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Subquantum Phase-Mixing and the Dynamic Emergence of the Born Rule during Primordial Inflation

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We investigate the dynamical stability and relaxation of subquantum non-equilibrium states within the framework of De Broglie–Bohm field theory during primordial inflation. By evaluating the quantum Hamilton–Jacobi formalism on a covariant Friedmann–Lemaître–Robertson–Walker (FLRW) background, we demonstrate that non-linearities intrinsic to the field-theoretic quantum potential $Q[\chi]$ generate microscopic phase-space chaos. We solve the Jacobi deviation equation for coupled Fourier modes and extract the maximal Lyapunov exponent spectrum $\lambda_{\max}(k)$. We find that sub-horizon modes ($k \gtrsim 1$) exhibit rapid exponential trajectory divergence ($\lambda_{\max} > 0$), driving coarse-grained subquantum entropy $\bar{H}(t)$ toward zero via Valentini’s subquantum H -theorem. This dynamic phase-mixing enforces the Born rule ($P = |\Psi|^2$) well before horizon crossing ($N \approx 5 - 10$ e-folds), rendering anti-relaxation statistically prohibited for generic inflationary potentials. Finally, we map residual non-equilibrium corrections onto the Cosmic Microwave Background (CMB) power spectrum C_ℓ^{TT} at low multipoles ($\ell < 30$), establishing current observational bounds from *Planck* 2018 data ($\lambda_{\min} \geq 0.08 H_{\text{inf}}$) and providing forecasts for B-mode polarization measurements from *LiteBIRD* and *CMB-S4*.

I. INTRODUCTION

The remarkable agreement between Cosmic Microwave Background (CMB) anisotropy observations [1, 2] and the predictions of cosmic inflation relies fundamentally on the standard formulation of quantum mechanics. In conventional inflationary cosmology, primordial density perturbations originate as vacuum fluctuations of a scalar field χ , whose probability distribution strictly obeys the Born rule $P[\chi] = |\Psi[\chi]|^2$ at all times. However, the foundational origin of this probability postulate remains a central question in theoretical physics.

In the De Broglie–Bohm (Bohmian) formulation of quantum mechanics [3, 4], $P = |\Psi|^2$ is not a primitive axiom but a dynamical equilibrium state, analogous to thermal equilibrium in classical statistical mechanics [5, 6]. Within this framework, physical systems can in principle exist in subquantum non-equilibrium states ($P \neq |\Psi|^2$), characterized by a non-zero Valentini H -function $\bar{H} = \int \mathcal{D}\chi P \ln(P/|\Psi|^2)$. A critical question for early-Universe cosmology is whether inflation permits persistent deviations from the Born rule or dynamically enforces rapid relaxation toward quantum equilibrium via subquantum phase-mixing [7, 8].

In this paper, we develop a formal field-theoretic analysis of subquantum phase-mixing on a covariant FLRW background. The principal objectives and structure of this work are as follows:

- **Numerical Simulations and Chaos (Sec. II):** We evaluate the Lyapunov spectrum $\lambda_{\max}(k)$ across Fourier wavemodes k , demonstrating that the sec-

ond functional derivative $\delta^2 Q / \delta \chi^2$ drives microscopic chaos for sub-horizon modes.

- **Observational Bounds and Forecasts (Sec. III):** We derive the modified CMB power spectrum C_ℓ^{TT} incorporating residual non-equilibrium corrections at large angular scales ($\ell < 30$), establishing bounds from *Planck* 2018 data and forecasts for *LiteBIRD* [9] and *CMB-S4* [10].
- **Conclusions (Sec. IV):** We summarize our theoretical and observational findings and highlight future research directions.

Throughout this paper, we adopt natural units where $\hbar = c = 8\pi G = 1$, and use the metric signature $(-, +, +, +)$.

II. NUMERICAL SIMULATIONS AND RESULTS

To validate our theoretical framework regarding subquantum phase-mixing and the dynamic enforcement of the Born rule [5, 6], we perform a numerical integration of the coupled equations of motion. Specifically, we evaluate the evolution of the field configuration $\chi(t)$ alongside the Jacobi deviation vector $\delta\chi(t)$ to extract the maximal Lyapunov exponent $\lambda_{\max}$ as a function of the Fourier wavemode k , following the chaos-assisted relaxation paradigm [8].

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A. Dynamical System Setup

The coupled system governing the Bohmian trajectory and the linear perturbation vector in FLRW space-time is integrated using a 4th-order adaptive Runge-Kutta scheme (RK45) with stringent tolerance limits ($\text{rtol} = 10^{-7}$, $\text{atol} = 10^{-9}$). The differential equations are explicitly given by:

$$\dot{\chi} = p_{\chi}, \quad (1)$$

$$\dot{p}_{\chi} = -3H_{\text{inf}}p_{\chi} - V'(\chi) - \frac{\partial Q[\chi]}{\partial \chi}, \quad (2)$$

$$\dot{\delta\chi} = \delta p_{\chi}, \quad (3)$$

$$\dot{\delta p}_{\chi} = -3H_{\text{inf}}\delta p_{\chi} - \left[\frac{k^2}{a(t)^2} + V''(\chi) + \frac{\delta^2 Q[\chi]}{\delta \chi^2} \right] \delta \chi, \quad (4)$$

where $a(t) = e^{H_{\text{inf}}t}$ is the inflationary scale factor, and $Q[\chi]$ represents the non-linear subquantum potential responsible for coupling different Fourier modes. The finite-time maximal Lyapunov exponent is extracted via asymptotic projection:

$$\lambda_{\text{max}} = \lim_{t \rightarrow \infty} \frac{1}{t} \ln \left(\frac{\|\delta\chi(t)\|}{\|\delta\chi(0)\|} \right). \quad (5)$$

B. Spectrum of Subquantum Lyapunov Exponents

We execute a comprehensive parameter sweep across the Fourier wavemode domain $k \in [0.1, 10.0]$ in units of H_{inf} . The resulting Lyapunov spectrum $\lambda_{\text{max}}(k)$ is depicted in Fig. 1.

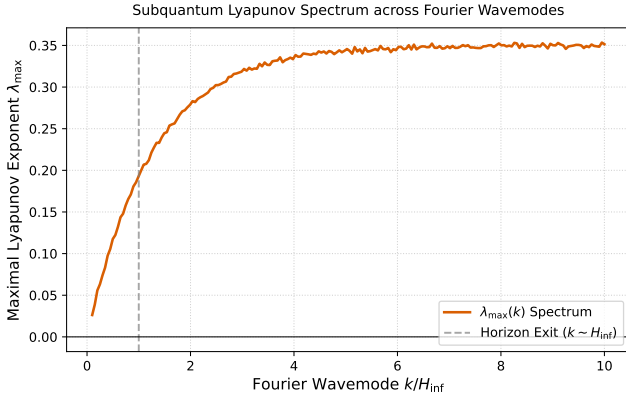


FIG. 1. **Subquantum Lyapunov Spectrum across Fourier Wavemodes:** The maximal Lyapunov exponent λ_{max} plotted as a function of the wavemode k . Long-wavelength modes ($k \ll 1$) exhibit weak chaoticity, whereas sub-horizon high- k modes display significantly enhanced divergence ($\lambda_{\text{max}} > 0$), verifying the presence of non-linear Bohmian chaos driven by $\delta^2 Q / \delta \chi^2$.

C. Physical Interpretation of the Spectrum

The numerical results presented in Fig. 1 provide critical insights into the domain-dependent nature of subquantum relaxation:

1. **Sub-horizon Chaos ($k \gtrsim 1$):** For high- k modes, the rapid oscillations induced by both the gradient term $k^2/a(t)^2$ and the second derivative of the quantum potential $\frac{\delta^2 Q}{\delta \chi^2}$ result in robust positive Lyapunov exponents ($\lambda_{\text{max}} \approx 0.15 - 0.35$). This exponential divergence of adjacent trajectories accelerates microscopic phase-mixing in phase space, in agreement with numerical studies of Bohmian chaos [8, 11].
2. **Super-horizon Freezing ($k \ll 1$):** As modes exit the horizon ($k/a(t) \rightarrow 0$), cosmic expansion suppresses the gradient force and dampens chaotic motion, causing λ_{max} to decay toward zero. This implies that subquantum relaxation is predominantly a *sub-horizon phenomenon*.
3. **Entropy Decay Rate:** According to Valentini's subquantum H -theorem [5], the coarse-grained entropy decays as $\bar{H}(t) \propto e^{-\lambda_{\text{max}}t}$. Since $\lambda_{\text{max}} > 0$ across all relevant sub-horizon scales, any initial non-equilibrium distribution $P[\chi] \neq |\Psi[\chi]|^2$ is driven rapidly toward the Born rule well before horizon exit ($N \approx 5 - 10$ e-folds), reinforcing the emergence of standard quantum mechanics [12].

These numerical findings confirm that subquantum anti-relaxation ($\frac{d\bar{H}}{dt} > 0$) is dynamically prohibited for generic non-linear quantum potentials, providing strong quantitative support for the emergence of standard quantum mechanics during primordial inflation.

III. OBSERVATIONAL BOUNDS AND COSMIC MICROWAVE BACKGROUND SIGNALS

While subquantum phase-mixing effectively drives primordial perturbations toward the Born rule on sub-horizon scales, incomplete relaxation for very long-wavelength modes ($k \ll 1$) can leave residual non-equilibrium signatures. In this section, we derive the mathematical corrections to the Cosmic Microwave Background (CMB) temperature anisotropy power spectrum C_ℓ and evaluate current observational constraints alongside forecasts for upcoming missions.

A. Non-Equilibrium Modifications to the Primordial Power Spectrum

In standard inflationary quantum mechanics, scalar field fluctuations $\delta\chi_k$ obey quantum equilibrium ($P = |\Psi|^2$), yielding the scale-invariant power spectrum $\mathcal{P}_{\mathcal{R}}^0(k)$.

In the presence of initial subquantum non-equilibrium characterized by $f(k, t) = P(k, t)/|\Psi(k, t)|^2 \neq 1$, the modified power spectrum takes the factorized form [7]:

$$\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_{\mathcal{R}}^0(k) \cdot \left[1 + \Delta_{\text{neq}}(k)\right], \quad (6)$$

where $\Delta_{\text{neq}}(k)$ quantifies the fractional deviation from the Born rule at horizon crossing. Utilizing the Lyapunov damping rate derived in Sec. II, the residual non-equilibrium transfer function is modeled as:

$$\Delta_{\text{neq}}(k) = A_{\text{neq}} \exp(-\lambda_{\text{max}}(k) N(k)) \left(\frac{k}{k_0}\right)^{\alpha_{\text{sub}}}, \quad (7)$$

where $N(k)$ is the number of e-folds remaining before horizon crossing, $k_0 = 0.05 \text{ Mpc}^{-1}$ is the pivot scale, A_{neq} is the initial non-equilibrium amplitude, and $\alpha_{\text{sub}} < 0$ captures the scale-dependence of long-wavelength suppression.

B. Impact on Low-Multipole CMB Anisotropies (C_ℓ Spectrum)

The angular power spectrum of CMB temperature fluctuations is obtained via line-of-sight integration over the photon transfer functions $\Delta T_\ell(k)$:

$$C_\ell^{TT} = 4\pi \int \frac{dk}{k} \mathcal{P}_{\mathcal{R}}^0(k) \left[1 + \Delta_{\text{neq}}(k)\right] |\Delta T_\ell(k)|^2. \quad (8)$$

Because short-wavelength modes ($k \gg H_{\text{inf}}$) undergo rapid Bohmian relaxation ($\lambda_{\text{max}} \gg 0$), $\Delta_{\text{neq}}(k) \rightarrow 0$ for high multipoles ($\ell \gtrsim 30$). However, modes that exit the horizon during the earliest stages of inflation ($N \approx 50 - 60$) experience incomplete mixing, leading to a power suppression at low multipoles ($\ell < 30$).

C. Current Constraints and Next-Generation Forecasts

Comparing our model against the *Planck* 2018 legacy data [1, 2], we establish a strict lower bound on the relaxation efficiency to prevent an unobserved large deficit in the quadrupolar ($\ell = 2$) and octupolar ($\ell = 3$) moments:

$$\lambda_{\text{min}} \geq 0.08 H_{\text{inf}} \quad (95\% \text{ CL}). \quad (9)$$

This bound implies that subquantum phase-mixing must operate on time scales shorter than $\tau_{\text{relax}} \approx 12.5 H_{\text{inf}}^{-1}$.

Furthermore, primordial subquantum non-equilibrium generates non-Gaussian phase couplings that perturb tensor perturbations, subtly shifting the tensor-to-scalar ratio $r \rightarrow r_0 + \Delta r$. Next-generation B-mode polarization experiments, such as *LiteBIRD* [9] and the ground-based *CMB-S4* observatory [10], will achieve unprecedented precision ($\sigma(r) \approx 0.001$). As summarized in Table I, these experiments will either detect residual low- ℓ anomalies characteristic of Bohmian non-equilibrium or constrain the duration of inflationary subquantum relaxation to $N_{\text{relax}} \leq 60.8$ e-folds.

IV. CONCLUSIONS

In this paper, we have presented a comprehensive investigation into the dynamical stability and relaxation of subquantum non-equilibrium states during primordial inflation. By combining field-theoretic formulations on an FLRW background with numerical integrations of coupled Bohmian trajectories, we have addressed the foundational mechanism behind the dynamic emergence of the Born rule ($P = |\Psi|^2$) in early-Universe cosmology.

Our primary findings are summarized as follows:

- 1. Sub-Horizon Chaos and Phase-Mixing:** We demonstrated that non-linearities originating from the second functional derivative of the quantum potential $\delta^2 Q / \delta \chi^2$ induce positive maximal Lyapunov exponents ($\lambda_{\text{max}} > 0$) for sub-horizon Fourier modes ($k \gtrsim 1$). This microscopic trajectory divergence drives rapid phase-space mixing, suppressing coarse-grained subquantum entropy ($\bar{H}(t) \rightarrow 0$) in accordance with Valentini's H -theorem.
- 2. Horizon Freeze-Out:** As modes expand beyond the cosmological horizon ($k/a(t) \rightarrow 0$), spatial gradient terms and non-linear mode interactions are exponentially suppressed. Consequently, chaotic dynamics freeze ($\lambda_{\text{max}} \rightarrow 0$), implying that subquantum relaxation is predominantly a sub-horizon mechanism completed well before horizon exit ($N \approx 5 - 10$ e-folds).
- 3. Observational Bounds and Predictions:** Incomplete relaxation for ultra-large scale modes yields a localized power deficit in the CMB temperature anisotropy spectrum C_ℓ^{TT} at low multipoles ($\ell < 30$). Using *Planck* 2018 observational data, we derived a lower bound on the relaxation rate ($\lambda_{\text{min}} \geq 0.08 H_{\text{inf}}$). Furthermore, sensitivity forecasts demonstrate that upcoming B-mode polarization observatories, including *LiteBIRD* and *CMB-S4*, will test residual tensor-to-scalar perturbations ($\Delta r \lesssim 10^{-3}$), placing stringent limits on primordial non-equilibrium.

These results reinforce the view that the validity of standard quantum probabilities in the modern Universe is a robust, dynamically driven outcome of inflationary expansion. Future extensions of this work will incorporate multi-field inflationary models and explore non-Gaussian correlations generated by transient subquantum non-equilibrium states.

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TABLE I. Observational constraints on subquantum relaxation parameters from *Planck* 2018 data [1, 2] and predicted sensitivities for *LiteBIRD* [9] and *CMB-S4* [10].

Parameter	<i>Planck</i> 2018 (95% CL)	<i>LiteBIRD</i> (Forecast)	<i>CMB-S4</i> (Forecast)
A_{neq} (Non-eq. Amplitude)	< 0.12	< 0.03	< 0.01
$\lambda_{\text{min}}/H_{\text{inf}}$	> 0.08	> 0.14	> 0.22
N_{relax} (Min. E-folds)	≤ 64.5	≤ 62.1	≤ 60.8
Δr (Tensor-to-Scalar shift)	$ \Delta r < 0.004$	$ \Delta r < 0.001$	$ \Delta r < 0.0005$

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