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Biexciton Binding and Radiative Lifetimes in Strongly Oblate Semi-Ellipsoidal Quantum Dots

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Abstract

We investigate the ground exciton and biexciton states in a hard-wall, strongly oblate semi-ellipsoidal GaAs quantum dot. Their energies are evaluated variationally using correlated trial states constructed from analytical single-particle wave functions, together with a ground-orbital-matched importance transformation and adaptive quasi-Monte Carlo integration. A noise-aware coordinate pattern search with two-budget verification is used to optimize the variational parameters along intersecting axial and lateral geometry scans. Because the effective-mass hard-wall model is intended to isolate geometrical effects, the principal results are qualitative trends rather than absolute predictions for a particular fabricated dot. The axial dimension controls the absolute confinement energy much more strongly than the lateral dimension, whereas enlargement along either direction weakens the biexciton binding. The central binding estimate remains positive throughout the sampled range but is not resolved from zero at the widest lateral geometry. Increasing either semiaxis enhances the exciton coincidence overlap and oscillator strength and therefore shortens the calculated radiative lifetimes. The lateral dependence remains pronounced where the axial dependence approaches saturation. These results distinguish the dominant axial control of the confinement-energy scale from the important lateral control of weak biexciton binding and radiative strength.

Keywords: semi-ellipsoidal quantum dot; exciton; biexciton; binding energy; variational method; quasi-Monte Carlo integration; radiative lifetime

1 Introduction

Semiconductor quantum dots combine discrete, atom-like carrier spectra with optical and electronic properties that can be tailored through composition, size, and shape. This tunability supports applications in displays and lighting, lasers, sensing, communications, energy conversion, bioimaging, and quantum information [1]. In quantum-photonics applications, individual dots can act as sources of single photons and photon pairs [2]. The radiative biexciton–exciton cascade produces time-correlated photon pairs [3, 4] and can generate polarization-entangled photons [5]. The biexciton binding energy sets the spectral separation of the two cascade transitions, while the exciton and biexciton radiative lifetimes set their emission time scales and influence the attainable photon-generation rate.

The energy of a confined biexciton is not fixed by a single Coulomb scale. Electron–hole attraction competes with electron–electron and hole–hole repulsion, while the carrier masses and confinement geometry determine how strongly the four particles can rearrange. Consequently, the sign and magnitude of the binding energy can vary strongly with the material and dot structure [6–8]. This sensitivity makes the biexciton a useful probe of few-particle correlations, but it also makes its numerical evaluation demanding: the binding energy is a small difference between two much larger correlated energies.

Strongly oblate dots are especially suitable for separating the roles of axial and lateral confinement. Their two semiaxes provide independent geometrical controls, while the large separation between axial and in-plane energy scales permits an adiabatic treatment of the single-particle states. Biexciton structure and optical response have consequently been examined in strongly oblate [9], prolate [10], and III-nitride [11] quantum dots, as well as under structured Bessel and needle beams [12, 13]. Ellipsoidal confinement has also been explored through magnetobiexcitons [14], nonlinear biexciton optics [15], relativistic fine-structure corrections [16], narrow-band quantum layers [17], interband absorption in prolate and oblate dots [18, 19], and donor states [20]. Against this broad ellipsoidal literature, semi-ellipsoidal calculations have addressed electronic and excitonic states, binding and recombination energies [21], single-particle states and interband absorption [22], and, more recently, perturbative exciton binding, absorption, and photoluminescence in a hard-wall lens-shaped model [23]. They have not treated the correlated biexciton together with its radiative lifetime.

The purpose of this work is to determine how the lateral semiaxis a and the axial semiaxis c control the correlated exciton and biexciton energies, biexciton binding, exciton coincidence overlap, and radiative lifetimes in a strongly oblate semi-ellipsoidal GaAs quantum dot. We restrict the calculation to the single-particle ground configuration and evaluate correlated variational states using a state-matched importance transformation, adaptive quasi-Monte Carlo integration, and a two-phase variational search. Two intersecting geometry scans vary one semiaxis at a time to distinguish axial and lateral trends. The idealized hard-wall effective-mass model is intended to isolate those geometrical effects rather than reproduce a particular fabricated dot. Accordingly, the principal results are the qualitative dependences on dot geometry; the absolute numerical values should be interpreted within the assumptions of the model. Particular attention is paid to whether the small biexciton binding remains resolved at the available numerical precision.

2 Model and theoretical framework

The interacting exciton and biexciton ground-state energies are obtained with the variational method: trial correlation factors are optimized by minimizing the corresponding Hamiltonian expectation values. We first establish the single-particle confinement states on which these trial states are built.

We consider a GaAs hard-wall semi-ellipsoidal quantum dot with cylindrical coordinates $\mathbf{r} = (r, \phi, z)$ and confinement potential

$$V_{\text{conf}}(r, z) = \begin{cases} 0, & r^2/a^2 + z^2/c^2 \leq 1 \text{ and } z \geq 0, \\ \infty, & \text{otherwise,} \end{cases} \quad c/a \ll 1. \quad (1)$$

The in-plane and axial semiaxes are a and c , respectively, as illustrated in Fig. 1.

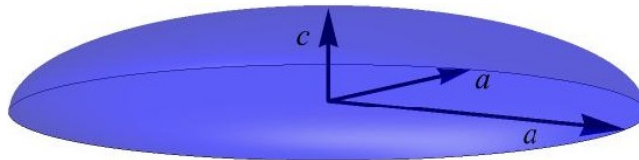


Figure 1: Semi-ellipsoidal quantum-dot geometry, with lateral semiaxis a and axial semiaxis c .

Lengths and energies are expressed in the effective electron Bohr radius and Rydberg,

$$r_B = \frac{4\pi\epsilon_0\epsilon_r\hbar^2}{m_e e^2}, \quad E_R = \frac{\hbar^2}{2m_e r_B^2}. \quad (2)$$

Once expressed in these units, the confinement and correlation contributions depend on the material only through the effective-mass ratio $\eta = m_e/m_h$. The GaAs band gap and the numerical calibration needed to form the total energies are introduced below with the radiative parameters.

Because the axial confinement is much stronger than the in-plane confinement, the single-particle problem is treated adiabatically [22]. At fixed r , the available height is

$$z_{\max}(r) = c\sqrt{1 - r^2/a^2}, \quad (3)$$

and the normalized ground-state wavefunction for the fast axial motion is

$$\chi_1(z; r) = \sqrt{\frac{2}{z_{\max}(r)}} \sin \left[\frac{\pi z}{z_{\max}(r)} \right]. \quad (4)$$

For a carrier of mass m_q with $q = e, h$ the axial eigenenergy acts as the adiabatic potential for the slow planar motion,

$$U_q(r) = \frac{m_e}{m_q} \frac{\pi^2}{z_{\max}^2(r)} = \frac{m_e}{m_q} \frac{\pi^2}{c^2} \left(1 + \frac{r^2}{a^2} + O(r^4/a^4) \right), \quad (5)$$

where lengths and energies are in the effective units of Eq. (2). Thus, after subtracting the constant axial energy, the planar harmonic-oscillator Hamiltonian is

$$H_{q,\parallel} = \frac{m_e}{m_q} \left[-\nabla_{\parallel}^2 + \frac{\pi^2 r^2}{a^2 c^2} \right]. \quad (6)$$

Writing $\lambda = \pi/(ac)$, the normalized ground-state wavefunction for the in-plane motion and the corresponding axial and in-plane ground-state energies are

$$\varphi_{q,\parallel}(r) = \sqrt{\frac{\lambda}{\pi}} e^{-\lambda r^2/2}, \quad \varepsilon_{q,z} = \frac{m_e}{m_q} \frac{\pi^2}{c^2}, \quad \varepsilon_{q,\parallel} = \frac{m_e}{m_q} \frac{2\pi}{ac}. \quad (7)$$

The total ground-state confinement energy of carrier q is therefore

$$E_q = \varepsilon_{q,z} + \varepsilon_{q,\parallel} = \frac{m_e}{m_q} \left(\frac{\pi^2}{c^2} + \frac{2\pi}{ac} \right). \quad (8)$$

The normalized single-particle ground-state wavefunction is therefore $\psi_q(r, \phi, z) = \chi_1(z; r)\varphi_{q,\parallel}(r)$. Thus the uncorrelated exciton and biexciton energies, including one and two band-gap contributions, respectively, are

$$E_X^{(0)} = E_g + E_e + E_h, \quad E_{XX}^{(0)} = 2E_g + 2E_e + 2E_h = 2E_X^{(0)}. \quad (9)$$

With the one-particle basis established, we construct the correlated exciton and biexciton states in turn. We denote electron coordinates by 1, 2 and hole coordinates by a, b ; the ground exciton uses coordinates 1 and a . In the effective units of Eq. (2), its Hamiltonian is

$$H_X = E_g + h_e(1) + h_h(a) - \frac{2}{r_{1a}}, \quad h_e\psi_e = E_e\psi_e, \quad h_h\psi_h = E_h\psi_h. \quad (10)$$

The uncorrelated state $\Psi_{0X} = \psi_e(1)\psi_h(a)$ is therefore an eigenstate of $E_g + h_e + h_h$ with energy $E_X^{(0)} = E_g + E_e + E_h$. Electron-hole correlation is introduced through

$$\Psi_X = \Psi_{0X} F_X, \quad F_X = \exp(-\alpha_X r_{1a}), \quad (11)$$

where $\alpha_X > 0$ is varied independently at each geometry. The variational energy is evaluated from

$$E_X = \frac{\langle \Psi_X | H_X | \Psi_X \rangle}{\langle \Psi_X | \Psi_X \rangle}. \quad (12)$$

The denominator is

$$N_X = \int |\Psi_{0X}|^2 F_X^2 d\tau = \int |\Psi_{0X}|^2 e^{-2\alpha_X r_{1a}} d\tau. \quad (13)$$

Using the single-particle eigenvalue equations in Eq. (10) and integrating the kinetic terms by parts gives

$$E_X = E_X^{(0)} + \frac{1}{N_X} \int |\Psi_{0X}|^2 \left[|\nabla_1 F_X|^2 + \frac{m_e}{m_h} |\nabla_a F_X|^2 - \frac{2F_X^2}{r_{1a}} \right] d\tau. \quad (14)$$

Since $|\nabla_1 r_{1a}| = |\nabla_a r_{1a}| = 1$, the kinetic contribution generated by F_X is

$$K_X(\alpha_X) = \alpha_X^2 \left(1 + \frac{m_e}{m_h} \right). \quad (15)$$

Defining the Coulomb numerator as

$$V_X = \int |\Psi_{0X}|^2 F_X^2 \left(-\frac{2}{r_{1a}} \right) d\tau \quad (16)$$

reduces Eq. (14) to

$$E_X = E_X^{(0)} + \Delta E_X, \quad \Delta E_X = K_X + \frac{V_X}{N_X}. \quad (17)$$

For each dot geometry, the remaining integrals N_X and V_X are evaluated numerically.

The same construction extends to the biexciton. Its Hamiltonian contains two band-gap contributions and the complete two-electron–two-hole interaction,

$$\begin{aligned} H_{XX} &= 2E_g + h_e(1) + h_e(2) + h_h(a) + h_h(b) + V_{XX}, \\ V_{XX} &= 2 \left(\frac{1}{r_{12}} + \frac{1}{r_{ab}} - \frac{1}{r_{1a}} - \frac{1}{r_{1b}} - \frac{1}{r_{2a}} - \frac{1}{r_{2b}} \right). \end{aligned} \quad (18)$$

For the biexciton we use the four-parameter ansatz employed by Takagahara [6],

$$\Psi_{XX} = \Psi_{0XX} F_{XX}, \quad \Psi_{0XX} = \psi_e(1)\psi_e(2)\psi_h(a)\psi_h(b), \quad (19)$$

with the exchange-symmetric correlation factor

$$\begin{aligned} F_{XX} &= G(P + Q), \\ G &= r_{ab}^\gamma e^{-\delta r_{ab}}, \\ P &= \exp[-\alpha(r_{1a} + r_{2b}) - \beta(r_{1b} + r_{2a})], \\ Q &= \exp[-\beta(r_{1a} + r_{2b}) - \alpha(r_{1b} + r_{2a})]. \end{aligned} \quad (20)$$

The four variational parameters $(\alpha, \beta, \gamma, \delta)$ control the direct and cross electron–hole pairings and the hole–hole correlation.

To write the kinetic numerator compactly, define $\mathbf{g}_i^P = \nabla_i \ln(GP)$ and $\mathbf{g}_i^Q = \nabla_i \ln(GQ)$. For electron coordinates the derivatives of G vanish, whereas for hole coordinates they contain the logarithmic derivative of G in Eq. (20). Unlike the exciton kinetic contribution, the biexciton kinetic numerator has no analytical form and is written as

$$K_{XX} = \int |\Psi_{0XX}|^2 G^2 \left[\sum_{i=1,2} |P\mathbf{g}_i^P + Q\mathbf{g}_i^Q|^2 + \frac{m_e}{m_h} \sum_{j=a,b} |P\mathbf{g}_j^P + Q\mathbf{g}_j^Q|^2 \right] d\tau. \quad (21)$$

The energy is then

$$E_{XX} = E_{XX}^{(0)} + \Delta E_{XX}, \quad \Delta E_{XX} = \frac{K_{XX} + \langle \Psi_{XX} | V_{XX} | \Psi_{XX} \rangle}{\langle \Psi_{XX} | \Psi_{XX} \rangle}. \quad (22)$$

In the numerical calculation, the kinetic contribution in Eq. (21) and the potential contribution are combined and evaluated in a single integral. No direct/cross or particle-exchange multiplicities are introduced in the final estimator.

We define the biexciton binding energy as

$$E_{\text{bind}} = 2E_X - E_{XX} = 2\Delta E_X - \Delta E_{XX}. \quad (23)$$

With this convention $E_{\text{bind}} > 0$ denotes a bound biexciton. The zeroth-order part, including the band-gap terms, cancels exactly, making Eq. (23) a small difference of correlation corrections.

The total exciton energy also determines the photon-energy scale of the radiative response. Following Ref. [9], we calculate the exciton and biexciton radiative lifetimes from

$$\tau_X = \frac{2\pi\epsilon_0 m_e c_0^3 \hbar^2}{\sqrt{\epsilon_r} e^2 E_X^2 f_X}, \quad \tau_{XX} = \frac{\tau_X}{4}. \quad (24)$$

The relation $\tau_{XX} = \tau_X/4$ follows the biexciton decay-channel convention of Ref. [9]; it is not an independently optimized quantity.

Evaluation of τ_X requires the oscillator strength, given by

$$f_X = \frac{E_P}{E_X} |M_X|^2, \quad E_X = E_g + E_e + E_h + \Delta E_X, \quad (25)$$

Equation (25) is evaluated in physical units rather than effective atomic units, with both E_X and E_P expressed in electronvolts. The remaining quantity M_X is the electron-hole coincidence overlap. At coincidence the exciton Jastrow factor in Eq. (11) is unity, and because the electron and hole ground-state wavefunctions are matched in the present model, the normalized overlap reduces to

$$M_X = \frac{1}{\sqrt{N_X}}, \quad |M_X|^2 = \frac{1}{N_X}. \quad (26)$$

Thus the lifetime can be obtained from the already calculated exciton norm; no additional oscillator-strength integral is required.

The GaAs parameters are $\epsilon_r = 12.8$, $m_e = 0.067m_0$, and $m_h = 0.45m_0$ [24], which gives $r_B \simeq 10.11$ nm and $E_R \simeq 5.564$ meV. The same ϵ_r is used for Coulomb screening and for the dielectric factor in Eq. (24). For the radiative-lifetime calculation we additionally take $E_g = 1.5192$ eV and $E_P = 22.71$ eV, following the optical model of Ref. [9]. We study the seven geometries listed in Table 1. The common point (5, 1) links a scan of axial confinement to a scan of lateral confinement.

Table 1: Quantum-dot semiaxes used in the axial and lateral scans, in effective Bohr radii.

Scan	a	c
Axial	5	0.5, 1, 1.5, 2
Lateral	3, 5, 7, 10	1

3 Results and discussion

The correlation corrections and total energies are collected in Table 2 of the Appendix. Throughout the figures, lengths are reported in nanometres. Total energies are given in electronvolts, whereas correlation corrections and binding energies are given in millielectronvolts, using the GaAs calibration after Eq. (26). The total exciton energy contains one band-gap contribution and the total biexciton energy contains two, as defined in Eq. (9). Because the zeroth-order confinement energies and band-gap terms are analytical, each correlation correction has the same numerical integration

error as its corresponding total energy. Although these rough errors are retained in the table, they are too small to be visible on the scale of the biexciton-energy curves in Fig. 2. The numerical values are reported for transparency and reproducibility, but the physically relevant conclusions are the comparative geometry trends within the idealized model rather than absolute predictions for a particular quantum dot.

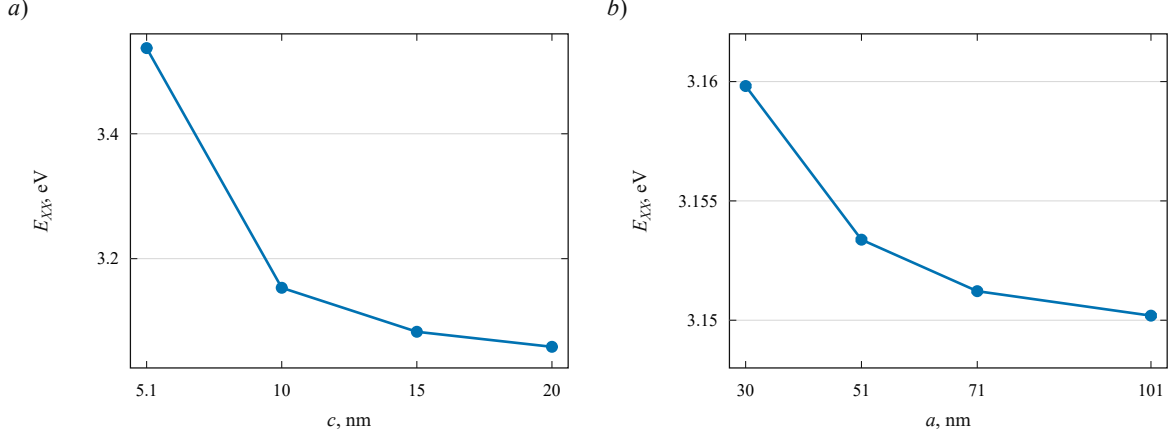


Figure 2: Total biexciton energy as a function of (a) the axial semiaxis c at fixed $a = 50.55$ nm and (b) the lateral semiaxis a at fixed $c = 10.11$ nm. The energy includes two GaAs band-gap contributions and is shown in electronvolts. Lines are guides to the eye, and numerical error bars are smaller than the symbols at the displayed scales.

Along the axial scan, E_{XX} decreases steeply and nonlinearly, with most of the change occurring between the two most strongly confined geometries. This behavior reflects the strong sensitivity of the confinement energy to the short semiaxis in the oblate regime.

The lateral scan is much flatter: enlarging the in-plane semiaxis produces only a modest reduction in E_{XX} . The axial dimension is therefore the dominant geometrical control of the absolute biexciton energy over the sampled range, consistent with the separation of axial and lateral energy scales in Eq. (7).

While the band-gap terms set the absolute energy scale, they cancel from the biexciton binding energy in Eq. (23), allowing the net four-particle correlation advantage to be read directly from Fig. 3.

The central binding energy is positive throughout the sampled geometries and decreases monotonically when either semiaxis is increased (Fig. 3). Thus relaxing axial or lateral confinement reduces the net correlation advantage of the four-particle state relative to two isolated excitons. The lateral scan approaches the zero-binding threshold more closely than the axial scan.

The binding is well separated from zero at every geometry except the widest lateral dot. There, the numerical-resolution estimate is comparable to the small central binding energy. Because that estimate omits systematic integration bias and uncertainty in the optimum, we classify the point as marginally positive rather than claiming firmly resolved binding. No sign change is demonstrated within the studied range.

Because the zeroth-order part, including the band-gap contributions, cancels in Eq. (23), the origin of this trend is exposed by comparing the two correlation corrections in Fig. 4. Both become less negative as the dot is enlarged, while the separation between $2\Delta E_X$ and ΔE_{XX} narrows. That separation is precisely E_{bind} , so the convergence of the two curves accounts directly for the suppression of binding, particularly along the lateral scan.

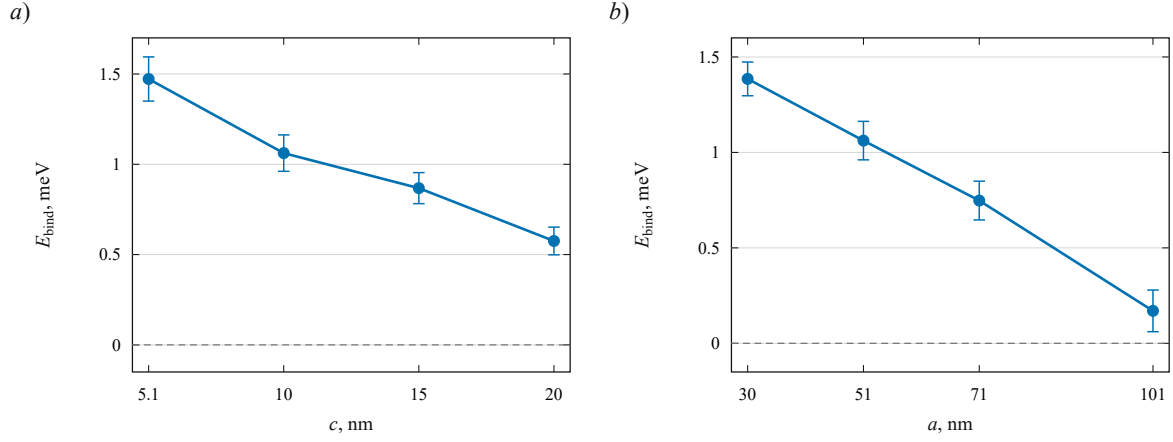


Figure 3: Binding energy $E_{\text{bind}} = 2E_X - E_{XX}$ as a function of (a) c at fixed $a = 50.55$ nm and (b) a at fixed $c = 10.11$ nm, in millielectronvolts. Positive values correspond to a bound biexciton. The dashed line marks zero binding. Error bars are rough first-order propagations of the adaptive-integration estimates, not statistical confidence intervals.

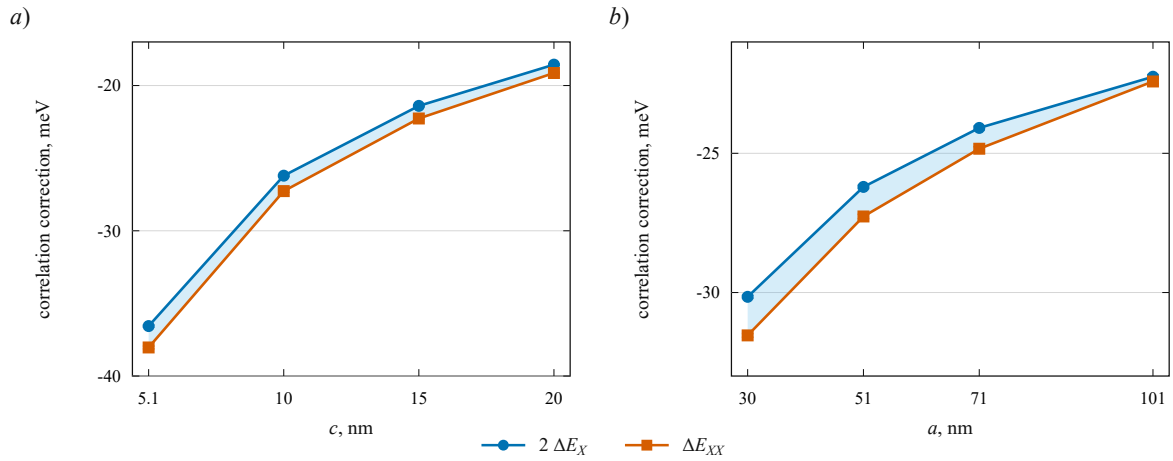


Figure 4: Comparison of the two-exciton correction $2\Delta E_X$ with the biexciton correction ΔE_{XX} as functions of (a) c at fixed $a = 50.55$ nm and (b) a at fixed $c = 10.11$ nm, in millielectronvolts. Their vertical separation equals the binding energy. Lines are guides to the eye; numerical error bars are smaller than the plotted symbols at this scale.

The energetic trends are accompanied by systematic changes in the optimized correlations. The complete biexciton parameters are reported in Table 3 of the Appendix and summarized graphically in Fig. 5. The exciton parameter α_X generally decreases along the axial scan and increases along the lateral scan. For the biexciton, the optimized cross-pair exponent β becomes very small at the larger axial sizes. The composite quantity γ/δ , which sets the maximum of the isolated factor $r_{ab}^\gamma e^{-\delta r_{ab}}$, increases smoothly as either dot dimension is enlarged. This indicates that the hole-hole factor shifts toward larger characteristic separations in larger dots.

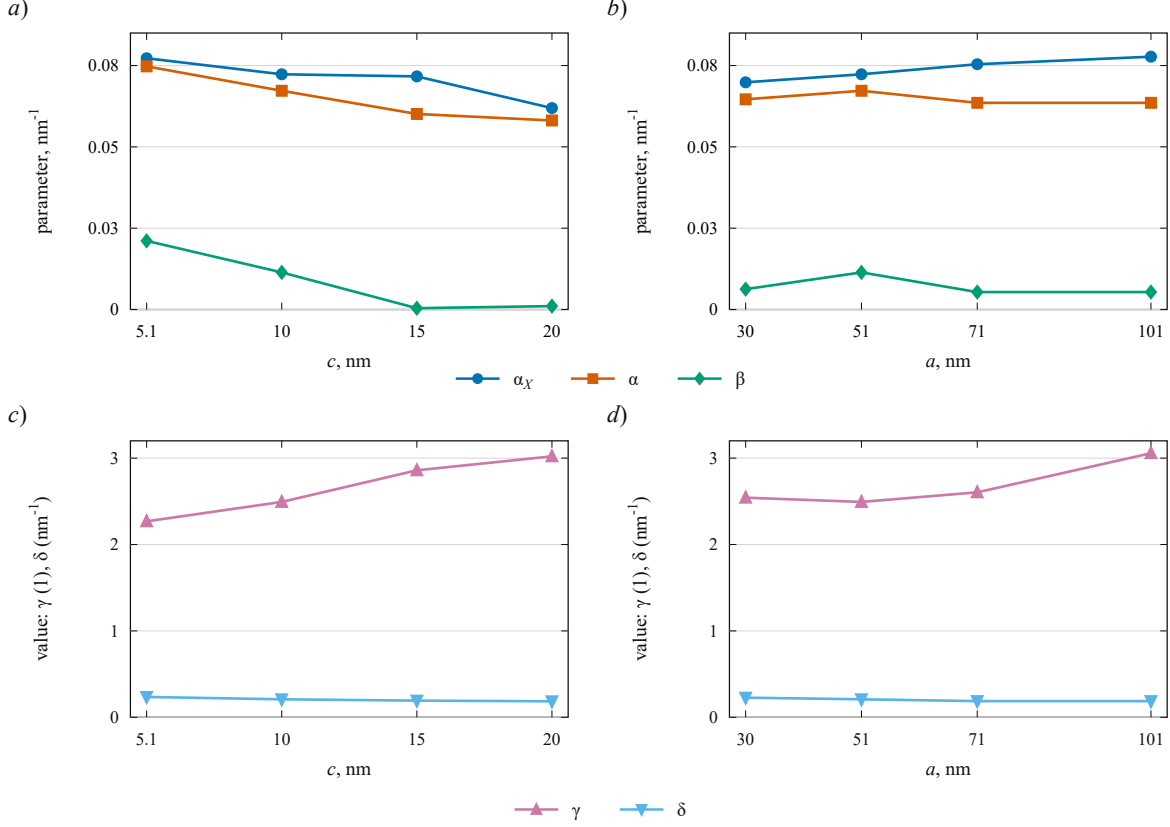


Figure 5: Optimized variational parameters along the axial scan at fixed $a = 50.55$ nm and the lateral scan at fixed $c = 10.11$ nm. Panels (a) and (b) show α_X , α , and β in nm^{-1} ; panels (c) and (d) show γ (dimensionless) and δ in nm^{-1} . Lines are guides to the eye.

These parameter-level trends should be interpreted more cautiously than the energies. The objective is numerically rough, the second-phase search is local, and different parameter combinations can produce energies separated by less than the practical quadrature resolution. The parameters characterize the retained variational minimum but are not claimed to be unique.

The coincidence overlap is shown in Fig. 6, while the derived optical quantities are listed in Table 4 of the Appendix. The overlap increases as either dot dimension is enlarged. It tends toward saturation at the larger axial sizes, whereas it continues to rise across the lateral scan. The oscillator strength follows the same behavior.

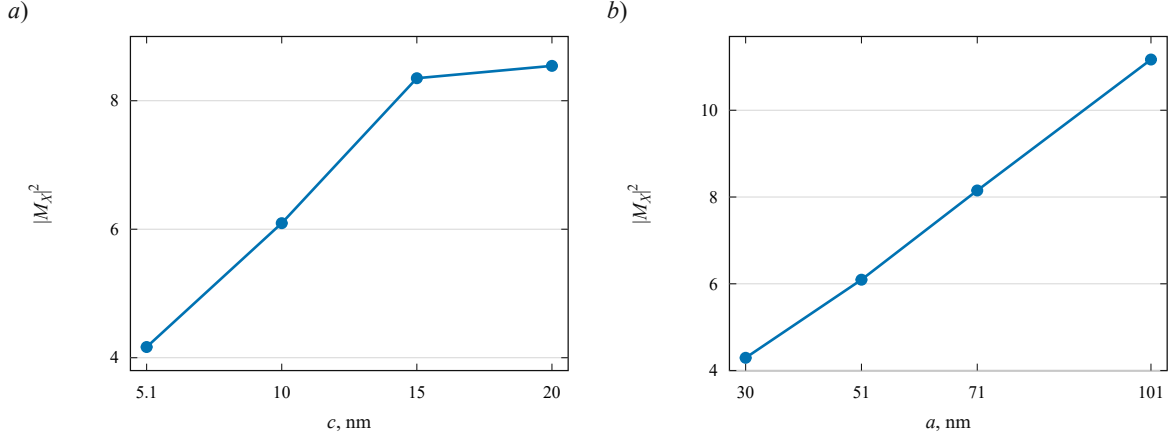


Figure 6: Squared exciton coincidence overlap $|M_X|^2$ as a function of (a) c at fixed $a = 50.55$ nm and (b) a at fixed $c = 10.11$ nm. Lines are guides to the eye, and numerical error bars are smaller than the plotted symbols.

Together with the total exciton energy, this increasing overlap determines the radiative time scale.

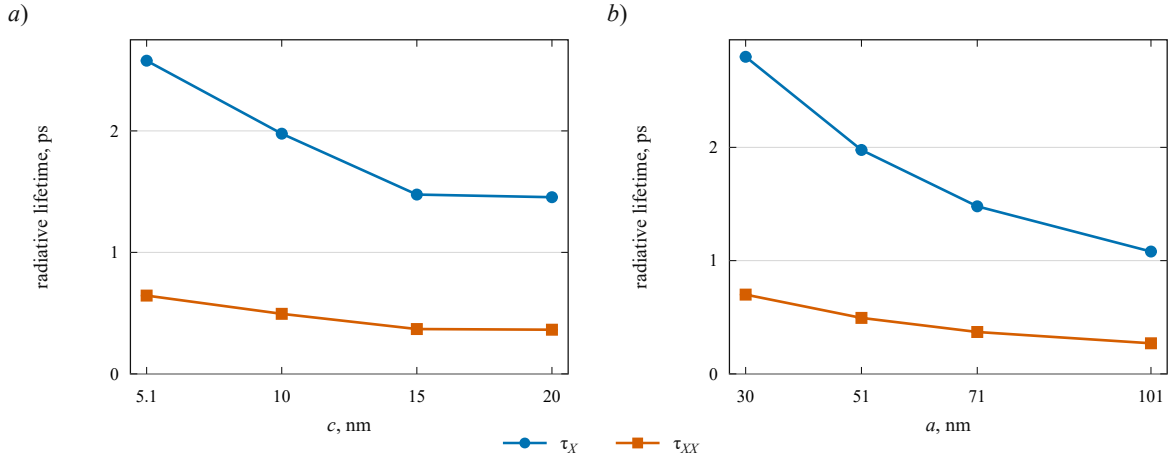


Figure 7: Exciton and biexciton radiative lifetimes as functions of (a) c at fixed $a = 50.55$ nm and (b) a at fixed $c = 10.11$ nm. The values use the total optical transition energy, including the band gap, and the coincidence overlap. The adopted decay-channel convention gives $\tau_{XX} = \tau_X/4$. The GaAs parameters follow Eqs. (24) and (25). Propagated error bars are smaller than the plotted symbols.

Both radiative lifetimes decrease as the dot is enlarged (Fig. 7). Along the axial scan, the decrease weakens and becomes nearly saturated for the largest dots. Along the lateral scan, the decline remains pronounced across the sampled range. The biexciton lifetime follows the same geometry dependence at one quarter of the exciton value.

Relaxing confinement decreases E_X , which alone would tend to increase the lifetime. The simultaneous increase in $|M_X|^2$ is stronger, however. Since substitution of Eq. (25) into Eq. (24) gives $\tau_X \propto (E_X |M_X|^2)^{-1}$, the overlap enhancement dominates and the lifetime decreases.

These trends can be placed alongside earlier calculations for related geometries. The present results are qualitatively consistent with calculations for full strongly oblate ellipsoidal dots, where the semiaxes tune the correlated energies and binding and the exciton overlap sets the radiative scale [9, 15]. A direct numerical comparison is not appropriate, however, because the semi-ellipsoidal

boundary changes the axial single-particle state, and the present four-particle trial function and full integration protocol differ from those earlier calculations.

The model assumes impenetrable barriers, bulk effective masses, static dielectric screening for the energy calculation, and an adiabatic single-particle ground state. Finite band offsets, dielectric mismatch, strain, atomistic symmetry, and phonon-assisted decay are outside the present scope. The lifetime calculation follows the optical model and decay-channel convention of Ref. [9], with the GaAs parameters given after Eq. (26). It should therefore be interpreted as the radiative lifetime within that model rather than as a complete experimental decay time.

These approximations can shift the absolute energies, binding energies, and lifetimes of a real structure. The principal outcome is therefore the qualitative separation of axial and lateral geometry trends within a single consistent model, not a quantitative prediction for a specific fabricated dot.

Numerically, the reported bars quantify only first-order propagation of the adaptive integrator’s internal errors. The smooth trends and the positive binding away from the widest lateral dot are robust on that scale, but a higher-resolution targeted calculation would be required to determine firmly whether the marginal endpoint is bound or antibound.

4 Conclusions

We have calculated the ground exciton and biexciton states of a strongly oblate semi-ellipsoidal GaAs quantum dot over intersecting axial and lateral geometry scans. A ground-orbital-matched importance transformation, a full global-rotation-fixed biexciton estimator, and the second-phase search with two-budget verification were combined to control the numerical sensitivity of the small binding energy.

Within the hard-wall effective-mass model, the main conclusions are qualitative. The axial dimension controls the absolute confinement-energy scale much more strongly than the lateral dimension. Enlarging the dot along either direction weakens the biexciton binding, with the lateral scan approaching the zero-binding threshold most closely. The central binding estimate remains positive across the sampled range, but its sign is not resolved at the widest lateral geometry.

Increasing either semiaxis also enhances the exciton coincidence overlap and oscillator strength. This enhancement outweighs the reduction of optical transition energy and shortens both calculated radiative lifetimes. The lateral dependence remains pronounced across the sampled range, whereas the axial dependence tends toward saturation.

The numerical values should therefore be interpreted as results of a controlled model comparison rather than as absolute predictions for a particular fabricated dot. Their principal contribution is to identify distinct geometrical controls: the short axial dimension sets the dominant confinement-energy scale, while lateral enlargement provides an effective route for suppressing weak biexciton binding and enhancing radiative strength. Extensions to finite barriers, dielectric mismatch, strain, and atomistic confinement are needed for material-specific quantitative predictions. A targeted higher-resolution calculation is also required to determine the sign of the near-zero binding at the widest lateral geometry.

Author contributions

Visualization, D.G.; Software, D.G.; Writing—original draft, D.G.; Validation, Y.B.; Supervision, E.K.; Writing—review & editing, all authors.

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Data availability

The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

Declarations

Ethics and consent to participate

Not Applicable.

Consent to publish

Not applicable.

Competing interests

The authors declare no competing interests.

Ethics and biosecurity

Not Applicable.

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A Supplementary numerical information

This appendix collects the numerical tables supporting the main-text figures. Table 2 gives the correlation corrections, total energies, and binding energies.

Table 2: Exciton and biexciton correlation corrections and binding energy in millielectronvolts, with the total energies in electronvolts and the semiaxes in nanometres. The total energies include one and two GaAs band-gap contributions, respectively. Errors are rough propagated numerical integration estimates.

a (nm)	c (nm)	ΔE_X (meV)	E_X (eV)	ΔE_{XX} (meV)	E_{XX} (eV)	E_{bind} (meV)
50.55	5.06	-18.28 ± 0.04	1.76934 ± 0.00004	-38.03 ± 0.10	3.53721 ± 0.00010	1.47 ± 0.12
50.55	10.11	-13.10 ± 0.03	1.57722 ± 0.00003	-27.27 ± 0.08	3.15338 ± 0.00008	1.06 ± 0.10
50.55	15.17	-10.70 ± 0.03	1.54189 ± 0.00003	-22.27 ± 0.06	3.08292 ± 0.00006	0.87 ± 0.08
50.55	20.22	-9.28 ± 0.03	1.52971 ± 0.00003	-19.14 ± 0.06	3.05884 ± 0.00006	0.57 ± 0.08
30.33	10.11	-15.08 ± 0.03	1.58060 ± 0.00003	-31.54 ± 0.07	3.15981 ± 0.00007	1.39 ± 0.09
70.77	10.11	-12.05 ± 0.03	1.57598 ± 0.00003	-24.84 ± 0.07	3.15122 ± 0.00007	0.75 ± 0.10
101.10	10.11	-11.12 ± 0.04	1.57518 ± 0.00004	-22.42 ± 0.08	3.15019 ± 0.00008	0.17 ± 0.11

Table 3 lists the final parameters retained by the second-phase search. The ratio γ/δ is also shown because it locates the maximum of the isolated hole-hole factor $r_{ab}^\gamma e^{-\delta r_{ab}}$.

Table 3: Final optimized parameters for the exciton and biexciton states. The semiaxes and γ/δ are in nanometres; α_X , α , β , and δ are in inverse nanometres, while γ is dimensionless.

a (nm)	c (nm)	α_X (nm ⁻¹)	α (nm ⁻¹)	β (nm ⁻¹)	γ (1)	δ (nm ⁻¹)	γ/δ (nm)
50.55	5.06	0.0772	0.0747	0.0211	2.268	0.2357	9.62
50.55	10.11	0.0723	0.0672	0.0114	2.493	0.2086	11.95
50.55	15.17	0.0717	0.0601	0.0004	2.859	0.1932	14.80
50.55	20.22	0.0619	0.0581	0.0010	3.021	0.1854	16.29
30.33	10.11	0.0698	0.0646	0.0063	2.542	0.2274	11.18
70.77	10.11	0.0754	0.0635	0.0054	2.604	0.1876	13.88
101.10	10.11	0.0777	0.0635	0.0054	3.057	0.1876	16.30

The coincidence overlap and the optical quantities derived from it are listed in Table 4.

Table 4: Coincidence overlap, oscillator strength, and radiative lifetimes using the GaAs parameters given after Eq. (26). The semiaxes are in nanometres; $|M_X|^2$ and f_X are dimensionless, and τ_X and τ_{XX} are in picoseconds. Numerical-resolution estimates are omitted because they are small compared with the geometry-dependent variations.

a (nm)	c (nm)	$ M_X ^2$ (1)	f_X (1)	τ_X (ps)	τ_{XX} (ps)
50.55	5.06	4.167	53.48	2.578	0.644
50.55	10.11	6.094	87.75	1.977	0.494
50.55	15.17	8.351	122.99	1.476	0.369
50.55	20.22	8.543	126.82	1.454	0.364
30.33	10.11	4.294	61.70	2.800	0.700
70.77	10.11	8.151	117.45	1.479	0.370
101.10	10.11	11.169	161.02	1.080	0.270