

Kalman filter-based error prediction with forgetting factor iterative learning control scheme

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Abstract

Aiming at core technical bottlenecks such as production accuracy degradation, large convergence errors, and insufficient interference suppression capabilities induced by random and non-repetitive interference and other uncertain factors in industrial intelligent manufacturing scenarios, this paper innovatively proposes a Kalman filter (KF)-based iterative learning control with forgetting factor (ILCFF, ILCFF-KF) composite error prediction algorithm that integrates feedback correction strategies. First, based on the supervector theory, the predictive ILCFF-KF controller is constructed using a functional hierarchical architecture, and a mechanism for dynamically updating the reference trajectory in real time is designed simultaneously. The trajectory adaptive optimization is achieved through real-time feedback correction of the prediction error, and the efficiency of the error adjustment is fully utilized. Secondly, the algorithm applies the KF error prediction function along the iterative axis, thereby accurately focusing on suppressing and compensating for system output errors. This design not only effectively weakens the degrading impact of real-time random interference on system performance but also significantly reduces the online calculation complexity of the algorithm and provides lightweight support. Subsequently, this paper clarifies the monotonic convergence conditions of the algorithm through a norm-theoretic derivation and provides key parameter optimization criteria to achieve a collaborative guarantee of rapid system convergence and stable operation in industrial scenarios. Finally, experimental verification shows that the proposed ILCFF-KF algorithm shows significant advantages in both anti-interference performance and control accuracy.

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1 Introduction

Iterative Learning Control (ILC) is a type of intelligent control that is mainly used in industrial processes with repetitive movements. It can achieve high-precision tracking of the desired trajectory of the system through multiple iterative movements [1]. To achieve rapid system convergence, Arimoto proposed the ILCFF method [2] to suppress disturbance caused by system output interference and improve system output stability, and it has been widely studied and applied [3–6]. However, the fixed

047 forgetting factor in the traditional ILCFF method has significant limitations, which
048 will cause the tracking error to fail to converge to zero [7–9], and the error will eas-
049 ily increase after multiple iterations [10]. In order to solve this problem, subsequent
050 research proposed a variable ILCFF method [11–13] related to time and iteration num-
051 ber, but its parameters are set based on experience and are difficult to adjust. When
052 the controlled system has model uncertainty, unmodeled dynamics, or random envi-
053 ronmental interference, the control performance and robustness of the algorithm often
054 decrease significantly, making it difficult to adapt to diverse industrial scenarios. More
055 importantly, existing ILCFF methods usually require multiple iterations of learning,
056 resulting in low iteration rates and weak suppression of non-repetitive interference.
057 Although the theoretical research on ILCFF continues to progress, its problems, such
058 as slow convergence speed and large convergence error [14, 15], have not been fun-
059 damentally solved, making it difficult to meet the urgent demand for efficient and
060 high-precision control in modern intelligent manufacturing.

061 With the rapid development of modern intelligent manufacturing technology, the
062 complexity and sophistication of automated production processes have increased sig-
063 nificantly. However, the error influencing factors in the actual industrial environment
064 show the characteristics of multi-source and coupling. In particular, random and non-
065 repetitive interference have become the core bottleneck restricting the improvement of
066 manufacturing accuracy. As a core indicator to measure manufacturing quality, pro-
067 duction accuracy is directly related to the mechanism of various error factors [16].
068 Therefore, conducting research on production accuracy optimization control under
069 random or non-repetitive interference scenarios has important engineering practice
070 value for breaking through the accuracy bottleneck of intelligent manufacturing and
071 improving the stability and reliability of industrial production [17].

072 In ILC research, predictive ILC algorithms have become an important branch due
073 to their advantages in predictive performance and their significant improvement in
074 estimation accuracy of the target state [18–20]. Among them, the ILC method fused
075 with the Kalman Filter (KF-ILC) is an important direction in the ILC prediction
076 algorithm [21]. The prediction-correction mechanism and optimal state estimation
077 capability of KF can effectively make up for the inherent shortcomings of ILC in pro-
078 cessing non-repetitive interference [22, 23], effectively suppressing non-repetitive noise
079 interference (especially random interference), reducing the impact of noise interfer-
080 ence on the system, and also providing theoretical support for the effective filtering
081 of random noise [24]. Therefore, the KF-ILC method has received increasing atten-
082 tion. Current KF-ILC research is mainly carried out along two technical directions.
083 First, the gain optimization type has made certain progress in enhancing the anti-
084 interference ability of iterative learning, mainly by introducing KF design learning
085 gain matrix, but most methods [25–28] still have deficiencies in computational effi-
086 ciency. Second, the error-compensation type uses a KF to estimate the system error
087 and embeds it into the iterative learning process, thereby achieving effective suppres-
088 sion of random interference and ensuring stable operation of the controlled system.
089 Along this direction, Liu et al. [29] proposed a class of enhanced Kalman filter itera-
090 tive learning control algorithms and applied them to the wafer stage. Qu Xiaofan et
091 al. [30] combined Kalman filtering with an iterative learning algorithm to propose an
092

improved predictive control method. Cao et al. [31] proposed a discrete-time iterative learning Kalman filtering scheme, which combines the Kalman filter with iterative learning control to suppress repeatable disturbances and random noise. In addition to conventional fixed-trajectory control systems, related fusion algorithms have also been extended to complex control scenarios such as variable-trajectory, stochastic, and non-linear systems. Oh et al. [32] proposed an adaptive iterative learning control scheme for discrete linear time-invariant stochastic systems with batch-varying reference trajectories. Lu Xingyuan et al. [33] proposed a model-free adaptive control strategy for interference suppression, combined with the Kalman filtering principle, enabling the system to suppress measurement disturbances and achieve better adaptability effectively. Li et al. [34] demonstrated the convergence of iterative learning control based on Kalman filtering for bounded noise. Regarding interference issues in the control of precision equipment, Zhou Wenju et al. [35] applied Kalman filtering on the iterative axis and proposed a camera control method combining iterative learning and Kalman filtering, effectively reducing the influence of interference. To further optimize the disturbance rejection capability of predictive control, Luan Xinyu et al. [36] proposed a forgetting-factor-based iterative learning control law based on the Kalman filter to address the deteriorating dynamic performance of nonlinear underactuated mechanical systems in disturbed environments. Cao et al. [37] proposed a method that combines iterative learning control with Kalman filtering for state estimation of repetitive process systems with norm-bounded uncertainty in the state and output matrices. However, the error-compensation method has clear limitations. Its control update law relies too heavily on historical error information and cannot respond immediately to real-time non-repetitive interference, resulting in significant residual effects. Moreover, the error convergence speed and ultimate accuracy have not reached the ideal level, and there is still room for significant improvement in anti-noise performance.

To address these problems, this paper proposes a new composite control algorithm that integrates KF and ILCFF to achieve collaborative optimization of control performance and anti-interference ability through functional layered design. In the algorithm, ILCFF is used to achieve high-precision tracking of the reference trajectory and to increase the system's iteration convergence speed, ensuring that the system quickly approaches the expected performance during repeated operations. At the same time, KF is applied to the iterative axis, focusing on the suppression and compensation of system output errors, which not only effectively weakens the negative impact of real-time random interference on the system but also greatly reduces the online computing complexity of the algorithm. Furthermore, this paper introduces the concept of supervector iterative learning control and relies on the error prediction and estimation capabilities of the KF algorithm on the iterative axis to construct a fast-convergent iterative learning control with a forgetting factor based on the Kalman filter (ILCFF-KF) composite algorithm. Through error information fusion and feedback correction in the iterative domain, this algorithm significantly enhances the system's robustness to non-repetitive interference, greatly improves the anti-interference performance, fundamentally guarantees the stability and reliability of system operation in complex industrial scenarios, and provides a practical technical solution for the high-precision and high-efficiency operation of intelligent manufacturing systems.

The rest of the article is structured as follows. Section 2 introduces the ILCFF prediction algorithm model. Section 3 presents the ILCFF-KF modeling method and all steps of controller design. In Section 4, convergence properties of the control law and the value basis of key parameters are provided. Section 5 presents comparative simulations of ILCFF and ILCFF-KF to verify the effectiveness of the proposed control strategy. Finally, concluding remarks are provided in Section 6.

2 Problem Description

We consider a class of standard state-space equations representing a discrete-time, linear, and time-invariant SISO system:

$$\begin{cases} x_j(i+1) = Ax_j(i) + Bu_j(i) + w_j(i) \\ y_j(i) = Cx_j(i) + v_j(i) \\ x_j(0) = x_0 \\ i = 0, 1, \dots, M, j = 0, 1, \dots \end{cases} \quad (1)$$

where $x_j \in R^n$, $u_j \in R^n$ and $y_j \in R^n$ are the state vector, input vector, and output vector, respectively; A, B, C are real number matrices of the corresponding dimensions; x_0 is the initial state of the system; j indicates the number of iterations; M represents the total number of sampling points in the iteration, and $i \in [0, M]$ represents the i th discrete point in one iteration, w_j, v_j are process noise and measurement noise.

Combined with Eq. (1), the following reasonable assumptions are made:

Assumption 1. *Under ideal conditions where noise is completely suppressed and the initial state is exactly replicated, i.e., $w_j(i) \equiv 0$, $v_j(i) \equiv 0$ and $x_j(0) = x_0$, the following expectation equation holds*

$$\begin{cases} x_d(i+1) = Ax_d(i) + Bu_d(i) \\ y_d(i) = Cx_d(i) \end{cases}$$

where $x_d(i)$ and $y_d(i)$ denote desired state and input, respectively.

Assumption 2. *The process noise and measurement noise are zero-mean Gaussian white noise, with covariances satisfying the following conditions*

$$E[w_j w_j^T] = Q, E[v_j v_j^T] = R, E[w_j v_j] = 0$$

Assumption 3. *The system's state and output disturbances are bounded. In addition, $\max_{1 \leq i \leq M} \|w_j(i)\| \leq b_w$ and $\max_{1 \leq i \leq M} \|v_j(i)\| \leq b_v$, where b_w and b_v are constants. This assumption is used in the subsequent analysis of convergence in norm.*

Assumption 4. *The eigenvalues of matrix A are all inside the unit circle $\rho(A) < 1$, B and C have proper dimensions, or at least the system is repeatable in the iterative domain.*

Suppose the expected trajectory $y_d(i)$ of the system is known, and as the number of iterations increases, the system output approaches the predicted trajectory, that is

$$\lim_{j \rightarrow \infty} y_d(i) - y_j(i) = 0 \quad (2)$$

$$e_j(i) = y_d(i) - y_j(i) \quad (3)$$

To mitigate system interference effectively, an open-loop PID-type ILCFF algorithm is employed for system control. Its iterative learning law is as follows

$$u_{j+1}(i) = \lambda u_j(i) + s_P e_j(i+1) + s_I \sum_{m=1}^{i+1} e_j(m) + s_D (e_j(i+1) - e_j(i)) \quad (4)$$

where λ represents the forgetting factor, whose value lies in the range $0 < \lambda < 1$. s_P , s_I , s_D are the control gains of ILCFF.

Then the vector form of the learning law is

$$U_{j+1} = \lambda U_j + L E_j \quad (5)$$

where $L = s_P I + s_I F_1 + s_D D$, $F_1 \in R^{M \times M}$ is the integration matrix (all lower triangles are 1), $D = I - S$ is the downshift matrix, and S is the downshift matrix: $S_{k,l} = 1$. If $k = l + 1$, otherwise 0.

According to the control law Eq. (4), the update of the ILCFF (Iterative Learning Feedforward) control quantity directly depends on the system's output measurements; that is, the control input correction is driven by the output error. In the absence of measurement disturbances or repetitive disturbances, this scheme can make the system output error asymptotically converge to 0. However, when the system is subject to non-repetitive random disturbances, because the control law cannot effectively distinguish the source of the error, the output error will converge to a nonzero constant and cannot be completely eliminated. This shows that random disturbances significantly reduce the control accuracy and robustness of the conventional ILCFF scheme.

3 ILCFF algorithm based on Kalman filter error estimator

This section first draws on the Kalman filter design method to derive and propose a Kalman filter error estimator using system I/O data. Then, based on this Kalman filter error estimator, a random disturbance suppression control scheme is given.

3.1 Estimator modeling method

For ease of analysis, we introduce the super-vector notation, stacking the scalars at all time points into an $M \times 1$ column vector. The matrix $Y_j, U_j, \tilde{E}_j, E_j, Y_d$ can be defined

231 as follows

$$\begin{aligned}
232 & \\
233 & Y_j = [y_j(1) \ y_j(2) \ \cdots \ y_j(M)]^T \\
234 & U_j = [u_j(0) \ u_j(2) \ \cdots \ u_j(M-1)]^T \\
235 & \\
236 & \tilde{E}_j = [\tilde{e}_j(1) \ \tilde{e}_j(2) \ \cdots \ \tilde{e}_j(M)]^T \\
237 & \\
238 & E_j = [e_j(1) \ e_j(2) \ \cdots \ e_j(M)]^T \\
239 & Y_d = [y_d(1) \ y_d(2) \ \cdots \ y_d(M)]^T \\
240 &
\end{aligned} \tag{6}$$

241 Define $a \in R^M$, $\psi \in R^{M \times M}$ as follows:

$$\begin{aligned}
242 & \\
243 & \forall a = [a_1 \ a_2 \ \cdots \ a_M]^T \\
244 & \\
245 & \\
246 & \psi(a) = \begin{bmatrix} a_1 & 0 & 0 & \cdots & 0 \\ a_2 & a_1 & 0 & \cdots & 0 \\ a_3 & a_2 & a_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_M & a_{M-1} & a_{M-2} & \cdots & a_1 \end{bmatrix} \\
247 & \\
248 & \\
249 & \\
250 & \\
251 &
\end{aligned} \tag{7}$$

252 among them, the operator ψ is a lower-triangular Toeplitz matrix with respect to a .

253 Define the following matrices

$$254 \quad I = \psi(\alpha), G = \psi(g), G_w = \psi(g_w) \tag{8}$$

255 where $\alpha = [1 \ 0 \ \cdots \ 0]^T$, $g = [g_1 \ g_2 \ \cdots \ g_M]^T$, $g_w = [C \ CA \ \cdots \ CA^{M-1}]^T$.

259 Furthermore

$$260 \quad g_k = CA^{k-1}B, k = 1, 2, \cdots, M \tag{9}$$

261 From formula Eq. (1), the actual and measured outputs can be obtained as

$$262 \quad Y_j = GU_j + g_x x_0 + G_w W_j \tag{10}$$

$$263 \quad \tilde{Y}_j = Y_j + V_j \tag{11}$$

264 where

$$265 \quad g_x = [CA \ CA^2 \ CA^3 \ \cdots \ CA^M]^T \tag{12}$$

266 The true tracking error is

$$267 \quad E_j = Y_d - Y_j = Y_d - GU_j - g_x x_0 - G_w W_j \tag{13}$$

The corresponding measurement error is

$$\tilde{E}_j = Y_d - \tilde{Y}_j = E_j - V_j \quad (14)$$

Substituting Eq. (5) into Eq. (13), we obtain the error for the next iteration

$$\begin{aligned} E_{j+1} &= Y_d - GU_{j+1} - g_x x_0 - G_w W_{j+1} \\ &= Y_d - G(\lambda U_j + LE_j) - g_x x_0 - G_w W_{j+1} \\ &= \lambda(Y_d - GU_j - g_x x_0) - GLE_j + (1 - \lambda)(Y_d - g_x x_0) - G_w W_{j+1} \end{aligned} \quad (15)$$

It can be seen from Eq. (10) that $Y_d - GU_j - g_x x_0 = E_j + G_w W_j$, substitute to get

$$E_{j+1} = \lambda(E_j + G_w W_j) - GLE_j + (1 - \lambda)(Y_d - g_x x_0) - G_w W_{j+1} \quad (16)$$

where the error evolution matrix

$$G_c = \lambda I - GL \quad (17)$$

Based on this, Eq. (16) can be rearranged as

$$E_{j+1} = G_c E_j + c + W_j^{KF} \quad (18)$$

where

$$c = (1 - \lambda)(Y_d - g_x x_0) \quad (19)$$

$$W_j^{KF} = \lambda G_w W_j - G_w W_{j+1} \quad (20)$$

Furthermore, $\lambda G_w W_j - G_w W_{j+1}$ is zero-mean noise, and the covariance is

$$\begin{aligned} Q' &= E(W_j^{KF} W_j^{KF T}) \\ &= E[(\lambda G_w W_j - G_w W_{j+1})(\lambda G_w W_j - G_w W_{j+1})^T] \\ &= (\lambda^2 + 1)G_w Q G_w^T \end{aligned} \quad (21)$$

Therefore, based on Eq. (14) and Eq. (18), assuming that the error dynamics are approximately linear, the following state-space model will be used for Kalman filter design

$$\begin{cases} E_{j+1} = G_c E_j + c + W_j^{KF} \\ \tilde{E}_j = E_j + V_j \end{cases} \quad (22)$$

where $H = I$, $HE_j + V_j$ and $HE_j - V_j$ are statistically equivalent, W_j^{KF} and V_j are equivalent to Gaussian white noise.

3.2 Interference suppression ILCFF algorithm based on KF

This section presents an iterative learning control scheme for disturbance suppression based on the Kalman filter error estimator, combining the KF and ILCFF methods to control systems in the presence of random disturbances.

Based on the modeling process and Eq. (22), the Kalman filter error recursion formula is

$$\bar{E}_j = G_c \hat{E}_{j-1} + c \quad (23)$$

$$\bar{P}_j = G_c \hat{P}_{j-1} G_c^T + Q' \quad (24)$$

$$K_j = \bar{P}_j H (H^T \bar{P}_j + R')^{-1} \quad (25)$$

$$\hat{E}_j = \bar{E}_j + K_j (\tilde{E}_j - H \bar{E}_j) \quad (26)$$

$$\hat{P}_j = \bar{P}_j - K_j \bar{P}_j \quad (27)$$

Remark 1. According to Eqs. (18), (26) and (27), under steady-state conditions (assuming $K_j \rightarrow K$), it can be derived

$$\Phi_{j+1} = (I - K)(G_c \Phi_j + W_j^{KF}) - K V_{j+1} \quad (28)$$

According to the stability theory of discrete Kalman filtering, if the steady-state filtering error dynamics matrix $A_e = (I - K)G_c$ is stable, then $\rho(A_e) < 1$ and the mean square of the estimation error is bounded. In particular, when the process noise covariance Q' and the measurement noise covariance R are sufficiently small, the expected value of $\|\Phi_j\|$ can be made sufficiently small. The effects of these two parameters on the system will be discussed further in Section 4.

In the presence of noise, the actual control law uses the estimated error $\hat{E}(j)$ (provided by the Kalman filter error estimator) instead of the true error $E(j)$; then the vector form of the ILCFF learning law will be

$$U_{j+1} = \lambda U_j + L \hat{E}_j \quad (29)$$

Taken together with the above content, the Kalman filter error estimator Eqs. (23)-(27) and the ILCFF learning law Eq. (29) constitute a complete control scheme.

Algorithm 1 ILCFF-KF Composite Control Strategy

Require: $u_0, y_0, t_f, M, j_{\max}, Q, R$ **Ensure:** $y_j(i), e_j(i), u_j(i)$

```
1: Initialize  $j = 0$ , apply initial control  $U_0$  to system Eq. (1)
2: for  $j = 1$  to  $j_{\max}$  do
3:   for  $i = 0$  to  $M - 1$  do
4:     Measure system output  $y_j(i)$  from Eq. (1)
5:     Compute tracking error  $e_j(i) = y_d(i) - y_j(i)$  according to Eq. (3)
6:   end for
7:   Construct supervectors  $E_j, \tilde{E}_j, U_j, Y_j, Y_d$  as in Eq. (6)
8:   Compute prior error estimate  $\hat{E}_j^-$  using Kalman filter recursion Eqs. (23)-(25)
9:   Update covariance  $P_j$  and gain  $K_j$  via Eqs. (26)-(27), obtain posterior estimate  $\hat{E}_j$ 
10:  Update control input using ILCFF law with estimated error:  $U_{j+1} = \lambda U_j + L \hat{E}_j$  as in Eq. (29)
11:  Apply  $U_{j+1}$  to system Eq. (1) for next iteration
12:  Record and store  $y_j(i)$  and  $e_j(i)$ 
13: end for
14: return  $U_j, y_j, e_j$  for all iterations
```

4 Convergence Analysis

Define the estimation error as

$$\Phi_j = E_j - \hat{E}_j \quad (30)$$

then

$$\hat{E}_j = E_j - \Phi_j \quad (31)$$

Substitute Eq. (31) into Eq. (16) get

$$E_{j+1} = G_c E_j + GL \Phi_j + c + W_j^{KF} \quad (32)$$

In this chapter, the stability of the ILCF-KF algorithm in the presence of random noise is examined.

Theorem 1. *Under the action of the iterative learning law as shown in Eq. (4), if system Eq. (1) satisfies*

$$|\lambda - \mathbf{CB}(s_P + s_I + s_D)| < 1 \quad (33)$$

then the system output settles on a bounded value.

Proof. The detailed proof process of this theorem is available in [3]. $\square$

415 To analyze the contractivity of the error evolution matrix, the following theorem
416 is presented.

417 **Theorem 2.** Suppose $\mathbf{A} = \mathbf{L} - \mathbf{N} \in \mathbb{R}^{n \times n}$ indicates the splitting of a non-singular
418 matrix, and $\mathbf{b} \in \mathbb{R}^n$. Assuming that L is non-singular, the iteration, $\mathbf{L}\mathbf{x}^{(k+1)} =$
419 $\mathbf{N}\mathbf{x}^{(k)} + \mathbf{b}$, converges to $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$ for all starting n -vectors, $\mathbf{x}(0)$, if and only if
420 $\rho(\mathbf{L}^{-1}\mathbf{N}) < 1$.

421 *Proof.* The detailed proof process of this theorem is available in [3]. □
422

423 **Corollary 1.** Under the conditions of Theorem 1, if the system satisfies

$$424 \quad \quad \quad \|G_c\| = \gamma < 1 \quad (34)$$

$$425 \quad \quad \quad \|Q\| \rightarrow 0 \quad (35)$$

426
427 then the estimation error is bounded.

428 *Proof.* The steady-state estimation error covariance matrix P_∞ satisfies the algebraic
429 Riccati equation

$$430 \quad \quad \quad P_\infty = G_c P_\infty G_c^T + Q' - G_c P_\infty G_c^T (G_c P_\infty G_c^T + R')^{-1} G_c P_\infty G_c^T \quad (36)$$

431 The upper bound of the expected norm of the estimation error can be defined as

$$432 \quad \quad \quad \varepsilon = E[\|\Phi_j\|] \leq \sqrt{\text{tr}(P_\infty)} \quad (37)$$

433 From Eq. (36) can be known

$$434 \quad \quad \quad P_\infty \leq G_c P_\infty G_c^T + Q' \quad (38)$$

435 Taking the matrix norm on both sides of the above equation gives

$$436 \quad \quad \quad \|P_\infty\| \leq \frac{\|Q'\|}{1 - \|G_c\|^2} \quad (39)$$

437 Then

$$438 \quad \quad \quad \varepsilon \leq \sqrt{\text{tr}(P_\infty)} \leq \sqrt{M} \|P_\infty\| \leq \sqrt{\frac{M}{1 - \gamma^2}} \|Q'\| \quad (40)$$

439 Furthermore, from Eq. (21) we obtain

$$440 \quad \quad \quad \|Q'\| \leq (\lambda^2 + 1) \|G_w\|^2 \|Q\| \quad (41)$$

Therefore

$$\varepsilon \leq \sqrt{\frac{M(\lambda^2 + 1) \|G_w\|^2 \|Q\|}{1 - \gamma^2}} \quad (42)$$

Thus, when $\|Q\| \rightarrow 0$, then $\varepsilon \rightarrow 0$ (by choosing sufficiently small physical process noise). $\square$

Remark 2. In real systems, if the process noise is much smaller than the measurement noise and the Kalman filter can provide high-precision estimates, the bounded norm of the estimation error Φ_j is included in the total noise term for processing (see the proof of Theorem 3 for details).

When the prediction error decreases as the number of iterations increases, it indicates that the prediction algorithm is converging, thereby ensuring the algorithm's accuracy and effectiveness. Therefore, it can be concluded that the monotonic convergence of the prediction error is a sufficient condition for the effectiveness of the prediction algorithm. The proof of the convergence of the prediction algorithm is as follows

Theorem 3. Consider a closed-loop system consisting of system Eq. (1), the ILCPF update law Eq. (4), and the Kalman filter Eqs. (23)-(27), if it satisfies

$$\|G_c\| = \gamma < 1 \quad (43)$$

$$E[\|\Phi_j\|] \leq \varepsilon, \quad \forall j \quad (44)$$

$$E[\|W_j^e\|] \leq \sigma_n \quad (45)$$

then the expected norm of the tracking error satisfies bounded convergence

$$E[\|E_{j+1}\|] \leq \gamma E[\|E_j\|] + \beta, \quad \forall j \geq 0,$$

where $\beta = \|GL\| \varepsilon + \|\mathbf{c}\| + \sigma_n$.

Proof. Compute the norm of Eq. (32) on both sides and apply the triangle inequality

$$\|E_{j+1}\| \leq \|G_c\| \|E_j\| + \|GL\| \|\Phi_j\| + \|\mathbf{c}\| + \|W_j^{KF}\|. \quad (46)$$

Taking the expectation from conditions Eqs. (43), (44) and (45), can get

$$\begin{aligned} E[\|E_{j+1}\|] &\leq \gamma E[\|E_j\|] + \|GL\| \varepsilon + \|\mathbf{c}\| + \sigma_n \\ &\leq \gamma E[\|E_j\|] + \beta. \end{aligned} \quad (47)$$

By repeatedly deriving this inequality, we obtain

$$E[\|E_j\|] \leq \gamma^j E[\|E_0\|] + \frac{1 - \gamma^j}{1 - \gamma} \beta, \quad (48)$$

Then it is expected that the error norm eventually converges to a bounded region

$$\begin{aligned} \limsup_{j \rightarrow \infty} E[\|E_j\|] &\leq \frac{\beta}{1 - \gamma} \\ &\leq \frac{\|GL\| \sqrt{\frac{M(\lambda^2 + 1)\|G_w\|^2\|Q\|}{1 - \gamma^2}} + \|c\| + \sigma_n}{1 - \gamma} \end{aligned} \quad (49)$$

Therefore, the system converges uniformly. $\square$

Based on a comprehensive analysis of the above conditions, as $\|G_c\|$ decreases further, the system's internal observation mechanism can effectively identify and suppress the effects of external disturbances, resulting in a faster convergence rate. When Q is smaller and $\lambda \rightarrow 1$, then $\varepsilon \rightarrow 0$ and $\lambda \rightarrow 0$, the final error bound becomes smaller. At the same time, when the norm of the KF's prediction error is bounded and much smaller than the norm of the current tracking error (i.e., $\|\tilde{E}_j\| \ll \|E_j\|$), incorporating a Kalman filter into the algorithm can effectively improve the accuracy of the system state estimate. Therefore, it can be concluded that, provided the above conditions are met, the algorithm proposed in this paper can significantly improve the system's monotonic convergence. This conclusion not only provides an important theoretical basis for the design of iterative learning control algorithms but also offers clear guidance and a theoretical foundation for the optimization and adjustment of parameters in practical applications.

5 Simulation

To verify the effectiveness of the KF-based disturbance suppression ILCFF scheme, this section presents a simulation comparison of the KF-based ILCFF scheme with the conventional ILCFF scheme in the tracking problem under random noise levels.

This paper simulates the object with the following simulation examples

$$\begin{cases} x_j(i+1) = A_D x_j(i) + B_D u_j(i) + w_j(i) \\ y_j(i) = C_D x_j(i) + v_j(i) \end{cases}$$

$$i = 0, 1, \dots, 20, j = 0, 1, \dots, 40$$

$$\text{where } A_D = \begin{bmatrix} 0.8187 & 0 & 0 \\ 0.4526 & 0.0097 & 0 \\ 0.0023 & 0.01 & 0.1 \end{bmatrix}, B_D = \begin{bmatrix} 0 \\ 0.0197 \\ 0.0211 \end{bmatrix}, C_D = [0 \ 0 \ 0.6].$$

The expected output trajectory is

$$y_d = 10 \left[1 + \sin \left(\frac{2\pi i}{M} - \frac{\pi}{2} \right) \right]$$

Where the sampling time is $T = 0.05$ s, the iteration period is $t_f = 1$ s, and initial state of the system is $x_0 = [0 \ 0 \ 0]^T$.

Randomly take the control gain [38] that meets the convergence condition as

$$S_P = 7, S_I = 10.5, S_D = 80$$

Take the forgetting factor $\lambda = 0.98$.

For random Gaussian noise, the covariance of w_j is taken as 1.5, and that of v_j is taken as 1, the noise distribution generated is shown in Figure 1.

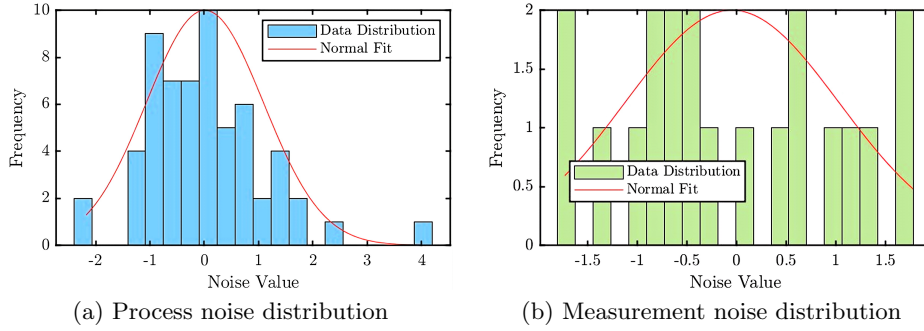
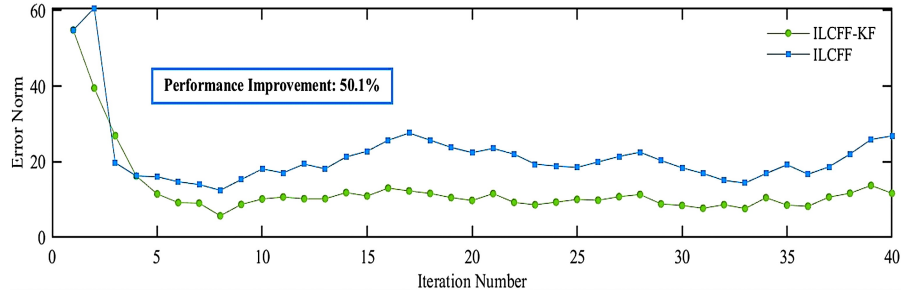


Fig. 1 Distribution map of noise characteristics

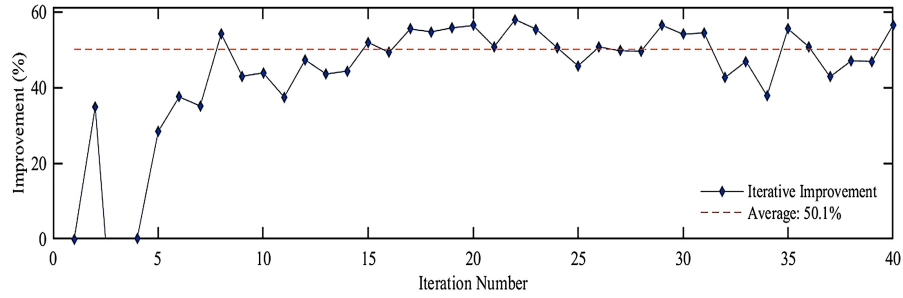
With the same control gain, the convergence characteristics of the ILCFF-KF and ILCFF algorithms were compared. The output error norms of the two algorithms are shown in Figure 2. The system converges quickly after the fifth iteration. The ILCFF-KF algorithm guarantees rapid system convergence while effectively suppressing random interference.

To compare KF's ability to suppress system output error fluctuations, the steady-state output error norm data from the 8th to the 40th iteration were selected for processing, as shown in Table 1. When the system reaches steady state, the algorithm's output-tracking error without the KF fluctuates wildly. In contrast, the algorithm with KF can effectively reduce fluctuations in the output tracking error due to random noise.

According to the performance indicators of the ILCFF-KF algorithm presented in Table 1, the proposed method significantly improves the system's average accuracy, stability, and suppression of fluctuations. When KF is used for prediction and error correction, the correction process plays a vital role in reducing the mean-squared error.



(a) Convergence performance comparison



(b) ILCFF-KF induced performance improvement

Fig. 2 Comparison of output error norms

Table 1 Comparison of Disturbance Suppression Capabilities.

Error norm indicator	ILCFF	ILCFF-KF	Improvement percentage
Mean	20.2201	10.0841	50.13%
Variance	14.1239	2.8765	79.63%
Standard deviation	3.7582	1.6960	54.87%
Sum of squares of deviations	451.9659	92.0483	79.63%

Figure 3 compares the distributions of the error norm after the system output reaches steady state, clearly illustrating the steady-state error trend and the corresponding performance optimization path. According to the analysis, the error norm estimated by the ILCFF-KF algorithm converges gradually. Additionally, the error norm of the ILCFF-KF system is consistently lower than that of the ILCFF system, and the improvement percentage remains positive. This demonstrates that the ILCFF-KF algorithm effectively accounts for interference between process noise and measurement noise by accurately estimating errors, thus improving overall system performance.

To demonstrate the superiority of the ILCFF-KF algorithm, the average errors obtained after the system reached steady state (from the 8th to the 40th iteration) were used for comparison. The specific results are shown in Table 2.

Table 2 compares the average errors of different algorithms during the steady-state stage (from the 8th to the 40th iteration). Specifically, the average error of the ILCFF algorithm was 20.2201, while the ILCFF-KF algorithm achieved a significantly

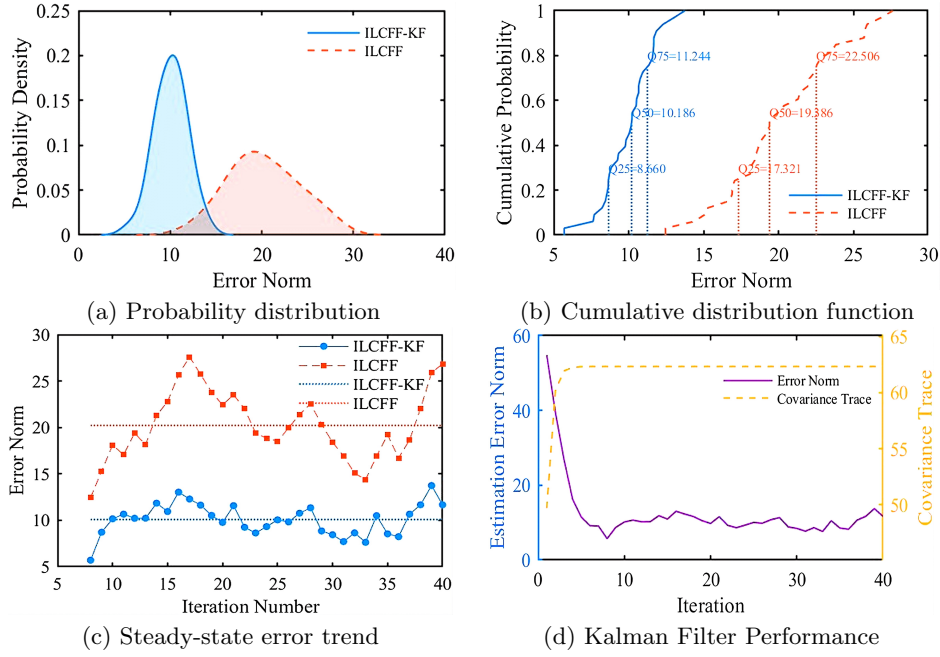


Fig. 3 Comparison of error norm distribution

Table 2 Comparison of Steady-state Average Errors.

	ILCFF	ILCFF-KF
Steady-state average error	20.2201	10.0841

lower average error of 10.0841, and the improvement percentage reached 50.1%. This result shows that the ILCFF-KF algorithm substantially reduces the steady-state error by integrating the Kalman filter's error-estimation capability, thereby significantly improving the system's output accuracy and stability. This improvement not only optimizes the system's dynamic performance but also enhances its robustness in complex environments, further demonstrating the significant advantages of the ILCFF-KF algorithm.

Based on the analysis of the steady-state average error, the notable improvement in system performance from the ILCFF-KF algorithm is shown in Figure 4.

To thoroughly evaluate the interference suppression performance of the Kalman filter error estimator, this article compares the mean, standard deviation, variance, and median of the error norm. Multiple independent Monte Carlo simulation experiments with different random seeds were conducted to assess the steady-state control performance of the two control strategies (ILCFF and ILCFF-KF) under various noise conditions. This approach helps characterize the algorithm's generalization ability and robustness to different noise samples. The results show that the ILCFF-KF algorithm achieves a smaller error norm and a more concentrated error-fluctuation interval; in

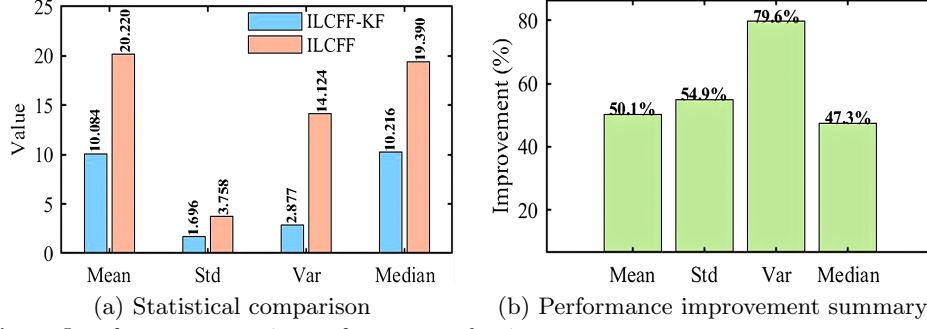


Fig. 4 Interference suppression performance evaluation

terms of average and maximum error metrics, the Kalman filter demonstrates a superior error suppression effect, indicating significantly better anti-interference capability compared to the traditional ILCFF algorithm. Additionally, while effectively reducing tracking errors, the KF also significantly improves the system's operational stability.

Since the system is subject to random noise, the results of a single simulation are not statistically representative. Therefore, this paper conducts $M = 50$ independent Monte Carlo comparison experiments, demonstrating the tracking performance relative to the desired trajectory and the time-dependent curve of the control input u , respectively. As shown in Figure 5, the tracking trajectory of ILCFF-KF almost perfectly follows the desired trajectory, with a significantly narrower standard deviation band than that of the standard ILCFF. Furthermore, the control input of ILCFF-KF is smooth and changes gradually; high-frequency oscillations are significantly suppressed, the standard deviation band is extremely narrow, and the control accuracy is markedly superior to that of the standard ILCFF, significantly enhancing the system's operational stability. Combined with Table 3, it can be seen that both the mean steady-state error norm and the tracking RMSE are reduced by more than 50% compared to the standard ILCFF. These results fully validate the significant effectiveness and superiority of the Kalman filter-based error prediction strategy in addressing the challenges of iterative learning control under random disturbances.

Table 3 Comparison of Disturbance Suppression Capabilities (Mean of 50 Monte Carlo mean $\pm$ std).

Algorithm	ILCFF	ILCFF-KF	Improvement percentage
Mean steady-state error	19.9839 ± 4.2916	9.6397 ± 1.6033	$51.50\% \pm 2.38\%$
RMSE	4.5960 ± 0.9543	2.2023 ± 0.3440	52.08%

From the above analysis, it is clear that the ILCFF-KF algorithm effectively mitigates the impact of random noise interference, enhances noise suppression, improves trajectory tracking, reduces errors, and achieves significantly better control performance than the ILCFF algorithm.

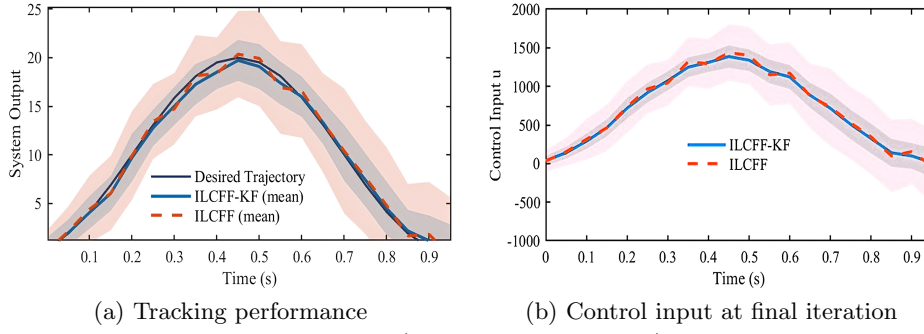


Fig. 5 Monte Carlo comparative analysis(mean $\pm$ std, 50 MC runs)

6 Conclusion

This paper proposes an ILCFF algorithm based on Kalman filtering for an LTI system affected by random noise. The algorithm fully utilizes KF's advantages in state estimation and optimizes the control vector for the next iteration by estimating the system's actual error in real time, thereby enhancing the system's output performance. Additionally, this article analyzes the system parameter boundary conditions and the convergence of the composite control algorithm and the Kalman filter error estimator in terms of noise suppression. It verifies the performance of the proposed scheme through rigorous simulation. Through the collaborative mechanism of these two components, this composite control algorithm not only retains the fast convergence features of iterative learning feedforward control but also effectively reduces the deviation between the system output and the desired trajectory. It demonstrates strong robustness to observation noise and parameter disturbances, has low implementation costs, and is easy to apply. This solution offers broad industrial application prospects and provides new ideas and methods for in-depth research and application of the "KF and ILCFF" composite control algorithm across various systems.

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