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Comparative Volatility Dynamics of Indian SME and ESG Indices: Persistence, Asymmetry, and Investment Implications

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Abstract

The present study compares risk–return characteristics, market performance and volatility dynamics of two thematic indices of India ; NSE ESG and SME Emerge . The analysis under the study was carried out across three calendar periods; Pre-Covid (2017-2019), Covid and Post- Covid (2020-2024) and combined study period (2017-2024). The study also made an attempt to identify most suitable volatility model as applicable for each of the two indices. Further, the suitable model as identified was tested for volatility persistence and asymmetry. The impact of Covid-19 on volatility performance was also studied under the study by incorporating a separate dummy variable. The results revealed similar return and risk profiles across both indices for entire study period except for SME Index revealing higher average return during post Covid. Analysis of volatility dynamics validated VARMA-GARCH(1,1) as optimal choice and further tests on this model demonstrated both indices exhibiting pronounced volatility persistence with gradual decay, however asymmetric response was only visible for SME index. Study implications contradict a general perception of SME risk being significantly higher while absence of asymmetry for ESG makes ESG an strong investment alternative during high volatility conditions

Keywords: Persistence, Asymmetry, ESG, SME Emerge Index, VARMA-GARCH Model

JEL Classification : C58, C22, C51

1. Introduction

Post Covid, the investment landscape in financial markets has undergone a significant transformation with the shift being clearly visible from traditional broad-based stock indices to more thematic indices. The thematic indices especially the ones focusing towards rising sectors are viewed by investors as having potential to outperform the broad based indices especially with respect to returns and therefore a noticeable shift in terms of growing preference for analysing such indices is also visible amongst researchers.

Thus with these consideration in mind, a study has been designed which makes an attempt to compare two thematic indices; NSE ESG Index and NSE SME Emerge Index with respect to return and volatility dimensions as driven by broader market index; NSE Nifty 500.

The choice of the two indices is primarily driven by diverse nature of companies included in two indices in terms of size, sectoral focus, and investment philosophy. The first index; NSE SME Emerge Index lists small sized companies aiming to provide high growth opportunities, however the general perception amongst investors is that same comes at a slightly higher risk. This was also seen in a study by Tudor et.al (2014) which reported higher returns with a slightly higher risk for smaller stocks as compared to stocks belonging to large cap portfolio on Romanian Market. In another study (Dharma Putra, 2020) reported superior performance of SME Index as compared to large caps when popular Sharpe ratio technique was employed for stock markets of UK, Indonesia and South Africa.

The second thematic index considered is the NSE ESG Index which broadly selects large and reputed companies, typically from NSE 100 Platform mainly considering three dimensions viz. Environment, Social, and Governance. The companies listed on this platform are the ones which usually adopt high level of governance and concern for environment.

Thus on comparative basis although the SME Emerge Index includes Companies which are comparatively smaller than the ones included in NSE ESG Index, the contribution of this sector towards economic development is quite substantial The SME sector is viewed as a major employment generator and contributes approximately 7 out of 10 new jobs created in the country, while at the global level the figure stood at 50 % (Thukral, et al., 2025).

Despite its growing relevance, the SME index platform is often regarded by investors as volatile, illiquid, and disclosure-unfriendly. The sector also has a reputation of performing poorly on sustainability parameters although this perception may not fully reflect the ground realities especially during the recent past. On the other hand, some recent studies have shown that the sector is more resilient than many other sectors during crisis situations (Koporcic, et al., 2025).

Thus a closer look at two indices included in the study reveal that both SME Emerge and ESG indices place some restrictions on inclusion of companies in their respective indices, whereas SME Emerge restricts companies on the basis of market capitalization and includes only companies classified as Small and Medium Enterprises, the ESG index has eligibility constraints and excludes companies violating principles related to climate change mitigation or human rights.

Hence considering the diverse nature of the two indices, it was decided to develop a study which could explore and compare the return and volatility performance of both the indices by

linking these further to the broader market index; the NSE Nifty 500. Thus by juxtaposing sustainability oriented investments (ESG) with small business entrepreneurship (SME) segments of the market, the paper aims to provide a deeper insight into the landscape of responsible and small entrepreneurship oriented investing.

Our discussion on existing studies begin with review of some studies on SME Indices, however only a few studies could be identified which were oriented towards SME indices. A study by Thukral, et al., (2025) where they tested for inter-relation between India's Nifty 50 and Nifty SME Emerge indices with the results showing bi-directional volatility transmission amongst two indices. Then a study by Abdellah, (2024) made a comparison of announcement of pandemic related information on large and SME companies listed on Morocco Exchange and found that SME companies suffered more losses in terms of returns with large companies being more resilient. Almost similar result was obtained by Hasnawati,(2020) where the study could detect higher returns for large companies during the crisis as compared to small companies, although returns were positive on both fronts. On the other hand, a somewhat different picture was seen in a study by Tudor et al., (2014) where they reported presence of size effect in Romanian Stock Markets i.e. smaller stocks outperformed larger stocks when risk adjusted performance was computed for both type of stocks.

Moving to studies on ESG Indices, research on these indices primarily revolves around comparing performance of ESG Indices to conventional indices. Studies including Bauer et al., (2005); Schroder, (2004) find ESG indices underperforming conventional indices while others including Statman (2000); Hamilton et al., (1993) proving their outperformance. On the other hand, there is somewhat consensus about these funds performing better than broader markets during a crisis (Chiappini and Palma, 2021; Leite & Cortez, 2015) . For risk assessment dimension, Amel-Zadeh and Serafeim, (2017) found such funds showed less of market risk as compared to other indices which enabled such funds to reduce the impact of negative externalities. For studies on asymmetric impact on ESG Index , a study by Sharma et al., (2024) failed to detect such an impact due to variation in broader market index; NSE 500. This result of no asymmetric impact was seen under both TGARCH and EGARCH approaches as applied under the study.

1.1 Research Gap

Thus, if we compare the research undertaken on above two sets of indices, we find that while ESG indices have been extensively studied in the domain of sustainable finance with evidence of such funds performing well during crisis, no clear indication was visible regarding the SME indices. One possible reason for the same could be paucity of research on SME indices thereby resulting in research on such indices remaining largely underexplored. With respect of performance, the available studies on the SME indices present a contradictory picture thus making it difficult to arrive at any concrete conclusion pertaining to their return performance and volatility dimension.

Hence considering the above, the current study was developed to fulfil the broader objective of making a comparison between these two indices, NSE ESG and NSE SME Emerge on dimensions including risk–return characteristics, return dependency, volatility behavior, and market performance. Our preliminary findings could not find any similar study being carried out with same objective of making a comparative analysis of sustainable indices performance vis-à-vis SME based growth indices. This therefore becomes the research gap and novelty of the present study. Then, to get a better insight such a comparative risk-return assessment, the

study not only considers combined study period (2017-2024) but also includes two sub periods; Period I as Pre-Covid (2017-19) and Period II as Covid & Post Covid (2020-2024). The data considered is log transformed eight years daily closing prices for these variables. Using GARCH Methodology, the study first tries to find out the appropriate volatility model for each of the two indices followed by test of persistence of the two models. The asymmetric impact of broader market; NSE 500 on the two indices is also considered under the study.

The rest of the paper is structured as follows: Section 2 provides the model construction and methodology employed, Section 3 gives the study results in detail while Section 4 concludes the study. The section also provides policy implications and limitations of the study and finally we have the concluding section, Section 5 providing the references.

2 Methodology

2.1 Model Construction

The methodology under the study revolves around five models; Model I, II and III (2.2 to 2.4) belong to GARCH Family Group , while remaining two models IV and V (2.5 and 2.6) test for the asymmetric impact.

Our analysis starts with a simple VAR Model, although VAR has its own problems including endogeneity and simultaneous problems but the same VAR Model is known to work very well when combined with volatility models, it was decided to include this model in our study. It was also decided to make the model more exogenous by including only relevant equations (two out of three initially under VAR framework) in the model.

However, even after this exercise we still faced diagnostic issues of serial correlation in the model , the same was detected at later lags when ‘Q’ statistics test was conducted. We then modified the same by adding an MA term which is commonly introduced for serial correlation present at later lags as seen for our model, the same was introduced for all the models.

Furthermore, keeping in view the study objective we modified our models to a VAR-GARCH Model. VAR-GARCH model served two purposes, first the model was technically superior to OLS, and second the model was capable of capturing volatility dynamics which was one of the requirement under the present study.

However as already mentioned earlier, with ‘Q’ test of serial correlation still showing serial correlation at later lags , the model was modified by incorporating an MA(1) term thus obtaining a modified model called VARMA-GARCH Model. The resultant model was found to be free from serial correlation and became the model for our subsequent analysis. Then the model also had another term in the volatility equation; a Dummy Variable ($D_{1,t}$) for Covid-19,

where we define Dummy as $D_{1,t} = \begin{cases} 1 & \text{if date falls in period 2020} \\ 0 & \text{if date falls in any other year} \end{cases}$

Further going along with the objective of our study to develop best possible volatility model for our two indices, we decided to develop three Model Variants, all belonging to the VARMA-GARCH family. These three variants differed with respect to number of ARCH and GARCH terms with details given as under:-

2.2 Model I: VARMA-GARCH (1,0) Mean and Volatility Models

Model I is our basic ARCH (1) Model where volatility of the error term is modelled as a function of only one lag of the previous error term i.e. $h_t = f(e_{1,t-1}^2)$ and $h_t = f(e_{2,t-1}^2)$ for our two models respectively.

$$\text{NSE ESG}_t = \beta_{1.1} + \beta_{2.1} \text{NSE ESG}_{t-1} + \beta_{3.1} \text{NSE SME}_{t-1} + \beta_{4.1} \text{NSE Nifty 500}_{t-1} + \beta_{5.1} e_{1,t-1,(\text{NSE ESG})} + e_{1,t} \dots (\text{i})$$

$$h_{1.1, \text{NSE ESG}_t} = \alpha_{1.1} + \alpha_{2.1} e_{1,t-1}^2 + \alpha_{3.1} D_{1,t} \dots (\text{ii})$$

$$e_{1,t} \sim \text{iid } N(0, h_{1.1,t})$$

$$\text{NSE SME}_t = \beta_{1.2} + \beta_{2.2} \text{NSE SME}_{t-1} + \beta_{3.2} \text{NSE ESG}_{t-1} + \beta_{4.2} \text{NSE Nifty 500}_{t-1} + \beta_{5.2} e_{2,t-1, \text{NSE SME}} + e_{2,t} \dots (\text{iii})$$

$$h_{1.2, \text{NSE SME}_t} = \alpha_{1.2} + \alpha_{2.2} e_{2,t-1}^2 + \alpha_{3.2} D_{1,t} \dots (\text{iv})$$

$$e_{2,t} \sim \text{iid } N(0, h_{1.2,t})$$

For the above models, eq.(i) is the NSE ESG mean equation for Model I. The equation is built as an AR(1) Model with coefficients being $\beta_{2.1}$, $\beta_{3.1}$ and $\beta_{4.1}$ with $\beta_{5.1}$ as the coefficient of the MA(1) term. Eq.(ii) is the volatility equation having $\alpha_{1.1}$ as constant term and $\alpha_{2.1}$ as the ARCH term. The volatility model also includes $\alpha_{3.1}$ which is the coefficient of Covid-19 dummy. We construct eq.(iii) as mean equation for NSE Emerge Index which is built along the same lines as eq.(i). Then eq.(iv) is built on same lines as eq.(ii)

2.3 Model II: VARMA-GARCH (2,0) Mean and Volatility Models

Model II extends our Model I and includes two ARCH terms i.e. $h_t = f(e_{3,t-1}^2, e_{3,t-2}^2)$ and $h_t = f(e_{4,t-1}^2, e_{4,t-2}^2)$ for our two models respectively

$$\text{NSE ESG}_t = \beta_{1.3} + \beta_{2.3} \text{NSE ESG}_{t-1} + \beta_{3.3} \text{NSE SME}_{t-1} + \beta_{4.3} \text{NSE Nifty 500}_{t-1} + \beta_{5.3} e_{3,t-1,(\text{NSE ESG})} + e_{3,t} \dots (\text{v})$$

$$h_{1.3, \text{NSE ESG}_t} = \alpha_{1.1} + \alpha_{2.1} e_{3,t-1}^2 + \alpha_{3.1} e_{3,t-2}^2 + \alpha_{4.1} D_{1,t} \dots (\text{vi})$$

$$e_{3,t} \sim \text{iid } N(0, h_{1.3,t})$$

$$\text{NSE SME}_t = \beta_{1.4} + \beta_{2.4} \text{NSE SME}_{t-1} + \beta_{3.4} \text{NSE ESG}_{t-1} + \beta_{4.4} \text{NSE Nifty 500}_{t-1} + \beta_{5.4} e_{4,t-1, \text{NSE SME}} + e_{4,t} \dots (\text{vii})$$

$$h_{1.4, \text{NSE SME}_t} = \alpha_{1.4} + \alpha_{2.4} e_{4,t-1}^2 + \alpha_{3.4} e_{4,t-2}^2 + \alpha_{4.4} D_{1,t} \dots (\text{viii})$$

$$e_{4,t} \sim \text{iid } N(0, h_{1.4,t})$$

2.4 Model III: VARMA-GARCH (1,1) Mean and Volatility Models

Model III is a standard GARCH(1,1) Model with variance equation having all three, typical of any GARCH Model; a Constant, square of the error term of previous period (ARCH term) & previous periods variance of the error term (GARCH term). The said Model which was later found to be ideal model for our further analysis was also considered for testing volatility persistency (memory). A high volatility persistence is a tendency where volatility (if high today) continues to remain high (tomorrow) and can be known by computing the sum of ARCH and GARCH terms. If the sum is close to '1', the persistence is high, if much less than '1' and close to '0', it suggests less persistence and tendency of shocks to fade away at a fast pace.

$$\text{NSE ESG}_t = \beta_{1.5} + \beta_{2.5} \text{NSE ESG}_{t-1} + \beta_{3.5} \text{NSE SME}_{t-1} + \beta_{4.5} \text{NSE Nifty 500}_{t-1} + \beta_{5.5} e_{5,t-1}(\text{NSE ESG}) + e_{5,t} \dots (\text{ix})$$

$$h_{1.5, \text{NSE ESG}_t} = \alpha_{1.5} + \alpha_{2.5} e_{5,t-1}^2 + \alpha_{3.5} h_{1.5, \text{NSE ESG}_{t-1}} + \alpha_{4.5} D_{1,t} \dots (\text{x})$$

$$e_{5,t} \sim \text{iid } N(0, h_{1.5,t})$$

$$\text{NSE SME}_t = \beta_{1.6} + \beta_{2.6} \text{NSE SME}_{t-1} + \beta_{3.6} \text{NSE ESG}_{t-1} + \beta_{4.6} \text{NSE Nifty 500}_{t-1} + \beta_{5.6} e_{6,t-1, \text{NSE SME}} + e_{6,t} \dots (\text{xi})$$

$$h_{1.6, \text{NSE SME}_t} = \alpha_{1.6} + \alpha_{2.6} e_{6,t-1}^2 + \alpha_{3.6} h_{1.6, \text{NSE SME}_{t-1}} + \alpha_{4.6} D_{1,t} \dots (\text{xii})$$

$$e_{6,t} \sim \text{iid } N(0, h_{1.6,t})$$

2.5 Model IV: Testing Asymmetry through TGARCH- in VARMA-GARCH (1,1) Volatility Models

All three Models viz. Models I, II and III as discussed are constructed to have a symmetric impact i.e. positive shock and negative shock to the error term has the same impact on volatility of the dependent variable. On the other hand, empirical evidence from financial markets has shown that negative shock (which is primarily due to bad news) has a much greater impact on volatility than equivalent positive shock (Christie, 1982; Nelson, 1991)

In order to empirically test the same on our models, we apply Threshold GARCH or T-GARCH technique which includes incorporating a multiplicative dummy term in the volatility component of the GARCH model. Thus the model has two Dummy terms, first for Covid-19 and second as a Threshold Dummy. This Threshold Dummy is included in model as either $D_{2,t-1}$ or $D_{3,t-1}$ for our two volatility models (eq. xiv and xvi) and takes value as '1' if the shock (e_{t-1}) is negative and '0' otherwise. Further to keep the impact as absolute, the study considers the squared error term; $e_{7,t-1}^2$ and $e_{8,t-1}^2$ for the two models. A significant Dummy term would result in coefficient of this term λ_1 or λ_2 being added to $\alpha_{3.7}$ or $\alpha_{3.8}$ thus proving significant difference to exist between a negative and positive shock or presence of asymmetry.

$$\text{NSE ESG}_t = \beta_{1.7} + \beta_{2.7} \text{NSE ESG}_{t-1} + \beta_{3.7} \text{NSE SME}_{t-1} + \beta_{4.7} \text{NSE Nifty 500}_{t-1} + \beta_{5.7} + e_{7,t-1,(\text{NSE ESG})} + e_{7,t} \dots (\text{xiii})$$

$$h_{1.7, \text{NSE ESG}_t} = \alpha_{1.7} + \alpha_{2.7} D_{1,t} + (\alpha_{3.7} + \lambda_1 D_{2,t-1}) e_{7,t-1}^2 + \alpha_{4.7}$$

$$h_{1.7, \text{NSE ESG}_{t-1}} \dots \dots (\text{xiv})$$

$$e_{7,t} \sim \text{iid } N(0, h_{1.7,t})$$

$$\text{NSE SME}_t = \beta_{1.8} + \beta_{2.8} \text{NSE SME}_{t-1} + \beta_{3.8} \text{NSE ESG}_{t-1} + \beta_{4.8} \text{NSE Nifty 500}_{t-1} + \beta_{5.8} + e_{8,t-1, \text{NSE SME}} + e_{8,t} \dots (\text{xv})$$

$$h_{1.8, \text{NSE SME}_t} = \alpha_{1.8} + \alpha_{2.8} D_{1,t} + (\alpha_{3.8} + \lambda_2 D_{3,t-1}) e_{8,t-1}^2 + \alpha_{3.8}$$

$$h_{1.8, \text{NSE SME}_{t-1}} \dots \dots (\text{xvi})$$

$$e_{8,t} \sim \text{iid } N(0, h_{1.8,t})$$

2.6 Model V: Testing Asymmetry through EGARCH- in VARMA-GARCH (1,1) Volatility Models

Model V, the EGARCH Model has been developed as an alternative test for asymmetry and aims at imposing non negative restrictions on parameters by introducing logarithm of the conditional variance and introducing the same in the variance equation. The variance equation in this Model has however the same three terms as seen in a typical GARCH Model viz. the constant term, the ARCH term and the GARCH term, the ARCH terms being $\left| \frac{e_{9,t-1}}{h_{9,t-1}} \right|$ and $\left| \frac{e_{10,t-1}}{h_{10,t-1}} \right|$ for eq. (xviii) and (xix) respectively while the GARCH term being $\log(h_{1.9, \text{NSE ESG}_{t-1}})$ and $\log(h_{1.10, \text{NSE ESG}_{t-1}})$ for the same two equations. The asymmetric term in the model is given as $\frac{e_{9,t-1}}{h_{9,t-1}}$ and $\frac{e_{10,t-1}}{h_{10,t-1}}$ respectively while $D_{1,t}$ representing the Dummy for Covid-19 in both models.

$$\text{NSE ESG}_t = \beta_{1.9} + \beta_{2.9} \text{NSE ESG}_{t-1} + \beta_{3.9} \text{NSE SME}_{t-1} + \beta_{4.9} \text{NSE Nifty 500}_{t-1} + \beta_{5.9} + e_{9,t-1,(\text{NSE ESG})} + e_{9,t} \dots (\text{xvii})$$

$$\log.h_{1.9, \text{NSE ESG}_t} = \alpha_{1.9} + \alpha_{2.9} \left| \frac{e_{9,t-1}}{h_{9,t-1}} \right| + \alpha_{3.9} \log(h_{1.9, \text{NSE ESG}_{t-1}}) + \alpha_{4.9} \frac{e_{9,t-1}}{h_{9,t-1}} +$$

$$\alpha_{5.9} D_{1,t} \dots (\text{xviii})$$

$$e_{9,t} \sim \text{iid } N(0, h_{1.9,t})$$

$$\text{NSE SME}_t = \beta_{1.10} + \beta_{2.10} \text{NSE SME}_{t-1} + \beta_{3.10} \text{NSE ESG}_{t-1} + \beta_{4.10} \text{NSE Nifty 500}_{t-1} + \beta_{5.10} + e_{10,t-1, \text{NSE SME}} + e_{10,t} \dots (\text{xix})$$

$$\log.h_{1.10, \text{NSE SME}_t} = \alpha_{1.10} + \alpha_{2.10} \left| \frac{e_{10,t-1}}{h_{10,t-1}} \right| + \alpha_{3.10} \frac{e_{10,t-1}}{h_{10,t-1}} +$$

$$\alpha_{4.10} \log(h_{1.10, \text{NSE ESG}_{t-1}}) + \alpha_{5.10} D_{1,t} \dots (\text{xx})$$

$$e_{10,t} \sim \text{iid } N(0, h_{1.10,t})$$

2.7 Model Diagnostics

To ensure the adequacy of our model, we apply two diagnostics; first being ARCH LM test, a variant of heteroscedasticity and checks for the presence of ARCH effects in the model and second, the 'Q' statistics which checks for serial correlation.

ARCH test is shown as eq (xxi), the test requires collecting residuals from AR(1) Model followed by an auxiliary equation of 'm' squared residuals which also follow an AR process;

$$e_t^2 = \alpha_0 + \rho_1 e_{t-1}^2 + \rho_2 e_{t-2}^2 + \dots + \rho_m e_{t-m}^2 + v_t \dots (xxi)$$

Null Hypothesis (H_0): $\rho_1 = \rho_2 = \dots = \rho_m = 0$ (no ARCH effect is present),

The study computes R^2 from model (xxi) and the value obtained is multiplied by the number of observations; the test follows $\chi^2_{(m)}$ distribution.

The second diagnostics is serial correlation and is represented by eq. (xxii) and is based upon 'Q' Statistics which is given as

$$Q_m = n \cdot \sum_{i=1}^m \rho_{u_{i,t}}^2 \dots (xxii); \text{ where } \rho_{u_t} = \frac{\text{cov}(u_t, u_{t-1})}{\sqrt{\sigma^2(u_{t-1}) \sigma^2(u_t)}};$$

Null Hypothesis (H_0): $\rho_{u_{1t}} = \rho_{u_{2t}} = \dots = \rho_{u_{mt}} = 0$ (no serial correlation). Null breaks down when some of the $\rho_{u_{it}}$ are not equal to 0.;

The test also follows Chi Square Distribution $Q_m \sim \chi^2$ and under the study we compute serial correlation at three different lags 1, 5 and 10.

The study also conducted test for stationarity of the variables, a prerequisite for any model building. The test applied was DF-GLS procedure as proposed by Elliott, Rothenberg, and Stock (1996) was employed for this purpose. The test was chosen due to its higher power relative to the conventional Augmented Dickey-Fuller (ADF) unit root test. Under the DF-GLS framework, the time series is first detrended using generalized least squares, after which the unit root test is performed on the detrended series through a regression that excludes both the intercept and the deterministic time trend, detrended unit root equation is given as eq. (xxiii).

$$\Delta \ddot{Y}_t = \alpha_1 \ddot{Y}_{t-1} + \sum_{i=1}^m \alpha_i \Delta \ddot{Y}_{t-i} + u_t \dots (xxiii)$$

For equation (xxiii), $\ddot{Y}_t$ denotes the detrended series, while $\Delta \ddot{Y}_{t-i}$ represents the augmentation terms and account for serial correlation. Since the data is detrended, the specification excludes an intercept term. The null hypothesis for stationarity test of Y_t , is specified as: $H_0: \alpha_1 - 1 = 0$ or equivalently $\alpha_1 = 1$, implying Y_t is non-stationary and has a unit root. The alternate hypothesis (H_a): $\alpha_1 < 1$, shows series is stationary.

3. Results of the Study

The section discusses the results of the study with supporting tables 1,2 and 3. Table 1 provides statistical description of returns for three indices included in the sample viz. NSE Nifty 500, NSE ESG and NSE SME Emerge indices. The statistical analysis has been carried out for three calendar periods; Pre-Covid (2017-2019), Covid and Post- Covid (2020-2024) and combined study period (2017-2024). The results indicate that the daily average returns were approximately similar across all three indices during the entire study period (2017–2024) while for post COVID-19 period, the SME index was seen exhibiting the highest average return. Regarding standard deviation ; a risk proxy parameter, no major differences were observed among the indices over the total study period while the post COVID-19 period showed higher risk levels for all indices compared to the pre COVID-19 period.

Furthermore, in terms of skewness, all the indices exhibited positive skewness during pre-covid period which became negative post covid, while in contrast, kurtosis which indicates presence of outliers was high for all indices both pre and post covid period and also for the entire period . This higher concentration of returns away from mean is typical of stock market returns as seen in many studies. Then, with both skewness and kurtosis jointly determining the distribution shape , the study compared the same with normal distribution using Jarque–Bera (JB) test and the results confirmed that none of the return distributions for any index, in any of the three periods, followed normality.

Moving to table 2 and 3, both tables have six columns each, columns 2,3 and 4 give results pertaining to Models I, II, and III showing a comparative assessment of three GARCH variant models, these are given separately for NSE ESG Index (Table 2) and NSE SME Emerge Index (Table 3). The two tables; 2 and 3 also make an attempt to capture volatility dynamics and short run persistence of volatility through these models. These tables further provide results for return dependency of respective thematic indices on benchmark NSE 500 index and also provide results pertaining to the test of asymmetry in volatility for which Model IV and V have been established under the study, results shown under column 5 and 6 of the two tables.

The findings from Table 2 indicate that among the three volatility specifications, Model III which is based upon GARCH(1,1) specification clearly outperformed the other two models (Models I and II). This was revealed by ARCH effects being statistically significant in both Model I and II volatility models which reflected that short term memory could not be removed even after fitting these two models. The solution targeting short run memory could only be achieved through Model III which was a VARMA-GARCH (1,1) Model. Hence VARMA-GARCH(1,1) shown as Model III in Table 2 may be considered as a superior volatility model out of the three models considered under the study and therefore was considered for further analysis. Furthermore, Model III also satisfied Diagnostics of Serial Correlation and no further residual ARCH Effects were seen as reported by the ARCH LM test results which was seen accepting the Null Hypothesis of No Serial Correlation. Similar findings pertaining to volatility characteristics showing the superiority of Model III were also obtained from Table 3 where similar exercise was carried out for another index; NSE SME Emerge.

The study also analysed the results from respective mean models under the GARCH framework which provided some insight about the return dependency of our variables. The results indicated that whereas return on ESG index was dependent upon the broader market index return; NSE Nifty 500 (Table 2), same was not true for SME Emerge Index (Table 3).

Yet another important result obtained from volatility models was persistency. A test of persistency of shocks ($\alpha + \beta < 1$) was carried out and it was seen that coefficients of ARCH and GARCH terms for both ESG and NSE SME Emerge Indices were totalling between 0.9 and 1 revealing high persistency with slow decay. The high short run persistency was also proved when the serial correlation was detected at later lags in these models. On the other hand, since the total of ARCH and GARCH terms was still less than '1' the conclusion was that the shock was not getting permanently inbuilt into the system which made the model stable and mean reverting.

Further, it was also noticed from the results that the coefficient of Dummy introduced for Covid-19 was highly significant for both the indices (ESG and SME) and for all the Model variants considered under the study. Then, since Dummy was introduced in the variance equation and not in the mean equation, it was therefore concluded that volatility dynamics of relevant index during Covid-19 period was different than that of remaining period of the study.

Tables 2 and 3 also provide results pertaining to test for asymmetric impact i.e. whether a positive shock on broader index (NSE 500) has equivalent (symmetric) impact on the volatility of the underlying index (ESG or SME) as a negative shock. These results shown as Column 5 (TGARCH) and Column 6 (EGARCH) clearly indicate that asymmetry was proved only for SME Emerge Index while the same was proved for NSE ESG Index.

4. Conclusion , Implications and Study limitations

To conclude, the study made an attempt to compare risk–return characteristics, market performance and volatility dynamics of two thematic indices namely NSE ESG and NSE SME Emerge Indices. The analysis under the study was carried out across three calendar periods; Pre-Covid (2017-2019), Covid and Post- Covid (2020-2024) and combined study period (2017-2024). The study also made an attempt to identify most suitable volatility model for each of the two indices. Further, the suitable model as identified was tested for volatility persistence and asymmetry. The impact of Covid-19 on volatility performance was also studied under the study by incorporating a separate dummy variable.

The study results revealed, NSE SME Emerge Index exhibiting the highest average return post Covid-19 while total and pre-covid period return was approximately similar for all indices. Interestingly, no major differences were observed amongst the indices in return standard deviations of each index, thus challenging the general perception of higher risk dimension for SME Indices. The return dependency of the two thematic indices clearly showed that whereas ESG was driven by broader market index NSE 500, the same was not true for SME emerge Index which showed no signs of movement due to movement of any other index except its own lagged values.

The analysis of volatility dynamics validated the VARMA-GARCH(1,1) model as the optimal choice with no residual ARCH effects. Then, further tests on this model demonstrated pronounced volatility persistence with gradual decay for both the indices, while retaining a finite memory structure as revealed by sum of ARCH and GARCH terms. In terms of asymmetric response, the same was somewhat different for two indices, whereas ESG Index revealed no asymmetry, this was clearly visible for SME index. The asymmetry results were found to be consistent across the two approaches T-GARCH and E-GARCH and matched the results obtained in another study by Sharma et al., (2024).

The study results obtained also have varied implications. The similarity in risk profiles across all indices suggest that the perception of the SME index as being significantly riskier was exaggerated and unfounded and these results also align with Koporcic, et al.(2025). Then non dependency of SME Index with respect to returns on NSE 500 reflects its independent movement to a large extent thus making the index a strong candidate for portfolio diversification.

Moving to volatility dimension, non-existence of asymmetric returns for ESG index reveals that this index may not react drastically as and when there is a fall in overall markets fall due to any bad news or an event. Some consider this as a sign of investor's maturity thereby making the ESG Index an interesting investment alternative during the period of crisis and high volatility conditions in the market. On the other hand, asymmetry which was clearly noticed for the SME index, makes the index vulnerable to declines in broader indices. The study actually found a very peculiar behavior of movement of NSE SME Emerge, the index had a negligible return relation but a very strong volatility dimension with respect to NSE Nifty 500. Amongst other results, high short run persistence and slow decay suggests delay in trend reversal, however since the impact of shocks was seen dying out completely, the result are much in line with findings of other studies.

Finally, before we could end, it becomes important to mention a couple of limitations of the present study. First, since entire analysis relies heavily on GARCH framework, overdependence on GARCH-type models might sometimes result in biased estimates when true volatility generation process deviates from such models. Many researchers have noticed such a phenomenon during major shocks or extreme market conditions which too have now become a regular and not rare feature during recent times. Another study limitation is inclusion of a Dummy Variable to control for Covid-19 shock, here too some economists bank for alternative tools which fall in the category of 'time varying' relation and include tools like DCC-GARCH, State Space Models including Kalman Filter, Markov and Threshold Models amongst others. Such techniques account for major shocks in the relation itself rather than studying the same through Dummy which is simply an inclusion of an additional variable. Thus, if a shock is not properly accounted for in the model not only the entire process of volatility generation gets impacted but it impacts subsequent model analysis including asymmetry response, shock persistency amongst others.

Table 1: Statistical Description of Returns on NSE SME EMERGE, NIFTY 100 ESG & NIFTY 500

Parameter	NSE SME Return Period-1 (2017-19)	NSE SME Return Period-2 (2020-24)	NSE SME Return Aggregate Period (2017-24)	Nifty 100 ESG Return Period-1 (2017-19)	Nifty 100 ESG Return Period-2 (2020-24)	NSE 100 ESG Return Aggregate Period (2017-24)	Nifty 500 Return Period-1 (2017-19)	Nifty 500 Return Period-2 (2020-24)	NSE 500 Return Aggregate Period (2017-24)
Mean	0.000	0.002	0.001	0.001	0.001	0.001	0.000	0.001	0.001
SD	0.008	0.010	0.009	0.008	0.012	0.010	0.008	0.012	0.010
Skewness	0.894	-0.719	-0.259	0.289	-1.685	-1.476	0.247	-2.100	-1.834
Kurtosis	12.175	3.864	5.596	3.350	21.532	22.623	3.256	22.014	22.837
Jarque-Bera(JB)	2686.867	145.814	578.304	14.036	18375.414	32503.833	9.527	19638.636	33589.372
Probability(JB)	0.000	0.000	0.000	0.001	0.000	0.000	0.009	0.000	0.000
CV	28.708	4.883	6.789	13.598	20.100	18.065	16.718	17.960	17.903
Observations	738.000	1243.000	1981.000	738.000	1243.000	1981.000	738.000	1243.000	1981.000

Table 2 : Comparative Analysis of Volatility Model Development for the variable NSE ESG Index (Values given are coefficient values with 'p' values in parenthesis)					
	Model I⁽ⁱ⁾	Model II⁽ⁱⁱ⁾	Model III⁽ⁱⁱⁱ⁾	Model IV^(iv)	Model V^(v)
Mean Equation					
Ret on ESG(-1)	1.134 (0.0206)	1.463 (0.000)	1.295(0.000)	Model IV is similar to Model III GARCH (1,1) Model plus inclusion of an asymmetric term	Model V is similar to Model III GARCH (1,1) Model plus inclusion of an asymmetric term
Ret on NSE SME(-1)	0.0006(0.977)	-0.008(0.678)	-0.001 (0.943)		
Ret on NSE 500(-1)	0.069(0.0231)	0.004(0.0432)	0.022 (0.0115)		
MA(1) Residual	-1.207(0.014)	-1.4066(0.00)	-1.2433(0.00)		
Variance Equation					
Constant	5.05E-05(0.000)	4.28E-05(0.000)	3.66E-06(0.00)		
ARCH (1)	0.3381(0.000)	0.277 (0.000)	0.125 (0.24)		
ARCH (2)	---	0.195 (0.395)	--		
GARCH term	---	---	0.828(0.18)		
Dummy for Covid	0.00013(0.000)	5.60E-05(0.000)	6.16E-06(0.00)		
Asymmetric term (<i>T GARCH /E GARCH</i>)	---	---	---	0.221(0.54)	0.156(0.13)
Model Diagnostics					
1. Serial Corr. (Q Stat)					
Lag 1	1.283(0.257)	0.340(0.560)	0.0478(0.827)		
Lag 5	6.266(0.281)	3.26(0.659)	4.4850(0.482)		
Lag 10	13.39(0.202)	6.83(0.741)	10.039(0.437)		
2. ARCH LM (F stat)	0.028(0.8651)	0.013(0.9084)	2.178362(0.1411)		
Note (1) * significant at 1 % level Note (2) Model I: VARMA-GARCH (1,0) Mean and Volatility Model; Model II : VARMA-GARCH (2,0) Mean and Volatility Model Model III: VARMA-GARCH (1,1) Mean and Volatility Model Model IV: TGARCH- in VARMA-GARCH (1,1) Volatility Model Model V : EGARCH- in VARMA-GARCH (1,1) Volatility Model					

Table 3 : Comparative Analysis of Volatility Model Development for the variable NSE SME Emerge Index (Values given are coefficient values with ‘p’ values in parenthesis)					
	Model I ⁽ⁱ⁾	Model II ⁽ⁱⁱ⁾	Model III ⁽ⁱⁱⁱ⁾	Model IV ^(iv)	Model V ^(v)
Mean Equation				Model IV is similar to Model III GARCH (1,1) Model plus inclusion of an asymmetric term	Model V is similar to Model III GARCH (1,1) Model plus inclusion of an asymmetric term
Ret on ESG(-1)	0.555609(0.0333)	-0.149343(0.1076)	-0.177789(0.0752)		
Ret on NSE SME(-1)	0.331506(0.000)	1.290129(0.000)	0.521114(0.000)		
Ret on NSE 500(-1)	0.141365(0.1445)	0.136355(0.1499)	0.191757(0.0553)		
Variance Equation					
Constant	6.34E-05(0.000)	5.49E-05(0.000)	3.61E-06(0.000)		
ARCH (1)	0.265676(0.000)	0.217759(0.000)	0.124393(0.392)		
ARCH(2)	--	0.161716(0.667)	---		
GARCH term			0.8430(0.0921)		
Dummy for covid	-1.98E-05(0.000)	-2.52E-05(0.000)	-1.56E-06(0.0024)		
Asymmetry Term (T-GARCH/E-GARCH)				0.121(0.000)	0.079(0.00)
Model Diagnostics					
1. Serial Corr. (Q Stat)					
Lag 1	0.043(0.612)	0.6513(0.420)	0.0108(0.917)		
Lag 5	7.432(0.19)	8.8781(0.114)	8.2026(0.145)		
Lag 10	11.492(0.097)	16.312(0.091)	17.131(0.072)		
2. ARCH LM (F stat)	0.307421(0.5793)	0.218026(0.6406)	0.268527(0.6044)		
Note (1) * significant at 1 % level Note (2) (i) Model I: VARMA-GARCH (1,0)Mean and Volatility Model; (ii) Model II :VARMA-GARCH (2,0) Mean and Volatility Model (iii) Model III: VARMA-GARCH (1,1) Mean and Volatility Model (iv) Model IV: TGARCH- in VARMA-GARCH (1,1) Volatility Model (v) Model V : EGARCH- in VARMA-GARCH (1,1) Volatility Model					

Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

During the preparation of this work, the author(s) used ChatGPT (free version, running OpenAI's GPT-4o mini model) in order to improve the language, readability, and grammatical flow of the manuscript. The AI was also used while deciding and finalizing the title of the paper. After using this tool/service, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the content of the publication

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