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Ordered Absorption Reveals Green–Kubo Memory Hidden from Survival Curves

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Can a binary survival measurement reveal dynamical memory? We prove that every stationary survival curve generated by a fixed state-dependent absorber is identical for a process and its time reverse. Two ordered pulses remove this obstruction: their joint survival probability gives an exact finite-amplitude lagged covariance, pulse reversal exposes the time-antisymmetric component, and the resulting Bernoulli probabilities bound path irreversibility. Spanning profiles reconstruct finite-state Markov dynamics. A held-out retrospective reanalysis of 20 independent optical-trap records shows a robust experimental time-antisymmetric correlation and recovers its bounded-profile counterpart from virtual survival probabilities. An independent two-state single-molecule model supplies a reversible null and finite-count reconstruction benchmark. These tests use experimental trajectories but emulate the proposed removals; direct physical pulse validation remains open.

Many experiments register only loss or survival although the underlying trajectory is inaccessible. Absorbing boundaries connect escape and transport [1–3], whereas Green–Kubo response is built from time correlations [4, 5]. Selective depletion is experimentally established [6], and controlled readout can resolve cross-spectral information [7, 8]. Two-pulse desorption spectroscopy is also established [9–11]; there the energetic pump pulses modify the excited dynamics, unlike the stochastic-removal/no-survivor-backaction model considered here. We ask what information is fundamentally absent from static survival and what minimal temporal control restores it.

Let X_t be stationary. A pulse with calibrated profile $0 \leq a \leq 1$ and amplitude $0 < u \leq 1$ removes a realization at time t with probability $ua(X_t)$. Conditional pulse randomizations are independent, and a pulse does not otherwise alter survivor dynamics. For a finite Markov model, let R be row stochastic, $\pi R = \pi$, $P = \text{diag } \pi$, and $R^\dagger = P^{-1}R^T P$.

For any fixed survival profile $D = \text{diag}(d_i)$ and integer $N \geq 1$,

$$\pi(DR)^N \mathbf{1} = \pi(DR^\dagger)^N \mathbf{1}. \quad (1)$$

The survival curve averages $\prod_{t=0}^{N-1} d(X_t)$, invariant under reversal of a stationary path. Equation (1) therefore extends beyond Markov processes to the complete finite-time absorption law. More observation time or stronger fixed loading cannot reveal stationary irreversibility.

Apply profile a at time zero and b at lag n :

$$\begin{aligned} S_{ab}(n) &= \mathbb{E}[(1 - ua_0)(1 - vb_n)], \\ S_a &= 1 - u\mathbb{E}a, \quad S_b = 1 - v\mathbb{E}b. \end{aligned} \quad (2)$$

Direct expansion gives

$$\text{Cov}(a_0, b_n) = \frac{S_{ab}(n) - S_a S_b}{uv}, \quad (3)$$

$$\begin{aligned} \text{Cov}(a_0, b_n) - \text{Cov}(b_0, a_n) \\ = \frac{S_{ab}(n) - S_{ba}(n)}{uv}. \end{aligned} \quad (4)$$

These identities are exact at finite amplitude and require no Markov, mixing, weak-pulse, or quasistationary approximation. Stationarity is needed only for the simplified second form; Supplemental Material gives the exact finite-record marginal correction [12].

Known profiles $F_{i\alpha} = f_\alpha(i)$ give

$$B(n) = F^T P R^n F. \quad (5)$$

If F has full row rank, choose U with $FU = I$; then

$$U^T B(n) U = P R^n, \quad R = P^{-1} U^T B(1) U. \quad (6)$$

Thus spanning controls identify a finite-state transition matrix under the Markov model. The minimum-norm inversion amplifies moment error by at most $\sigma_{\min}(F)^{-2}$ before division by rare stationary probabilities; nonspanning profiles can hide complete modes [12].

Time’s arrow can be certified without full reconstruction. For a stationary finite chain with bidirectional support,

$$\sigma = \sum_{ij} \pi_i R_{ij} \log \frac{\pi_i R_{ij}}{\pi_j R_{ji}} \geq \frac{1}{n} d_B(S_{ab}(n) \| S_{ba}(n)), \quad (7)$$

where d_B is Bernoulli relative entropy. This is data processing applied to forward and reversed path laws and implements the established coarse-observation principle [13]. Thermodynamic interpretation requires additional physical assumptions such as local detailed balance.

We tested these consequences on experimental records without relabeling virtual pulses as a new experiment. The primary dataset is the SwitchingTrap subset of the Dryad record accompanying the variance-sum-rule experiment [14, 15]: 20 independent 15-s records at 100 kHz, with ten, five, and five realizations at trap separations 18, 70, and 280 nm. For each record, the first 20% defines robust logistic profiles of bead position and force; the remaining 80% is held out. All 20 pass a transparent half-to-half mean/variance stationarity screen stated in Supplemental Material.

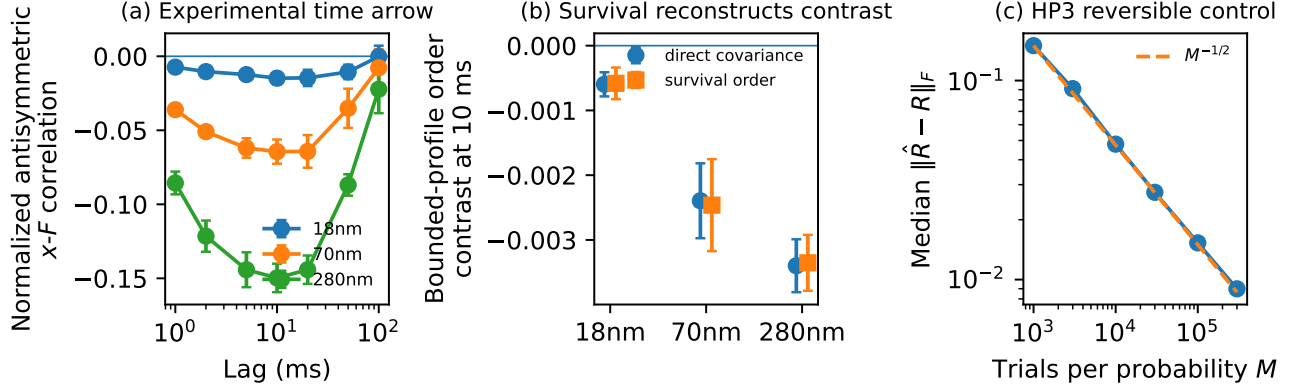


FIG. 1. Retrospective experimental-data tests. (a) Held-out normalized time-antisymmetric x - F correlation for 20 switching optical-trap records [14, 15]; error bars are 95% intervals across independent records. (b) With bounded profiles fixed using the first 20% of each record, direct covariance-order contrast and the stationary virtual-survival estimator $S_{ab} - S_{ba}$ agree at 10 ms; the exact finite-record correction removes marginal drift. (c) Survival-only transition reconstruction for the reversible two-state HP3 model inferred in Ref. [16, 17] follows $M^{-1/2}$. Panels (a,b) use experimental trajectories but computationally emulate removal; panel (c) is model-mediated.

At 10 ms, the normalized experimental antisymmetric correlation $[\text{Cov}(x_0, F_n) - \text{Cov}(F_0, x_n)]/[\text{sd } x \text{ sd } F]$ is -0.01479 [95% CI $-0.01889, -0.01069$], -0.06439 $[-0.07257, -0.05621]$, and -0.14974 $[-0.15936, -0.14011]$ for 18, 70, and 280 nm. The frozen bounded profiles give direct covariance-order contrasts -5.97×10^{-4} , -2.40×10^{-3} , and -3.40×10^{-3} ; the stationary survival-order estimates are -5.82×10^{-4} , -2.46×10^{-3} , and -3.35×10^{-3} . Endpoint-specific marginal correction restores exact agreement to machine precision. Using x -high and F -high indicator profiles on the same four-state endpoint coarse graining, the common Bernoulli survival channel obeys $d_B(S_{ab} \| S_{ba}) \leq D(P_{ij} \| P_{ji})$ in every record, directly checking information contraction without identifying either quantity as thermodynamic entropy production.

Finite counts expose the cost of this coarse readout. For the strongest 280-nm measured probabilities, independent virtual Bernoulli cohorts with simultaneous 95% Clopper–Pearson intervals [18] detect the order asymmetry in 0.0868, 0.4239, 0.6973, and 0.9633 of simulations at $M = 10^5, 3 \times 10^5, 5 \times 10^5, 10^6$ trials per order. The method restores identifiability but does not create information; any parameter-independent readout obeys Fisher-information contraction [19].

As an independent null, we use HP3 nsALEX photon data and the source authors’ fitted H2MM models [16, 17]. The uploaded HDF5 contains 6,768,183 photons over 3599.77 s; their model table selects two states with $k_{01} = 678.8685$ and $k_{10} = 103.1168 \text{ s}^{-1}$. Every stationary two-state chain is reversible. At 1 ms the reference matrix has $R_{01} = 0.470966$ and $R_{10} = 0.071537$ with detailed-balance residual 6.9×10^{-18} . Survival-only count reconstruction has median Frobenius error 0.1497 at $M = 10^3$, 0.0479 at 10^4 , and 0.0090 at 3×10^5 , while

the reversible order test has a conservative false-positive rate near 10^{-3} . Degrading $\sigma_{\min}(F)$ produces the predicted rapid instability [12]. This is a model-mediated benchmark, because the reference R is itself inferred from the experimental photons.

If $C(n) = \text{Cov}(\mathbf{f}_0, \mathbf{f}_n)$ is absolutely summable, the same measurements recover the Green–Kubo spectral kernel,

$$\mathcal{S}(\omega) = C(0) + \sum_{n \geq 1} [C(n)e^{-in\omega} + C(n)^T e^{in\omega}], \quad (8)$$

with the order contrast supplying its antisymmetric/imaginary cross component. Finite lag windows still require tail control.

Static survival is therefore exactly blind to stationary time reversal, whereas two calibrated ordered removals reveal lagged memory and projected current. Retrospective experimental trajectories show that the relevant asymmetry survives realistic dynamics and finite records, and an independent reversible molecular model provides a null and reconstruction benchmark. The remaining decisive step is physical implementation of the stochastic removals with independently prepared cohorts and controls proving no pulse-induced survivor dynamics.

Data and software availability. The exact analysis tables, deterministic reproduction code, source manifests, and copies of the third-party files used in the present reanalysis accompany the submission package. The authoritative public sources are Dryad DOI [15], Zenodo DOI [17], and the Hsp90 source record used only for the stationarity diagnostic [20]. Original repository terms govern third-party data. No new laboratory data were generated.

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- [1] P. Gaspard and J. R. Dorfman, Chaotic scattering theory, thermodynamic formalism, and transport coefficients, *Phys. Rev. E* **52**, 3525 (1995).
- [2] J. R. Dorfman and P. Gaspard, Chaotic scattering theory of transport and reaction-rate coefficients, *Phys. Rev. E* **51**, 28 (1995).
- [3] L. A. Bunimovich and C. P. Dettmann, Peeping at chaos: Nondestructive monitoring of chaotic systems by measuring long-time escape rates, *EPL* **80**, 40001 (2007).
- [4] M. S. Green, Markoff random processes and the statistical mechanics of time-dependent phenomena. ii. irreversible processes in fluids, *J. Chem. Phys.* **22**, 398 (1954).
- [5] R. Kubo, Statistical-mechanical theory of irreversible processes. i. general theory and simple applications to magnetic and conduction problems, *J. Phys. Soc. Jpn.* **12**, 570 (1957).
- [6] D. Axelrod, D. Koppel, J. Schlessinger, E. Elson, and W. Webb, Mobility measurement by analysis of fluorescence photobleaching recovery kinetics, *Biophysical Journal* **16**, 1055 (1976).
- [7] P. Szańkowski, M. Trippenbach, and Ł. Cywiński, Spectroscopy of cross correlations of environmental noises with two qubits, *Physical Review A* **94**, 012109 (2016).
- [8] J. Rojas-Arias, P. Stano, Y.-H. Wu, L. Camenzind, S. Tarucha, and D. Loss, Noise cross-correlations from single-shot measurements, *PRX Quantum* **7**, 020351 (2026).
- [9] F. Budde, T. F. Heinz, M. M. T. Loy, J. A. Misewich, F. de Rougemont, and H. Zacharias, Femtosecond time-resolved measurement of desorption, *Physical Review Letters* **66**, 3024 (1991).
- [10] S.-Y. Hong, P. Xu, N. R. Camillone, M. G. White, and N. Camillone, Adlayer structure dependent ultrafast desorption dynamics in carbon monoxide adsorbed on pd(111), *The Journal of Chemical Physics* **145**, 014704 (2016).
- [11] R. Bombín, A. S. Muzas, A. Serrano Jiménez, J. I. Juaristi, and M. Alducin, Asymmetry and coverage dependence in two-pulse correlation measurements of co photodesorption from pd(111): Insights from theory, *The Journal of Chemical Physics* **164**, 064709 (2026).
- [12] Supplemental material for ordered absorption reveals green-kubo memory hidden from survival curves (2026), URL will be inserted by the publisher.
- [13] É. Roldán and J. M. R. Parrondo, Estimating dissipation from single stationary trajectories, *Physical Review Letters* **105**, 150607 (2010).
- [14] I. Di Terlizzi, M. Gironella, D. Herráez-Aguilar, T. Betz, F. Monroy, M. Baiesi, and F. Ritort, Variance sum rule for entropy production, *Science* **383**, 971 (2024).
- [15] I. Di Terlizzi, M. Gironella, D. Herráez-Aguilar, T. Betz, F. Monroy, M. Baiesi, and F. Ritort, Data from: Variance sum rule for entropy production (2024).
- [16] P. D. Harris, A. Narducci, C. Gebhardt, T. Cordes, S. Weiss, and E. Lerner, Multi-parameter photon-by-photon hidden markov modeling, *Nature Communications* **13**, 1000 (2022).
- [17] P. D. Harris, A. Narducci, C. Gebhardt, T. Cordes, S. Weiss, and E. Lerner, Multi-parameter photon-by-photon hidden markov modeling dataset (2022).
- [18] C. J. CLOPPER and E. S. PEARSON, The use of confidence or fiducial limits illustrated in the case of the binomial, *Biometrika* **26**, 404 (1934).
- [19] D. Pollard, A note on insufficiency and the preservation of fisher information, in *Institute of Mathematical Statistics Collections* (Institute of Mathematical Statistics, 2013) pp. 266–275.
- [20] B. Vermeer, J. van Ossenbruggen, and S. Schmid, Single-molecule fret-resolved protein dynamics - from plasmid to data in six steps (2023).