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The Gravity of Love: Intrinsic Decay and Counteracting Effort in Romantic Relationships

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The Gravity of Love: Intrinsic Decay and Counteracting Effort in Romantic Relationships

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Abstract

Understanding how successful romantic relationships are sustained remains a central challenge in the social sciences. Although relationship satisfaction is known to decline over time, the extent to which this reflects an intrinsic decline versus maintenance effort by partners remains unclear. Here, we show that the observed relationship dynamics can be disentangled into two latent processes: an automatically driven process of decline, which we term the *gravity of love*, and the behavioral maintenance effort exerted by partners to counteract it. We combine a computational dynamic game model with a calibration procedure to reproduce cross-cultural trajectories of marital satisfaction from approximately 7,000 couples across 33 countries. The calibrated model reproduces the observed dynamics over three decades and predicts that, in free fall under the gravity of love—that is, in the absence of maintenance effort—relationship satisfaction would fall to one-half of its initial level within about four years. The estimated equilibrium effort remains persistently above the preferred level —that is, the level that minimizes effort-related dissatisfaction— indicating that successful relationships require sustained maintenance effort. Across the observed marital-satisfaction trajectories, the estimated maintenance effort remains approximately 16% above the preferred level. These findings provide a quantitative characterization of two latent processes underlying relationship dynamics and the balance between them, placing a central idea in relationship science on a quantitative footing. More broadly, this study illustrates how behavioral modeling and computational calibration can be combined with observational data to infer latent mechanisms that are not directly observable.

Introduction

Romantic relationships, and marriage in particular, are widely regarded as cultural universals and fundamental social institutions across human societies [1, 2]. Beyond their social significance, satisfying long-term relationships are consistently associated with better mental and physical health, increased longevity, and higher levels of subjective well-being [3–5]. Moreover, the contribution of marital quality to happiness has increased over recent decades [6], underscoring the growing importance of stable romantic partnerships for individual well-being.

Despite their importance, understanding how successful romantic relationships are sustained remains a central challenge in the social sciences [7, 8]. A substantial body of research has identified psychological and interpersonal processes that contribute to relationship maintenance, including commitment, responsiveness, adaptation, and maintenance behaviors [9]. However, less is known about how these processes combine over time to generate the observed trajectories of relationship satisfaction. Progress has been limited by the scarcity of long-term longitudinal data [10]. This scarcity is itself partly a consequence of the high rates of relationship dissolution observed across many Western societies [11]. At the same time, these persistently high divorce rates underscore the practical importance of understanding the mechanisms governing relationship dynamics.

One promising approach is to investigate relationship dynamics using computational models calibrated against empirical observations. Recent developments in computational social psychology have emphasized the value of such models for making behavioral theories explicit, generating quantitative predictions, and linking latent psychological mechanisms to observable social behavior [12]. In the context of romantic relationships, computational models make it possible to generate synthetic trajectories that can be compared with observed patterns of marital satisfaction, providing a framework for disentangling the processes underlying relationship dynamics.

A robust finding in relationship science is that relationship satisfaction tends to decline over time, even among otherwise successful couples [10, 13]. This pattern suggests the existence of an automatically driven process, which we conceptualize as the *gravity of love*: an intrinsic tendency for relationship satisfaction to decrease over time. Regardless of the intensity of initial romantic attachment, sustaining a successful relationship therefore requires behavioral adaptation and maintenance effort to counteract this process.

The need to counterbalance relationship decline has long been recognized in relationship science. Early cognitive approaches emphasized active relational maintenance [14], while later work described romantic relationships as being subject to a "second law of thermodynamics" [7] or a form of "default relational entropy" [9]. Evidence from neuroscience and evolutionary psychology further suggests mechanisms underlying the attenuation of romantic intensity over time, together with the substantial investment required to sustain pair bonds [15, 16]. Estimates of the duration of intense romantic attraction are typically on the order of a few years [15, 17], suggesting characteristic timescales that may also condition the long-term dynamics of relationship satisfaction.

Romantic relationships typically evolve through phases of ascent, peak experiences, and subsequent decline [18]. This study focuses on the post-peak phase, in which the observed relationship

trajectory reflects the combined effects of an intrinsic force of decline and the effort invested by partners to sustain the relationship over time. Because both are jointly expressed in the observed data, neither can be directly observed in isolation. The central objective of this study is to determine whether these latent processes can be quantitatively disentangled and characterized.

Several mathematical and computational approaches have been proposed to describe romantic relationship dynamics, ranging from dynamical systems models of couple interaction to behavioral and game-theoretic formulations that explicitly incorporate individual decision-making [7, 19–22]. While these studies have provided valuable theoretical insights, their ability to reproduce large-scale empirical patterns of marital satisfaction remains largely unexplored.

This study builds on a previously developed dynamic game framework [21, 22], in which individual behavioral responses are grounded in established principles of cognitive psychology [20]. We investigate whether the model can reproduce observed trajectories of marital satisfaction while providing a framework for inferring the underlying processes governing relationship dynamics. To address this question, we analyze a large cross-cultural dataset comprising more than 7,000 couples across 33 countries and five world regions [23]. The framework is combined with a new calibration procedure, termed *Learning by Playing*, to identify the model parameterization that best reproduces the observed trajectories of marital satisfaction.

The calibrated model accurately reproduces marital-satisfaction dynamics over a thirty-year period and provides a quantitative characterization of the latent processes underlying these trajectories. Although the empirical data indicate that successful relationships preserve more than 90% of their initial satisfaction after thirty years, the inferred dynamics imply that, in free fall under the gravity of love—that is, in the absence of maintenance effort—relationship satisfaction would fall to one-half of its initial level within about four years. Across the observed marital-satisfaction trajectories, maintenance effort is estimated to remain approximately 16% above the preferred level —the level that minimizes effort-related dissatisfaction— throughout the thirty-year period. This finding indicates that successful long-term relationships require sustained behavioral investment beyond partners' preferred level of effort.

A large body of research has emphasized the importance of maintenance processes in preserving relationship quality over time [24, 25]. This study extends that perspective by providing a quantitative characterization of the intrinsic process of relationship decline and the maintenance effort required to offset it. From this perspective, successful relationships are not naturally stable systems but dynamic systems sustained through continuous behavioral investment. More importantly, the findings provide a quantitative answer to a longstanding question in the relationship-maintenance literature: How much maintenance effort is required to sustain relationship quality over time?

The remainder of the paper presents the data, the dynamic game framework, and the Learning by Playing calibration procedure; reports the empirical results; and concludes with a discussion of the broader implications, limitations, and future directions of this research.

Materials and methods

This section presents the empirical, modeling, and computational framework underlying the study. The approach integrates three components: a large cross-cultural dataset of marital satisfaction, a dynamic game model of romantic relationships, and the calibration procedure, Learning by Playing, that links the theoretical model to the empirical data by identifying the parameterization that best reproduces the observed marital-satisfaction trajectory. The empirical data provide the observed trajectory of marital satisfaction, reflecting the combined effects of intrinsic decline and relationship-maintenance effort. The dynamic game model describes the interaction between these processes, while the calibration procedure estimates the model parameterization that best reproduces the observed trajectory. Once calibrated, the framework not only generates a synthetic approximation of the empirical trajectory but also estimates otherwise unobservable quantities, including maintenance effort, relationship values, and subjective well-being.

Data: observed trajectories of marital satisfaction

Figure 1 summarizes the relationship between marital satisfaction and relationship duration using the large cross-cultural dataset of Sorokowski et al. [23]. The smoothed trajectory, $x_{\text{data}}(t)$, represents the average evolution of marital satisfaction over a 30-year period and provides a continuous representation of the empirical data used in the calibration procedure described below.

The data were collected between 2012 and 2013 using the Kansas Marital Satisfaction Scale (KMSS) [26, 27]. The study includes 7,178 couples from 33 countries spanning five world regions, making it one of the largest cross-cultural datasets on marital satisfaction.

The KMSS comprises three questions assessing satisfaction with the marriage, the spouse, and the marital relationship, each rated on a seven-point scale ranging from 1 (very dissatisfied) to 7 (very satisfied). Following Sorokowski et al. [23], marital satisfaction (MMS) is defined as the average of the three responses and serves as the empirical proxy for relationship satisfaction in our model.

Relationship duration, measured in years, is used to reconstruct the trajectory of marital satisfaction. For each duration, we compute the average MMS and its standard error. The target trajectory $x_{\text{data}}(t)$ is then obtained by smoothing these observations using local polynomial regression (LOESS) [28], thereby capturing the dominant temporal pattern in the data.

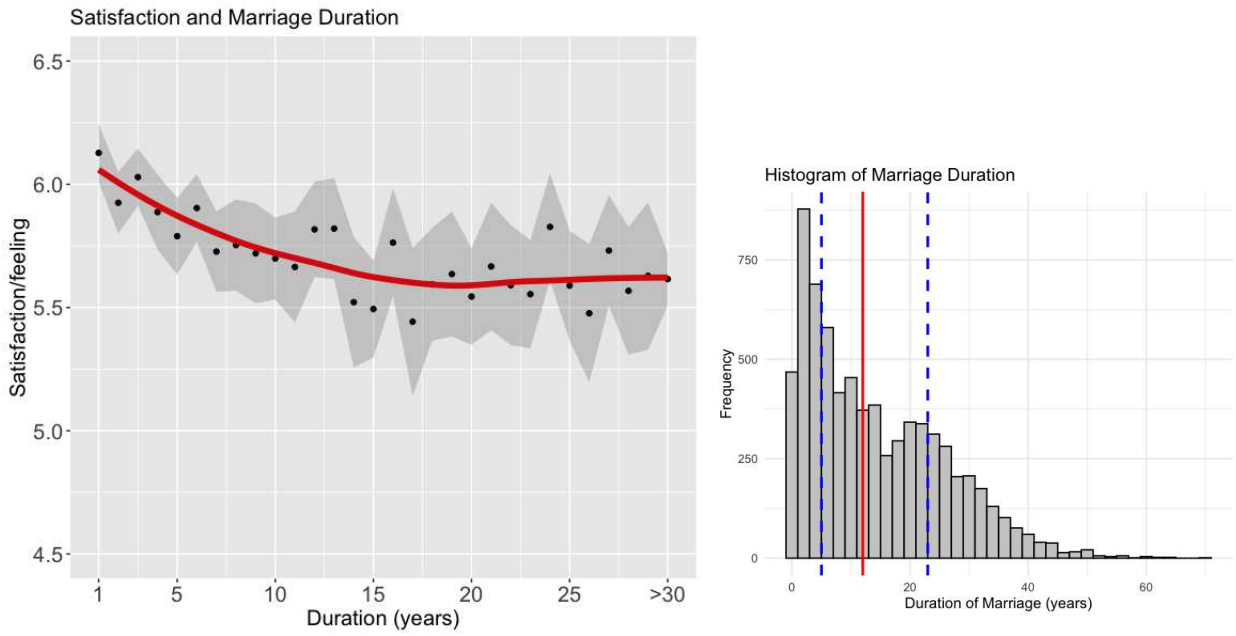


Figure 1: Marital satisfaction across the duration of marriage. The left panel shows average marital satisfaction (MMS) as a function of relationship duration in the cross-cultural dataset of Sorokowski et al. [23] ($N = 7,178$ couples from 33 countries). Points represent yearly averages, the shaded region indicates the 95% confidence interval of the population mean, and the red curve, $x_{\text{data}}(t)$, is a LOESS-smoothed estimate of the underlying trajectory. Marital satisfaction declines markedly during the early years of marriage before gradually approaching a plateau. The right panel shows the distribution of relationship duration in the sample; blue lines indicate the first and third quartiles, and the red line the median. The smoothed trajectory $x_{\text{data}}(t)$ serves as the calibration target for the computational framework developed in this study.

Several features of Figure 1 are noteworthy. Approximately 75% of the relationships in the sample have durations shorter than 25 years, resulting in wider confidence intervals at longer durations. Marital satisfaction begins at a relatively high level (6.12 on a 1–7 scale), declines substantially during the first decade of marriage, and then decreases more gradually before approaching a plateau. This pattern is consistent with previous studies reporting an early decline followed by partial stabilization of marital satisfaction over time [13, 29, 30].

The observed trajectory reflects the combined effects of relationship decline and partners' maintenance effort, neither of which can be directly observed in isolation. Accordingly, the smoothed trajectory $x_{\text{data}}(t)$ serves not only as a descriptive summary of the data but also as the target against which the proposed computational framework is calibrated to disentangle and characterize these latent processes.

A dynamic game model of romantic relationships

Relationship dynamics

We represent the state of a romantic relationship by a latent variable $x(t)$, which may be referred to as *feeling*, capturing the overall quality of the relationship at time t. Feeling evolves under the combined influence of two opposing forces: intrinsic decline and the effort invested by both partners

to sustain the relationship. Formally, the relationship state $x(t)$ obeys the stochastic differential equation

$$\mathrm{d}x(t) = [-rx(t) + a_1c_1(t) + a_2c_2(t)]\,\mathrm{d}t + \sigma(x(t))\,\mathrm{d}w, \tag{1}$$

where $c_i(t)$ denotes the effort exerted by partner i at time t, $a_i > 0$ measures the effectiveness of that effort, and $r > 0$ quantifies the tendency of relationship quality to decline in the absence of maintenance effort, that is, the strength of the gravity of love. The process $w(t)$ is a standard Wiener process [31], and the term $\sigma(x(t))\,\mathrm{d}w$ captures random shocks affecting relationship dynamics.

Setting $\sigma(\cdot) = 0$ gives the deterministic version of the model, in which Eq. (1) reduces to the ordinary differential equation

$$\dot{x}(t) = -rx(t) + a_1c_1(t) + a_2c_2(t). \tag{2}$$

Equations (1) and (2) should not be interpreted as literal descriptions of the psychological mechanisms underlying romantic relationships. Rather, they provide a parsimonious representation of the balance between deterioration and maintenance. The linear specification is analogous to relaxation models widely used in the natural and social sciences to describe systems subject to competing forces of decay and renewal. In the context of relationship dynamics, this formulation provides a mathematical representation of what Gottman and collaborators [7] termed the "second law of thermodynamics" of romantic relationships.

Costs, benefits, and behavioral preferences

At each point in time t, both partners evaluate the relationship by balancing the benefits of relationship quality against the psychological cost of maintaining it. For partner i, current well-being is represented as $U_i(x(t)) - D_i(c_i(t))$, where $U_i(x)$ represents the utility generated by relationship quality x, whereas $D_i(c_i)$ represents the psychological cost associated with exerting effort c_i.

The qualitative properties of these functions are grounded in well-established principles of cognitive psychology [20]. Relationship satisfaction increases well-being, but at a decreasing rate, reflecting the principle of diminishing marginal returns to subjective well-being. Conversely, exerting effort is beneficial up to a preferred level c_i^* but becomes increasingly costly beyond that point.

Consistent with these principles, utility functions are assumed to be increasing and concave, whereas disutility functions are convex and attain their minima at the preferred effort levels c_i^*. Following [32], we adopt the specification

$$\begin{cases} U_i = U_i(x; A_i, \alpha_i) = A_i(x+1)^{\alpha_i}, \quad \alpha_i \in (0,1),\ A_i > 0, \\ D_i = D_i(c_i; c_i^*, \beta_i) = \beta_i(c_i - c_i^*)^2 + \dfrac{(1-\beta_i)(c_i - c_i^*)^2}{1 + (c_i - c_i^*)^2}, \quad \beta_i > 0,\ c_i^* > 0. \end{cases} \tag{3}$$

which satisfies the psychological principles outlined above. Representative members of these utility and disutility families are displayed in panels (a)–(a') and (b)–(b'), respectively, of Figure 3.

The parameters $A_i, \alpha_i, c_i^*, \beta_i$ governing these families have direct behavioral interpretations, quantifying how individuals perceive relationship quality and experience the psychological costs of maintenance effort [32].

Forward-looking decision making and lifetime well-being

The final ingredient of the model concerns how effort decisions are made. A purely myopic individual would choose effort at time t by maximizing current well-being only, $U_i(x(t)) - D_i(c_i(t))$. Such behavior neglects a fundamental feature of dynamic decision problems: current decisions may influence future states and thereby affect future well-being. In romantic relationships, effort invested today influences relationship quality tomorrow, making purely myopic decisions generally suboptimal.

We therefore assume that both partners are forward-looking and view relationship maintenance as an intertemporal decision problem, choosing effort by balancing its immediate costs and benefits against its future consequences. To capture this behavior, we assume that each partner determines effort through feedback rules of the form $c_i(t) = S_i(x(t)),\ i = 1, 2$, where effort depends only on the currently perceived level of feeling. In optimal control theory, such decision rules are known as *feedback* or *closed-loop controls* [33]. Behaviorally, they capture the idea that partners continuously adapt their effort in response to the current state of the relationship.

The key question is how future consequences should be evaluated. A generic forward-looking adjustment does not necessarily lead to optimal long-run outcomes. In the present framework, the future component is determined through functions v_i, which measure the expected future well-being associated with the current state of the relationship. In the stochastic formulation, the equilibrium feedback rules are given by

$$\begin{cases} S_1^{\heartsuit}(y) = \arg\max_{c_1} \left\{ U_1(y) - D_1(c_1) + v_1'(y) \left(-ry + a_1 c_1 + a_2 S_2^{\heartsuit}(y) \right) + \frac{1}{2} v_1''(y)\, \sigma^2(y) \right\}, \\ S_2^{\heartsuit}(y) = \arg\max_{c_2} \left\{ U_2(y) - D_2(c_2) + v_2'(y) \left(-ry + a_1 S_1^{\heartsuit}(y) + a_2 c_2 \right) + \frac{1}{2} v_2''(y)\, \sigma^2(y) \right\}, \end{cases} \tag{4}$$

where the first two terms inside the brackets correspond to current well-being, whereas the remaining terms account for the expected contribution of current decisions to future well-being and for the effect of stochastic fluctuations in the relationship dynamics, respectively. More specifically, the expression inside the parentheses represents the expected change in relationship quality induced by the current effort decision, whereas $v_i'(y)$ measures the value, in terms of future well-being, of an additional unit of relationship quality at the current relationship state.

The functions v_i, $i = 1, 2$, known in optimal control theory as *value functions*, are determined by the stochastic Hamilton–Jacobi–Bellman (HJB) equations [33]

$$\begin{cases} \rho_1 v_1(y) = \max_{c_1} \left\{ U_1(y) - D_1(c_1) + v_1'(y) \left(-ry + a_1 c_1 + a_2 S_2^{\heartsuit}(y) \right) + \frac{1}{2} v_1''(y)\, \sigma^2(y) \right\}, \\ \rho_2 v_2(y) = \max_{c_2} \left\{ U_2(y) - D_2(c_2) + v_2'(y) \left(-ry + a_1 S_1^{\heartsuit}(y) + a_2 c_2 \right) + \frac{1}{2} v_2''(y)\, \sigma^2(y) \right\}, \end{cases} \tag{5}$$

where $\rho_i > 0$ denotes the intertemporal discount factor of partner i. Larger values of ρ_i place greater

emphasis on immediate well-being.

The value functions therefore provide the mechanism through which future consequences of current decisions are evaluated and incorporated into the equilibrium decision rules. More importantly, they do not merely introduce a forward-looking adjustment to current decision-making; they determine the feedback rules that maximize well-being over the lifetime of the relationship. Specifically, given an initial relationship state $x(0) = x_0$, the equilibrium effort rules maximize the total expected discounted well-being,

$$\mathbb{E}\left[\int_0^\infty e^{-\rho_i t}\left(U_i(x(t)) - D_i(c_i(t))\right)\mathrm{d}t \,\middle|\, x(0) = x_0\right]. \tag{6}$$

In the terminology of dynamic games, the pair of feedback rules $\left(S_1^\heartsuit(\cdot), S_2^\heartsuit(\cdot)\right)$ constitutes a *stationary feedback Nash equilibrium* [33]. Substituting these equilibrium rules into Eq. (1) yields, for each model parameter configuration, a stochastic prediction for the evolution of relationship quality, denoted by $\{x_s^\heartsuit(t)\}_{t\geq 0}$. Agreement between this predicted trajectory and the observed marital-satisfaction trajectory for some parameter configuration would provide indirect evidence that partners adjust their efforts in a manner broadly consistent with the forward-looking behavior implied by the HJB framework.

The deterministic framework is obtained by setting the volatility parameter σ equal to zero. The stochastic HJB equations then reduce to their deterministic counterparts, the expectation operator disappears from Eq. (6), and lifetime well-being is given by the discounted integral of instantaneous well-being along the deterministic trajectory. The resulting equilibrium trajectory $\{x_d^\heartsuit(t)\}_{t\geq 0}$ is the deterministic counterpart of the stochastic trajectory.

Calibration: inferring relationship dynamics from marital satisfaction data

The dynamic game models introduced above generate equilibrium trajectories of relationship quality and effort once the model parameters are specified. Computing these trajectories is, however, far from trivial. The stationary feedback Nash equilibrium must be obtained by numerically solving the coupled HJB equations. Throughout the analysis, we employ the RaBVIt-G algorithm developed in [21, 22], which computes approximate equilibrium solutions for both the deterministic and stochastic versions of the model.

The calibration problem considered here is the inverse counterpart of this computation. Rather than predicting relationship dynamics from a known parameter configuration, we seek the parameter values that best reproduce the observed marital-satisfaction trajectory described in Figure 1. In other words, the relationship dynamics are inferred from the observed data rather than predicted from known parameters. The unknown parameter vector

$$P = (r, a_1, a_2, \sigma, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2)$$

contains both relationship-dynamics parameters (r, a_1, a_2, σ) and behavioral parameters $\boldsymbol{\theta}_i =$

$(A_i, \alpha_i, \beta_i, c_i^*)$. The first group (r, a_1, a_2, σ) governs the relationship dynamics (see Eq. (1)), whereas the second group, $\boldsymbol{\theta}_i = (A_i, \alpha_i, \beta_i, c_i^*)$, contains the behavioral parameters of partner i (see Eq. (3)).

The deterministic and stochastic specifications are calibrated separately by identifying parameter vectors, $\hat{P}_d$ and $\hat{P}_s$, that minimize the discrepancy between the corresponding equilibrium trajectories computed by RaBVIt-G, $x^{\heartsuit}_{P_d}(t)$ and $x^{\heartsuit}_{P_s}(t)$, and the empirical trajectory $x_{\text{data}}(t)$. Formally,

$$\hat{P}_d = \arg\min_{P_d} \sum_{k=1}^{M} \left| x^{\heartsuit}_{P_d}(t_k) - x_{\text{data}}(t_k) \right|,$$

and

$$\hat{P}_s = \arg\min_{P_s} \mathbb{E}\left[\sum_{k=1}^{M} \left| x^{\heartsuit}_{P_s}(t_k) - x_{\text{data}}(t_k) \right| \right],$$

where $t_1, \ldots, t_M$ denote the observation times of the empirical trajectory. The empirical trajectory consists of $M = 30$ annual observations.

Calibration therefore links the theoretical models to the empirical data by identifying the parameter configurations for which the models best reproduce the observed relationship trajectory. The calibrated models can then be used to infer latent relationship mechanisms that are not directly observable from the data alone.

The calibration procedure is implemented using a tailored algorithm termed *Learning by Playing* (LBP). The algorithm alternates between a *Playing* phase, in which RaBVIt-G computes the equilibrium trajectories implied by the current parameter estimates, and a *Learning* phase, in which the parameter vectors are updated to reduce the discrepancy between the simulated and empirical trajectories. These two phases are repeated until convergence. The overall procedure is summarized in Figure 2, while algorithmic details and pseudocode are provided in Supplementary Note A. The resulting estimates $\hat{P}_d$ and $\hat{P}_s$ define the calibrated deterministic and stochastic model trajectories used throughout the remainder of the paper.

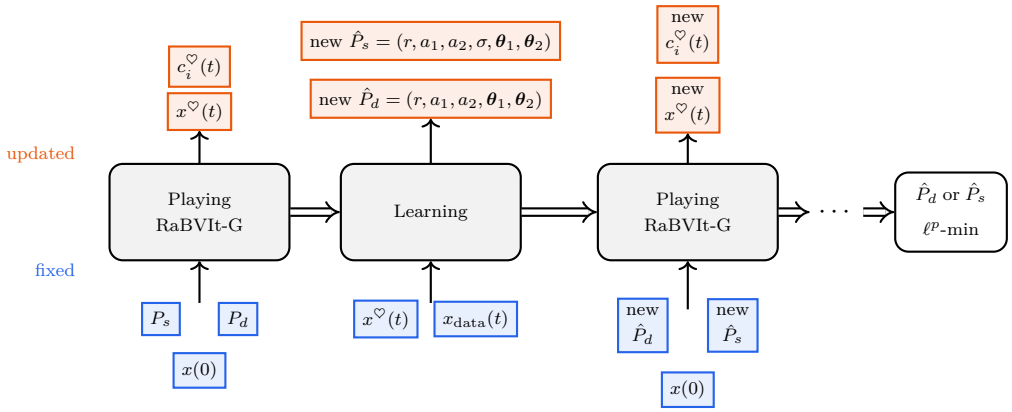


Figure 2: Learning by Playing (LBP) calibration scheme. The algorithm LBP alternates between *Playing*, where RaBVIt-G generates equilibrium relationship trajectories under the current parameter configuration, and *Learning*, where the parameters are updated to reduce the discrepancy between the model prediction and the observed marital satisfaction trajectory.

Results and discussion

The primary purpose of calibration is not simply to reproduce the observed trajectory of marital satisfaction, but to identify the latent mechanisms that best explain it. We first show that the calibrated deterministic and stochastic trajectories closely match the empirical data. This agreement then provides the basis for inferring quantities that are not directly observable, beginning with the intrinsic tendency of relationship satisfaction to decline over time—the gravity of love.

Quantifying the gravity of love

We begin with the benchmark case of *homogamous* couples, in which both partners are assumed to share identical preferences, effort costs, emotional efficiency, and intertemporal discounting. Accordingly, we impose $U_1 = U_2$, $D_1 = D_2$, $a_1 = a_2$, $\rho_1 = \rho_2$. Homogamy is a well-established concept in relationship science [34], referring to the tendency of individuals to form relationships with similar partners. The benchmark specification considered here adopts a particularly strong form of homogamy, providing a natural reference point against which more heterogeneous relationship structures are subsequently compared.

Calibration is performed using the Learning by Playing procedure described in the previous section. Throughout, the discount factors are fixed at $\rho_1 = \rho_2 = 0.1$, representing a moderate degree of impatience. Additional computational details are provided in Supplementary Note B.

Table 1 reports the parameter estimates obtained for both the deterministic and stochastic versions of the model. Figure 3 illustrates the corresponding calibrated behavioral structure and the resulting fit to the empirical data.

Table 1: Parameter estimates obtained through the *Learning by Playing* calibration procedure. The table reports the parameter configurations that best reproduce the observed trajectory of marital satisfaction for the deterministic and stochastic versions of the RaBVIt-G model. The estimated parameter vectors are $P_d = (r, a_1, a_2, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2)$ and $P_s = (r, a_1, a_2, \sigma, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2)$, where (r, a_1, a_2, σ) are the relationship-dynamics parameters and $\boldsymbol{\theta}_i = (A_i, \alpha_i, \beta_i, c_i^*)$ denotes the behavioral parameter vector of partner i. In particular, r measures the rate of intrinsic relationship decline, a_i quantifies the effectiveness of relationship-maintaining effort, σ captures emotional uncertainty, and $\boldsymbol{\theta}_i$ characterizes the evaluation of relational rewards and effort costs. Under the homogamy assumption, the symmetry constraints $a_1 = a_2$ and $\boldsymbol{\theta}_1 = \boldsymbol{\theta}_2$ are imposed throughout. The last two columns report the correlation between the empirical marital satisfaction trajectory and the corresponding model prediction, together with the associated calibration error, ℓ_1.

Model	r	a_1=a_2	c_1^*=c_2^*	σ	A_1=A_2	α_1=α_2	β_1=β_2	Corr.	ℓ_1
Deterministic	0.1862	1.0784	0.4170	0	1.10	0.8430	1.45	0.82	2.10
Stochastic	0.1857	1.0454	0.4257	0.0654	1.20	0.8385	1.40	0.82	2.09

Panels (c) and (c') compare the observed trajectory of marital satisfaction with the calibrated feeling trajectory generated by the model. Agreement is remarkably close throughout the entire thirty-year period. Both the deterministic and stochastic specifications successfully reproduce the characteristic empirical pattern: a relatively rapid decline during the early years of marriage followed by gradual stabilization. Consistent with this visual evidence, Table 1 reports correlations above 0.8

and nearly identical ℓ_1 errors for both formulations, indicating a close correspondence between the observed and model-generated trajectories.

These results show that the dynamic game framework captures the principal empirical regularity documented in the marital-satisfaction literature. More importantly, the successful reproduction of the observed trajectory indicates that the observed dynamics are consistent with the forward-looking behavioral adaptation implied by the HJB framework. Rather than reflecting passive emotional decay alone, the calibrated equilibrium suggests that partners continuously adjust their maintenance efforts in response to changes in relationship quality, producing the characteristic long-term trajectory observed in the data.

Among the estimated parameters, the decay coefficient r is of particular interest because it quantifies the gravity of love. Across deterministic and stochastic specifications, the estimated values are nearly identical, with $r \approx 0.186$. Additional robustness analyses reported in Supplementary Note B show that virtually identical estimates of the decay parameter are obtained under alternative utility and disutility specifications. Despite substantial differences in functional form and parameterization, the estimated values remain within a narrow range, indicating that the inferred magnitude of r is robust to the specific preference specification adopted.

The estimated value r admits a simple behavioral interpretation. Since the free-fall dynamics satisfy $\dot{x} = -rx$, the corresponding half-life is $t_{1/2} = \ln(2)/r$. Thus, in the absence of maintenance effort —that is, in free fall under the gravity of love— relationship satisfaction would fall to one-half of its initial level within approximately four years. This timescale is remarkably close to independent estimates of the duration of limerence or intense romantic attraction, which typically attenuate within approximately 18 months to three years [15, 17]. Although relationship satisfaction and limerence are distinct constructs, the similarity of these independently derived timescales suggests that both phenomena may be governed by a common temporal scale underlying the progressive weakening of romantic attachment.

The remaining calibrated parameters characterize how partners perceive relationship benefits and maintenance costs. Panels (a)–(b') of Figure 3 display the corresponding utility and disutility functions. Although their qualitative shapes are determined by the psychological principles embedded in the model, calibration identifies the parameter values that best reproduce the observed marital-satisfaction trajectory. The close agreement between the deterministic and stochastic estimates indicates that these inferred behavioral characteristics are robust to random emotional fluctuations. In the adopted specification, α_i governs the sensitivity of subjective well-being to relationship quality, whereas β_i determines how rapidly maintenance effort becomes psychologically costly [32]. Consequently, calibration provides quantitative estimates of both the dynamic parameters governing relationship evolution and the psychological characteristics underlying relationship maintenance.

The counteracting force: the cost of sustaining successful relationships

The previous section quantified the intrinsic force driving the decline of relationship satisfaction. We now turn to the complementary question: how much effort is required to counteract this force

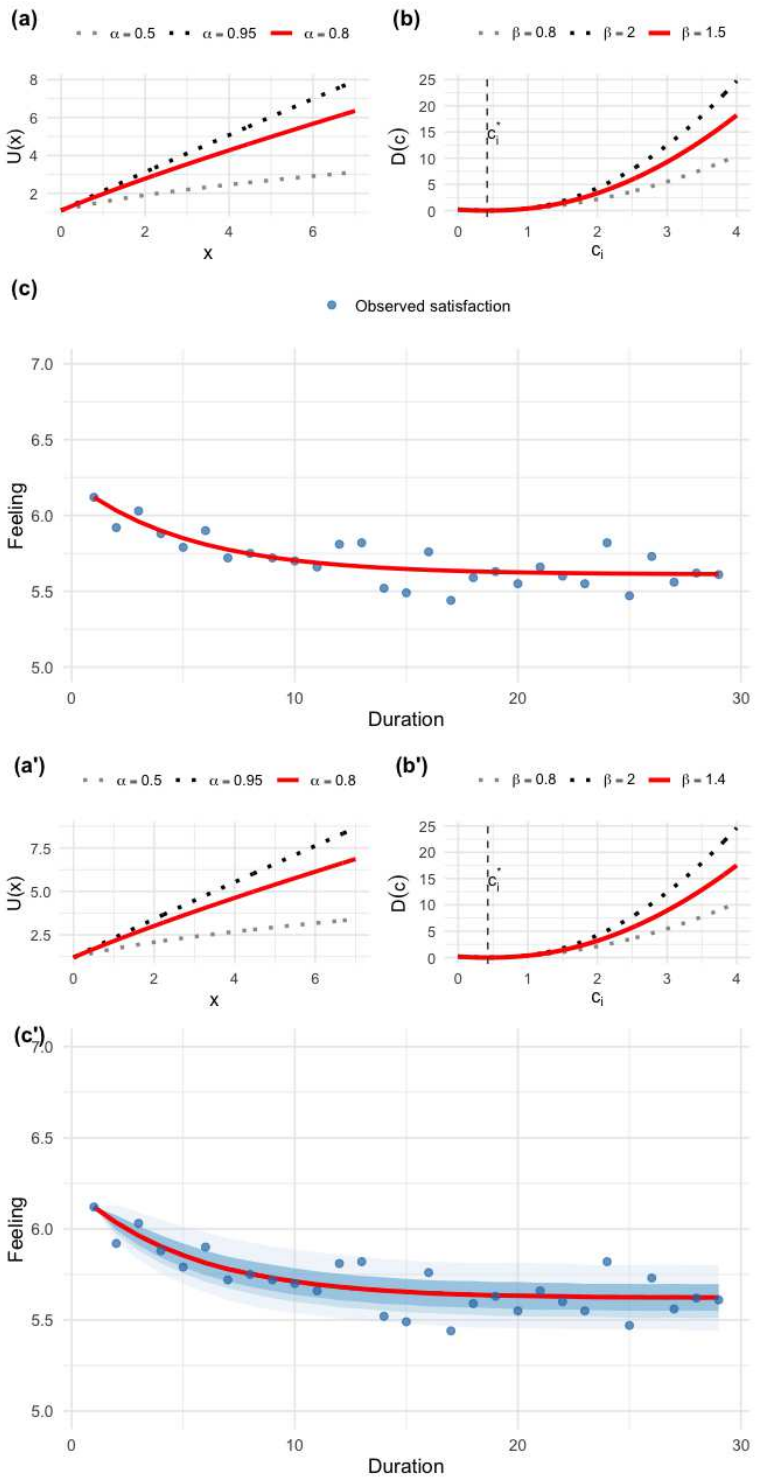


Figure 3: Replicating marital satisfaction trajectories and inferring latent behavioral mechanisms. Panels (c) and (c') compare the observed marital-satisfaction trajectory with the fitted *feeling* trajectory generated by the calibrated deterministic and stochastic models, respectively. Both formulations reproduce the empirical pattern of an initial decline followed by gradual stabilization. The calibration procedure simultaneously identifies behavioral mechanisms underlying relationship dynamics from the data. Panels (a) and (a') show the calibrated utility functions, while panels (b) and (b') display the corresponding disutility-of-effort functions for the deterministic and stochastic formulations. Together, these functions characterize the preferences and effort costs that are most consistent with the observed marital-satisfaction data.

and sustain a successful long-term relationship?

Figure 4 reports the equilibrium maintenance effort implied by the calibrated model. Effort is expressed relative to the preferred effort level c_i^*, namely, $\frac{S_i^{\heartsuit}(x)}{c_i^*}$, where c_i^* denotes the effort level that minimizes effort-related dissatisfaction. Values greater than one therefore indicate that relationship maintenance requires effort beyond the individual's preferred effort level.

Several observations emerge. First, the estimated effort remains systematically above the preferred level throughout the entire duration of the relationship. This finding implies that successful long-term relationships are not naturally self-sustaining systems. Rather, maintaining high levels of relationship satisfaction requires a persistent compensatory effort that continuously counteracts the intrinsic tendency of relationship satisfaction to decline.

Second, the required effort increases over time. Although relationship satisfaction declines only moderately over the thirty-year period considered here, the calibrated model suggests that preserving this relatively stable trajectory requires progressively greater maintenance effort.

Third, the stochastic formulation consistently predicts higher effort levels than the deterministic model. When emotional fluctuations and uncertainty are incorporated into the dynamics, maintaining a given level of relationship satisfaction requires additional compensatory effort. This finding suggests that emotional uncertainty itself imposes an additional burden on relationship maintenance, requiring greater compensatory effort to sustain the same level of relationship quality.

The estimated effort gap is also noteworthy. In both formulations, equilibrium effort exceeds the preferred effort level by approximately 16% throughout most of the relationship. Previous theoretical and computational analyses predicted that successful long-term relationships require effort persistently above the preferred level, implying an *effort gap* between the effort individuals would ideally like to exert and the effort needed to sustain relationship quality over time [21, 22]. The present study provides empirical support for this prediction by showing that the effort gap emerges naturally when the model is calibrated to large-scale observations of marital satisfaction. More importantly, the calibration provides the first quantitative estimate of its magnitude. In the present dataset, the inferred effort gap is approximately 16%, indicating that equilibrium maintenance effort exceeds the preferred effort level by about one-sixth.

Taken together, these findings place the balance between decline and compensatory effort into quantitative perspective. While the calibrated dynamics imply a characteristic half-life of relationship satisfaction of approximately four years in the absence of compensatory effort, the empirical data indicate that successful marriages preserve more than 90% of their initial satisfaction after thirty years. The calibrated model therefore suggests that this remarkable long-term stability can be explained only by the sustained maintenance effort exerted by both partners over time.

The calibrated model also reveals two distinct notions of relationship-maintenance cost. The first is an objective (or dynamical) cost, measured by the amount of effort required to sustain relationship satisfaction over time. The second is a subjective cost, captured by the disutility functions $D_i(\cdot)$, which quantify how burdensome that effort is perceived to be by the individuals involved. The model therefore distinguishes between the behavioral demands imposed by relationship maintenance

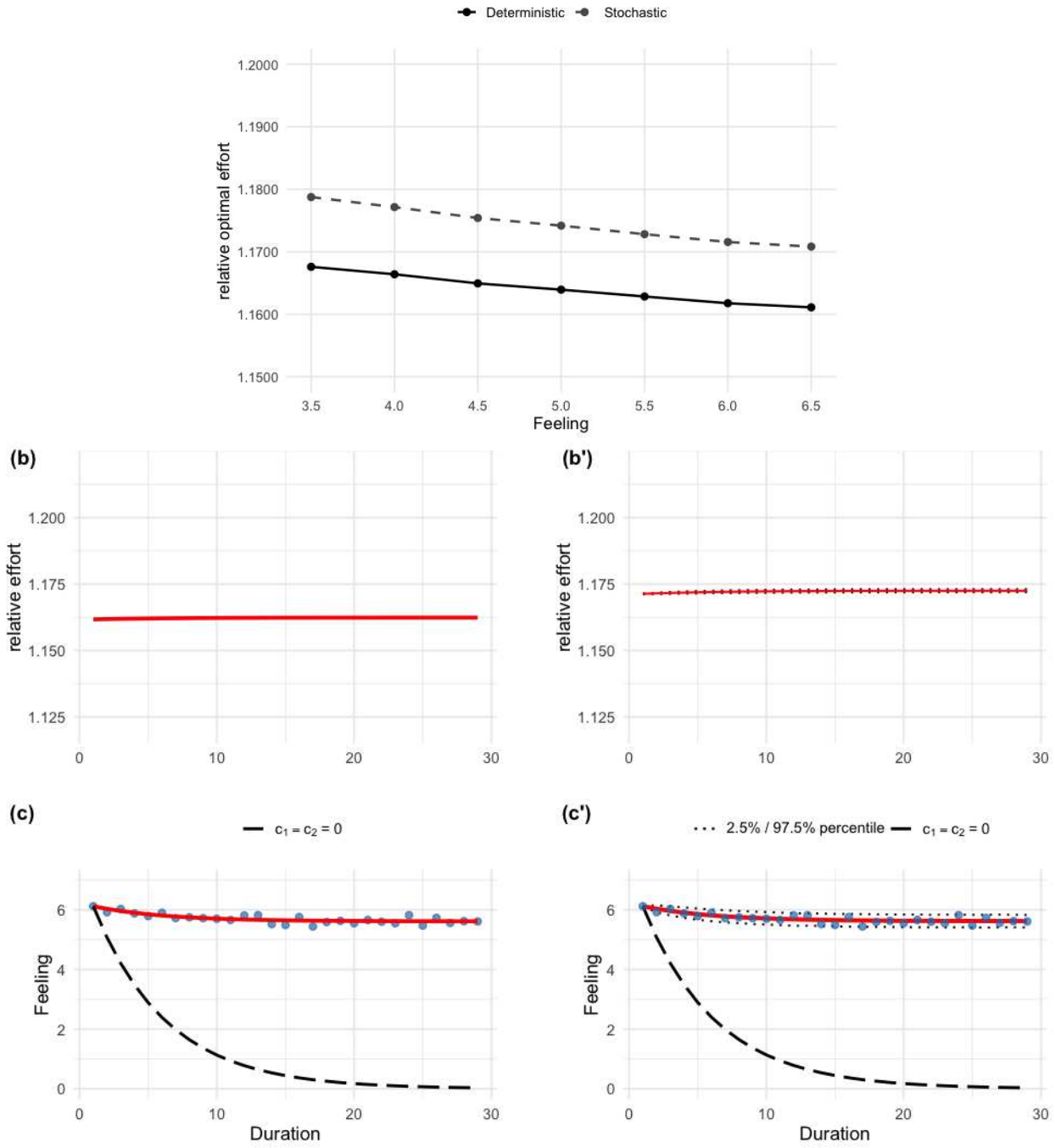


Figure 4: The counteracting force: subjective and objective dimensions of relationship-maintenance effort. Panel (a) reports the normalized equilibrium effort ($S_i^{\heartsuit}(x)/c_i^*$) for the deterministic and stochastic formulations. Values greater than one indicate effort beyond the individual's preferred level, capturing the *subjective* dimension of maintenance effort. Panels (b) and (b') report the corresponding equilibrium effort trajectories over time. Across both formulations, effort remains above the preferred level and gradually increases, quantifying the *objective* dimension of maintenance effort. In the stochastic specification, fan charts represent simulated quantiles of the equilibrium effort distribution. Panels (c) and (c') compare the calibrated marital-satisfaction trajectory with the counterfactual trajectory obtained when no maintenance effort is exerted ($c_1 = c_2 = 0$). The latter corresponds to the action of the *gravity of love* in isolation. The contrast between the two curves illustrates the extent to which sustained maintenance effort is required to preserve relationship satisfaction over time.

and the psychological burden associated with meeting those demands. This distinction provides a possible explanation for why couples facing similar relational challenges may differ substantially in their experienced well-being and long-term relationship resilience.

Relational asymmetry: multiple pathways to successful relationships

The analyses presented thus far have focused on the benchmark homogamous couple. However, the close agreement between the calibrated model and the observed trajectory does not imply that successful long-term relationships are characterized by a unique behavioral configuration. An important question is therefore whether the same empirical trajectory can arise from alternative forms of relational heterogeneity.

To address this issue, we perform a series of counterfactual calibration exercises that relax the benchmark homogamy assumption. These experiments serve two complementary purposes. First, they allow us to investigate whether alternative behavioral structures are observationally consistent with the data. Second, they reveal which aspects of the underlying behavioral structure are identifiable from a single observed trajectory and which remain observationally equivalent.

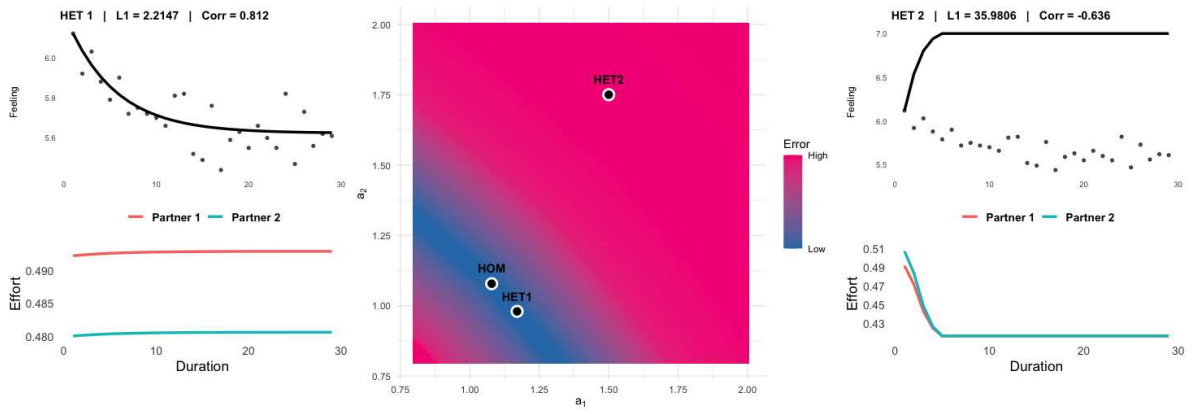


Figure 5: Multiple pathways to successful relationships in the (a_1, a_2) parameter space. The contour plot reports the ℓ_1 prediction error obtained after recalibrating the model for alternative combinations of the efficiency parameters (a_1, a_2), with blue and red regions indicating low and high prediction errors, respectively. The point HOM corresponds to the homogamous benchmark introduced earlier. A narrow low-error locus emerges around parameter combinations satisfying approximately $a_1 + a_2 \approx 2.2$, indicating that the data identify an aggregate emotional-efficiency requirement for successful long-term relationships. Although the individual efficiency contributions of the two partners cannot be uniquely identified, the aggregate efficiency with which effort is converted into relationship quality is tightly constrained by the observed marital-satisfaction trajectory. The locus therefore reveals a continuum of heterogamous parameterizations that remain observationally consistent with the data while sharing the same aggregate emotional efficiency. Points HET1 and HET2 illustrate, respectively, a heterogamous parameterization consistent with the data and a configuration that produces a substantially poorer fit. The accompanying panel reports the corresponding equilibrium effort levels, expressed relative to the preferred effort level c_i^*. The quantities L1 and Error both denote the ℓ_1 prediction error defined in the calibration section.

Remarkably, Figure 5 reveals that the observed marital-satisfaction trajectory can be reproduced not only by the homogamous benchmark but also by a continuum of heterogamous configurations

satisfying approximately $a_1 + a_2 \approx 2.2$. This finding shows that the empirical trajectory identifies an aggregate behavioral constraint rather than a unique decomposition of behavioral traits across partners.

Under the homogamous benchmark ($a_1 = a_2 \approx 1.1$), both partners contribute equally to transforming maintenance effort into improvements in relationship quality. More generally, however, the empirical data do not require equal individual efficiencies. If one partner is less effective at converting maintenance effort into relationship quality, the other can compensate by being proportionally more effective, provided that their combined emotional efficiency remains close to the value identified by the calibration.

The notion of heterogamy considered here is intentionally narrow. To isolate the effects of behavioral asymmetry, all parameters are fixed at their benchmark values except the emotional-efficiency coefficients a_1 and a_2, which measure the effectiveness with which each partner's effort contributes to relationship quality. The objective is to determine whether asymmetric behavioral structures can reproduce the observed marital-satisfaction trajectory as accurately as the benchmark homogamous specification.

The existence of multiple successful calibrations should therefore not be interpreted as a structural indeterminacy of the model. Rather, it reflects the limited information contained in a single observable. While the calibration uniquely identifies the aggregate efficiency with which the couple transforms maintenance effort into relationship quality, it does not uniquely determine how that efficiency is distributed between partners. In other words, the data identify a collective behavioral property of the relationship as a whole, while allowing multiple allocations of that property across individuals.

At the same time, Figure 5 makes clear that not all behavioral configurations are observationally equivalent. Prediction errors increase sharply outside the narrow admissible region $a_1 + a_2 \approx 2.2$, indicating that only specific combinations of behavioral parameters are consistent with the data. Although the calibration does not uniquely determine the individual emotional efficiencies of the two partners, it places strong constraints on their admissible combinations. This finding reinforces the conclusions of the previous sections by showing that the close agreement between model and data is not a generic consequence of the model structure, but rather the result of satisfying well-defined behavioral constraints identified by the calibration.

Conclusions

Understanding the mechanisms that sustain successful long-term romantic relationships remains a central challenge in the social sciences. In this work, we combined a dynamic game model of romantic relationships with empirical calibration to reproduce large-scale empirical patterns of marital satisfaction. This approach allows observed relationship trajectories to be translated into quantitative estimates of otherwise unobservable psychological mechanisms and behavioral characteristics.

Three main findings emerge from the analysis. First, the calibrated model reproduces large-scale empirical trajectories of marital satisfaction and yields a robust estimate of the gravity of love—the intrinsic tendency of relationship satisfaction to decline in the absence of compensatory effort. Across alternative specifications, the inferred dynamics imply a characteristic half-life of relationship satisfaction of approximately four years. This estimate is broadly consistent with independent evidence concerning the duration of limerence and intense romantic attraction, suggesting that the calibrated dynamics may capture a fundamental temporal scale underlying the attenuation of romantic attachment.

Second, the analysis provides the first quantitative estimate of the maintenance effort required to sustain relationship satisfaction in the face of decline. While successful marriages preserve more than 90% of their initial satisfaction after thirty years, the calibrated dynamics indicate that this stability requires a persistent maintenance effort approximately 16% above the preferred effort level. In this sense, the results place a central idea in relationship science into quantitative perspective: long-term relationship stability is not a passive state, but the outcome of sustained maintenance effort over time [7, 8]. More generally, these findings are consistent with influential theoretical perspectives emphasizing the role of adaptive processes in sustaining long-term relationship quality [35].

Third, although the observed marital-satisfaction trajectory constitutes the only target of the estimation procedure, the calibrated model recovers otherwise unobservable behavioral properties of successful long-term relationships. In particular, the heterogamy analysis identifies aggregate behavioral characteristics of the couple as a whole. Together, these findings extend previous theoretical analyses [21, 22] by showing that the proposed behavioral mechanisms are not only internally consistent but also quantitatively consistent with large-scale empirical observations.

The findings reported here should be interpreted in light of the assumptions underlying the model. The analysis relies on cross-sectional rather than longitudinal observations of marital satisfaction. Although longitudinal panel data would permit a more direct assessment of individual relationship dynamics, the robustness of the calibrated estimates across alternative model specifications suggests that the inferred gravity of love reflects a stable population-level process. If so, it should be recoverable from both longitudinal and appropriately aggregated cross-sectional observations. More generally, the available data impose natural limits on the degree of behavioral inference. Despite relying on a single population-level trajectory of marital satisfaction, the calibration recovers several latent behavioral characteristics of successful long-term relationships. However, because this trajectory reflects the couple's aggregate behavior, it cannot uniquely determine the behavioral characteristics of each partner. Richer sources of information—including partner-level, behavioral, demographic, or longitudinal data—could extend the present framework to identify individual-level behavioral heterogeneity.

Several directions for future research naturally follow from the present results. One extension would be to move beyond relationship satisfaction and investigate crises, separation processes, and marital dissolution within the same dynamic framework. At the methodological level, richer longitudinal and partner-level data would permit a more direct assessment of the proposed dynamics

while helping to identify the behavioral heterogeneity underlying successful long-term relationships. Incorporating additional sources of heterogeneity also represents an important avenue for future research, as behavioral differences across multiple parameters may substantially alter equilibrium relationship dynamics.

Another direction concerns the measurement of relationship-maintenance effort. The present analysis suggests the existence of a persistent gap between the effort individuals would ideally prefer to exert and the effort required to sustain relationship quality over time. Future work could combine the present calibration framework with measurements of the utility and disutility associated with relationship maintenance, allowing these latent behavioral quantities to be estimated more directly across relatively homogeneous populations. Such analyses could reveal how the behavioral demands of successful long-term relationships vary across cultural, socioeconomic, demographic, and psychological contexts, thereby providing a stronger empirical foundation for relationship science and future interventions.

Overall, the present study demonstrates the value of combining behavioral theory, mathematical modeling, and empirical calibration to investigate mechanisms underlying successful long-term romantic relationships. More broadly, it illustrates how formal behavioral models, when calibrated against large-scale empirical data, can transform observable trajectories into estimates of otherwise unobservable quantities.

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Data availability

The empirical data analyzed in this study were originally published by Sorokowski et al. [23]. Information on access to the original dataset is provided in that publication.

Code availability

The code used to implement the computational framework and calibration procedure will be deposited in a public repository before publication.

Competing interests

The authors declare no competing interests.

Author contributions

J.M.R. Conceptualization; Formal analysis; Funding acquisition; Investigation; Methodology; Project administration; Supervision; Validation; Writing–original draft; Writing–review & editing. **J.H.C.** Conceptualization; Formal analysis; Funding acquisition; Investigation; Methodology; Software; Data curation; Validation; Visualization; Writing–original draft.

References

[1] W. R. Jankowiak and E. F. Fisher. A cross-cultural perspective on romantic love. *Ethnology*, 31(2):149–155, 1992.

[2] S. Coontz. *Marriage, A History*. Viking, New York, 2005.

[3] J. Weimann, A. Knabe, and R. Schöb. *Measuring Happiness: The Economics of Well-Being*. The MIT Press, Cambridge, Massachusetts, 2015.

[4] D. Carr. The linked lives principle in life course studies: classic approaches and contemporary advances. In *Social Networks and the Life Course: Integrating the Development of Human Lives and Social Relational Networks*, pages 41–63. Springer, 2018.

[5] S. Pelzman. The socio political demography of happiness. *New Working Paper Series*, 331, 2023.

[6] C. M. Proulx, H. M. Helms, and C. Buehler. Marital quality and personal well-being: a meta-analysis. *J. Marriage Fam.*, 69:576–593, 2007.

[7] J. M. Gottman, J. D. Murray, C. C. Swanson, R. Tyson, and K. R. Swanson. *The Mathematics of Marriage: Dynamic Nonlinear Models*. MIT Press, Cambridge, MA, 2005.

[8] E. J. Finkel. *The All-or-Nothing Marriage: How the Best Marriages Work*. Penguin Random House, New York, 2017.

[9] E. J. Finkel. Romantic relationships. In *The Handbook of Social Psychology*. Situational Press, 6 edition, 2025.

[10] C. M. Proulx, A. E. Ermer, and J. B. Kanter. Group-based trajectory modeling of marital quality: a critical review. *J. Fam. Theory Rev.*, 9:307–327, 2017.

[11] R. Schoen and V. Canudas-Romo. Timing effects on divorce: 20th century experience in the United States. *J. Marriage Fam.*, 68(3):749–758, 2006.

[12] F. Cushman. Computational social psychology. *Annu. Rev. Psychol.*, 75:625–652, 2024.

[13] J. L. Bühler, S. Krauss, and U. Orth. Development of relationship satisfaction across the life span: a systematic review and meta-analysis. *Psychol. Bull.*, 147(10):1012–1053, 2021.

[14] A. T. Beck. *Love Is Never Enough: How Couples Can Overcome Misunderstandings, Resolve Conflicts, and Solve Relationship Problems Through Cognitive Therapy*. Harper & Row, New York, 1988.

[15] H. Fisher. *Why We Love: The Nature and Chemistry of Romantic Love*. Henry Holt, 2004.

[16] R. I. M. Dunbar. *The Science of Love and Betrayal*. Faber and Faber, London, 2012.

[17] D. Tennov. *Love and Limerence: The Experience of Being in Love*. Stein and Day, 1979.

[18] P. W. Eastwick, E. J. Finkel, and J. A. Simpson. Relationship trajectories: a meta-theoretical framework and theoretical applications. *Psychol. Inq.*, 30(1):1–28, 2019.

[19] S. Rinaldi, F. Della Rossa, F. Dercole, A. Gragnani, and P. Landi. *Modeling Love Dynamics*, volume 89 of *World Scientific Series on Nonlinear Science, Series A*. World Scientific Publishing, Singapore, 2016.

[20] J.-M. Rey. A mathematical model of sentimental dynamics accounting for marital dissolution. *PLoS ONE*, 5(3):e9881, 2010.

[21] J. Herrera de la Cruz and J.-M. Rey. Controlling forever love. *PLoS ONE*, 16(12):e0260529, 2021.

[22] J. Herrera de la Cruz and J.-M. Rey. A computational stochastic dynamic model to assess the rise of breakup in a romantic relationship. *Math. Methods Appl. Sci.*, 48(7):7727–7744, 2025.

[23] P. Sorokowski et al. Marital satisfaction, sex, age, marriage duration, religion, number of children, economic status, education, and collectivistic values: data from 33 countries. *Front. Psychol.*, 8:1199, 2017.

[24] D. J. Canary and L. Stafford. Relational maintenance strategies and equity in marriage. *Commun. Monogr.*, 59(3):243–267, 1992.

[25] L. Stafford. Measuring relationship maintenance behaviors: critique and development of the revised relationship maintenance behavior measure. *J. Soc. Pers. Relat.*, 28(2):278–303, 2011.

[26] C. W. Nichols, W. R. Schumm, K. L. Schectman, and C. C. Grigsby. Characteristics of responses to the Kansas Marital Satisfaction Scale by a sample of 84 married mothers. *Psychol. Rep.*, 53(2):567–572, 1983.

[27] W. R. Schumm et al. Kansas Marital Satisfaction Scale (KMS). *J. Fam. Ther.*, 29:307–324, 2000.

[28] J. Fan and I. Gijbels. *Local Polynomial Modelling and Its Applications*. Chapman & Hall, London, 1996.

[29] J. Van Laningham, D. R. Johnson, and P. Amato. Marital happiness, marital duration, and the U-shaped curve: evidence from a five-wave panel study. *Soc. Forces*, 79(4):1313–1341, 2001.

[30] P. R. Amato and S. L. James. Changes in spousal relationships over the marital life course. In *Social Networks and the Life Course: Integrating the Development of Human Lives and Social Relational Networks*, pages 139–158. Springer, 2018.

[31] H. Kushner and P. G. Dupuis. *Numerical Methods for Stochastic Control Problems in Continuous Time*, volume 24. Springer Science & Business Media, New York, 2013.

[32] J. Herrera de la Cruz and J.-M. Rey. Emotional trade-offs in successful romantic relationships: a differential game approach. *Mathematics*, 13(23):3745, 2025.

[33] E. J. Dockner, S. Jorgensen, N. Van Long, and G. Sorger. *Differential Games in Economics and Management Science*. Cambridge University Press, 2000.

[34] M. K. Whyte. *Dating, Mating, and Marriage*. Routledge, New York, 2018.

[35] B. R. Karney and T. N. Bradbury. The longitudinal course of marital quality and stability: a review of theory, method, and research. *Psychol. Bull.*, 118(1):3–34, 1995.

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