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Prediction Model for the Dispersion Pattern of Pre-Controlled Fragments from a Multi-Layer Ring Casing

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Abstract : Aiming at the prediction of fragment dispersion pattern of pre-controlled fragments under explosive loading, this paper investigates the influencing factors and a prediction model for the projection angle of fragments from a multi-layer ring casing. Through numerical simulations using AUTODYN-3D software, the effects of various parameters such as different explosive types and charge dimensions on the fragment dispersion of the multi-layer ring casing are analyzed. A prediction model for the fragment projection angle is proposed based on the physical mechanism of fragment deflection driven by axial velocity gradient. A new prediction framework is established by introducing the concept of fragment stabilization time and an axial position correction factor, followed by experimental validation. The results show that the new model accurately captures the characteristics of the fragment projection angle, which is negative near the initiation end, positive near the non-initiation end, and exhibits large angular gradients at both ends while becoming gentle in the middle. The error between the model predictions and experimental data is less than 15%, and the model is applicable to various pre-controlled or pre-formed fragment warheads. The study improves the accuracy and universality of predicting the fragment projection angle under explosive loading.

Keywords: detonation wave; metal casing; fragment dispersion model; internal explosive loading; numerical simulation;

1. Introduction

Pre-controlled fragmentation warheads are an important component of various modern munitions. Among the existing pre-controlled techniques, the multi-layer ring casing structure has been widely adopted due to its advantages such as controllable axial fragment dimensions, good mass uniformity, and low manufacturing cost^[1-4]. The terminal effectiveness of this type of warhead is directly determined by the fragment dispersion behavior under explosive loading^[5]. Therefore, accurately understanding the fragment dispersion law of the multi-layer ring casing under detonation loading is an important engineering problem in the interdisciplinary field of energetic materials application and explosion mechanics, and can also provide guidance for safety analysis of sympathetic detonation of cased explosives in military and industrial applications..

Although research on the dispersion law of pre-controlled fragments from multi-layer ring casings is limited, the dispersion law of natural fragments has been relatively well established. The calculation of the fragment projection angle requires the initial velocity. Regarding the study of initial velocity, since the proposal of the Gurney formula ^[6], numerous researchers have conducted extensive investigations and achieved many results, including the Zulkoski formula ^[7] and the Charron model ^[8] that account for the effect of axial rarefaction waves, the Huang formula ^[9] fitted based on the Gurney formula and experimental data, the Gao model ^[10] which corrects the effect of axial rarefaction waves on the initial velocity distribution of fragments by reducing the effective charge mass, as well as the Liu formula ^[11] and the Bi formula ^[12] fitted for calculating the axial distribution of initial fragment velocity of warheads with different length-to-diameter ratios. In addition, there are studies that use machine learning to determine the initial fragment velocity, such as the work by Noh et al. ^[13].

Regarding the projection angle of fragments, Taylor ^[14] first proposed a theoretical analysis method for the fragment projection angle of a cylindrical charge detonated at one end and established the Taylor formula. To improve the prediction accuracy of the fragment projection angle, many scholars ^{[15][16][17]} have modified the Taylor formula. Wang et al. ^[18] considered the effect of fragment acceleration and added a correction term related to acceleration to the Taylor formula.

$$\delta = \frac{V_0}{2D_s} \sin \alpha_e - \frac{1}{2} \tau_e V' \quad (1)$$

In the formula, τ_e is the fragment acceleration time, and V' is the derivative of the fragment velocity along the warhead axis. Although this formula is more accurate than the Taylor formula, it has significant limitations because τ_e and V' are difficult to obtain.

Felix et al. ^{[19][20]} established a model for calculating the fragment projection direction based on the expansion shape of a cylindrical casing under detonation loading.

$$\begin{cases} \delta = \left(\frac{180}{\pi}\right) \left(\frac{1}{\lambda}\right) \tan^{-1} \left(-0.33 \times 4 \left(\frac{(x-M)^3}{M^4}\right)\right) + \theta & (0 \leq x \leq M) \\ \delta = \left(\frac{180}{\pi}\right) \left(\frac{1}{\lambda}\right) \tan^{-1} \left(-0.33 \times 4 \left(\frac{(x-M)^3}{(1-M)^4}\right)\right) + \theta & (M \leq x \leq M1) \end{cases} \quad (2)$$

In the formula, λ is the casing expansion coefficient measured at the initiation end; x is the axial position relative to the detonation end; M is assumed to be 0.6, meaning that both the casing expansion and the fragment velocity reach their maximum values at the relative axial position of 0.6. In this formula, λ and M are both obtained by assumption, and the research object is a cylindrical casing made of a specific material. Therefore, its applicability to the multi-layer ring casing studied in this paper requires further investigation.

Jiang et al. [21] considered the effect of axial rarefaction on the fragment dispersion distribution and obtained a formula for calculating fragment dispersion with high accuracy by fitting experimental data.

$$\frac{\theta}{2} = \left[-0.138 + \sinh\left(\frac{x}{d} 0.409\right) \right] \cdot \left[4.415 + \sinh\left(\frac{1.47(L-x)}{d}\right) \right] \cdot \frac{180}{\pi} \cdot \tan^{-1} \cdot \left(\frac{V_0}{2D_s} \cdot \sin \alpha \right) \quad (3)$$

In the formula, $\sinh(x) = (e^x - e^{-x})/2$, Although this formula reduces the prediction error of the fragment projection angle at the initiation end, making the calculation more accurate, the research model is mainly based on spherical pre-formed fragments with a liner, and its compatibility with pre-controlled fragments from a multi-layer ring casing requires further validation.

The aim of this paper is to investigate the dispersion law of pre-controlled fragments from a multi-layer ring casing through numerical simulation, and to propose a new model capable of accurately predicting the fragment projection angle for this type of warhead based on theoretical derivation, taking advantage of the determined axial characteristic dimensions of the multi-layer ring casing fragments. The constant terms in the model are determined through numerical simulation. Finally, the accuracy of the model and its universality in predicting the fragment projection angles of other types of fragment warheads are verified by experimental data.

2. Numerical Simulation Study

This section uses numerical simulation to analyze the dispersion characteristics of pre-controlled fragments from a multi-layer ring casing and the effects of various parameters on the fragment projection angle. The coupling relationships between the projection angle and factors such as axial groove depth, casing material, length-to-diameter ratio, ring height, and explosive type are investigated.

2.1 Computational Scheme and Model

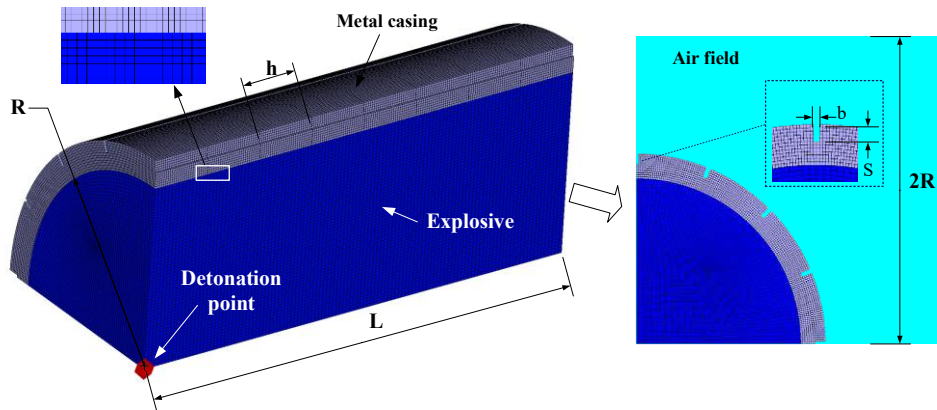


Fig. 1. Numerical Simulation Model

In this study, a quarter model of the multi-layer ring casing warhead was established for numerical simulation using AUTODYN-3D software [22][23]. As shown in Fig. 1, the explosive charge has a radius of R (diameter D) and a length of L ; the height of each ring layer is h , and the total height of the casing is also L . Grooves with rectangular cross-section are machined on the outer surface of the casing, with a groove width of b and a groove depth of S . The axial relative groove depth s is defined as the ratio of the groove depth S to the casing thickness, and the casing thickness is 6 mm.

A total of 21 computational cases are employed in this numerical simulation study to investigate the effects of various factors on the fragment projection angle of the multi-layer ring casing warhead: length-to-diameter ratio (Cases 1–9), ring height (Cases 1–7, 10–13), groove depth (Cases 3, 14, 15), explosive type (Cases 16–18), and casing material (Cases 3, 19–21). The detailed data for each case are listed in Table 1.

Table 1. Numerical Simulation Cases

Cases	L/D	h/L	s	Explosive type	Casing material
1	0.625	0.25	0.42	Comp B	Steel 45
2	0.9375	0.1667	0.42	Comp B	Steel 45
3	1.25	0.125	0.42	Comp B	Steel 45
4	1.5625	0.1	0.42	Comp B	Steel 45
5	1.875	0.0833	0.42	Comp B	Steel 45
6	2.5	0.0625	0.42	Comp B	Steel 45
7	2.0	0.125	0.42	Comp B	Steel 45
8	1.667	0.125	0.42	Comp B	Steel 45
9	1.0	0.125	0.42	Comp B	Steel 45
10	1.25	0.067	0.42	Comp B	Steel 45
11	1.25	0.1	0.42	Comp B	Steel 45
12	1.25	0.167	0.42	Comp B	Steel 45
13	1.25	0.25	0.42	Comp B	Steel 45
14	1.25	0.125	0.17	Comp B	Steel 45
15	1.25	0.125	0.92	Comp B	Steel 45
16	2.5	0.0833	0.42	Comp B	Steel 45
17	2.5	0.0833	0.42	TNT	Steel 45
18	2.5	0.0833	0.42	PBX9502	Steel 45
19	1.25	0.125	0.42	Comp B	Steel 4340
20	2.5	0.0833	0.42	Comp B	Steel 4340
21	1.25	0.25	0.42	Comp B	Steel 4340

In the numerical simulation, the casing is discretized with Lagrange elements, while the air and explosive are filled in the Eulerian domain. Based on a comprehensive analysis of Refs. [22][24][25], it is found that the casing mesh size generally ranges from 0.2 mm to 1.0 mm,

and the explosive mesh size is relatively uniform, typically 0.8 mm. Therefore, Case 3 is adopted to conduct a mesh convergence analysis for this study. Five different casing mesh sizes (0.3 mm, 0.45 mm, 0.6 mm, 0.75 mm, and 0.9 mm) are set as mesh1 through mesh5, and the most appropriate mesh size is selected in combination with the simulation time.

The results are shown in Fig. 2. According to Fig. 2 (b), mesh 2 is selected as the mesh size for the numerical simulation, i.e., the casing mesh size is 0.45 mm and the Eulerian domain mesh size is 0.8 mm.

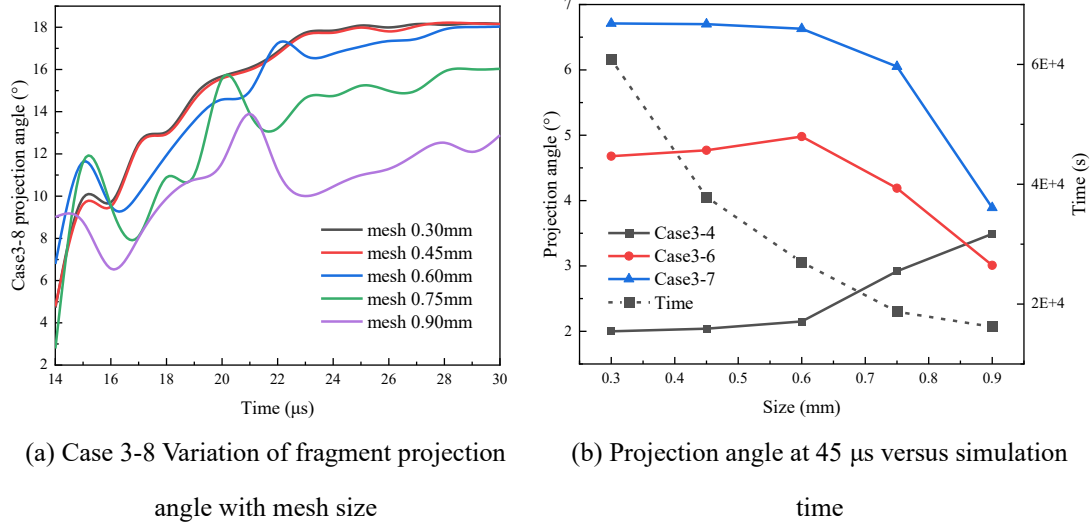


Fig. 2. Mesh convergence analysis

2.2 Material Model

In this study, the multi-layer ring casing is made of 45 steel and 4340 steel [26], and its material model employs the Johnson-Cook strength model, which is given as:

$$\sigma_y = [A_1 + B_1(\bar{\epsilon}^p)^n](1 + C \ln \dot{\epsilon}^*)[1 - (T^*)^m] \quad (4)$$

In the formula, σ_y is the flow stress, in MPa; A_1 is the yield stress, in MPa; B_1 is the hardening strength, in MPa; $(\bar{\epsilon}^p)$ is the equivalent plastic strain; n is the strain hardening exponent; C is the strain rate sensitivity factor; $\dot{\epsilon}^*$ is the relative equivalent plastic strain rate; $T^* = (T - T_r) / (T_m - T_r)$, where T represents the absolute temperature, T_r is the room temperature, and T_m is the melting temperature, in K; m is the thermal softening exponent. The specific parameters are listed in Table 2.

The Johnson-Cook failure model can reflect the effects of strain rate, stress triaxiality and temperature, and effectively describe the failure and fracture process of the casing. The failure strain can be expressed as:

$$\epsilon^f = (D_1 + D_2 \cdot \exp D_3 \sigma^*)(1 + D_4 \ln \dot{\epsilon}^*)(1 - D_5 T^*) \quad (5)$$

In the formula, ϵ^f is the failure strain; σ^* is the ratio of pressure to effective stress; D_1 - D_5 is the failure parameter, and its specific values are shown in Table 3.

Table 2. Johnson-Cook model parameters of the casing [27][28]

Material	ρ_0 ($\text{g}\cdot\text{cm}^{-3}$)	A_1 (MPa)	B_1 (Pa)	n	C	m	T_m (K)	T_r (K)
steel 45	7.83	553.1	600.8	0.234	0.0134	1.00	1733	300
steel 4030	7.83	792	510	0.26	0.014	1.03	1793	300

Table 3. Johnson-Cook failure parameters of the casing [27][28]

Material	D_1	D_2	D_3	D_4	D_5
steel 45	0.06	3.31	-1.96	0.0018	0.58
steel 4030	0.05	3.44	-2.12	0.002	0.61

The explosive adopts the JWL equation of state to describe the expansion process of detonation products. The JWL equation is shown as follows:

$$P = A_2 \left(1 - \frac{\omega}{R_1 V}\right) e^{-R_1 V} + B_2 \left(1 - \frac{\omega \eta}{R_2}\right) e^{-R_2 V} + \frac{\omega E}{V} \quad (6)$$

In the formula, P is the pressure of the explosive, in Mbar; V is the volume of the reaction products; E is the internal energy of the explosive per unit volume; A_2 is the pressure coefficient for the high-pressure region; B_2 is the pressure coefficient for the medium-pressure region.

In the numerical simulation, the specific parameters of the explosives are sourced from the material library of AUTODYN. The parameters of the three types of explosives are shown in Table 4:

Table 4. Explosive Material Parameters

Material	P ($\text{g}\cdot\text{cm}^{-3}$)	D ($\text{m}\cdot\text{s}^{-1}$)	A (Mbar)	B (Mbar)	R_1	R_2	ω
Comp B	1.717	7980	8.52	0.18	4.6	1.3	0.38
TNT	1.63	6930	3.738	0.0375	4.15	0.9	0.35
PBX9502	1.895	7710	4.603	0.0954	4.0	1.7	0.48

2.3 Numerical Simulation Results and Analysis of Influencing Factors

The formation process of fragments from the multi-layer ring casing under detonation loading is shown in Fig. 3. Due to the segmented nature, the axial casing separates directly and disperses toward both ends as the detonation proceeds, which is similar to the formation of pre-formed fragments.

It can be observed from Fig. 3 that during fragmentation, the fragments not only translate away from the axis but also rotate towards the ends, and this phenomenon is more pronounced near the ends. The overall velocity direction of the fragments changes with the fragment rotation within 0–36 μs . However, during the interval of 36–48 μs , although the fragment attitude still changes, the overall velocity direction remains essentially unchanged, indicating that the

velocity direction of fragments varies with their rotation at the initial stage of driving and tends to stabilize with increasing time.

It can also be observed from Fig. 3 that the fragment velocity exhibits a distribution characteristic of being low at both ends and high in the middle along the axial direction. To investigate the underlying causes, taking Case 7 as an example, Fig. 4 presents the pressure contours on the inner surface of the casing at the initial stage of detonation and the pressure histories at typical observation points (observation points 1–8 are moving observation points located on the inner surface of the casing; observation points 9–16 are fixed observation points in the explosive).

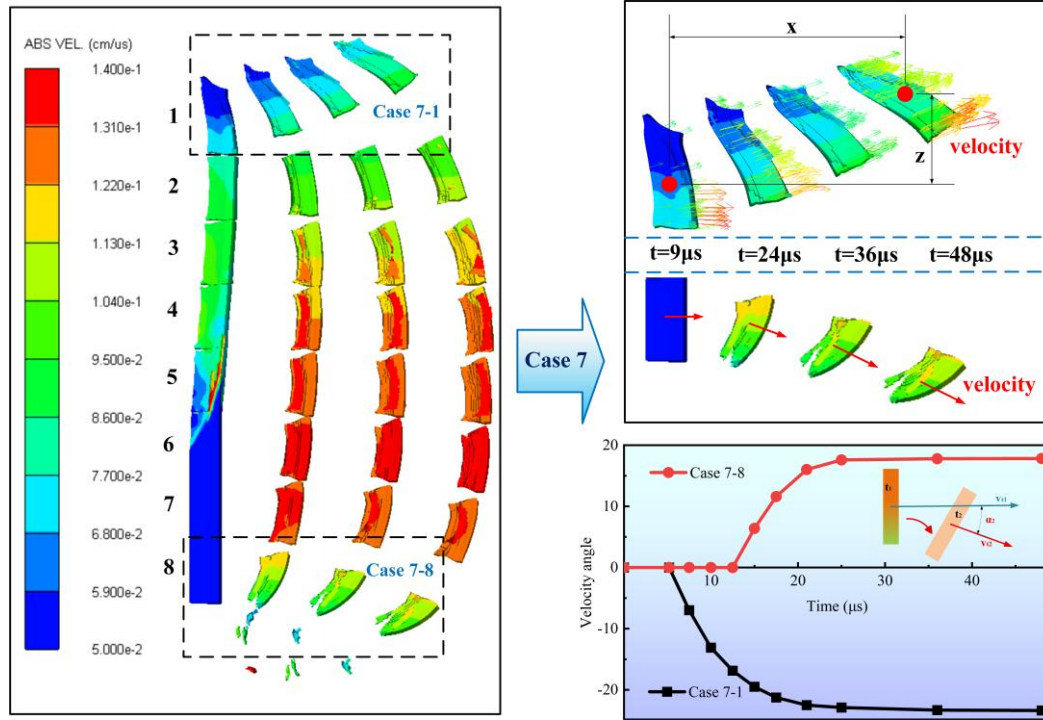


Fig. 3. Formation process of fragments from the multi-layer ring pre-controlled casing

It can be seen from Fig. 4 that for a cylindrical warhead with single-end single-point initiation, the pressure on the inner wall varies significantly along the axial direction: it reaches a maximum of approximately 23.765 GPa at the initiation end and a minimum of approximately 21.003 GPa near the non-initiation end, with a difference of about 11.6%. The pressure acting on the casing from the detonation wave generally tends to be slightly higher near the initiation end and slightly lower near the non-initiation end. This difference arises from the varying incidence and reflection ratios of the C-J detonation wave on the casing at different axial positions; that is, as the axial position approaches the non-initiation end, the incidence angle increases, resulting in a certain degree of decrease in the pressure acting on the inner surface.

Therefore, the factor that truly dominates the differences in fragmentation characteristics

(velocity and angle) at different axial positions is not the difference in peak pressure itself, but rather the effect of the axial rarefaction wave. As shown by the impulse per unit area obtained by integrating the overpressure in Fig. 4, the impulse along the axial direction also exhibits a distribution of being low at both ends and high in the middle, with the maximum value occurring at the 6th fragment (approximately $5.065 \text{ Mbar} \cdot \mu\text{s}$) and the minimum value at the initiation end (approximately $2.3289 \text{ Mbar} \cdot \mu\text{s}$). This is because the axial rarefaction wave reduces the duration of overpressure acting on the casing, thereby weakening the driving effect on the fragments at both ends, which in turn affects their velocity magnitude and subsequent dispersion behavior.

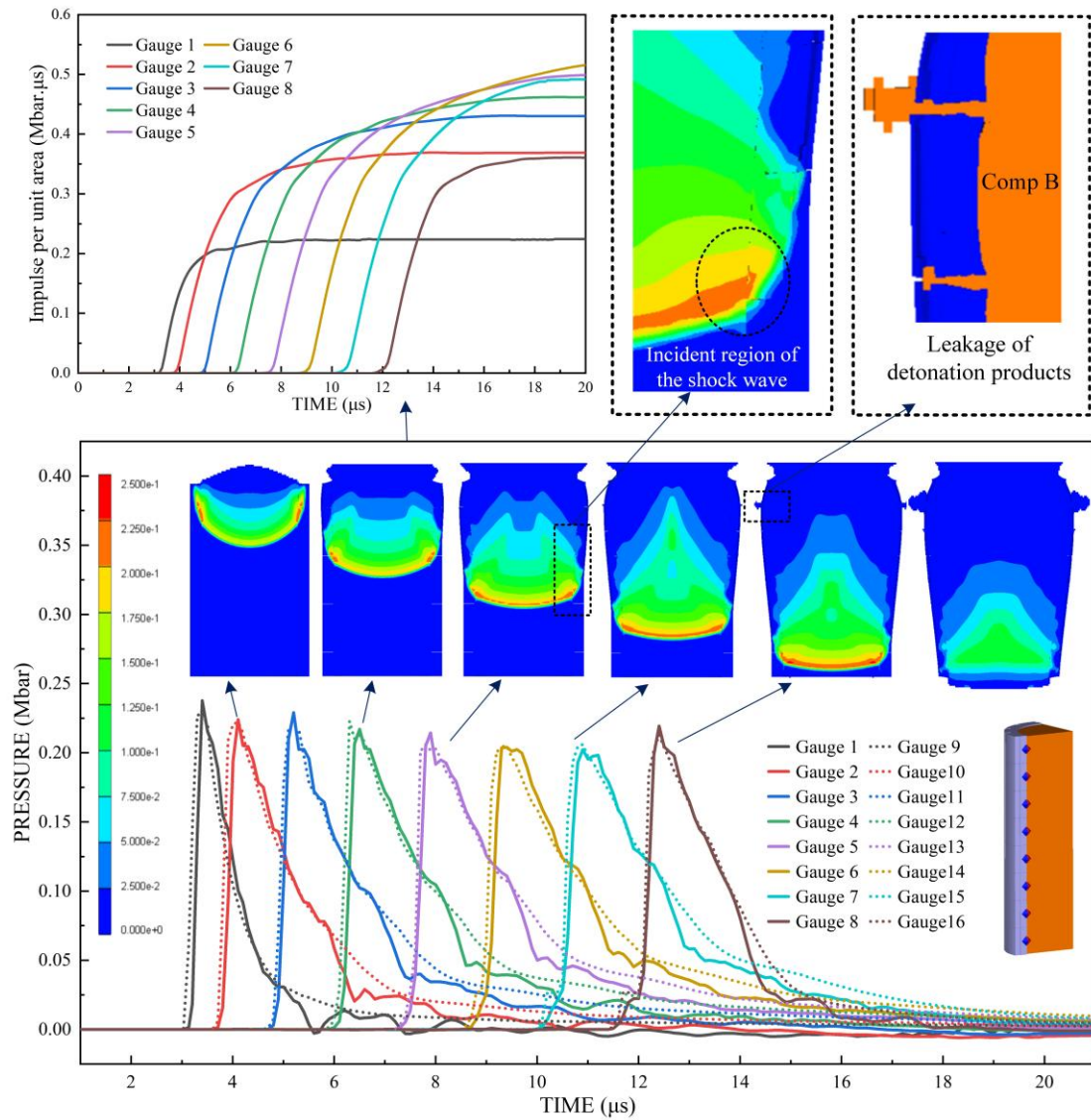


Fig. 4. Pressure contours at the initial stage of detonation and pressure-time curves at the observation points

Fig. 5 shows the effects of ring height and material on fragment formation and dispersion,

presenting the fragment morphologies and velocity contours of Cases 3, 11–13, and 21 at 42 μ s. At this moment, the fragment velocity reaches its maximum value and the projection angle has stabilized. Among these cases, Cases 11–13 vary the ring height based on Case 3, and Case 21 varies the material based on Case 13. It can be observed from the figure that under various conditions, the initial fragment velocity is higher in the middle and lower at both ends. Moreover, when the ring height is excessively large, the ring undergoes axial fragmentation, altering the fragment formation pattern [24][29][30].

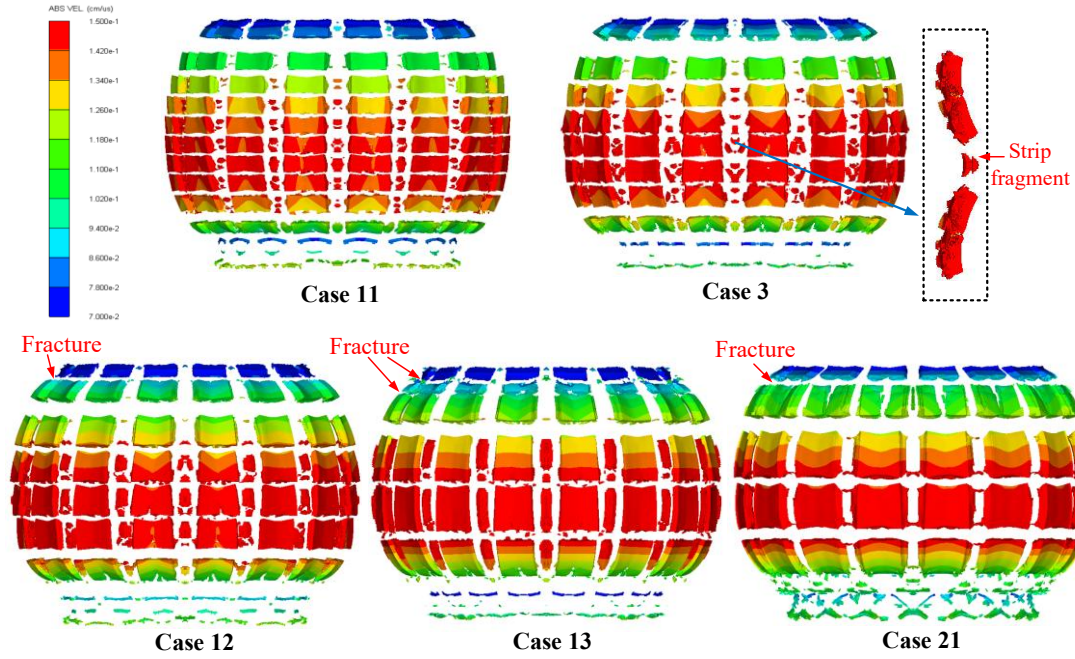


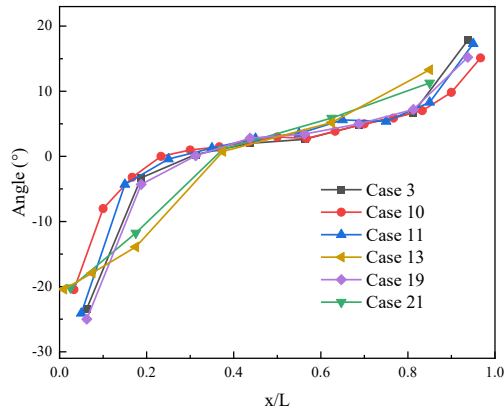
Fig. 5. Effects of ring height and material on fragment formation and dispersion

Fig. 6 shows the simulation results of the projection angle and velocity for different operating conditions. Specifically, Fig. 6 (a) includes Cases 3, 10–13, 19, and 21, which are used to analyze the effects of ring height and casing material on the fragment projection angle. It can be observed from the figure that when the ring does not undergo axial fracture, a smaller ring height results in a smaller fragment projection angle, and this phenomenon is more pronounced at the initiation end and the non-initiation end but less obvious in the middle section. Moreover, there is almost no difference in the fragment projection angle between 4340 steel and 45 steel. The reason is that the projection angle is dominated by the charge loading gradient; as shown in Fig. 3 and Fig. 6. the plastic flow of the casing does not change the velocity direction difference. Therefore, for the multi-layer ring casing warhead, if different steels are used as the casing material, the casing material does not alter the fragment projection angle.

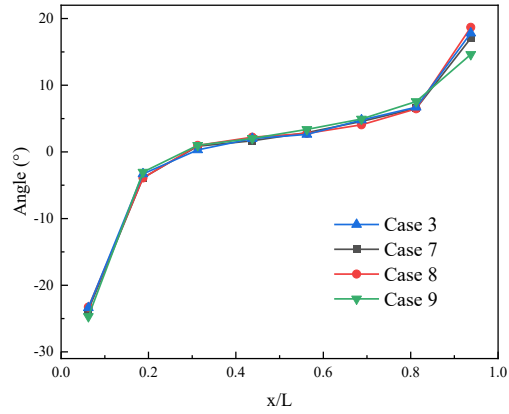
Fig. 6 (b) includes Cases 3 and 7–9, with the warhead diameter as the variable, showing the effect of warhead diameter on the projection angle. Combined with the results from Ref.

[36] and fig. 6 (b), it can be concluded that the warhead diameter has essentially no effect on the fragment projection angle.

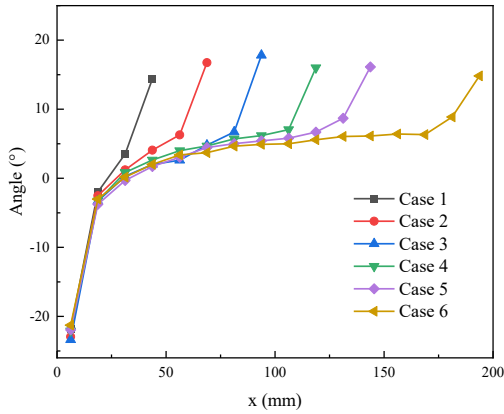
Fig. 6 (c) includes Cases 1–6, illustrating the effect of warhead length on the fragment projection angle. It can be seen from the figure that under different length-to-diameter ratios, the fragments at both the initiation end and the non-initiation end exhibit a large projection angle, and the magnitude of this angle does not change significantly with the length-to-diameter ratio. Thus, for the multi-layer ring warhead, the fragment projection angle is decoupled from the length-to-diameter ratio.



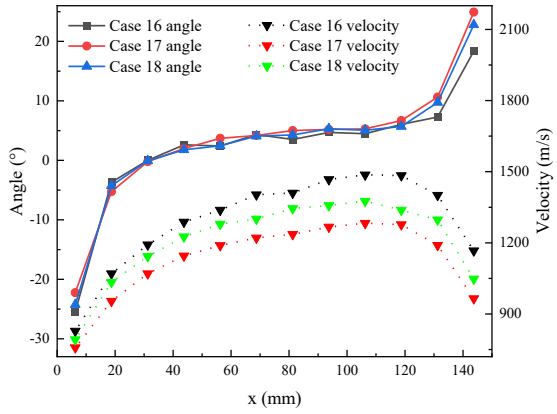
(a) Effect of ring height and material on projection angle



(b) Effect of warhead diameter on projection angle



(c) Effect of warhead length on projection angle



(d) Effect of explosive on velocity and projection angle

Fig. 6. Simulation results of projection angle and velocity under different operating cases

Fig. 6 (d) includes Cases 16–18, used to analyze the effects of explosive type on the fragment projection angle and fragment velocity. It can be seen from the figure that when different explosives are used, the fragment velocity increases significantly with the detonation velocity of the explosive. The initial fragment velocity generally exhibits a large axial gradient near the two ends, while the variation of velocity along the axis becomes gentle in the middle section. Near the initiation end, the axial velocity gradients caused by the three explosives differ considerably, whereas near the non-initiation end the differences are smaller. Overall, the

velocity is positively correlated with the detonation velocity. Regarding the fragment projection angle, under the three conditions, the projection angles at the initiation end and in the middle section are relatively close, but there are certain differences at the non-initiation end, which are negatively correlated with the detonation velocity. Therefore, it can be inferred that the fragment projection angle has a certain coupling relationship with the detonation velocity of the explosive and the velocity gradient induced by the explosive.

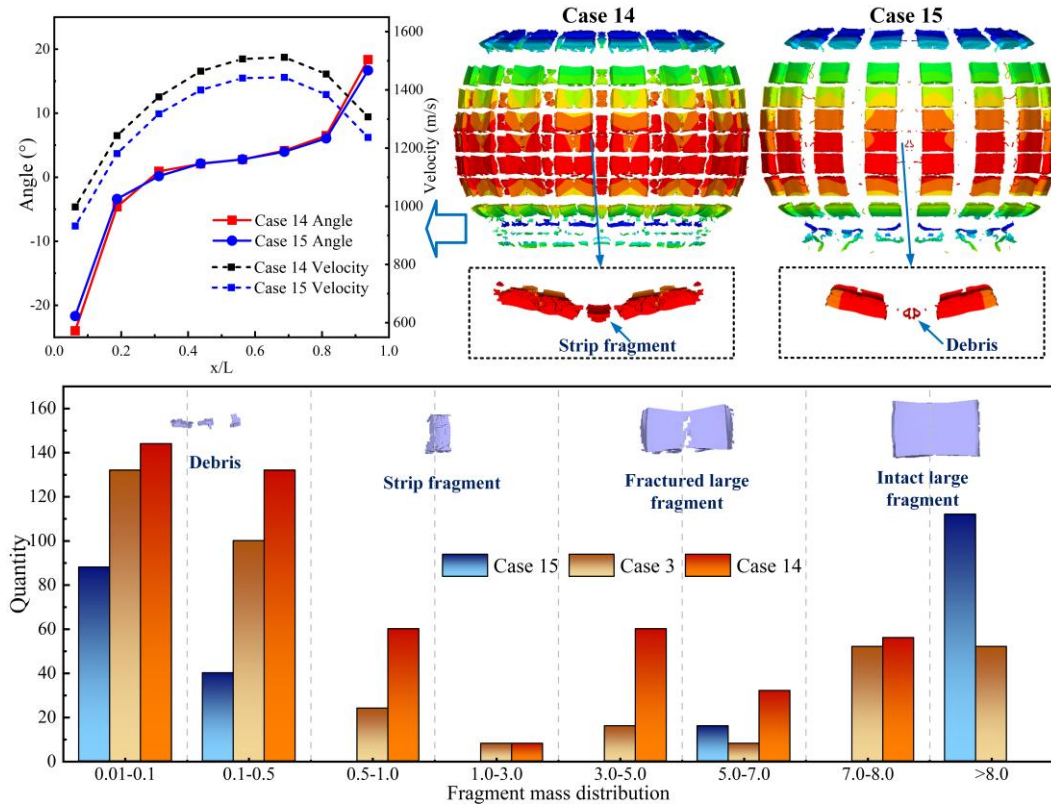


Fig. 7. Effect of groove depth on fragment formation and dispersion

Fig. 7 illustrates the effects of groove depth on fragment formation and dispersion. Unlike spherical cased charges which exhibit uniform fracture^[31], cylindrical cased charges tend to form elongated fragments during detonation because the hoop strain is much larger than the axial strain. Therefore, for the grooved multi-layer rings, in terms of fragment formation, under different groove depths, the ring fragments mainly fracture at the groove positions. When the groove depth is relatively large, as in Case 15, the ring fractures directly along the grooves to form complete large fragments, with metal debris of mass less than 0.5 g generated at the groove positions and the non-initiation end. When the groove depth is moderate, as in Case 3, shear fractures occur on both sides of the grooves, and in addition to the complete large fragments, strip-shaped fragments are formed at the groove positions. When the groove depth is further reduced, as in Case 14, the size of the strip-shaped fragments increases, and some large fragments fracture at their mid-positions, resulting in overall fragment dimensions and masses

smaller than those in Case 15.

Regarding fragment dispersion, analysis of Fig. 6 and Fig. 7 shows that when the relative groove depth increases from 0.17 (Case 14) to 0.91 (Case 15), the fragment velocity increases by only about 4.7%, and the difference in the fragment projection angle is also within 5%, which is mainly caused by the change in fragment velocity. Therefore, the groove depth does not directly affect the fragment projection angle.

From a mechanistic perspective, the reason why the groove depth does not directly affect the projection angle is that, although the multi-layer ring casing is pre-controlled in the circumferential direction by grooving, it behaves as a pre-formed fragment in the axial direction. The debonding strength of the adhesive bonding between the rings is several orders of magnitude lower than the overpressure of the detonation, so the rings have essentially no mutual influence during detonation wave propagation. Moreover, when the detonation wave uniformly loads the casing, the circumferential loading on the rings is consistent. Therefore, the longitudinal grooves only affect the fracture of each individual ring layer and the formation of fragments, while their influence on the separation and deflection of fragments in the same axial column is very small.

In summary, the fragment projection angle of the multi-layer ring casing is decoupled from factors such as axial groove depth, casing material, and length-to-diameter ratio, while it is negatively correlated with ring height and coupled with the detonation velocity of the explosive and the axial velocity gradient.

3. Fragment Projection Angle Model Considering Axial Gradient -Driven Deflection

In this section, based on the laws and data obtained from numerical simulations, the applicability of existing velocity and projection angle models for predicting the fragment dispersion of a multi-layer ring casing is examined through theoretical analysis. Furthermore, a fragment projection angle model considering axial gradient-driven deflection is established according to the characteristic that the fragment velocity direction varies with the deflection driven by the axial velocity gradient.

3.1 Applicability of Existing Models

Fig. 8 shows a comparison between the calculation results of existing formulas for fragment initial velocity and projection angle and the numerical simulation results. Compared with the simulation results, among the existing prediction formulas for fragment initial velocity and projection angle, the research on initial velocity is more mature and generally accurate. As

can be seen from Fig. 8 (b), except for the Gurney formula which ignores the effect of axial rarefaction waves, the errors of the other velocity formulas relative to the numerical simulation results along different axial positions are all within 20%. Among them, the Gao formula shows the best agreement with the present study, with very small errors at the initiation end and in the middle section (basically less than 5%), and the error at the non-initiation end is also within 15%. Therefore, the Gao formula is selected as the reference for fragment initial velocity in this study. The Gao formula is given as follows, where β_{ex} is the charge-to-metal mass ratio of the warhead.

$$v(x) = \sqrt{2E} \cdot \sqrt{\frac{\beta \cdot f(x)}{1 + 0.5\beta \cdot f(x)}}, f(x) = \left(1 - 0.689e^{-\frac{0.953x}{D}}\right) \left(1 - 0.385e^{-\frac{2.3(L-x)}{D}}\right) \quad (7)$$

Regarding the fragment projection angle, existing models are mainly derived from further derivation or modification of the Taylor formula. As shown in Fig. 9 (a), the Taylor formula is derived theoretically and explains the fragment projection direction from the perspective of detonation wave propagation. It can be used to calculate the fragment projection angle of a cylindrical charge detonated at one end, and is expressed as:

$$\frac{\theta}{2} = \arcsin\left(\frac{V_0}{2D_{ex}} \sin \alpha\right) \quad (8)$$

In the formula, $\theta/2$ is the angle between the fragment projection direction and the normal of the casing ($^\circ$), V_0 is the initial projection velocity of the fragment, D_{ex} is the detonation velocity of the explosive, α is the angle between the detonation wave propagation direction and the normal of the casing ($^\circ$).

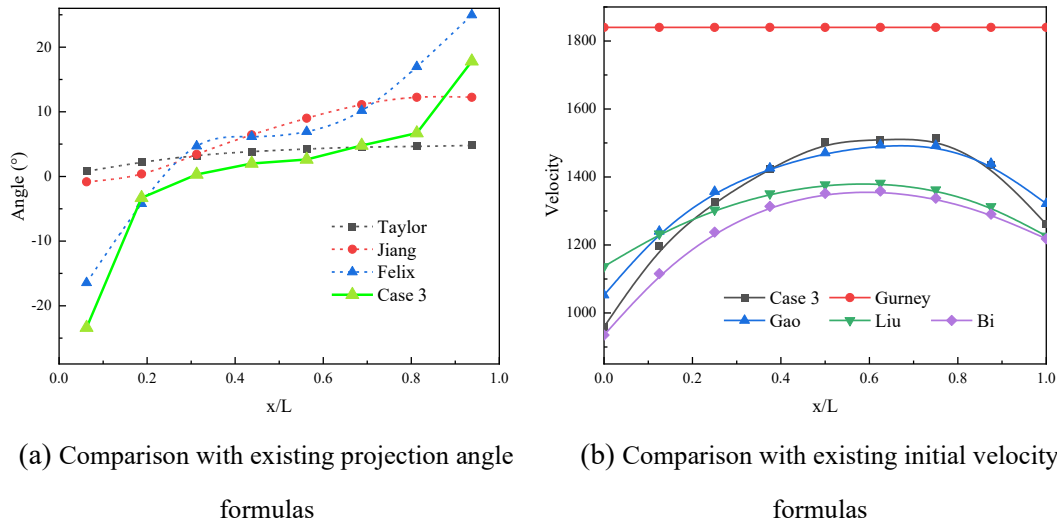


Fig. 8. Validation of the accuracy of existing models

However, as can be seen from Fig. 8 (a), the projection angles calculated by the Taylor formula are always positive, which does not conform to the characteristic that “fragments at the

upper end disperse upward while those at the lower end disperse downward” shown in Fig. 3-6. Therefore, the Taylor formula is inaccurate in predicting the projection angles near the two ends of the multi-layer ring casing, mainly because this assumption neglects the effect of axial rarefaction waves. Other existing theoretical or empirical formulas also give unsatisfactory predictions for the fragment projection angles of the multi-layer ring casing. Among them, the Taylor formula and the Jiang formula exhibit large prediction errors at both ends of the warhead; for example, the relative errors between the calculated values and the simulation data at the non-initiation end are -73.4% and -31.3%, respectively. Although the Felix formula gives slightly smaller errors at the two ends (-29.6% and 36.1%, respectively), it yields a relatively large error in the middle section, and its overall calculated results tend to be too high.

3.2 Effect of axial gradient-driven deflection

For a multi-layer ring casing, the axial rarefaction wave creates an axial pressure gradient, which in turn leads to an axial velocity gradient, as shown in Fig. 9 (b). Under the influence of the axial rarefaction wave, the fragment velocity is low at both ends of the casing and high in the middle. This phenomenon causes the fragments to acquire a deflection velocity driven by the axial gradient during dispersion, thereby affecting their projection angle. The angle formed by this velocity is denoted as γ .

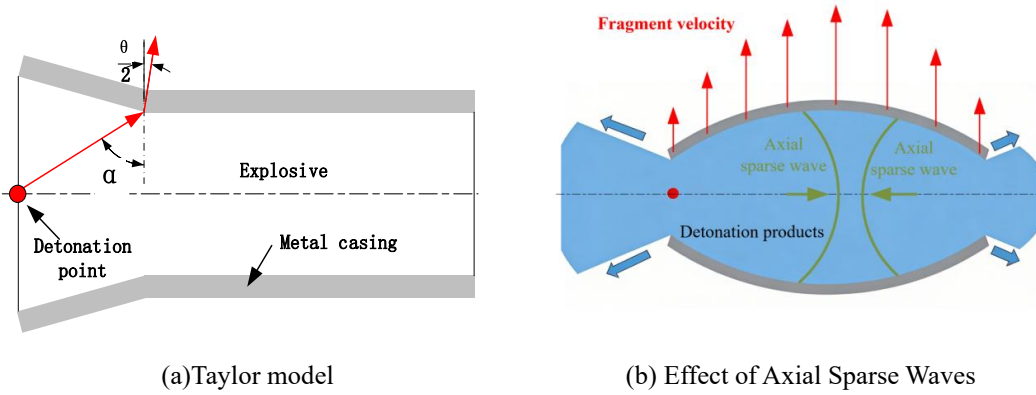


Fig. 9. Taylor model and the effect of axial sparse waves

It follows that the fragment projection angle β is actually determined jointly by the Taylor angle $\theta/2$ formed by the direct action of the detonation wave and the deflection angle γ driven by the axial fragment gradient. Moreover, the deflection angle driven by the axial gradient can be calculated only when the fragment size is determined. Most existing formulas are used to predict the projection angle of natural fragments. Since the size of natural fragments cannot be accurately estimated, the effect of axial gradient-driven deflection is not considered. In contrast, the axial dimensions of the fragments from the multi-layer ring casing used in this paper are determined, so the effect of axial gradient-driven deflection on fragment dispersion can be taken into account, further improving the accuracy of the model.

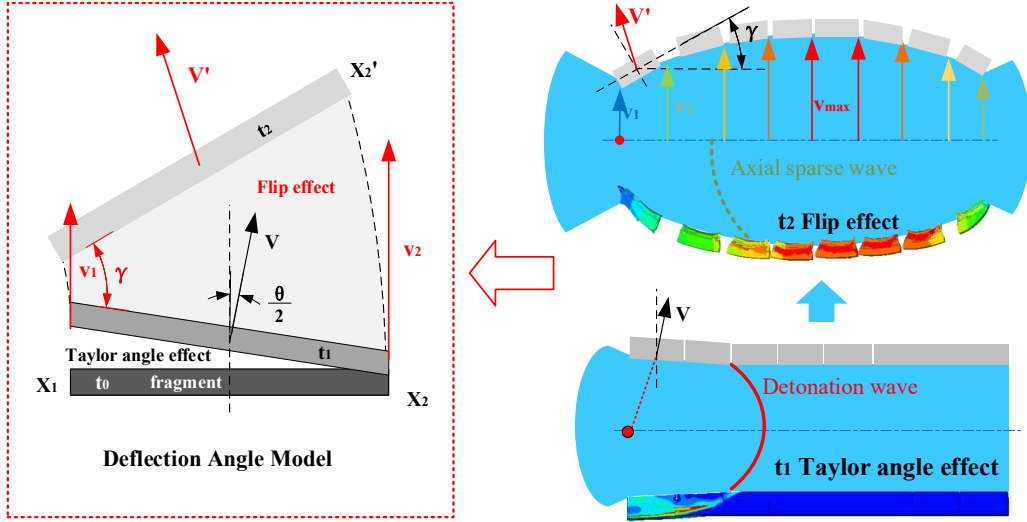


Fig. 10. Projection Angle Model Considering Axial Gradient-Driven Deflection

To establish a model that can more accurately predict the fragment projection angle, this section derives a fragment projection angle model considering axial gradient-driven deflection based on the influence of axial gradient-driven fragment velocity direction deflection on the fragment dispersion law.

Because the axial characteristic dimensions of fragments produced by the multi-layer ring casing are essentially determined, the positions of the upper and lower edges of each ring fragment are fixed. As shown in Fig. 10, taking the first ring fragment as an example, the velocities at the upper point x_1 and the lower point x_2 of the fragment are different. Since the axial deformation of the fragment is small and the deformation velocity is much lower than the fragment dispersion velocity, the deformation does not significantly affect the projection angle. Therefore, this model assumes the fragment to be a rigid body. A detailed analysis of a single-ring fragment is shown in Fig. 10. The velocity difference between point x_1 and point x_2 causes the fragment to deflect toward the end with the lower velocity as it moves away from the axis of the projectile. The rotation law can be described by treating point x_1 as undergoing translation, while the other points rotate about x_1 with an angular velocity ω , $\omega = (V_2 - V_1) / (\overline{x_1 x_2})$. The deflection angle of the fragment can then be expressed by Equation (9), where V'_1 is the instantaneous velocity at point x_1 , and V'_2 is the instantaneous velocity at point x_2 .

$$\gamma = \arctan \left(\frac{\int \delta(V'_2 - V'_1) dt}{\overline{x_1 x_2}} \right) \quad (9)$$

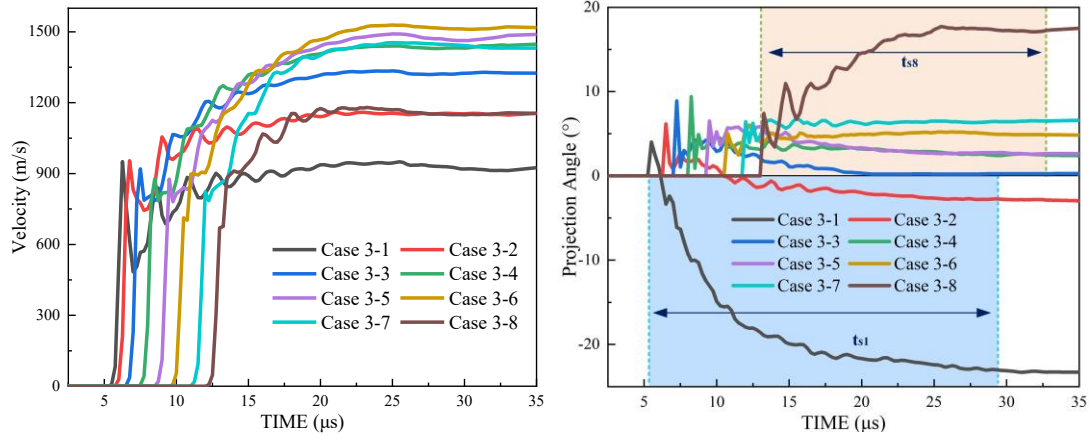
According to the fragment velocity dispersion law ^{[32][33]}, at the initial stage of detonation, the fragments are driven by the detonation wave to reach a relatively high velocity close to the initial velocity within a short time, and then slowly accelerate under the influence of the

detonation products until reaching the maximum velocity. However, numerical simulation analysis reveals that although the velocity magnitude can reach its maximum within a short period (approximately 2–5 μs), its direction continues to change until stabilizing after a longer period (approximately 15–25 μs), as shown in Fig. 10. Therefore, following the assumption of the Taylor formula, it can be assumed that point x_1 instantly reaches its final velocity V_1 and point x_2 instantly reaches its final velocity V_2 . The projection angle driven by the axial gradient can then be approximated by Equation (10).

$$\gamma = \arctan \left(\frac{(V_2 - V_1) \left(T - \frac{\sqrt{x^2 + (0.5D)^2}}{D_{ex}} \right)}{h} \right) \quad (10)$$

In the formula, $h = \overline{x_1 x_2}$, is the axial characteristic dimension of the fragment, i.e., the ring height in this paper; and T is the time when the fragment finally stabilizes in its dispersion.

Equation (10) requires the final stabilization time T of the fragment to calculate the deflection angle γ driven by the axial gradient. However, T is not a constant for casings of different dimensions and must be obtained through numerical simulation or experiment. Therefore, this section investigates the effect of time by analyzing the numerical simulation results of different operating conditions.



(a) Variation of fragment velocity with time

(b) Variation of projection angle with time

Fig. 11. Time required for fragments to reach maximum initial velocity and stable projection angle

As shown in Fig. 11, the fragment velocity magnitude reaches its maximum within a relatively short time, but the velocity direction has not yet stabilized. For the convenience of investigation, the concept of fragment stabilization time t_s is introduced, defined as the time from the onset of fragment motion to the stabilization of its velocity direction. When the fragment has been driven for a time longer than t_s , the projection angle no longer changes significantly under the influence of various factors.

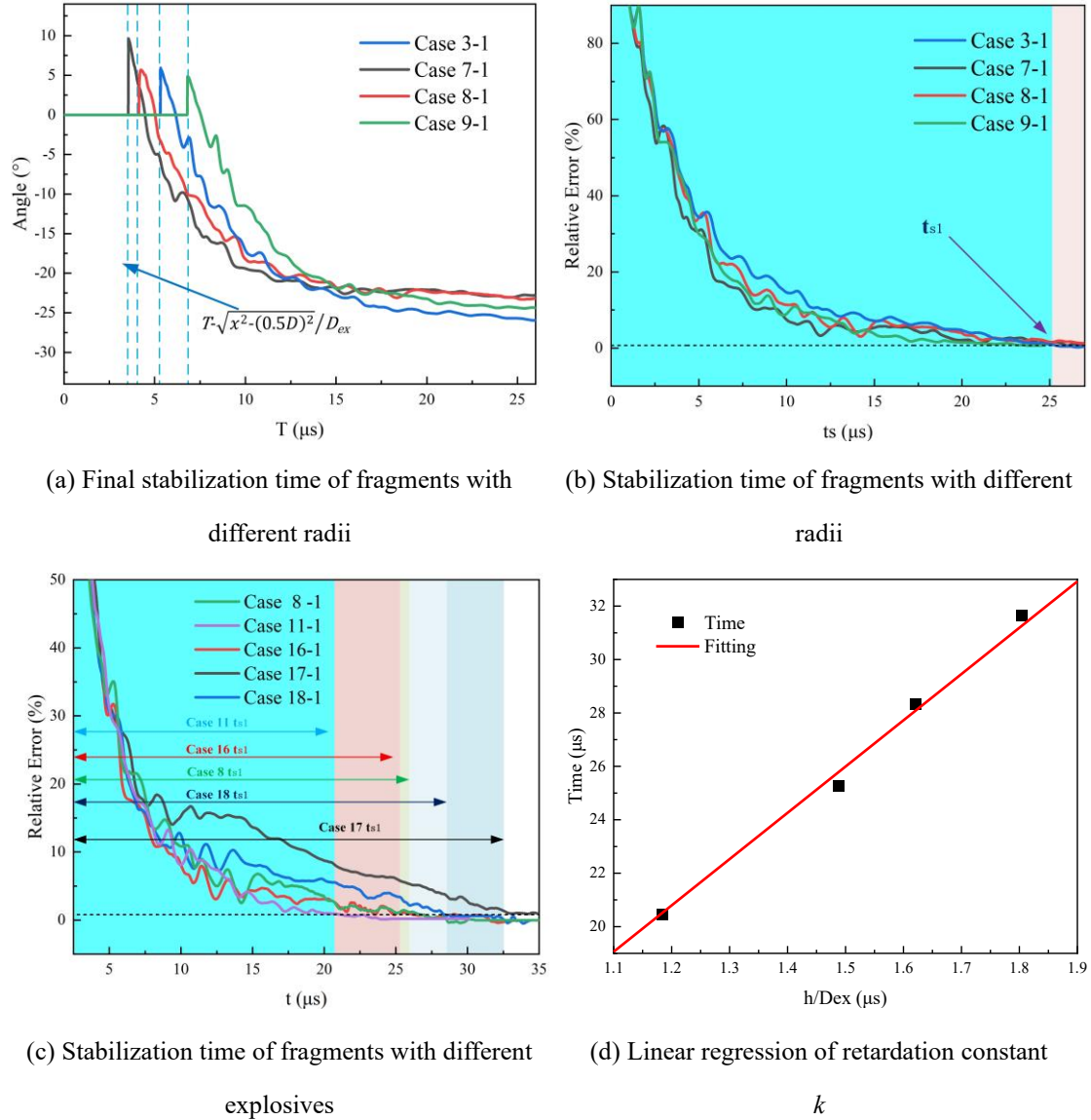


Fig. 12. Analysis of the time for fragments to reach stable projection angle

Fig. 12 shows the time for fragments to reach a stable projection angle under different operating conditions. As shown in Fig. 12 (a), according to numerical simulations, although the warhead diameter and length-to-diameter ratio vary considerably under these conditions, the time t_s for fragments to stabilize their flight is very close, approximately 25 μs as shown in Fig. 12 (b). However, as seen in Fig. 12 (c), the ring height and explosive type have a significant effect on the fragment stabilization time. The reason is that the formation of the deflection angle driven by the axial gradient is mainly attributed to the axial rarefaction wave after explosive detonation, and the velocity of the axial rarefaction wave is related to the detonation velocity of the explosive itself. Moreover, its influence on the axial gradient-driven deflection angle is also positively correlated with the action time of the axial rarefaction wave. Therefore, the fragment stabilization time should be related to the ratio of ring height to detonation velocity

of the explosive, and when the ring height tends to zero, the stabilization time t_s also tends to zero. Hence, the two are proportional to each other, i.e.:

$$t_{s1} = k \cdot \frac{h}{D_{ex}} \quad (11)$$

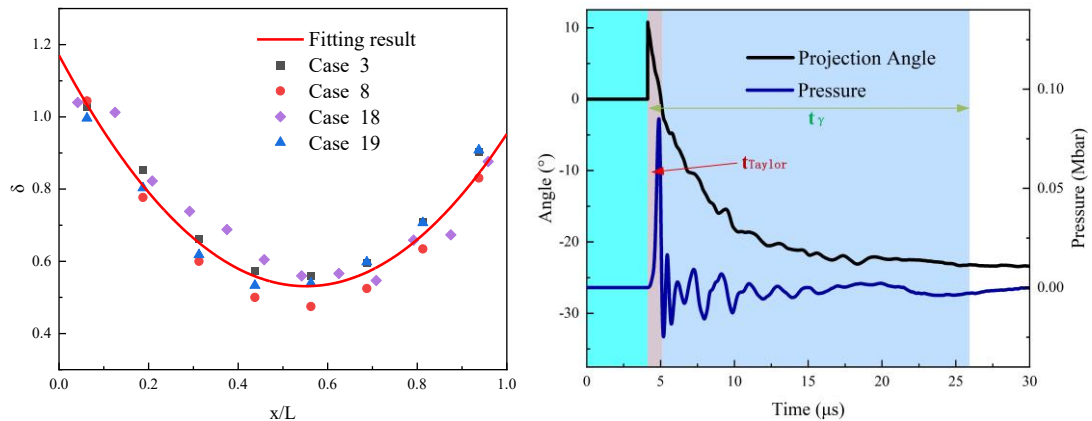
Therefore, the time term, $T \cdot \sqrt{x^2 - (0.5D)^2} / D_{ex}$, in the equation can be directly replaced by the actual stabilization time t_s . Substituting Equation (11) into Equation (10) yields the formula for calculating the axial gradient-driven deflection angle without needing to compute T , i.e., Equation (12).

$$\gamma = \arctan \left(k \cdot \frac{(V_2 - V_1)}{D_{ex}} \right) \quad (12)$$

In the formula, k represents the retardation coefficient of the casing, which is the ratio of the influence time of the axial rarefaction wave on the casing to the driving time of the detonation wave on the casing. It is a dimensionless constant. Therefore, a linear regression analysis was performed based on the data of different explosives, and the results are shown in Fig. 11 (d), with $k=17.3281$.

3.3 Axial Correction and Coupling Analysis

Since the fragment is influenced by multiple factors such as the detonation wave, rarefaction wave, and detonation products at different axial positions, its stabilization time t_s also varies. Based on the comprehensive analysis in Fig. 11 and Fig. 12, Equation (11) is used to characterize the stabilization time t_{s1} of fragments near the initiation end. However, the factors affecting the fragment stabilization time include both the ring height and the rarefaction wave effect, and the rarefaction wave effect changes with axial position, generally exhibiting a trend of being large at both ends and small in the middle. To accurately characterize the stabilization time t_s , this study also uses an axial correction factor $\delta(x/L)$ to modify the effect of axial position on the time.



(a) Correction coefficient for axial positions

(b) Decoupling analysis

Fig. 13. Axial correction fitting and coupling analysis

Fig. 13 shows the stabilization time at different axial positions under various warhead diameter and length conditions with a fixed ring height. It can be seen from Fig. 13 (a) that the variation trend of the stabilization time t_s along the axial position is basically consistent and the data are similar across different operating conditions, exhibiting a trend of being large at both ends and small in the middle, which is consistent with the previous speculation. Therefore, a quadratic function is used to fit the stabilization time t_s , and the axial correction factor $\delta(x/L)$ is obtained as shown in Equation (13).

$$\delta\left(\frac{x}{L}\right) = 2.0996 \cdot \left(\frac{x}{L}\right)^2 - 2.3174 \cdot \left(\frac{x}{L}\right) + 1.1705 \quad (13)$$

Therefore, the formula for calculating γ considering the axial position can be expressed as Equation (14).

$$\gamma = \arctan\left(k \cdot \frac{(V_2 - V_1)}{D_{ex}} \left(2.0996 \left(\frac{x}{L}\right)^2 - 2.3174 \left(\frac{x}{L}\right) + 1.1705\right)\right) \quad (14)$$

Regarding the coupling between the Taylor angle and the γ angle, they should be decoupled. As shown in Fig. 13 (b), there is a temporal difference between the formation of the axial gradient-driven deflection angle and the action time of the shock wave overpressure. According to the definition of the Taylor angle, it is mainly formed by the action of the shock wave; whereas the axial gradient-driven deflection angle is formed under the influence of the rarefaction wave behind the wave front. These two processes have different time scales and do not interfere with each other. Moreover, the axial gradient-driven deflection angle does not vary with the Taylor angle, which further proves that they are decoupled and independent variables. Therefore, the fragment projection angle β can be regarded as the direct sum of γ and the Taylor angle $\theta/2$, where the first part of the formula represents the Taylor angle and the second part represents the axial gradient-driven deflection angle. Combining Equation (8) with Equation (14), the fragment projection angle β can be expressed as:

$$\beta = \arcsin\left(\frac{V_0}{2D_{ex}} \sin \alpha\right) - \arctan\left(k \cdot \frac{(V_2 - V_1)}{D_{ex}} \left(2.0996 \left(\frac{x}{L}\right)^2 - 2.3174 \left(\frac{x}{L}\right) + 1.1705\right)\right) \quad (15)$$

In the formula, β is the fragment projection angle; D_{ex} is the detonation velocity of the explosive, in m/s; V_1 is the velocity of the upper edge of the fragment, and V_2 is the velocity of the lower edge of the fragment, calculated by Equation (7), in m/s; $k=17.3281$ is the driving time constant; x is the axial position of the fragment center, L is the length, and D is the warhead diameter, in m.

Fig. 14 shows a comparison between the calculation results of Equation (15) and the simulation results under different operating conditions. It can be seen from Fig. 13 that the calculation results of Equation (15) exhibit the same trend as the simulation data under various

429 conditions, and the agreement is good.

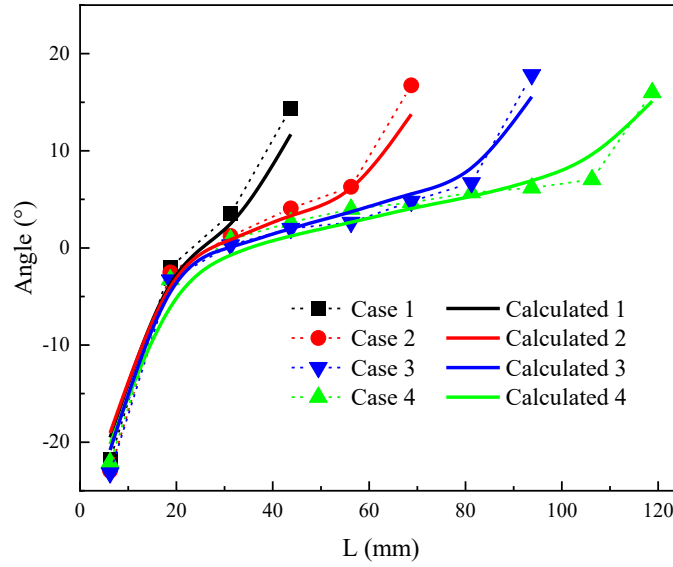


Fig. 14. Matching results of the formula with different simulation cases

4. Experiment and Accuracy Validation

To verify the correctness of the simulation method and the obtained fragment projection angle in the previous chapter, as well as the accuracy of Equation (15), experiments were conducted in this study. The actual fragment velocities and projection angles under selected operating conditions were measured and compared with the numerical results from the simulations and the formula to validate the accuracy of both the simulation and the formula.

4.1 Experimental Cases and Methods

Table 5. Parameters of Experimental Cases

Experimental group	L (mm)	D (mm)	h (mm)	s	Explosive type	Casing material
#1	100	50	12.50	0.42	Comp B	Steel 45
#2	100	50	6.66	0.42	Comp B	Steel 45

A total of two groups of experiments were designed in Table 5. For Group #1, the parameters of the multi-layer ring casing were designed according to Case 7, and all parameters were consistent with the simulation data of Case 7. This group was used to validate the correctness of the simulation method and results in Section 2. For Group #2, the operating conditions were different from all the 21 simulation cases used to derive the model, and the data from this group were used to validate the accuracy of Equation (15). In addition, both Group #1 and Group #2 varied the ring height, thereby verifying the inference in Section 3 that the driving time expressed by Equation (11) is proportional to the ring height and affects the axial gradient-driven deflection angle, and consequently influences the projection angle.

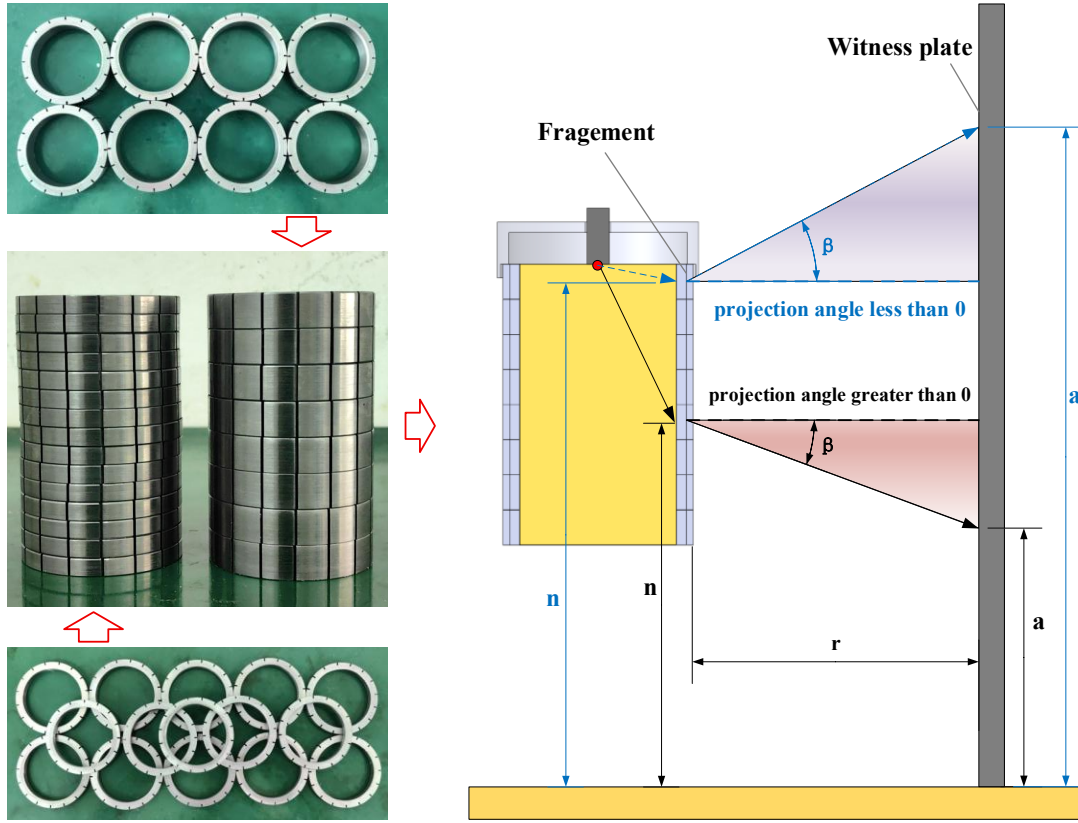


Fig. 15. Structure of the experimental warhead and method for measuring the projection angle

The rings used in the experiments and the assembled projectile body are shown in Fig. 15. The multi-layer rings were fixed together by adhesive bonding. After assembly, the projectile body was mounted on a projectile support. The fragment projection angle in the experiments was measured by an inverse trigonometric function, and the specific method is illustrated in Fig. 15.

As shown in Fig. 15, n is the distance from the fragment to the ground, a is the distance from the hole left by the fragment to the ground, and r is the distance between the warhead and the target. According to the definition, fragment dispersion toward the non-initiation end is considered as a positive projection angle. Therefore, the experimentally obtained fragment projection angle can be calculated using Equation (16).

$$\beta = \arctan\left(\frac{n-a}{r}\right) \quad (16)$$

Fig. 16 shows the layout of the experimental site. The experimental setup mainly includes a witness plate, a velocity measurement screen, and a high-speed camera. The witness plate is a steel plate 2 m in height and 2 mm in thickness, used to record the vertical impact positions of fragments after they reach the target area. By measuring the positions of the holes on the witness plate, the projection angle can be calculated. The velocity measurement screen is used to measure the fragment velocity, which can be cross-validated with the simulation data. The

high-speed camera and its protective shelter are placed behind the witness plate to record the sequence of fragments passing through the target, facilitating the identification of which fragment layer corresponds to each hole on the witness plate. Moreover, the high-speed camera can record both the initiation time and the times at which multiple fragments pass through the target. By using the time and the fragment flight distance, the velocity can be calculated, which not only provides a circular validation with the velocity measurement screen but also allows determination of the velocities of fragments at different axial positions.

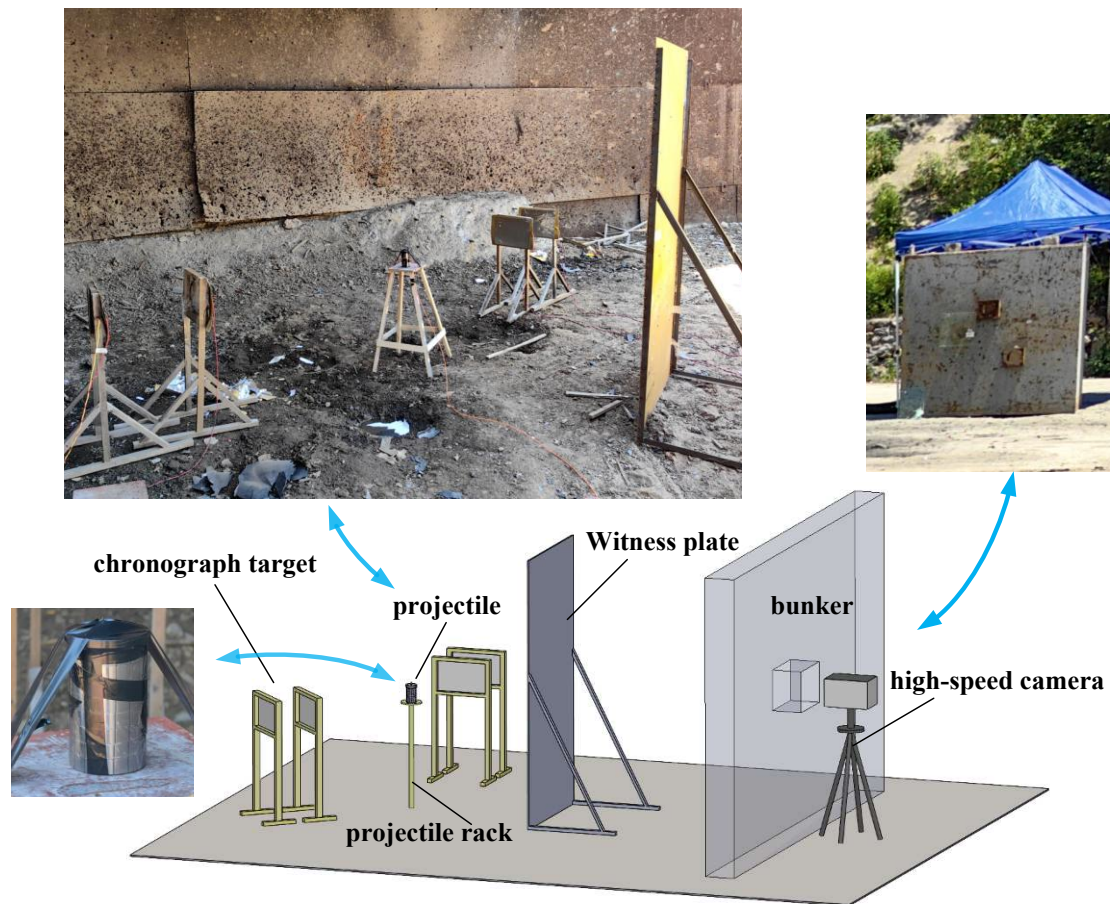


Fig. 16. Design and Layout of the Experimental Site

4.2 Experimental Results and Validation

Fig. 17 shows the fragment transit time and position distribution recorded by the high-speed camera, documenting the transit times and sequence of fragments at different positions.

Fig. 18 (a) presents the holes on the witness plate left by the fragments, recording the perforation positions of the fragments. Fig. 18 (b) shows the binarization processing results of the witness plate holes for experimental groups #1 and #2, presenting the perforation results and position distributions of the fragments for groups #1 and #2, respectively. Fig. 18 (c)

presents the fragment type analysis. Based on the witness plate results and the high-speed camera records, the fragment projection angles and velocities can be measured using the method described in the previous section.

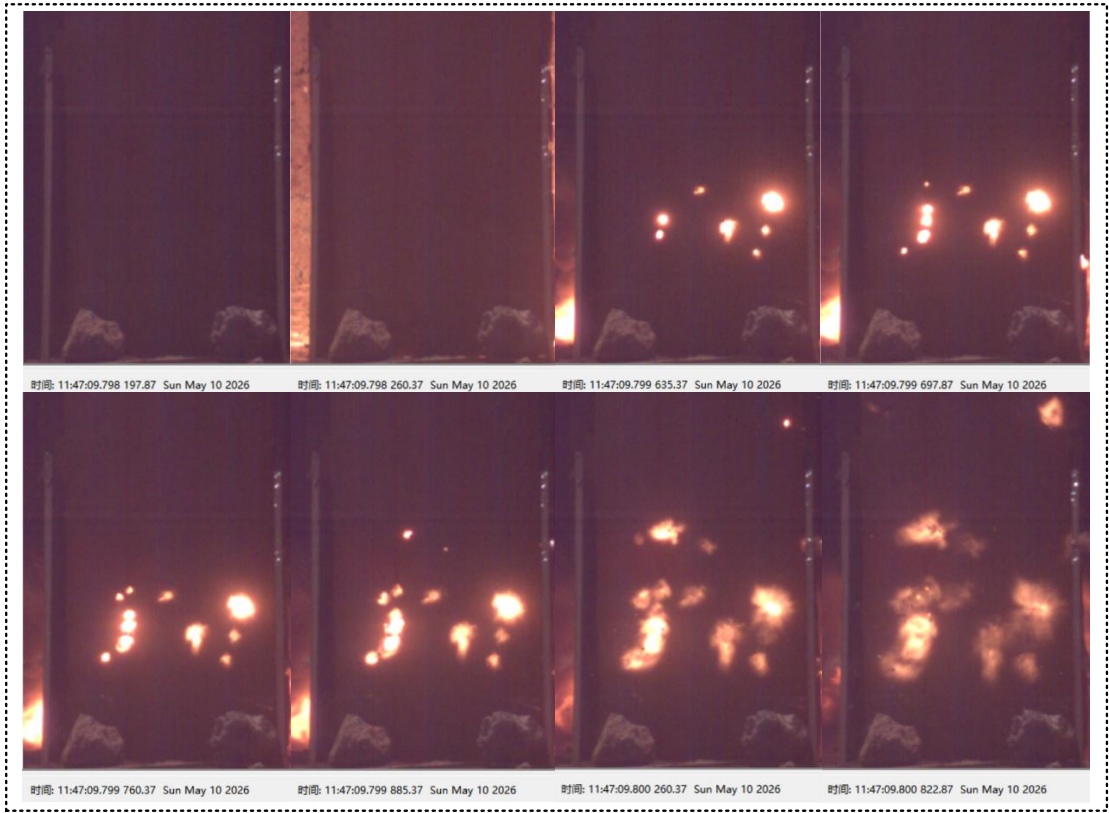


Fig. 17. High-speed camera photography results

Table 6 presents the comparative analysis between the experimental results and simulation for Group #1, and the experimental data obtained are shown in Table 6.

Table 6. Comparative analysis of experimental results and simulation for Group #1

Fragment No.	Simulation velocity (m/s)	Simulation projection angle (°)	Experimental velocity (m/s)	Experimental projection angle (°)
#1-1	810	-23.70	/	-20.32
#1-2	1001	-3.88	980	-4.68
#1-3	1153	0.86	1156	1.38
#1-4	1244	1.97	1200	3.53
#1-5	1296	2.89	1312	5.03
#1-6	1344	4.57	1314	6.79
#1-7	1300	6.52	1261	9.08
#1-8	1019	17.05	/	13.85

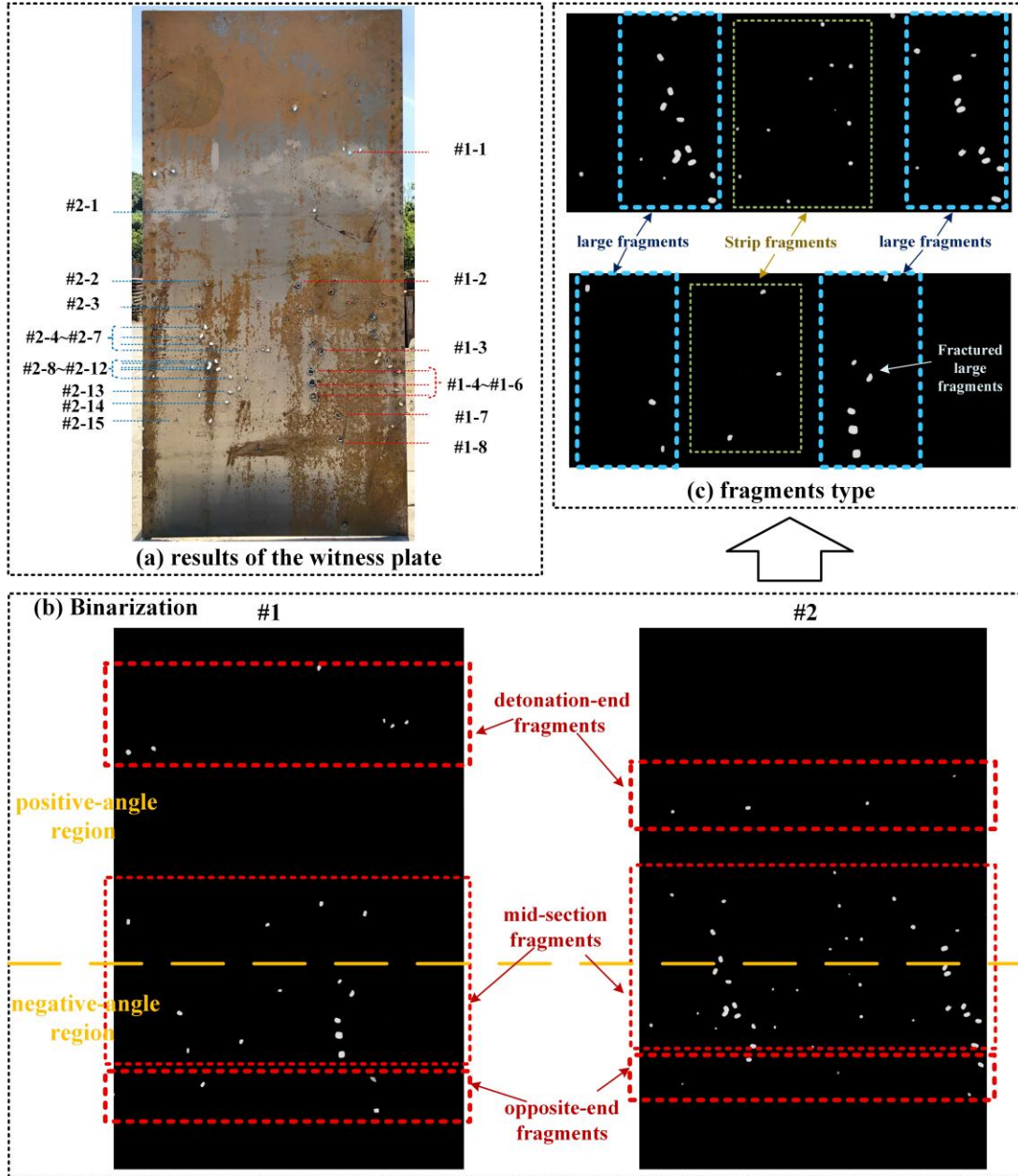


Fig. 18. Analysis of Witness Plate Results

Analysis of Fig. 18 shows that for both Group #1 and Group #2, the fragment projection angles are large at the initiation end and the non-initiation end, while relatively small in the middle section, which is consistent with the simulation results. Moreover, as can be seen from Fig. 18 (c), the hole sizes vary significantly. The two distinct rows of relatively large regular holes are left by the pre-controlled large fragments, while the small holes between the two rows of large fragments are left by the strip-shaped fragments formed at the groove positions. This phenomenon is also consistent with the simulation results.

Fig. 19 presents a comparison between the experimental and simulation results for Group #1. As shown in Fig. 19 and Table 6, the projection angle of fragments from the multi-layer

ring casing is negative near the initiation end and positive near the non-initiation end, ranging from -20.32° to 13.85° . This trend is consistent with the simulation data. Numerically, the maximum deviation in fragment projection angle between the simulation and experiment is less than 3.38° , and the minimum deviation is 0.8° . Thus, the feasibility of the simulation method is verified, and the accuracy of the simulation data in Section 2 is validated.

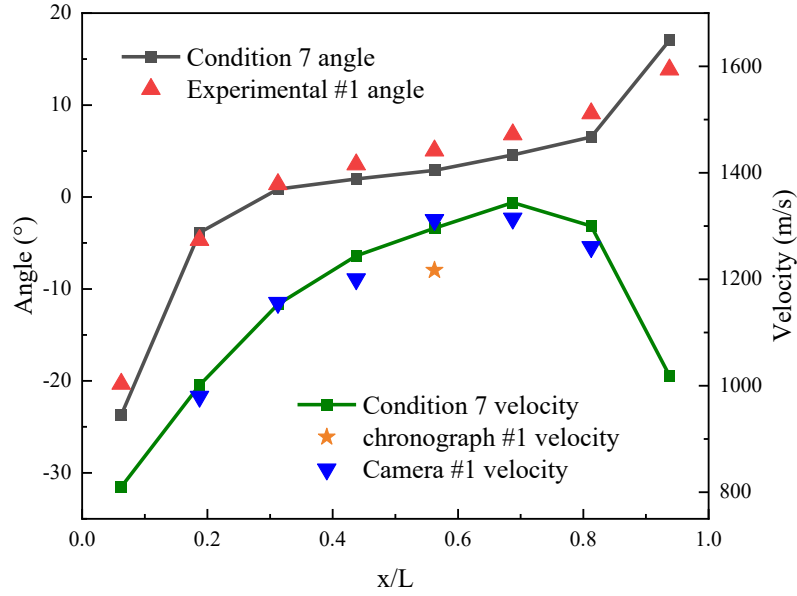
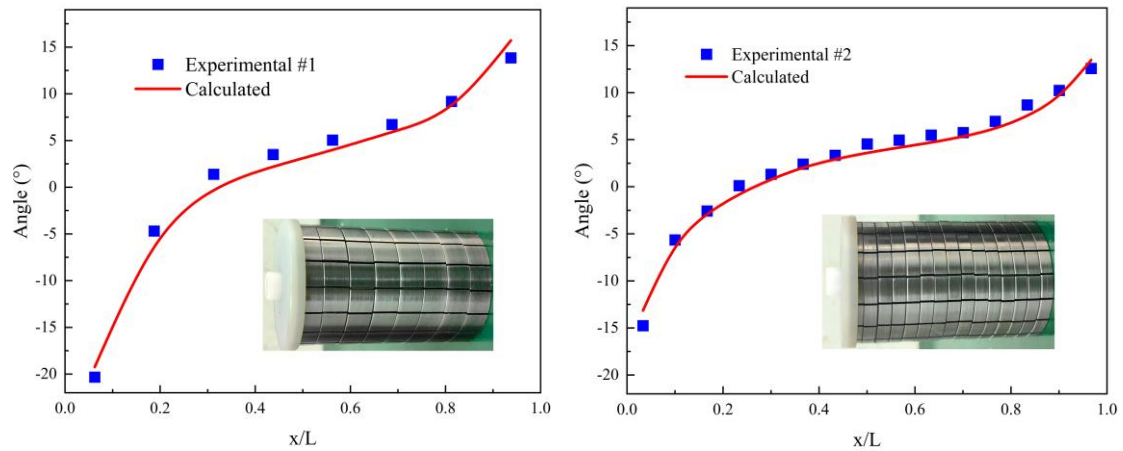


Fig. 19. Comparison of experimental and numerical simulation results for Group #1

Fig. 20 shows a comparison between the calculated results of Equation (15) and the experimental results of Groups #1 and #2, which is used to validate the accuracy of the model in predicting the fragment projection angle. Unlike the previous comparison in Fig. 19, which was used to validate the simulation against the Group #1 experiment for Case 7, the Group #2 experiment shown in Fig. 20 is mainly used to verify the accuracy of the calculated results of the formula.



(a) Validation of the formula with Group #1

Validation of the formula with Group #2

Fig. 20. Validation of the formula accuracy with experiments

Table 7. Comparison and error of calculated results with experimental data

Experimental group	Fragment No.	Experimental results (°)	Calculated results (°)	Relative error
#1	#1-1	-20.33	-19.24	-5.35%
	#1-6	6.71	5.90	-12.12%
	#1-7	9.19	8.79	-4.34%
	#1-8	13.85	15.72	13.50%
#2	#2-1	-14.76	-13.14	-11.03%
	#2-3	-2.58	-2.93	13.89%
	#2-7	3.35	2.92	-12.73%
	#2-11	5.76	5.36	-6.90%
	#2-15	12.57	13.47	7.11%

Table 7 presents a comparative analysis between the calculated results of Equation (15) and the experimental data. The comparison shows that the calculated results of Equation (15) are in good agreement with the experimental results of both groups. The absolute relative errors at the initiation end and the middle section for Groups #1 and #2 are both less than 15%, which is much smaller than the -73.4% error of the Taylor formula and the -31.3% error of the Jiang formula. Therefore, Equation (15) can accurately reflect the fragment projection angle of multi-layer ring casing warheads of different sizes, and its prediction accuracy for the projection angle of the multi-layer ring casing fragments used in this study is superior to that of the Taylor formula and the Jiang formula.

Regarding the issue that the error at the non-initiation end of Group #1 is larger than that at the initiation end, it is speculated that a large number of tiny fragments are generated during the explosion at the non-initiation end, and the unloading wave caused by their fracture leads to a change in the projection angle. Therefore, based on the data analysis in Table 7, Equation (15) exhibits high accuracy.

4.3 Validation of model universality

Following the derivation in Section 3, Equation (15) is applicable not only to the pre-controlled fragment warhead with a multi-layer ring casing, but also to all cylindrical warheads for which the fragment dimensions are known or can be estimated. Therefore, to validate the universality of the formula, it is compared with the experimental data of Predebon^[34] and Jiang^[21]. The experiment by Predebon used a warhead with 15 layers of square pre-formed fragments, with a warhead diameter of approximately 40 mm and a length of about 82 mm. The experiment by Jiang used a warhead with spherical pre-formed fragments

(diameter 5 mm) and a liner, with a warhead diameter of 56 mm and a length of 112 mm.

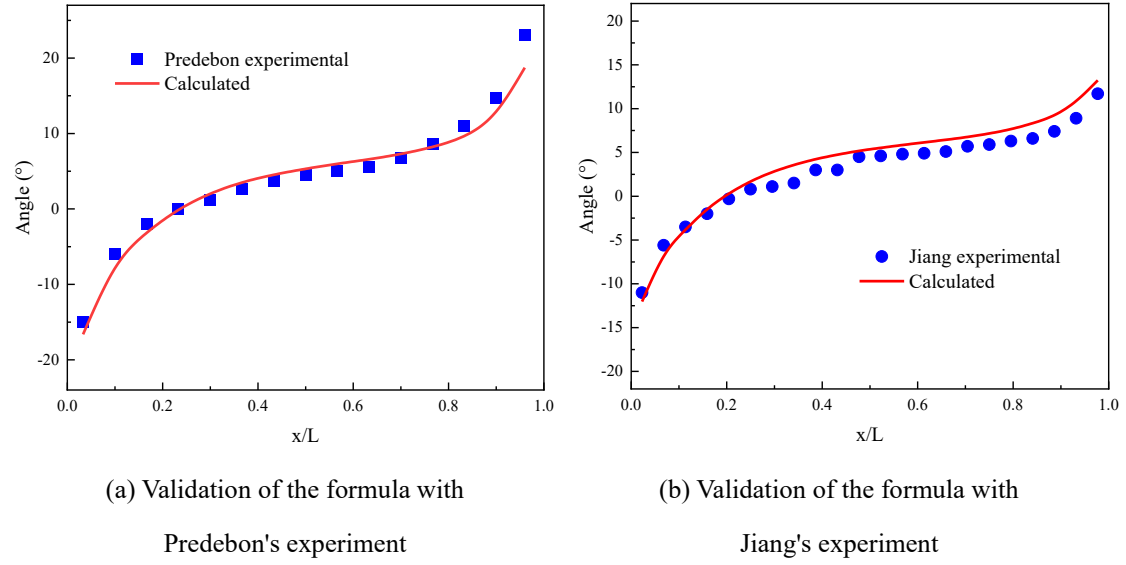


Fig. 21. Validation of the formula universality

Fig. 21 shows the calculation results of Equation (15) for the above two experiments. By comparing with the experimental data, it can be seen that the formula predicts the projection angle of the square pre-formed fragments in Predebon's experiment with good accuracy, with a maximum error of less than 10%. For the spherical pre-formed fragments with a liner in Jiang's experiment, the prediction accuracy and trend are also quite accurate, with an error of about 5% near the initiation end. The error near the non-initiation end is slightly larger than that at the initiation end, but it is still within 15%, and the formula basically captures the trend of the projection angle varying along the axial position.

In summary, Equation (15) exhibits good universality and is applicable to most conventional cylindrical warheads (with moderate wall thickness; the model may not be applicable when the wall thickness is extremely large) for which the axial characteristic dimensions of fragments are known or can be estimated, including multi-layer ring pre-controlled fragments as well as square and spherical pre-formed fragments.

5. Conclusions

This study focuses on the projection angle law of fragments from a multi-layer ring casing warhead. Through numerical simulation, experimental validation, and theoretical derivation, the key factors affecting the fragment projection angle are systematically analyzed, and a universal prediction model is proposed. The main conclusions are as follows:

(1) Based on the deflection characteristics of fragments from the multi-layer ring casing, the relationship among the fragment stabilization time, the ring height, and the detonation velocity of the explosive is analyzed. A prediction model suitable for calculating the fragment

projection angle of a multi-layer ring casing warhead is established. The prediction error of this model for the multi-layer ring casing warhead is less than 15%.

(2) The distribution law of the fragment projection angle of the multi-layer ring casing warhead is obtained. The fragment projection angle exhibits a characteristic distribution along the axial direction, being large at both ends and small in the middle. It is clarified that the warhead length, ring height, explosive type, and the resulting velocity gradient are the main factors affecting the projection angle, while the use of different steel materials for the casing, warhead diameter, and axial groove depth have no significant effect on the projection angle.

(3) The new model has good engineering universality. The prediction error for square pre-formed fragments is less than 10%, and the prediction error for spherical pre-formed fragments with a liner is less than 15%. The model can be extended to all cylindrical warheads for which the axial characteristic dimensions of the fragments are known, effectively compensating for the weak universality of existing models.

The axial gradient-driven deflection angle model established in this study provides a reliable theoretical basis for the structural optimization design and precise regulation of the lethal range of multi-layer ring casing warheads, and has engineering guiding significance for improving the design of explosive and casing matching in modern munitions.

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