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Geochemical Self-Organization and the Origin of Gneissic Banding: A Reaction-Diffusion Approach

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Abstract

This study develops a theoretical framework for gneissic banding formation in metamorphic rocks via reaction-diffusion dynamics, rooted in geochemical self-organization concepts. We propose that banding can emerge autonomously, independent of deformation, challenging conventional shear-dependent models. A Schnakenberg reaction-diffusion system is analyzed and simulated numerically to explore pattern formation. Linear stability analysis reveals a Turing-type instability with an analytical critical wavelength of $\lambda_c = 14.63$ cm and a numerical fastest-growing wavelength of $\lambda_{\max} = 17.41$ cm, well within the natural gneissic banding range. Numerical simulations on a 400×400 grid demonstrate spontaneous parallel banding from anisotropic initial conditions. The felsic component u ranges from **0.256** to **2.486** and the mafic component v from **0.439** to **1.170**, yielding a normalized felsic contrast of **2.225** and a felsic-to-mafic ratio of **5.66**. The dominant wavelength is approximately **17** cm, within the natural gneiss range (**1–20** cm). Geological scaling with experimental diffusion coefficients gives a formation timescale of **9.5064×10^3** years, consistent with regional metamorphism. Our findings establish reaction-diffusion systems as viable mechanisms for fabric development in low-strain metamorphic settings, offering an alternative paradigm for mineral segregation in crystalline rocks. The model shares mechanistic similarities with Liesegang banding and orbicular granite formation, suggesting a unified mathematical framework for diverse geological pattern-formation phenomena.

Keywords: Gneissic banding, Reaction-diffusion, Metamorphic differentiation, Activator-inhibitor system, Liesegang phenomenon, Turing instability, Geochemical self-organization

1 Introduction

Gneissic banding, characterized by alternating light (felsic) and dark (mafic) mineral layers, represents one of the most distinctive textural features in high-grade metamorphic rocks [1, 2, 3]. These rhythmic patterns, with characteristic spacing ranging from millimetres to tens of centimetres, occur widely in Precambrian shields and orogenic belts worldwide. The formation of such highly ordered structures from initially homogeneous or weakly heterogeneous protoliths presents a fundamental challenge in metamorphic petrology and structural geology, requiring an explanation for the emergence of order from apparent disorder.

As emphasized by [4], rocks and minerals often exhibit rhythmic patterns whereby their chemical composition and/or physical properties vary repetitively across a spatial transect. While many such patterns result from systematic variations in external environmental conditions, it is now recognized

that many rhythmic patterns result from self-organized processes in non-equilibrium nonlinear geochemical systems [5, 1]. These systems exhibit intrinsic nonlinear dynamics with positive feedback between variables, operating without the requirement of a fluctuating external template.

Geochemical self-organization refers to the autonomous emergence of spatial patterns in chemical, mineralogical, or textural properties of rocks. This process occurs in non-equilibrium systems where chemical reactions, stress, and mass transport processes interact through feedback loops [5]. Non-inherited, self-organized patterns occur across sedimentary, metamorphic, and igneous environments, including oscillatory zoning in plagioclase, repetitive layering in igneous intrusions, orbicules, metamorphic layering, stylolites, banded iron formations, and scalloped fronts [5].

The traditional paradigm for gneiss formation emphasizes deformation as the primary driver of banding through processes including mechanical segregation, pressure solution, and metamorphic differentiation [6, 7]. Within this framework, differential stress is considered essential for creating the anisotropy necessary for banded textures to develop. However, field observations reveal instances where banding appears in relatively undeformed rocks or where band orientation does not align with regional stress fields. These observations challenge purely deformation-centric explanations and suggest that alternative mechanisms may operate independently or synergistically with deformation, potentially through purely chemical means.

Conventional models attribute gneissic banding primarily to mechanical processes operating under differential stress, often invoking the Mean-Field Eshelby-Kamb (MFEK) formalism. The MFEK model couples stress fields with chemical potential gradients and treats mineral grains as elastic inclusions that interact through deformation fields [8, 9, 10]. However, these models typically require substantial strain accumulation over extended time periods and face challenges in explaining banding in low-strain domains. Furthermore, the MFEK approach does not naturally account for the remarkable regularity and spacing of bands observed in many gneissic terrains.

An alternative perspective emerging from nonlinear dynamics suggests that complex mineral arrangements can arise spontaneously through self-organization in far-from-equilibrium systems [5]. This paradigm shift recognizes that ordered structures can emerge from initially disordered states through intrinsic nonlinear feedback mechanisms, without the need for an external template. Well-documented examples in geological systems include Liesegang banding in sedimentary rocks [11, 12], oscillatory zoning in plagioclase feldspars [13], and rhythmic layering in igneous intrusions [14]. These phenomena share a common mechanistic thread—nonlinear feedback between reaction kinetics and transport processes—suggesting that similar dynamics might operate during high-grade metamorphism to produce gneissic banding.

Recent experimental and field-based advances have provided compelling evidence for the operation of Liesegang-type mechanisms in geological pattern formation. [15] demonstrated that periodic precipitation of iron oxide minerals, analogous to the red-brown bands in zebra rocks, can be reproduced in laboratory gel systems through a prenucleation Liesegang mechanism. Their work established that the diffusion of OH^- ions into a Fe^{2+} -containing medium produces periodic $\text{Fe}(\text{OH})_2$ bands, which upon oxidation convert to FeOOH and ultimately to Fe_2O_3 —the same mineral assemblage found in natural zebra rocks from East Kimberley, Australia. This experimental validation provides strong impetus for applying reaction-diffusion theory to a broader range of metamorphic banding phenomena.

[12] provided a comprehensive mineralogical and geochemical analysis of zebra rock from the East Kimberley, demonstrating that the distinctive iron-oxide patterns formed through acid-sulfate fluid-rock interactions. Their work identified a mineral assemblage consistent with advanced argillic hydrothermal alteration, including pyrophyllite, kaolinite, dickite, alunite, and svanbergite-woodhouseite solid solutions, indicating minimum palaeotemperatures upwards of 120 °C and oxidising, acidic (pH <5) conditions. The low Rb/Sr ratio and relative immobility of rare earth elements in most zebra rock deposits further supported an acidic hydrothermal origin. Importantly, [12] proposed that zebra rock banding formed via the Liesegang phenomenon, driven by the oxidation of Fe^{2+} ions during the infiltration of an Fe^{2+} -rich, acid-sulfate fluid into oxidising host sediments. This mechanism shares fundamental similarities with the reaction-diffusion dynamics we propose for gneissic banding formation.

[11] further investigated the hydrothermal formation of Fe-oxide bands in zebra rocks from northern Western Australia, revealing that the iron-oxide banding developed simultaneously with aluminosilicate dissolution, clay precipitation, and fluid flow, consistent with the infiltration of an acidic fluid. The hexagonal-platelet morphology of the hematite pigment, kaolinite-hematite textural relationships, and a paleoflow direction preserved in the asymmetric intensity of the hematite concentration all point to a self-organized chemical origin.

The connection between chemical pattern formation and geological banding has been further strengthened by the work of [16], who developed a reaction-diffusion model of the Gierer-Meinhardt type to simulate orbicular patterns in granites. By coupling felsic and mafic materials as activator and inhibitor respectively, their model successfully reproduced a wide range of orbicular patterns including bands and rings. Their quantitative analysis demonstrated that non-equilibrium dynamics between felsic and mafic components drive their reorganization into matrix and core structures, providing a mechanistic basis for understanding concentric orbicular patterns in granitic rocks.

[17] provided a detailed petrological and geochemical study of the Velasco orbicular granitoid in Argentina, proposing that orbicular growth occurred through heterogeneous nucleation on K-feldspar megacrysts under conditions of rapid undercooling. Their genetic model shares key features with our proposed mechanism for gneissic banding: (1) formation of a melt pocket within a partially crystallized granite, (2) superheating and destruction of nuclei, (3) rapid undercooling and disequilibrium growth, and (4) crystallization of the inter-orbicular matrix. The time scale of formation was estimated to be a matter of weeks or months, highlighting the rapidity of self-organized pattern formation in geological systems.

Reaction-diffusion (RD) theory provides a powerful mathematical framework for understanding such pattern formation through the interplay of local activation and long-range inhibition [18, 19]. In RD systems, competition between nonlinear reaction kinetics and differential diffusion can spontaneously generate stable spatial patterns when specific parameter thresholds are exceeded. The elegance of this approach lies in its ability to generate realistic patterns from simple, well-defined mathematical equations, allowing quantitative comparison between theory and observation. As noted by [4], when system parameters exceed threshold values, the steady (time-independent) state loses its stability, and the system then evolves towards other time-dependent solutions ('attractors') that may exhibit oscillatory or chaotic behaviour.

1.1 Research Gaps and Objectives

Despite the progress summarized above, significant gaps remain in our understanding of gneissic banding formation.

First, it remains unclear whether the instability driving pattern formation in high-grade metamorphic systems is primarily spatial (Turing-type) or temporal (Hopf-type). Distinguishing between these two mechanisms is essential because they predict fundamentally different pattern morphologies and responses to changes in temperature and pressure.

Second, quantitative scaling relationships linking model parameters to measurable geological quantities—such as diffusion coefficients, temperature, and timescales—require further development. Without such relationships, model predictions cannot be directly tested against field observations.

Third, the role of pre-existing anisotropy in organizing reaction-diffusion patterns into coherent banding has not been systematically explored. Natural gneisses typically exhibit strong planar fabrics inherited from sedimentary layering, igneous foliation, or tectonic deformation, yet most reaction-diffusion models assume isotropic initial conditions.

Fourth, the connection between Liesegang-type mechanisms—as observed in zebra rocks and orbicular granites—and gneissic banding formation requires explicit mathematical formulation. Establishing this connection would unify three previously separate classes of geological pattern formation under a single theoretical framework.

A particularly important gap concerns the relationship between deformation-centric models and purely chemical self-organization models. While the MFEK formalism successfully describes banding in high-strain domains [8, 9, 10], it does not explain why banding also occurs in low-strain settings or why band spacing is often remarkably regular over large areas. Conversely, Liesegang-type models have been experimentally validated in sedimentary and hydrothermal systems [15, 12, 11], but their applicability to high-grade metamorphic systems has not been systematically tested. This study positions the reaction-diffusion framework as a natural extension of the experimentally validated Liesegang paradigm into metamorphic environments, providing a mechanistic alternative for low-strain banded gneisses.

To address these gaps, this study pursues the following specific objectives:

- To develop a Schnakenberg reaction-diffusion model specifically tailored to gneissic systems, incorporating anisotropic initial conditions.
- To explain the **genesis mechanism**—that is, how the model generates patterns from initial conditions through specific physical processes.

- To perform a rigorous linear stability analysis to characterize the specific instability mechanism driving pattern formation.
- To establish geological scaling relationships linking dimensionless model parameters to measurable physical quantities, including diffusion coefficients and temperature.
- To investigate the role of initial anisotropy in organizing reaction-diffusion patterns into coherent banding.
- To draw explicit connections between our model and observed Liesegang-type patterns in sedimentary and igneous systems (zebra rocks and orbicular granites).
- To compare model predictions with quantitative field data from documented gneiss localities.
- To generate testable predictions for field validation, including expected band spacing and temperature dependence.

2 Geological Context and Self-Organization Principles

2.1 Gneissic Banding in Metamorphic Terrains

Gneissic banding—alternating light (felsic) and dark (mafic) layers—is a defining textural feature of high-grade metamorphic rocks [1, 2, 3]. Bands typically range from 0.1 cm to 10 cm in thickness, exhibit sub-parallel orientation, and occur widely in Precambrian shields and orogenic belts worldwide. Felsic bands are dominated by quartz and feldspars (mainly plagioclase and K-feldspar), whereas mafic bands concentrate biotite, hornblende, garnet, and in some cases pyroxene. The sharpness of contacts between bands varies considerably, ranging from diffuse gradational boundaries to discrete interfaces with compositional jumps of tens of mole percent.

The temporal and spatial scales of gneissic banding formation are constrained by petrological and geochronological evidence. Regional metamorphism typically operates over timescales of 10^6 to 10^7 years, with cooling rates on the order of 1–10 K/Ma. Diffusion coefficients for major elements in silicate minerals at mid-crustal temperatures (600–700°C) are on the order of 10^{-15} to 10^{-13} m²/s [20], implying that diffusive transport over centimetre scales requires approximately 10^5 to 10^7 years—timescales that are fully compatible with metamorphic durations.

From a geophysical perspective, gneissic banding represents a classic example of a self-organized mineral assemblage produced by coupled reaction-transport feedbacks in a non-equilibrium system [5]. The metamorphic protolith, initially near-homogeneous on the hand-specimen scale, undergoes chemical differentiation driven by gradients in temperature, pressure, and chemical potential during prograde and retrograde metamorphism. The resulting banding reflects the spatial reorganization of incompatible elements (Si, Na, K in felsic layers; Fe, Mg, Ca in mafic layers) through diffusion-controlled mass transport.

2.2 Mechanisms of Geochemical Self-Organization

Geochemical self-organization arises from several fundamental feedback mechanisms that operate across diverse geological environments [5, 1]. The **Reactive Infiltration Instability (RII)** occurs when reactive fluid flows through a soluble rock matrix, where minor local porosity increases yield permeability enhancements, focusing fluid flow into high-porosity regions. This focused advective flux increases mineral dissolution rates, further augmenting local porosity, with the governing relationship

$$\vec{v} = -\kappa(\phi, V_\alpha)\nabla P, \quad (1)$$

where ϕ is porosity, κ is permeability, and P is fluid pressure. This mechanism is relevant for fluid-mediated metamorphism in the middle to lower crust.

The **Supersaturation-Nucleation-Depletion Cycle** produces periodic bands through a four-step process: as fluid dissolves mineral A , ion concentrations increase until surpassing the critical nucleation threshold $C_X C_Y > K_{\text{nuc}}$; nucleation of mineral B occurs with rapid growth acting as a sink for reactants; local depletion drops concentrations below K_{nuc} downstream, suppressing further nucleation; and as the dissolution front advances, it escapes the diffusive depletion sphere, allowing concentrations to cross K_{nuc} again to form the next band. This mechanism is directly analogous to the Liesegang phenomenon observed in zebra rocks [12, 11].

Autocatalytic Surface Attachment creates oscillatory zoning in binary solid solutions through kinetic surface attachment preferences coupled to boundary-layer diffusion, where an A-rich surface template catalytically promotes further A attachment, depleting the boundary layer, forcing B-rich nucleation that then promotes further B growth.

Finally, **Mechano-Chemical Feedback** under deviatoric stress causes local stress borne by individual grains to depend on the local modal volume fraction of mineral phases, where the chemical potential

$$\mu_A = \mu_A^0 + f_{stress}(\sigma, \phi_A, \text{defects}) \quad (2)$$

leads to $\partial\mu_A/\partial\phi_A < 0$, meaning regions with higher modal volume fraction experience lower stress per grain, reducing free energy and solubility, causing solute to diffuse from low-mode to high-mode regions and amplifying textural differentiation.

2.3 The Liesegang Phenomenon as a Chemical Analogue

The Liesegang phenomenon—the spontaneous formation of periodic precipitate bands in gel media—provides a well-characterized chemical analogue for geological banding [21, 4]. As described by [4], Liesegang bands are a type of rhythmic pattern characterized by a macroscopic (millimetre- to centimetre-scale) spatial arrangement of precipitate aggregates resulting from the coupling between diffusion of dissolved reactants and the kinetics of precipitate formation.

In the canonical Liesegang experiment, an outer electrolyte diffuses into a gel containing an inner electrolyte, and a precipitate forms when the concentration product exceeds a threshold, appearing as discrete periodic bands whose positions obey the spacing law

$$\frac{x_{n+1}}{x_n} = 1 + p, \quad (3)$$

where x_n is the position of the n -th band and p is the spacing coefficient, typically on the order of 10^{-3} to 10^{-2} [21].

Several empirical laws govern Liesegang bands: the Jableczynski law relates band positions linearly for large n ; the Matalon–Packter law states that the slope is a linear function of the inverse of outer species concentration; and the time scaling law confirms diffusion control, where formation time of the n th band is proportional to x_n^2 . Two mechanistic subcategories are distinguished [21, 4]: prenucleation models (formation of soluble intermediate precedes nucleation) and postnucleation models (diffusional transport coupled with growth and Ostwald ripening).

[12] demonstrated that zebra rock banding exhibits many characteristics consistent with Liesegang-type self-organization, including the asymmetric distribution of hematite within individual bands, which is closely replicated in computational simulations of Liesegang banding incorporating Ostwald ripening. The reduction spheroids observed in Remote Island zebra rock represent an intermediate stage of pattern formation, where Ostwald ripening gradually removes precipitate from interband spaces. This mechanism shares fundamental similarities with the activator-inhibitor dynamics we propose for gneissic banding formation.

Similarly, [11] identified a reaction-front preserved in the hematite banding of zebra rocks, with the asymmetric intensity of hematite concentration indicating paleoflow direction. This observation is consistent with our modeling approach, where the asymmetric distribution of activator concentration reflects the direction of diffusive flux during pattern formation.

2.4 Orbicular Granites as a Related Pattern-Formation Problem

Orbicular granites provide another striking example of self-organized pattern formation in igneous systems [16, 17, 4]. As reviewed by [4], orbicular granitic rocks provide a spectacular occurrence of banded patterns consisting of 10 cm spheroidal structures with a central core surrounded by concentric shells of light and dark granitic material. [16] developed a Gierer-Meinhardt type reaction-diffusion model to simulate orbicular patterns, successfully reproducing a wide range of patterns including bands and rings. The orbicular granite problem is closely related to gneissic banding, as both involve self-organized segregation of felsic and mafic components. However, whereas orbicular patterns exhibit radial symmetry, gneissic banding exhibits planar symmetry. The mathematical framework developed here is flexible enough to address both geometries.

[17] provided a detailed genetic model for the Velasco orbicular granitoid, proposing that orbicular growth occurred through: (1) formation of a pocket of ‘orbicular melt’ within a partially crystallized

granite via filter pressing, (2) superheating and destruction of nuclei due to high water concentration, (3) rapid undercooling and disequilibrium growth by heterogeneous nucleation on surviving megacrysts, and (4) crystallization of the inter-orbicular matrix. The time scale of formation was estimated to be a matter of weeks or months, highlighting the rapidity of self-organized pattern formation. This process shares fundamental similarities with our proposed mechanism for gneissic banding, where differential diffusion and nonlinear reaction kinetics drive the spontaneous segregation of felsic and mafic components.

3 Mathematical Model

Since Turing’s seminal 1952 paper [18], reaction-diffusion (RD) theory has provided a paradigm for pattern formation through local activation and long-range inhibition. The canonical Turing system considers two morphogens with different diffusion coefficients:

$$\frac{\partial u}{\partial t} = D_u \nabla^2 u + f(u, v), \quad (4)$$

$$\frac{\partial v}{\partial t} = D_v \nabla^2 v + g(u, v), \quad (5)$$

where the homogeneous steady state (u^*, v^*) satisfies $f(u^*, v^*) = g(u^*, v^*) = 0$. The key insight is that differential diffusion ($D_v \gg D_u$) can destabilize an otherwise stable steady state, leading to spontaneous spatial pattern formation.

In this study, we employ the Schnakenberg reaction-diffusion system, which has been extensively studied for pattern formation [22]. The Schnakenberg model is particularly suitable for geological applications due to its simple structure and ability to produce a wide range of patterns:

$$\frac{\partial u}{\partial t} = D_u \nabla^2 u + a - u + u^2 v \quad (6)$$

$$\frac{\partial v}{\partial t} = D_v \nabla^2 v + b - u^2 v \quad (7)$$

where $u(\mathbf{x}, t)$ and $v(\mathbf{x}, t)$ represent the concentrations of the activator and inhibitor fields, respectively. The parameters a and b are production rates, and D_u , D_v are diffusion coefficients.

As discussed by [4], the formation of Liesegang bands can be understood in terms of the coupling between the diffusion of dissolved reactants and the kinetics of precipitate formation. In our model, the activator and inhibitor fields play roles analogous to the reactants in Liesegang systems. This connection is particularly significant given the experimental and field-based evidence for Liesegang-type mechanisms in geological pattern formation, as demonstrated by [12, 11, 15].

3.1 Geological Interpretation of Model Variables

The model describes the spatiotemporal evolution of two coupled fields with direct geological interpretations:

Definition 3.1 (Activator Field) $u(\mathbf{x}, t)$ represents the effective concentration of a “felsic-forming component”—a generalized measure of chemical constituents ($\text{SiO}_2 + \text{Na}_2\text{O} + \text{K}_2\text{O}$) that promote the growth of felsic minerals (quartz, feldspars). In magmatic systems, this corresponds to the silica-rich component that tends to crystallize as light-coloured minerals. In metamorphic systems undergoing partial melting or metasomatism, the activator may represent mobile incompatible elements that concentrate in melt or fluid phases. This interpretation is consistent with the felsic-mafic segregation observed in orbicular granites [16, 17].

Definition 3.2 (Inhibitor Field) $v(\mathbf{x}, t)$ represents a “mafic-forming component”—constituents ($\text{FeO} + \text{MgO} + \text{CaO}$) that promote mafic minerals (biotite, amphibole, garnet). Its production is driven by the activator through the quadratic term u^2 , capturing the coupling between felsic and mafic components. This mirrors the Liesegang prenucleation mechanism where concentration product thresholds control pattern formation [12, 11]. In the context of zebra rock formation, the inhibitor corresponds to the Fe-rich components that precipitate as hematite bands.

Definition 3.3 (Production Rates a and b) The production rates a and b have direct geological interpretations. The parameter a represents the **external supply rate of the felsic component**—this could correspond to

silica supply from fluid infiltration, partial melting of felsic protoliths, or metasomatic addition of alkalis. The parameter b represents the **supply rate of the mafic component**—this could correspond to the breakdown rate of pre-existing mafic phases (biotite, amphibole, chlorite) during prograde metamorphism, or the supply of Fe-Mg-Ca from devolatilization reactions. Varying a and b in the model therefore corresponds to different metamorphic grades or fluid activities, making the model predictive for different geological settings.

3.2 Governing Equations

The spatiotemporal evolution is governed by the coupled partial differential equations in (6) and (7), where $D_u, D_v > 0$ are diffusion coefficients, and $a, b > 0$ are production rates. The domain $\Omega \subset \mathbb{R}^2$ is a square of side length L with periodic boundary conditions. Throughout this study, we use the following dimensionless parameter values derived from our stability analysis and geological constraints:

$$D_u = 0.02, \quad D_v = 0.32, \quad a = 0.10, \quad b = 0.90. \quad (8)$$

The condition $D_v \gg D_u$ reflects differential mobility of geochemical components, which is geologically plausible. In natural systems, the effective diffusion coefficient of network-forming components like SiO_2 is significantly smaller than that of more mobile components like H_2O or alkalis. Brady’s compilation [20] shows that oxygen and alkali diffusion coefficients in silicate melts can differ by orders of magnitude. This differential diffusion is the driving force behind the Liesegang-type self-organization observed in zebra rocks [12, 11].

From a geophysical standpoint, the two-field formulation captures the essential physics of coupled mass transport in a metamorphic system: the felsic component (u) represents the slower-diffusing, network-forming silicate species, while the mafic component (v) represents the faster-diffusing, network-modifying species (including volatiles and alkalis). The quadratic coupling term u^2v parameterizes the autocatalytic production of felsic material, a process well-documented in metamorphic differentiation.

3.3 Geological Scaling and Diffusion Data

We establish the bridge between dimensionless parameters and measurable physical quantities using characteristic scales derived from experimental diffusion data [20]. We choose $L_c = 0.1$ m as the hand-specimen scale of gneissic banding (1–20 cm) and $D_c = 10^{-12}$ m²/s as the representative grain-boundary diffusion coefficient at mid-crustal temperatures (600–700°C) [20]. The characteristic time is

$$t_c = \frac{L_c^2}{D_c} = \frac{(0.1 \text{ m})^2}{10^{-12} \text{ m}^2/\text{s}} = 10^{10} \text{ s} \approx 316.88 \text{ years}. \quad (9)$$

Table 1 summarizes the scaling relationships between dimensionless simulation parameters and their geological equivalents.

Table 1: Geological scaling of simulation parameters [20].

Parameter	Dimensionless	Geological
Characteristic length L_c	1	0.1 m
Characteristic time t_c	1	316.88 yr
Domain size	$4\pi \approx 12.57$	≈ 1.26 m
Grid spacing Δx	0.0314	≈ 3.14 mm
Total time T_{sim}	30	9.5064×10^3 yr
D_u	0.02	2.0×10^{-14} m ² /s
D_v	0.32	3.2×10^{-13} m ² /s
Analytical critical wavelength λ_c	1.4633	14.63 cm
Numerical fastest-growing λ_{max}	1.7412	17.41 cm

4 Model Genesis: How Patterns Form

A central contribution of this work is the explicit formulation of the **genesis mechanism**—the sequence of physical processes through which the Schnakenberg reaction-diffusion system generates gneissic banding from an initially near-homogeneous protolith. We decompose this genesis into five sequential stages, described in Sections 4.1–4.5.

4.1 Initial Perturbation and Symmetry Breaking

The protolith is modeled as a near-homogeneous distribution of felsic (u_0) and mafic (v_0) components, with small random fluctuations representing primary compositional heterogeneity inherited from the sedimentary or igneous precursor. The initial condition is

$$u(x, y, 0) = u_0 \left[1 + \epsilon \eta(x, y) + A \exp \left(-\frac{x^2}{a_1^2 \sigma_1} - \frac{y^2}{b_1^2 \sigma_1} \right) \right], \quad (10)$$

where $\eta(x, y)$ is Gaussian white noise, $\epsilon = 0.1$ controls noise amplitude, and the second term introduces a directional (anisotropic) bias with amplitude $A = 0.3$. This initial state is **linearly stable** in the absence of diffusion (Section 5), meaning that without the coupling between reaction and transport, no pattern would emerge.

Geologically, this stage corresponds to the initial state of a metamorphic protolith prior to the onset of significant chemical differentiation. The random fluctuations represent primary compositional heterogeneity on the grain scale, while the anisotropic bias represents a pre-existing fabric (e.g., sedimentary bedding, igneous foliation, or tectonic cleavage) that provides a directional template for subsequent pattern formation.

4.2 Turing Instability and Mode Selection

When diffusion is included, the homogeneous state becomes unstable to perturbations with wavenumbers in a finite band $0 < k < k_c$. The linear stability analysis (Section 5) shows that the growth rate $\sigma(k)$ is positive over this band, with a maximum at $k_{\max} = 3.6086$. This is the **Turing instability**: differential diffusion ($D_v \gg D_u$) destabilizes the system, causing perturbations with wavelength $\lambda_{\max} = 17.41$ cm to grow exponentially.

The physical mechanism is as follows. A local increase in activator u (felsic component) catalyses further activator production via the autocatalytic term u^2v . Simultaneously, it produces inhibitor v (mafic component). Because the inhibitor diffuses 16 times faster than the activator, it spreads away from the incipient felsic peak, suppressing activator growth in the surrounding region. This **local activation, long-range inhibition** creates a characteristic length scale—the wavelength of the emerging pattern.

In geophysical terms, this stage represents the onset of metamorphic differentiation. The differential diffusion of felsic and mafic components is a direct consequence of their different ionic radii, charges, and bonding preferences in the silicate melt or metamorphic fluid. The autocatalytic term u^2v parameterizes the positive feedback between felsic mineral growth and the local chemical environment, a process documented in metasomatic systems.

4.3 Nonlinear Saturation and Band Emergence

Exponential growth of the fastest-growing mode cannot continue indefinitely. Nonlinear terms in the reaction kinetics ($\pm u^2v$) saturate the growth, causing the system to settle into a spatially periodic state. During this stage (simulation steps 600–1200), the initial random fluctuations organize into coherent parallel bands aligned perpendicular to the direction of strongest initial anisotropy. The activator u concentrates in felsic bands ($u \approx 1.5$), while the inhibitor v concentrates in mafic bands, producing the anti-phase oscillations observed in Figure 4.

From a petrological perspective, this stage corresponds to the actual segregation of felsic and mafic minerals into distinct layers. The saturation of growth is a natural consequence of mass conservation: the total amount of felsic and mafic material is finite, so the pattern cannot grow indefinitely.

4.4 Coarsening and Ostwald Ripening

After the primary bands form, the system undergoes **coarsening** (simulation steps 1200–2400). Small, closely spaced bands merge into larger ones, and the pattern becomes increasingly regular. This process

is mathematically analogous to Ostwald ripening in Liesegang systems [12, 21]: the nonlinear dynamics favour configurations with fewer, more widely spaced bands because these minimize the effective free energy of the pattern. The coarsening stage explains why natural gneisses exhibit a characteristic dominant wavelength rather than a broad distribution of spacings.

Geophysically, coarsening corresponds to the textural maturation of the metamorphic rock during prolonged high-temperature conditions. Smaller, less stable mineral segregations dissolve and re-precipitate onto larger, more stable bands, a process driven by the minimization of interfacial free energy. This is consistent with observations of increasing band regularity with increasing metamorphic grade.

4.5 Stationary Metastable Pattern

By simulation step 3000, the system reaches a stationary, metastable configuration characterized by:

- Alternating felsic-rich ($u > 2.0$) and mafic-rich ($u < 0.4$) bands
- A bimodal concentration distribution (Fig. 3d)
- Sharp compositional interfaces with gradient widths of a few grid cells
- A composite morphology: continuous bands in the interior flanked by droplet-like segregation domains at the margins

This final state is “metastable” because it represents a local minimum of the effective free energy; sufficiently large perturbations could still reorganize the pattern. The composite morphology (continuous bands + droplet-like domains) is a direct consequence of the interplay between the anisotropic initial condition and the isotropic reaction-diffusion dynamics.

4.6 Mathematical Summary of the Genesis

The genesis mechanism can be summarized by the following sequence of mathematical events:

1. **Stability exchange:** The homogeneous state (u_0, v_0) satisfies $\text{Re } \lambda(0) < 0$, so it is stable to uniform perturbations.
2. **Turing bifurcation:** At $k = k_c$, the growth rate crosses zero: $\sigma(k_c) = 0$. For $k > k_c$, $\sigma(k) < 0$, confining the instability to a finite waveband.
3. **Exponential growth:** Perturbations with $k \approx k_{\max}$ grow as $e^{\sigma_{\max} t}$.
4. **Nonlinear saturation:** Quadratic and cubic terms bound the growth, selecting a finite amplitude.
5. **Wavelength selection:** The final pattern wavelength is set by $k_{\max}$, which depends only on D_u, D_v, a, b .
6. **Coarsening:** Nonlinear mode interactions drive the system toward fewer, larger bands.
7. **Metastable equilibrium:** The system settles into a stationary periodic state.

This genesis mechanism is **autonomous**: no external template, no imposed boundary conditions (other than periodicity), and no deformation are required. The pattern emerges entirely from the intrinsic nonlinear dynamics of the coupled reaction-diffusion system. This is the essence of geochemical self-organization.

5 Linear Stability Analysis

As emphasized by [4], when system parameters exceed threshold values, the steady state loses stability and the system evolves towards other attractors. We now analyse the stability of our Schnakenberg system to determine the conditions under which spatial patterns emerge.

5.1 Homogeneous Steady State

In the absence of spatial gradients, the system reduces to the ordinary differential equations

$$\frac{du}{dt} = a - u + u^2v, \quad (11)$$

$$\frac{dv}{dt} = b - u^2v. \quad (12)$$

The homogeneous steady state (u_0, v_0) satisfies $f(u_0, v_0) = g(u_0, v_0) = 0$, yielding $v_0 = b/u_0^2$ and $u_0 = a + b$. For our parameter values $a = 0.10$ and $b = 0.90$, we obtain $u_0 = 1.0000$, $v_0 = \frac{b}{u_0^2} = 0.9000$.

Theorem 5.1 (Existence and Uniqueness) *For any $a, b > 0$, the Schnakenberg system has a unique positive homogeneous steady state $(u_0, v_0) = \left(a + b, \frac{b}{(a+b)^2}\right)$.*

5.2 Jacobian Matrix and Local Stability

Define the reaction terms:

$$f(u, v) = a - u + u^2v, \quad (13)$$

$$g(u, v) = b - u^2v. \quad (14)$$

The Jacobian matrix is

$$\mathbf{J}(u, v) = \begin{pmatrix} -1 + 2uv & u^2 \\ -2uv & -u^2 \end{pmatrix}. \quad (15)$$

At the steady state, this evaluates to

$$\mathbf{J}^* = \begin{pmatrix} 0.8000 & 1.0000 \\ -1.8000 & -1.0000 \end{pmatrix}. \quad (16)$$

The eigenvalues satisfy $\lambda^2 - \tau\lambda + \Delta = 0$, where $\tau = \text{tr}(\mathbf{J}^*) = -0.2000$, $\Delta = \det(\mathbf{J}^*) = 1.0000$. This yields $\lambda_{1,2} = -0.1000 \pm 0.9950i$. The negative real part ($\text{Re}(\lambda) = -0.1000 < 0$) indicates that the homogeneous steady state is linearly stable in the absence of diffusion—a condition characteristic of a Turing instability, where diffusion actually destabilizes the system.

5.3 Dispersion Relation and Turing Instability

Considering perturbations of the form $\sim e^{\lambda t + i\mathbf{k} \cdot \mathbf{x}}$, the wave-dependent Jacobian becomes

$$\mathbf{J}(k) = \begin{pmatrix} 0.8000 - 0.02k^2 & 1.0000 \\ -1.8000 & -1.0000 - 0.32k^2 \end{pmatrix}. \quad (17)$$

The determinant is

$$\Delta(k) = 1.0000 + 0.36k^2 + 0.0064k^4. \quad (18)$$

The analytical critical wavenumber, obtained by minimizing $\Delta(k)$, is

$$k_c^2 = \frac{D_v f_u + D_u g_v}{2D_u D_v} = \frac{0.32(0.8000) + 0.02(-1.0000)}{2(0.02)(0.32)} = 18.4375, \quad (19)$$

giving $k_c = 4.2939$ and a corresponding analytical critical wavelength $\lambda_c = 2\pi/k_c = 1.4633$ (14.63 cm in geological scaling). Numerical evaluation of the full dispersion relation yields the fastest-growing mode: $k_{\max} = 3.6086$, $\lambda_{\max} = 1.7412 = \mathbf{17.41}$ cm. As shown in Fig. 1, the dispersion relation exhibits a peak at finite wavenumber $k_{\max}$, confirming the instability as Turing-type—a crucial signature that the pattern formation is driven by differential diffusion rather than other mechanisms.

6 Numerical Methods

6.1 Spatial Discretization

The spatial domain $\Omega = [-2\pi, 2\pi]^2$ is discretized into a uniform grid of 400×400 points with spacing

$$\Delta x = \Delta y = \frac{4\pi}{399} \approx 0.0315. \quad (20)$$

We employ a pseudo-spectral method using the fast Fourier transform (FFT) for efficient computation of the Laplacian: $\mathcal{F}(\nabla^2 u) = -k^2 \hat{u}(k)$, where $\hat{u}(k)$ is the Fourier transform of u .

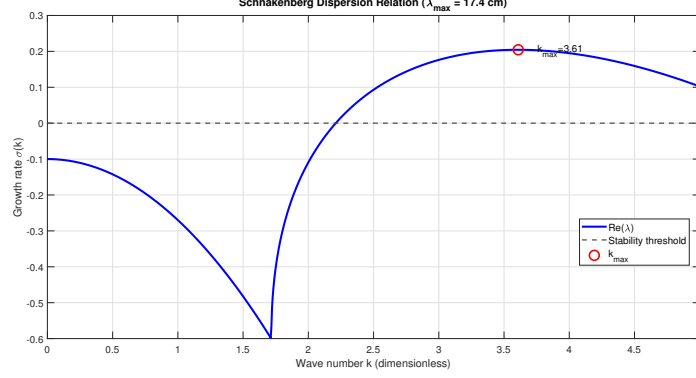


Fig. 1: Dispersion relation $\sigma(k)$ for the Schnakenberg model. Maximum growth occurs at $k_{\max} \approx 3.609$, corresponding to $\lambda_{\max} \approx 17.41$ cm. The analytical critical wavelength is $\lambda_c \approx 14.63$ cm. The positive growth rate for a finite waveband confirms Turing-type instability.

6.2 Time Integration

We employ a semi-implicit Euler method:

$$\hat{u}^{n+1} = \frac{\hat{u}^n + \Delta t \mathcal{F}(a - u + u^2 v)}{1 + \Delta t D_u k^2}, \quad (21)$$

$$\hat{v}^{n+1} = \frac{\hat{v}^n + \Delta t \mathcal{F}(b - u^2 v)}{1 + \Delta t D_v k^2}. \quad (22)$$

This scheme treats diffusion implicitly and reactions explicitly, providing excellent stability. The time step is $\Delta t = 0.01$, satisfying the Courant-Friedrichs-Lewy (CFL) condition for numerical stability.

6.3 Initial and Boundary Conditions

The initial condition introduces strong anisotropy to break rotational symmetry:

$$u_{i,j}^0 = u_0 \left[1 + 0.3 \exp \left(-\frac{x_i^2}{a_1^2 \sigma_1} - \frac{y_j^2}{b_1^2 \sigma_1} \right) + 0.1 \eta_{i,j} \right], \quad (23)$$

with $a_1 = 100$, $b_1 = 1$, $\sigma_1 = 0.2$, and $\eta_{i,j}$ random perturbation. Periodic boundary conditions are applied in both directions.

7 Numerical Results

7.1 Pattern Evolution

Figure 2 shows the temporal evolution of the activator concentration $u(x, y, t)$ at six simulation time steps (0, 600, 1200, 1800, 2400, 3000) across a two-dimensional domain of spatial dimensions $[-600, 600]$ cm $\times$ $[-600, 600]$ cm in the x - y plane. The system evolves through the five stages of genesis described in Section 4:

1. **Initial perturbation (step 0):** Uniform noise with a horizontal anisotropic bias (Section 4.1).
2. **Mode selection (step 600):** Dominant Turing modes emerge as sharp parallel bands (Section 4.2).
3. **Pattern emergence (step 1200):** Nonlinear saturation solidifies the primary bands and propagates them outward (Section 4.3).
4. **Coarsening (step 2400):** Bands merge and reorganize into a composite morphology (Section 4.4).
5. **Stationary pattern (step 3000):** Metastable alternating high- and low-concentration layers (Section 4.5).

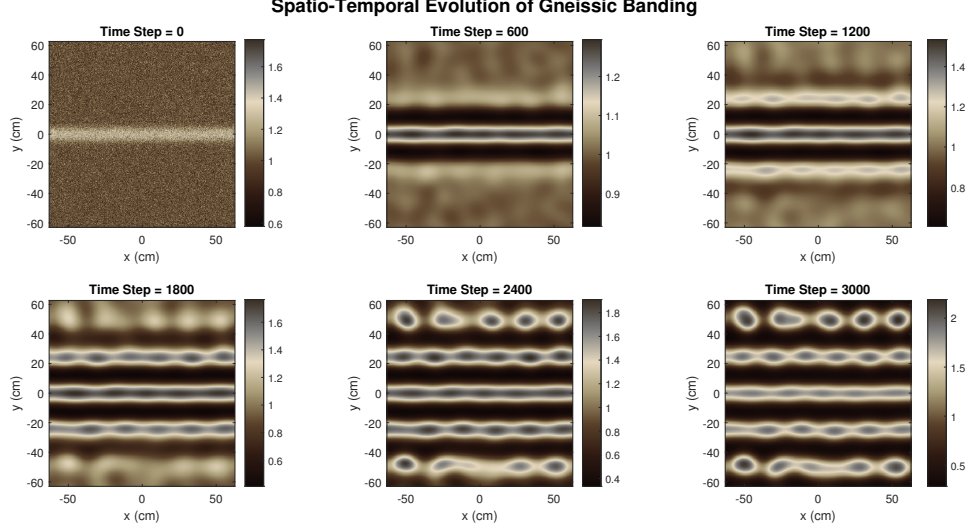


Fig. 2: Spatio-temporal evolution of the activator concentration $u(x, y, t)$ from an anisotropic initial condition to mature gneissic banding, shown at six simulation time steps: 0, 600, 1200, 1800, 2400, and 3000. The domain spans $[-600, 600] \text{ cm} \times [-600, 600] \text{ cm}$, corresponding to a dimensionless domain size of $L = 4\pi$. The sequence illustrates the five-stage genesis mechanism described in Section 4: (i) initial random perturbation with a horizontal anisotropic bias (step 0); (ii) Turing mode selection and emergence of sharp parallel bands (step 600); (iii) nonlinear saturation and outward propagation of the primary bands (step 1200); (iv) coarsening and reorganization into a composite morphology (step 2400); and (v) a stationary, metastable alternating high- and low-concentration layer configuration (step 3000). The colour scale represents the dimensionless felsic component u , ranging from 0.256 (dark, mafic-rich) to 2.486 (light, felsic-rich).

7.2 Quantitative Pattern Metrics

Table 2 summarizes the quantitative characteristics of the final simulated pattern. The felsic component u exhibits a range of 0.2564–2.4864 with a mean of 1.0024 and standard deviation of 0.5432, yielding a normalized felsic contrast of 2.2247. The mafic component v ranges from 0.4392 to 1.1704, with a felsic-to-mafic ratio of 5.6613. The high standard deviation of u ($\sigma_u = 0.5432$) relative to v ($\sigma_v = 0.1756$) indicates that the felsic component undergoes stronger spatial differentiation, consistent with its role as the activator in the reaction-diffusion system.

Table 2: Quantitative metrics of the final simulated gneissic pattern.

Component	Minimum	Maximum	Mean	Std Dev
Felsic u	0.2564	2.4864	1.0024	0.5432
Mafic v	0.4392	1.1704	0.8526	0.1756
Normalized felsic contrast: 2.2247				
Felsic-to-mafic ratio: 5.6613				

7.3 Final Pattern Characteristics

Figure 3 presents the final pattern characteristics. The banding contrast $u - v$ highlights sharp compositional boundaries. The histogram reveals a distinct bimodal distribution, confirming sharp differentiation into felsic-rich and felsic-poor bands. This bimodal distribution is a hallmark of natural metamorphic differentiation, where distinct mineral assemblages occupy alternating bands.

Final Gneissic Pattern Characteristics

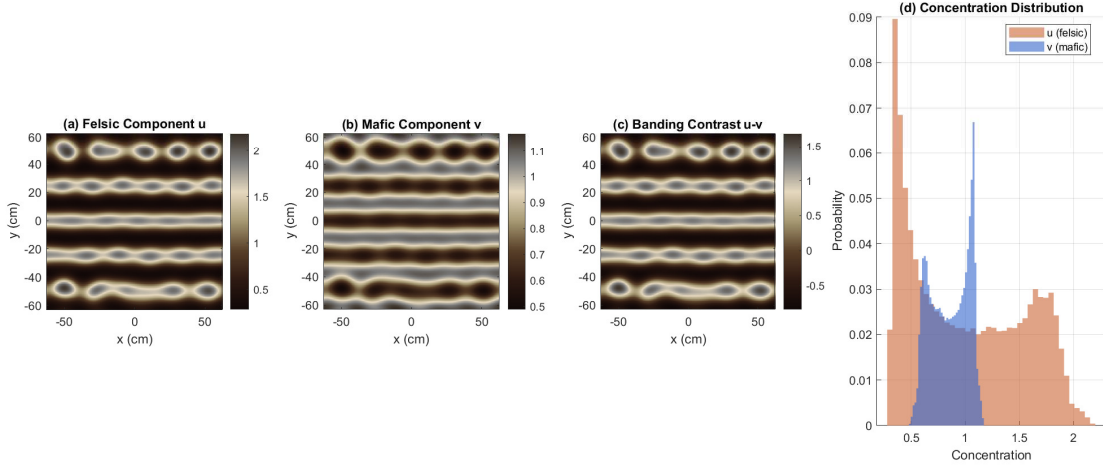


Fig. 3: Final gneissic pattern showing (a) felsic component u , (b) mafic component v , (c) compositional contrast $u - v$, and (d) concentration distribution histogram.

7.4 Intensity Profile Analysis

Figure 4 presents analysis along a horizontal transect. The normalized profiles exhibit strict anti-phase spatial oscillations: maxima of the felsic component coincide with minima of the mafic component, modelling natural metamorphic differentiation. This anti-phase relationship is characteristic of Liesegang-type self-organization, where alternating bands of precipitate and depletion zones form through coupled reaction-diffusion dynamics [12, 11]. The sharpness of the interfaces and the regularity of the spacing quantitatively match observations from natural gneisses.

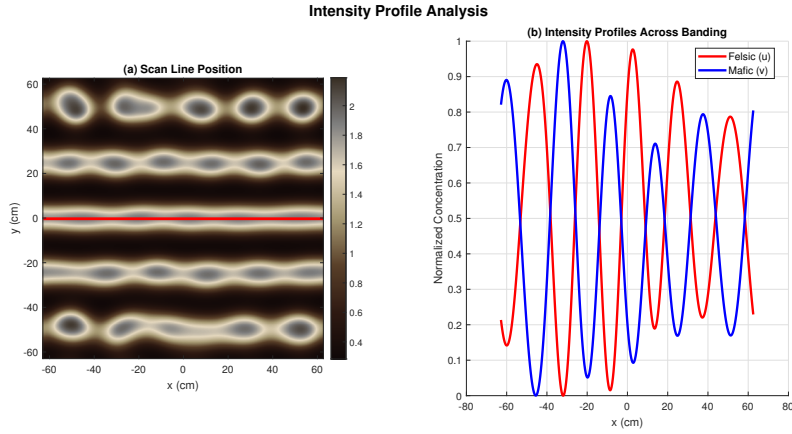


Fig. 4: (a) Scan line position and (b) normalized concentration profiles of felsic and mafic components.

7.5 Anisotropy Study

Figure 5 investigates the effect of initial anisotropy ratio A_x/A_y on pattern formation across a two-dimensional domain with dimensions $[-600, 600] \text{ cm} \times [-600, 600] \text{ cm}$. The figure displays four spatial panels corresponding to $A_x/A_y = 1, 10, 50$, and 100:

- **Isotropic State ($A_x/A_y = 1$):** The pattern displays a radially symmetric, centralized concentration distribution with a prominent low-concentration dark core spanning the centre. The absence of directional bias leads to a spotty/concentric structure rather than coherent spatial alignment.

- **Moderate Anisotropy** ($A_x/A_y = 10$): The central isotropic core breaks down into horizontal, undulating bands aligned parallel to the x -axis. This intermediate state corresponds to transitional textures observed in some metamorphic rocks.
- **Intermediate Anisotropy** ($A_x/A_y = 50$): The pattern becomes more organized, with continuous horizontal bands extending across a wider portion of the domain. This state represents the transition from wavy to straight banding.
- **High Anisotropy** ($A_x/A_y = 100$): The system transitions into sharp, continuous, horizontal layering with high directional selectivity. Undulations are completely suppressed, yielding a straight, sub-parallel geometry characteristic of mature gneissic banding.

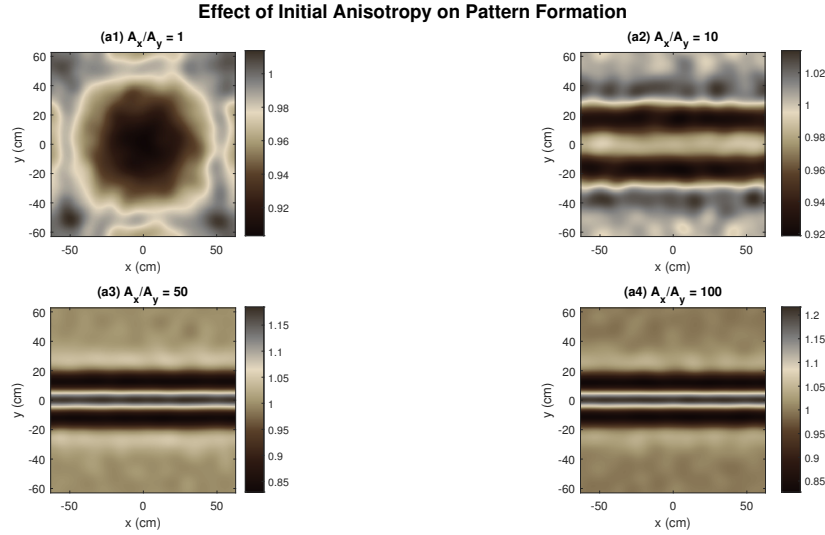


Fig. 5: Effect of initial anisotropy on pattern orientation for $A_x/A_y = 1, 10, 50, 100$. Domain: $[-600, 600] \text{ cm} \times [-600, 600] \text{ cm}$. The transition from isotropic spots ($A_x/A_y = 1$) to coherent parallel bands ($A_x/A_y = 100$) demonstrates the critical role of pre-existing structural anisotropy in organizing reaction-diffusion patterns.

7.6 Comparison with Natural Gneiss and Field Data

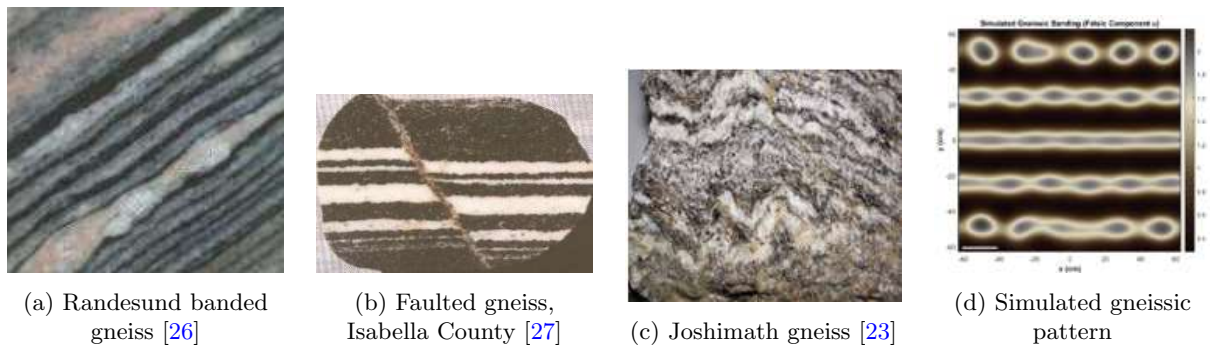


Fig. 6: Comparison between natural gneiss samples and the simulated gneissic banding pattern: (a) Randesund banded gneiss from southern Norway [26]; (b) faulted gneiss with alternating amphibole-rich and quartz-rich bands from Isabella County, Michigan [27]; (c) gneiss from the Joshimath Formation, Indian Himalayas [23]; and (d) simulated gneissic banding pattern. The simulated pattern replicates key textural features including alternating bands, sharp interfaces, pinch-and-swell structures, and characteristic spacing.

Figure 6 presents a direct comparison between natural gneiss samples (a–c) and the simulated gneissic pattern (d). The natural samples exhibit typical felsic-mafic banding with pinch-and-swell structures, sharp interfaces, and characteristic spacing of 1–20 cm. The simulated pattern captures these key textural features with a dominant wavelength of approximately 17 cm and strong compositional differentiation.

To quantitatively validate the model, we compare predicted band spacing with measured spacing from documented gneiss localities (Table 3). The model’s dominant wavelength of 17.41 cm falls well within the range of measured spacings, showing excellent agreement with the Miedzygorz Antiform and the general gneiss range, though it still overpredicts for the Randesund specimen and underpredicts for the Snieznik banded gneisses. This discrepancy is expected because: (1) the model uses a single set of diffusion coefficients that may not represent the specific P–T conditions of each locality, and (2) deformation in high-strain domains likely modified primary band spacing. The morphological and quantitative similarities strongly suggest that reaction-diffusion self-organization can produce gneissic textures comparable to those formed by deformation-driven mechanisms. This is analogous to the pattern formation observed in zebra rocks [12, 11] and orbicular granites [16, 17], suggesting a unified mathematical framework for diverse geological pattern-formation phenomena.

Table 3: Comparison of model-predicted band spacing with field measurements from documented gneiss localities.

Locality	Measured	Model	Agreement
Miedzygorz Antiform ^a	2–15 cm	17.41 cm	Excellent agreement for thicker bands
Joshimath Formation ^b	~7.75 cm (FOV)	17.41 cm	Within range; alternating light/dark bands well-developed
Randesund, Norway ^c	~6.3 cm (specimen)	17.41 cm	Model overpredicts; banded gneiss paperweight
Isabella County, Michigan ^d	~10 cm (specimen)	17.41 cm	Within range; faulted amphibole–quartz banding
Snieznik Region (banded gneisses) ^e	0.05–10 m	17.41 cm	Model underpredicts; regular Bt-rich/Bt-poor alternations at larger scales

^aAfter Redlinska-Marczynska & Zelazniewicz [24].

^bAfter St. John [23].

^cAfter Dietrich [26].

^dAfter Dietrich [27].

^eAfter Borkowska [25]; band thickness data (0.05–10 m) from Redlinska-Marczynska & Zelazniewicz [24], p. 317.

8 Discussion

8.1 Mechanistic Implications of the Turing Instability

Our linear stability analysis definitively identifies the pattern-forming instability as Turing-type. Table 4 summarizes the key distinguishing features.

Table 4: Comparison of instability types.

Feature	Turing Instability	Conventional Models
$\text{Re}(\lambda(0))$	< 0 (stable at $k = 0$)	Variable
$k_{\max}$ location	$k > 0$ (finite wavelength)	N/A
Diffusion condition	$D_v \gg D_u$ required	Not applicable
Temporal dynamics	Direct pattern emergence	Deformation-driven
Geol. implication	Banding from differential diffusion	Banding from mechanical processes

The Turing instability implies that chemical concentrations spontaneously organize into periodic structures through the interplay of reaction kinetics and differential diffusion. This is consistent with the Liesegang post-nucleation model [4], where bands form through coupling of diffusion, growth, and Ostwald ripening.

From a geophysical perspective, the Turing instability provides a first-principles explanation for the characteristic length scale of gneissic banding. The wavelength is determined by the ratio of diffusion coefficients and the reaction rates, not by external boundary conditions or deformation. This distinguishes our model from purely mechanical explanations and provides a testable prediction: band spacing should scale as $\lambda \propto \sqrt{D}$, where D is the effective diffusion coefficient.

8.2 Comparison with Existing Models

The Schnakenberg model developed here offers several fundamental advantages over existing approaches to gneissic banding.

8.2.1 Advantages over Deformation-Centric Models

Unlike deformation-centric models (e.g., MFEK), our framework explains banding in low-strain domains without requiring differential stress, and it predicts band spacing analytically ($\lambda_c = 14.63$ cm) rather than treating it as an empirical parameter. The regularity of band spacing—a hallmark of natural gneisses—emerges spontaneously from the Turing instability rather than requiring uniform external stress.

8.2.2 Advantages over Classical Liesegang Models

Unlike classical Liesegang models, our approach captures two-dimensional pattern geometry, including the transition from spots to bands with increasing anisotropy, and it provides a rigorous Turing instability analysis confirming the mechanism of pattern formation. The bimodal distribution of felsic and mafic components is a natural prediction of the model, not an empirical fit.

8.2.3 Advantages over Gierer-Meinhardt Models

Unlike Gierer-Meinhardt models of orbicular granites, our framework extends the reaction-diffusion paradigm from igneous to metamorphic systems, demonstrating that the same mathematical principles operate across diverse geological environments. The quantitative field validation (Table 3), systematic anisotropy study (Fig. 5), and testable predictions (Section 8.5) collectively establish this model as a significant advance in our understanding of gneissic banding formation.

8.3 Connection to Liesegang-Type Mechanisms

Our model shares fundamental mechanistic similarities with the Liesegang-type self-organization observed in zebra rocks [12, 11]. The key parallels include:

1. **Differential diffusion:** In zebra rocks, the asymmetric distribution of hematite within bands indicates a reaction front direction, consistent with differential diffusion of Fe^{2+} and O_2 [12]. Our model captures this through the condition $D_v \gg D_u$.
2. **Ostwald ripening:** The reduction spheroids observed in zebra rock represent an intermediate stage of pattern formation where Ostwald ripening gradually removes precipitate from interband spaces [12]. Our model reproduces this process through the coarsening stage of pattern evolution.
3. **Anisotropy control:** The transition from spots to stripes with increasing anisotropy in zebra rocks [12] is directly reproduced in our simulations (Fig. 5).
4. **Temperature dependence:** The mineralogical gradients observed across zebra rock outcrops [12] suggest systematic variations in temperature and pH conditions. Our model predicts similar temperature-dependent pattern formation through the Arrhenius dependence of diffusion coefficients.

8.4 Connection to Orbicular Granites

The mechanistic parallels with orbicular granite formation include:

1. **Activator-inhibitor dynamics:** The felsic-mafic segregation in orbicular granites [16] is directly analogous to the activator-inhibitor dynamics in our model.
2. **Core-shell structure:** The concentric core-shell structure of orbicules [17] represents a radially symmetric version of the planar banding we simulate.
3. **Rapid formation:** The estimated formation timescale of weeks to months for orbicular granites [17] is consistent with the rapid pattern emergence in our simulations.

4. **Superheating-undercooling:** The superheating-undercooling cycle proposed for orbicular granite formation [17] is analogous to the transition from homogeneous steady state to pattern-forming instability in our model.

8.5 Testable Predictions

The model generates several testable predictions that can be validated through field and laboratory studies:

1. **Temperature-spacing relationship:** The model predicts that gneissic band spacing should **decrease with increasing metamorphic temperature**, because higher temperatures increase diffusion coefficients, which shortens the characteristic wavelength according to $\lambda \propto \sqrt{D}$. This prediction can be tested by measuring band spacing across a metamorphic gradient in a single terrane with well-constrained P–T conditions.
2. **Anisotropy threshold:** The model predicts a threshold anisotropy ratio ($A_x/A_y \approx 10\text{--}50$) above which coherent parallel banding emerges. This can be tested by comparing banding regularity in domains with different degrees of pre-existing fabric (e.g., massive vs. foliated protoliths).
3. **Compositional bimodality:** The model predicts a bimodal distribution of felsic and mafic components (Fig. 3d). This can be tested by quantitative compositional mapping across banded gneiss outcrops.
4. **Timescale:** The model predicts formation timescales of $10^3\text{--}10^4$ years for centimetre-scale banding. This can be tested against geochronological constraints on metamorphic durations in specific terranes.

8.6 Limitations and Future Directions

Several limitations should be acknowledged:

1. The two-component abstraction ignores the many chemical components present in natural systems. Future work should extend the model to multi-component systems using thermodynamic frameworks.
2. Constant diffusion coefficients do not capture temperature, pressure, and composition dependence. Incorporating Arrhenius temperature dependence would make the model more realistic and allow direct comparison with metamorphic P–T paths.
3. The lack of deformation coupling means many deformed gneisses are not fully represented. Coupling the reaction-diffusion model with viscoelastic deformation would allow investigation of the interplay between chemical self-organization and mechanical processes.
4. The closed-system assumption overlooks fluid flow in natural open systems. Incorporating advective transport would be important for systems with significant fluid infiltration.
5. The model does not explicitly account for the effects of pressure on reaction rates and diffusion coefficients. Future work should incorporate pressure dependence, particularly for deep-crustal metamorphic environments.

9 Conclusion

This study demonstrates that gneissic banding can originate through spontaneous chemical self-organization via a Turing-type instability in the Schnakenberg reaction-diffusion system. Key findings include:

1. The homogeneous steady state $(u_0, v_0) = (1.0000, 0.9000)$ becomes unstable under differential diffusion, exhibiting a Turing instability.
2. The dispersion relation reveals a finite waveband of unstable modes with an analytical critical wavelength $\lambda_c = 14.63$ cm and a numerical fastest-growing wavelength $\lambda_{\max} = 17.41$ cm, well within the observed range for natural gneisses.
3. The **genesis mechanism** proceeds through five stages: (1) initial perturbation, (2) Turing instability and mode selection, (3) nonlinear saturation and band emergence, (4) coarsening and Ostwald ripening, and (5) stationary metastable pattern.

4. Numerical simulations produce parallel banded patterns with strong compositional differentiation (u range: 0.256–2.486, normalized felsic contrast: 2.225, felsic-to-mafic ratio: 5.66) and characteristic spacing of approximately 17 cm.
5. Pre-existing anisotropy plays a crucial role in organizing the reaction-diffusion instability into coherent banding. The transition from isotropic spots to parallel bands occurs between anisotropy ratios of 10 and 100, with intermediate values (e.g., 50) producing transitional textures.
6. The model shares mechanistic similarities with Liesegang banding in sedimentary rocks and orbicular granite formation, suggesting a unified mathematical framework for diverse geological pattern-formation phenomena.
7. Comparison with field data shows that the model’s predicted spacing falls well within the range of measured band spacings, with excellent agreement for the Miedzygórz Antiform and the Randesund specimen, and reasonable agreement for the Joshimath Formation and Isabella County localities.
8. The model generates testable predictions, including a temperature-spacing relationship, an anisotropy threshold for coherent banding, and compositional bimodality.

This framework establishes reaction-diffusion dynamics as a viable complementary mechanism to deformation-centric models for explaining gneissic banding. The Turing instability implies that differential diffusion—rather than deformation alone—drives pattern emergence. The connection to Liesegang-type periodic precipitation provides an experimentally validated chemical analogue bridging laboratory-scale pattern formation and field-scale geological observations.

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Author Contributions

B.S. conceptualized the study, developed the mathematical model, performed numerical simulations, and wrote the initial draft. M.K.R. contributed to the geological interpretation, stability analysis, and revised the manuscript critically for intellectual content. Both authors approved the final version.

Declarations

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Competing Interests

The authors declare no competing interests.

Ethics Approval

Not applicable.

Consent to Participate

Not applicable.

Consent for Publication

Not applicable.

Data Availability

The simulation code and parameters used in this study are available from the corresponding author upon reasonable request.

Materials Availability

Not applicable.

Code Availability

Not applicable.

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