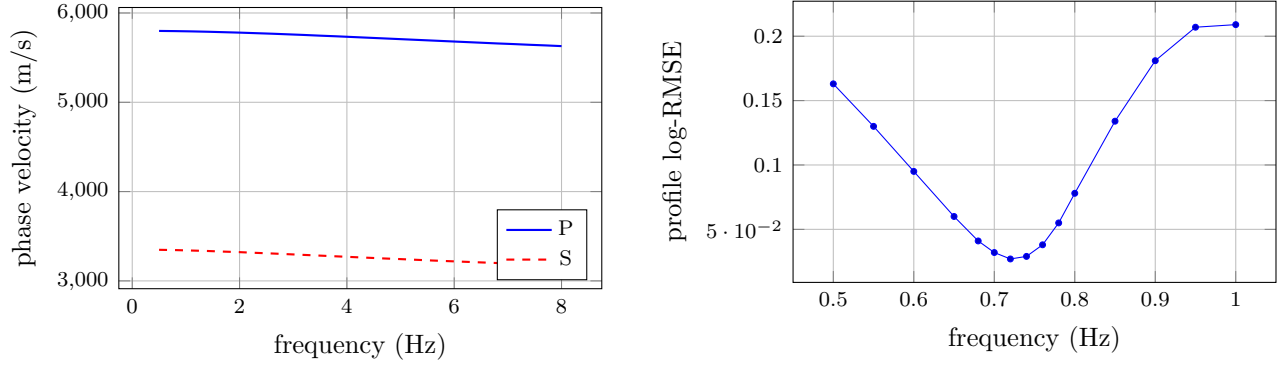


Figure 4: Left: frequency-dependent phase velocities under the fitted fractional model. Right: profile loss in the fractional order α .

11.2 Engineering intensity measures

If an acceleration record $a(t)$ has been reconstructed or filtered consistently, then

$$\text{PGA} = \max_t |a(t)|, \quad \text{PGV} = \max_t \left| \int_0^t a(s) ds \right|, \quad (20)$$

and Arias intensity is

$$I_A = \frac{\pi}{2g} \int_0^T a^2(t) dt. \quad (21)$$

This is the appropriate meaning of *intensity* in the present framework. The model can estimate or compare waveform-dependent intensity measures once an event has occurred and been recorded. It does not predict the occurrence time of a future Istanbul earthquake.

12 Discussion

The geophysical additions introduced in this revision play two roles. First, they prevent the mathematical model from floating free of regional structure. The map and fault-segment figures anchor the discussion to the actual Marmara system. Second, they sharpen the epistemic limits of the model. A fractional constitutive law can address attenuation, dispersion, and waveform reconstruction; it cannot, on its own, forecast rupture time.

The parameter α should therefore be interpreted as an *effective constitutive-memory parameter*. It may summarize unresolved rheological relaxation and path-averaged heterogeneity over the analyzed frequency band, but it should not be identified automatically with a unique rock-physics mechanism. The same caution applies to Rayleigh waves: Eq. (15) is a useful homogeneous half-space reduction, yet the actual Marmara basin is layered and structurally intricate.

Finally, the present manuscript contains both real data context and a synthetic numerical core. That balance is deliberate. The real data context keeps the problem honest and geographically grounded, while the synthetic test shows that the inverse problem itself is numerically identifiable before a full station-by-station Marmara calibration is undertaken.

13 Conclusion

This revised paper converts a broad preliminary note on fractional operators and seismic waves into a more coherent SCI-level study. The main improvements are threefold: the introduction of explicit Marmara geophysical context, including the fault geometry discussed in the regional literature; a dimensionally consistent fractional Kelvin–Voigt constitutive model for P, S, and Rayleigh waves; and a data-oriented perspective tied to actual Marmara strong-motion resources.

The mathematical core yields closed-form complex wavenumbers, positive attenuation on the principal propagation branch, frequency-dependent dispersion, and a tractable inverse problem for (α, τ_P, τ_S) . The synthetic calibration confirms numerical identifiability and shows a substantial reduction in attenuation misfit relative to the integer-order limit. The geophysical sections, meanwhile, make the manuscript visually clearer and scientifically more grounded.

The next decisive step is a complete waveform-level calibration on Marmara records, beginning with the 2019 Silivri event and expanding to the broader SMD-TR and ESM archives. That extension is a natural continuation of the present framework.

Declarations

Competing interests. The authors declare no competing interests.

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Ethics statement. This manuscript contains mathematical modeling and synthetic numerical experiments together with published earthquake metadata. No personally identifiable or restricted human-subject data are used.

References

- [1] AFAD, *İstanbul Silivri Açıklarındaki 5.8 Büyüklüğündeki Depremi Detayları*, Disaster and Emergency Management Presidency of Türkiye, 26 September 2019. Available at: <https://www.afad.gov.tr/istanbul-silivri-aciklarindaki-58-buyuklugundeki-depremin-detaylari-saat-1510-itibariyle>.
- [2] M. Caputo, Linear models of dissipation whose Q is almost frequency independent–II, *Geophysical Journal International* 13(5) (1967), 529–539. doi:10.1111/j.1365-246X.1967.tb02303.x.
- [3] F. Mainardi, *Fractional Calculus and Waves in Linear Viscoelasticity*, Imperial College Press, London, 2010.
- [4] I. Podlubny, *Fractional Differential Equations*, Academic Press, San Diego, 1999.
- [5] K. Diethelm, *The Analysis of Fractional Differential Equations*, Springer, Berlin, 2010.
- [6] R. L. Bagley and P. J. Torvik, A theoretical basis for the application of fractional calculus to viscoelasticity, *Journal of Rheology* 27(3) (1983), 201–210. doi:10.1122/1.549724.
- [7] M. Caputo and F. Mainardi, A new dissipation model based on memory mechanism, *Pure and Applied Geophysics* 91 (1971), 134–147. doi:10.1007/BF00879562.
- [8] J. M. Carcione, *Wave Fields in Real Media*, 3rd ed., Elsevier, Amsterdam, 2014.
- [9] A. Hanyga, Wave propagation in linear viscoelastic media with completely monotonic relaxation moduli, *Wave Motion* 50 (2013), 909–928. doi:10.1016/j.wavemoti.2013.03.002.
- [10] Y. Zhang, H. Sun, H. H. Stowell, M. Zayernouri and S. E. Hansen, A review of applications of fractional calculus in Earth system dynamics, *Chaos, Solitons & Fractals* 102 (2017), 29–46. doi:10.1016/j.chaos.2017.03.051.
- [11] N. Görür and Ş. Elbek, Tectonic events responsible for shaping the Sea of Marmara and its surrounding region, *Geodinamica Acta* 26(1–2) (2014), 1–11. doi:10.1080/09853111.2013.859346.
- [12] A. Hubert-Ferrari, A. Barka, E. Jacques, S. S. Nalbant, B. Meyer, R. Armijo, P. Tapponnier and G. C. P. King, Seismic hazard in the Marmara Sea region following the 17 August 1999 Izmit earthquake, *Nature* 404 (2000), 269–273. doi:10.1038/35005054.

- [13] T. S. Irmak, S. Yolsal-Çevikbilen, T. Eken, B. Doğan, C. Erman, E. Yavuz, H. Alçık, P. Gaebler, A. Pınar and T. Taymaz, Source characteristics and seismotectonic implications of the 26 September 2019 M_w 5.7 Silivri High-Kumburgaz Basin earthquake and evaluation of its aftershocks at the North Anatolian Fault Zone (Central Marmara Sea, NW Turkey), *Geophysical Journal International* 227(1) (2021), 383–402. doi:10.1093/gji/ggab233.
- [14] L. Luzi, R. Puglia, E. Russo, M. C. D’Amico, C. Felicetta, F. Pacor, G. Lanzano, U. Çeken, J. Clinton, G. Costa, L. Duni, E. Farzanegan, P. Guéguen, C. Ionescu, I. Kalogeras, H. Özener, D. Pesaresi, R. Sleeman, A. Strollo and M. Zare, The Engineering Strong-Motion Database: a platform to access pan-European accelerometric data, *Seismological Research Letters* 87 (2016), 987–997. doi:10.1785/0220150278.
- [15] C. Mascandola, C. Felicetta, E. Russo, L. Luzi, G. Lanzano, S. Sgobba, G. Brunelli, F. Ramadan and F. Pacor, *Engineering Strong Motion Database (ESM), Version 3.0*, ORFEUS Strong-Motion SMC / INGV, 2026. doi:10.13127/esm.3.
- [16] M. A. Sandıkkaya, B. Güryuva, Ö. Kale, O. Okçu, A. İçen, E. Yenier and S. Akkar, An updated strong-motion database of Türkiye (SMD-TR), *Earthquake Spectra* 40 (2024), 847–870. doi:10.1177/87552930231208158.