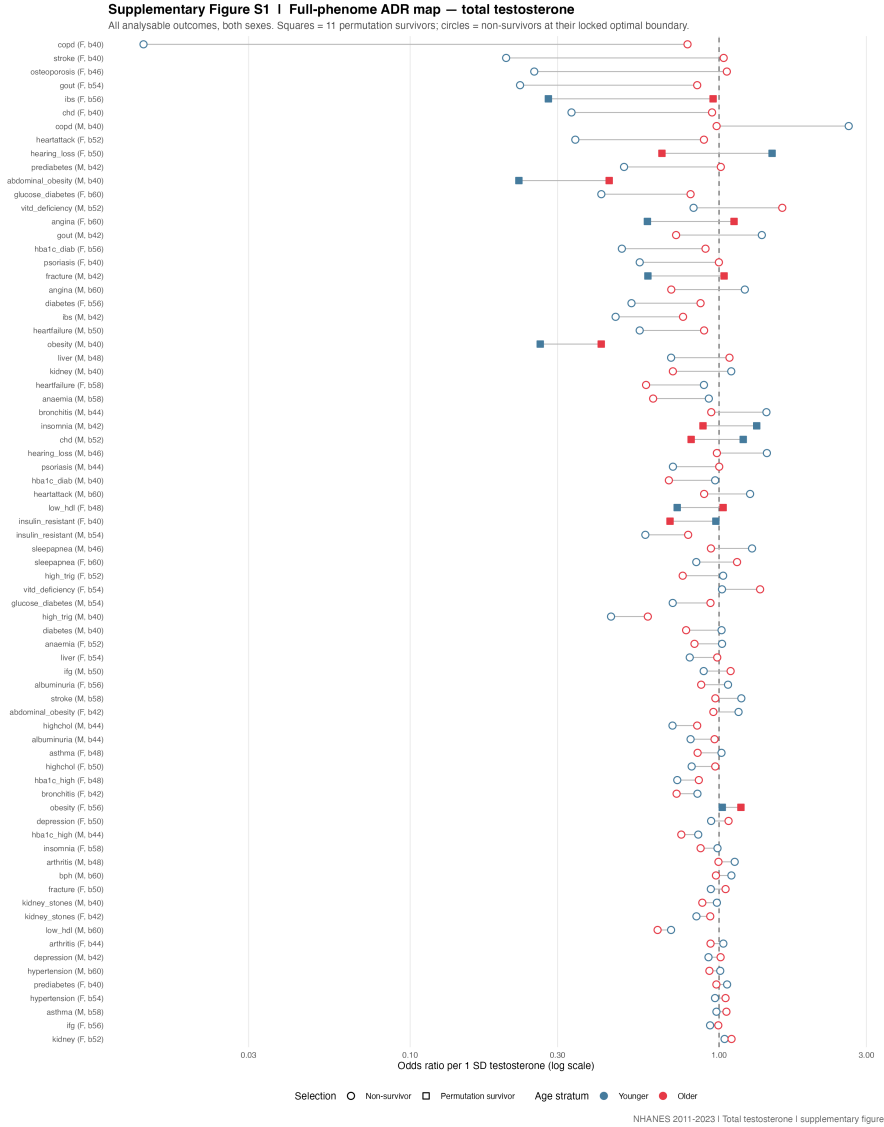


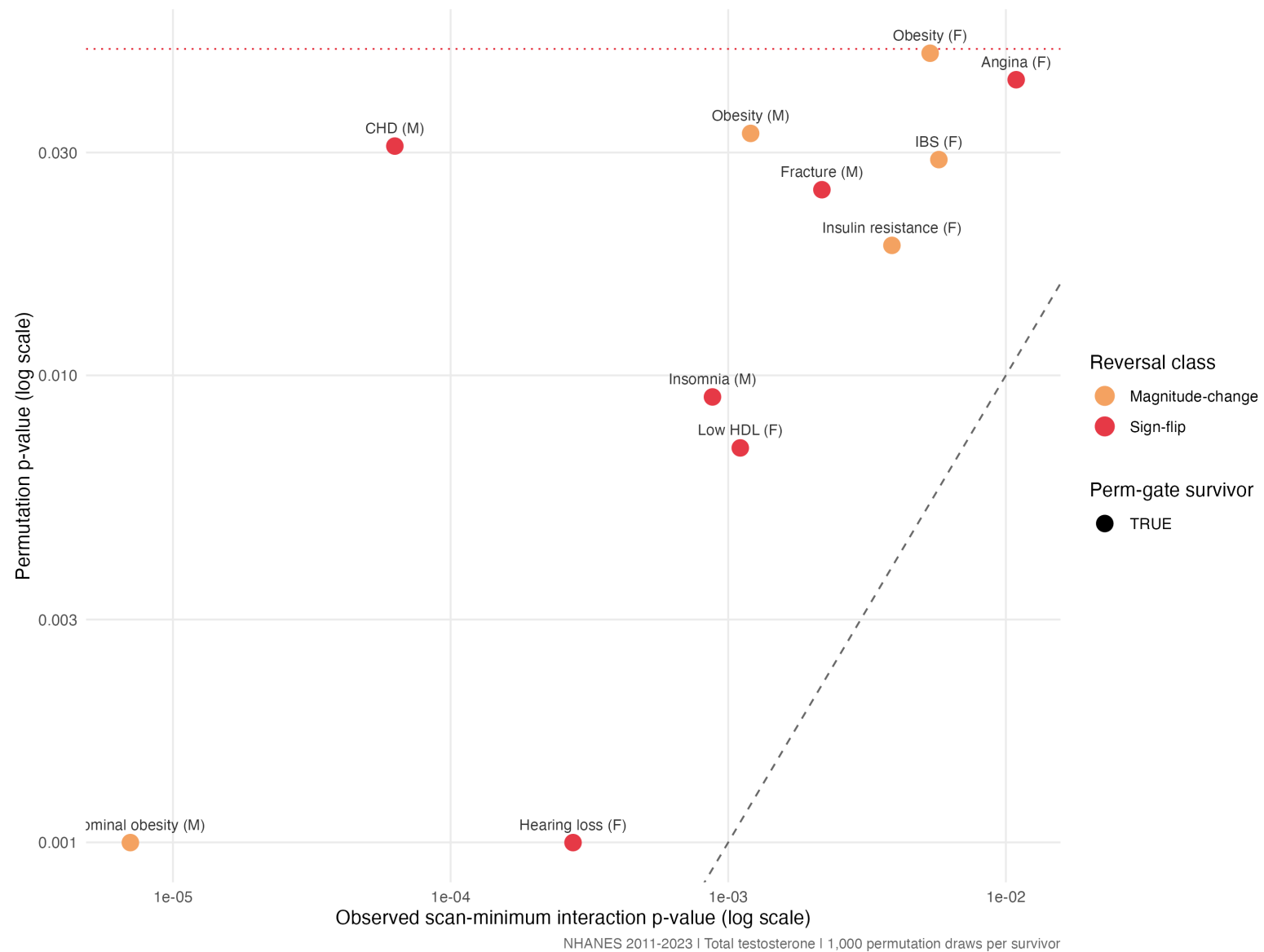
1. Supplementary Figures

Age-dependent directional reversal of total testosterone across the NHANES phenome



Supplementary Figure S1. Full-phenome age-dependent reversal map for total testosterone — all analysable outcomes, both sexes, showing young and old odds ratios at each optimal boundary (main-text figures show the top permutation-ranked findings only).

Each point = one surviving reversal. Red dotted = $p = 0.05$ permutation threshold. Diagonal = no permutation inflation.

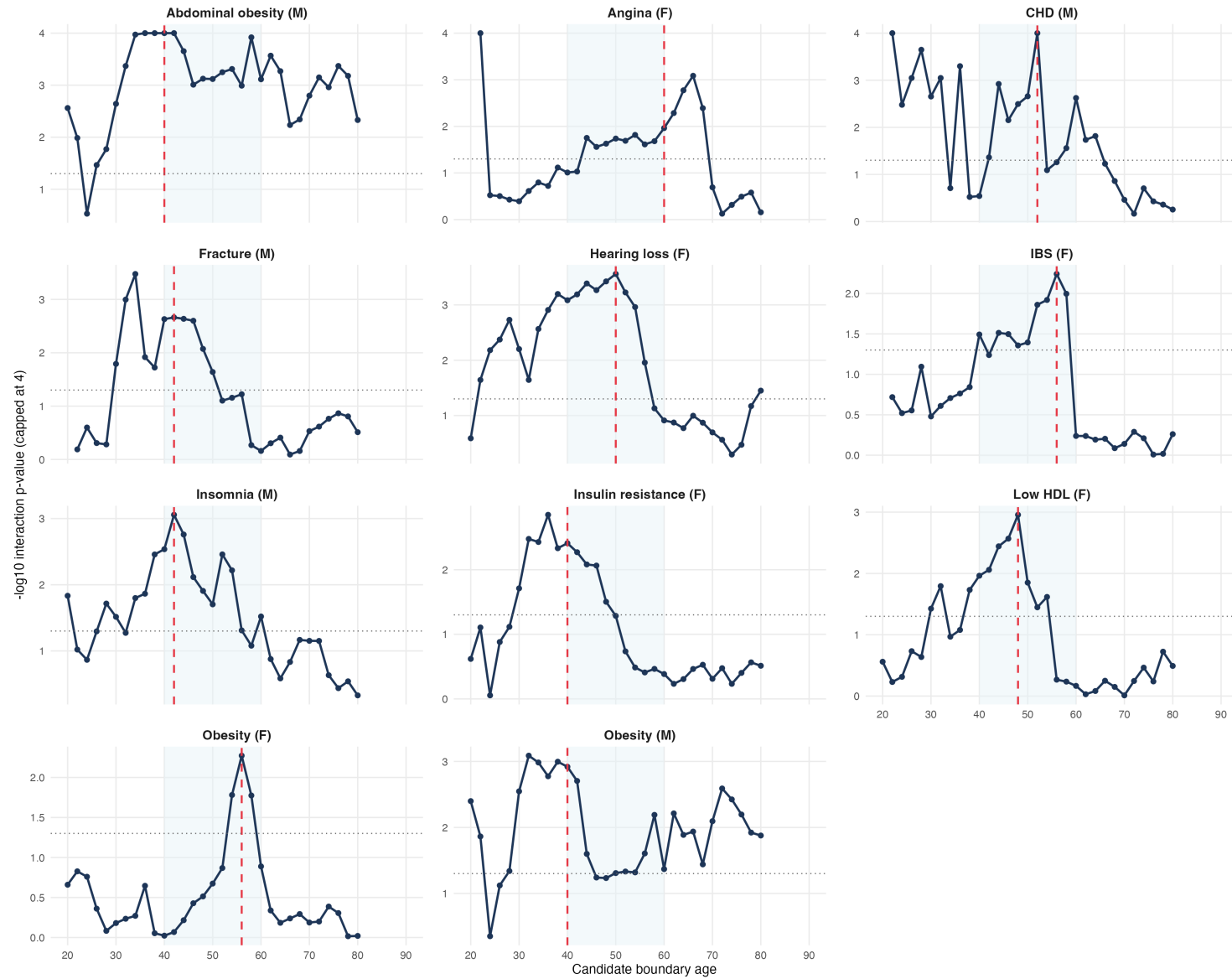


NHANES 2011-2023 | Total testosterone | 1,000 permutation draws per survivor

Supplementary Figure S2. Permutation null distributions for each surviving reversal, with the observed scan-minimum statistic marked.

Supplementary Figure S3 | Full-range 20-90 threshold scan

Interaction significance across candidate boundaries. Shaded = 40-60 primary window. Red dashed = locked boundary. Dotted = $p < 0.05$.

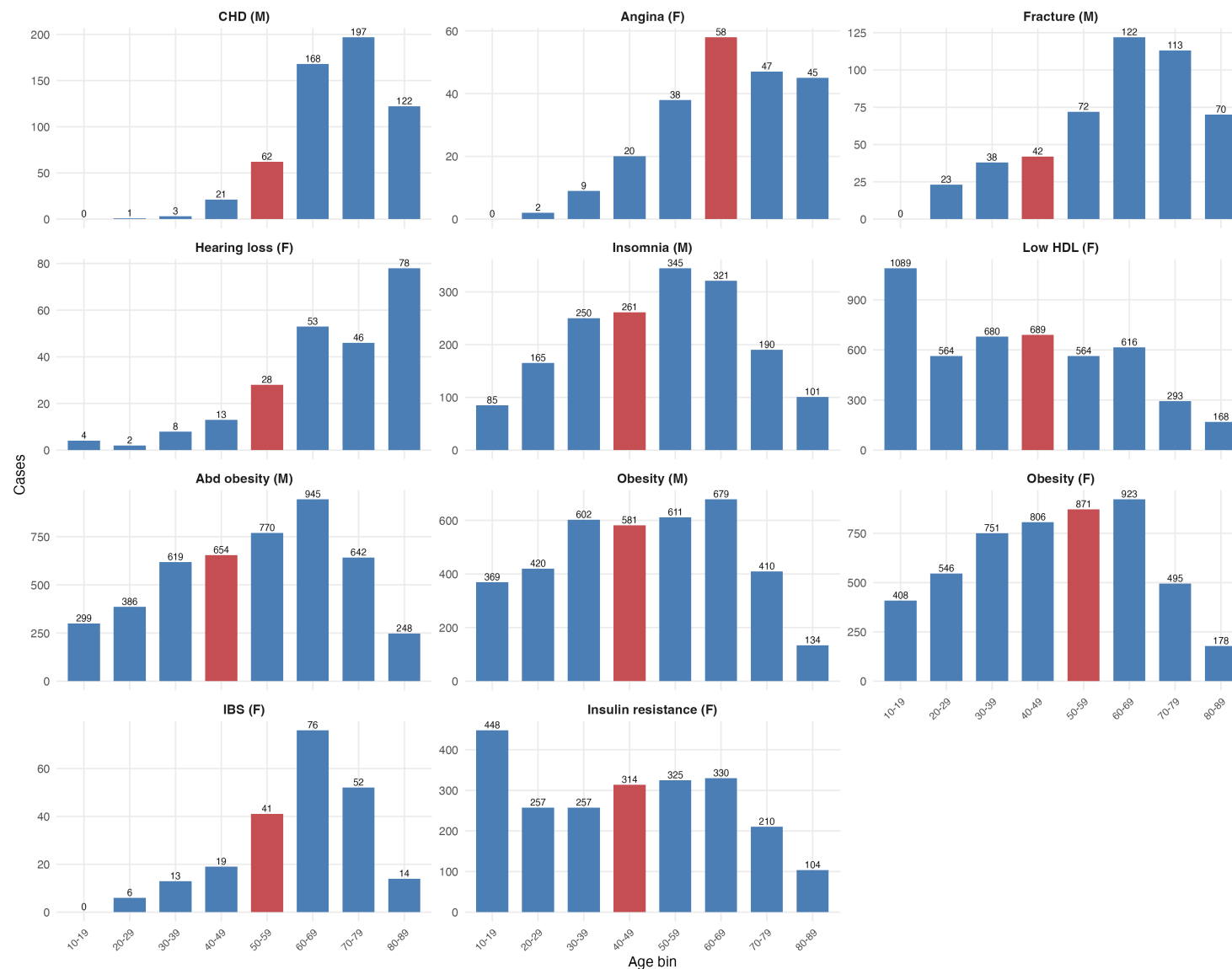


NHANES 2011-2023 | Total testosterone | supplementary sensitivity

Supplementary Figure S3. Full-range 20-90 threshold-scan sensitivity for boundary placement, compared against the 40-60 primary window.

SF12 | Case counts per 10-year age bin for 11 permutation survivors

Red = bin containing the ADR boundary. Includes 10-19 to show pre-adult sparsity.

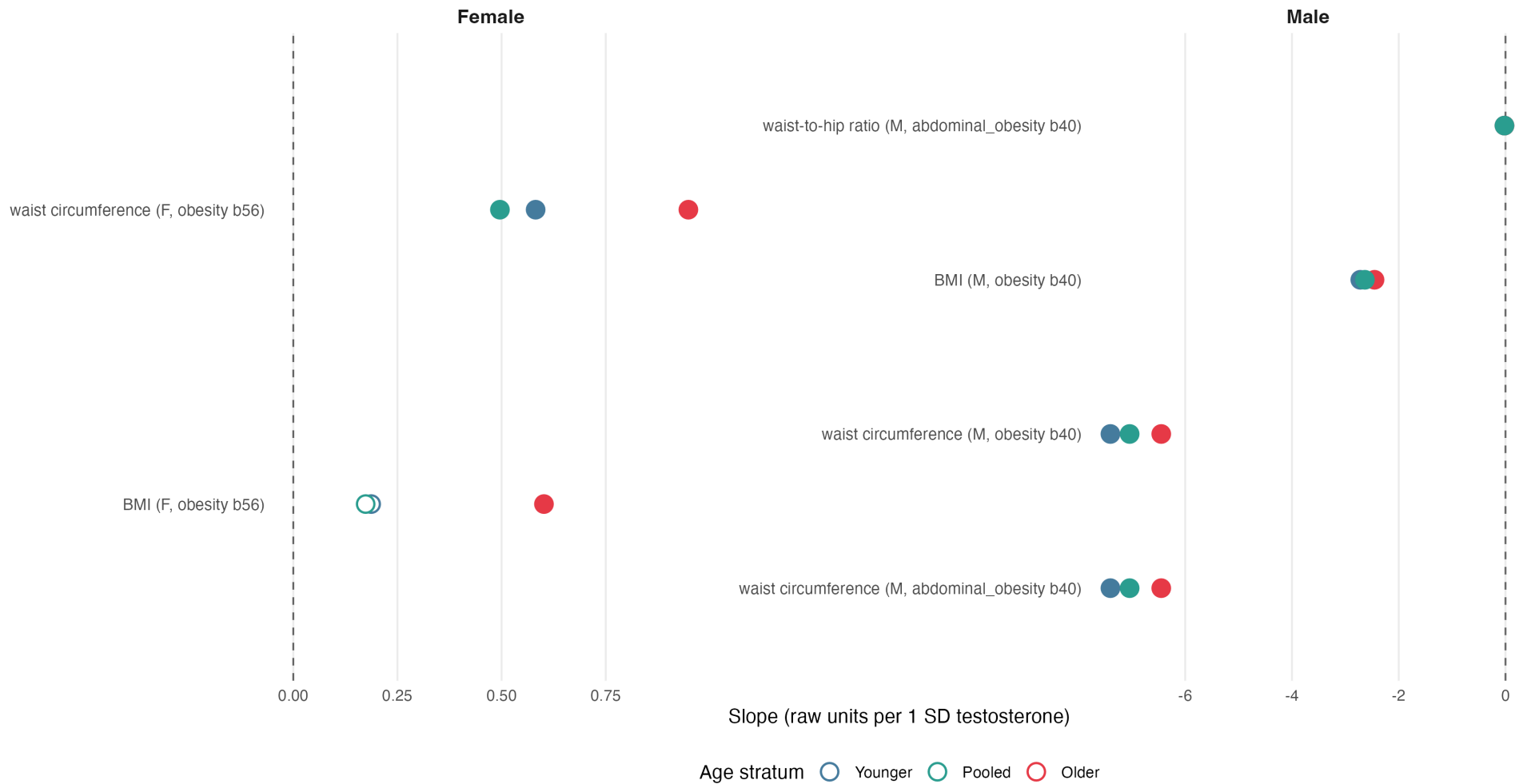


NHANES 2011-2023 | Total testosterone | 2026-09-09

Supplementary Figure S4. Case counts per 10-year age bin for the eleven permutation-surviving reversals.

Supplementary Figure S4 | Anthropometric severity confirmations

Waist, BMI, and waist-to-hip ratio slopes by age stratum. Filled = $p < 0.05$.

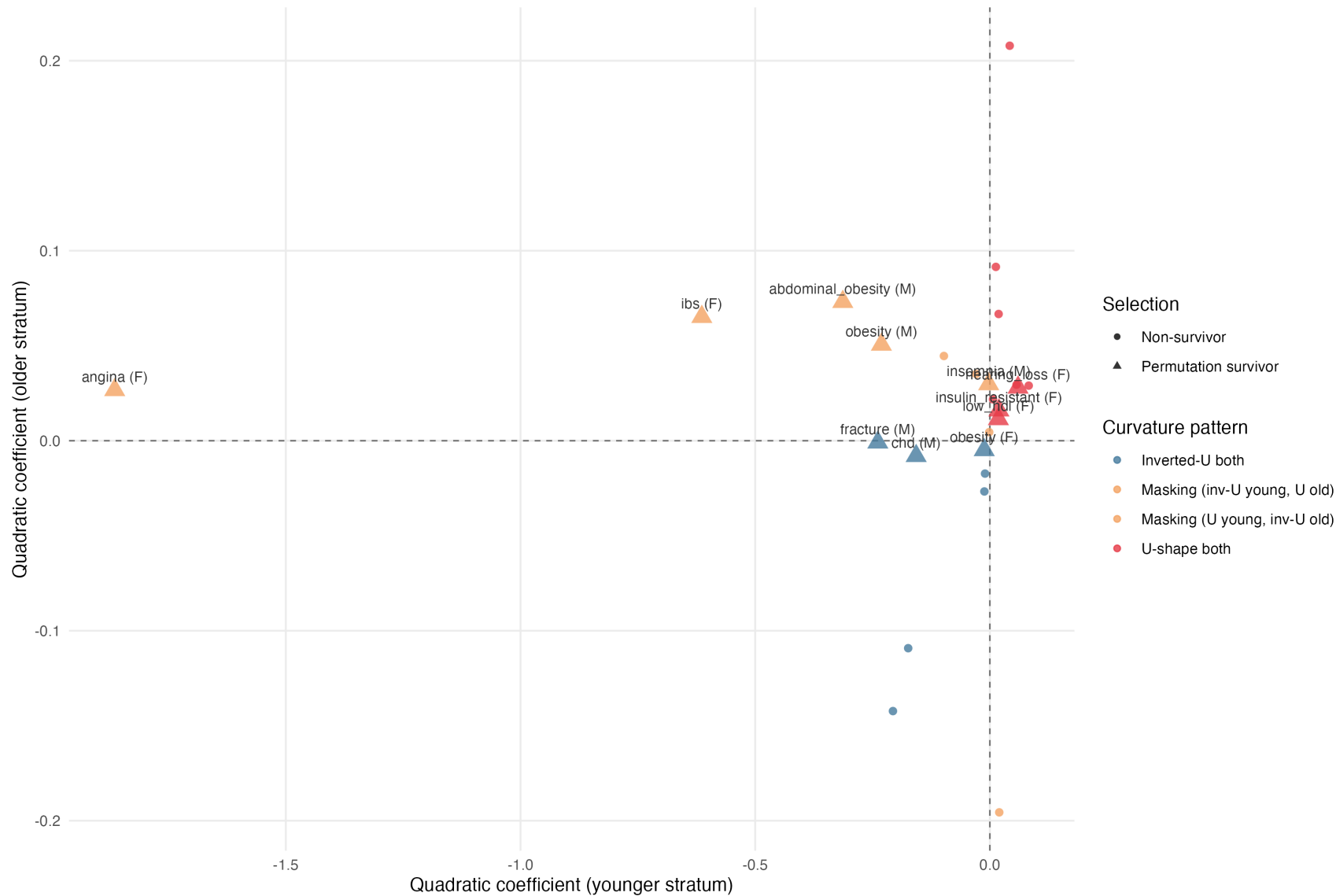


NHANES 2011-2023 | Total testosterone | complementary to Figure 4

Supplementary Figure S5. Anthropometric severity confirmations — waist circumference, BMI, and waist-to-hip ratio slopes by age stratum for the five survivors carrying anthropometric markers.

Supplementary Figure S5 | Curvature map

Young-stratum vs old-stratum quadratic coefficient (b2). Triangles = permutation survivors.

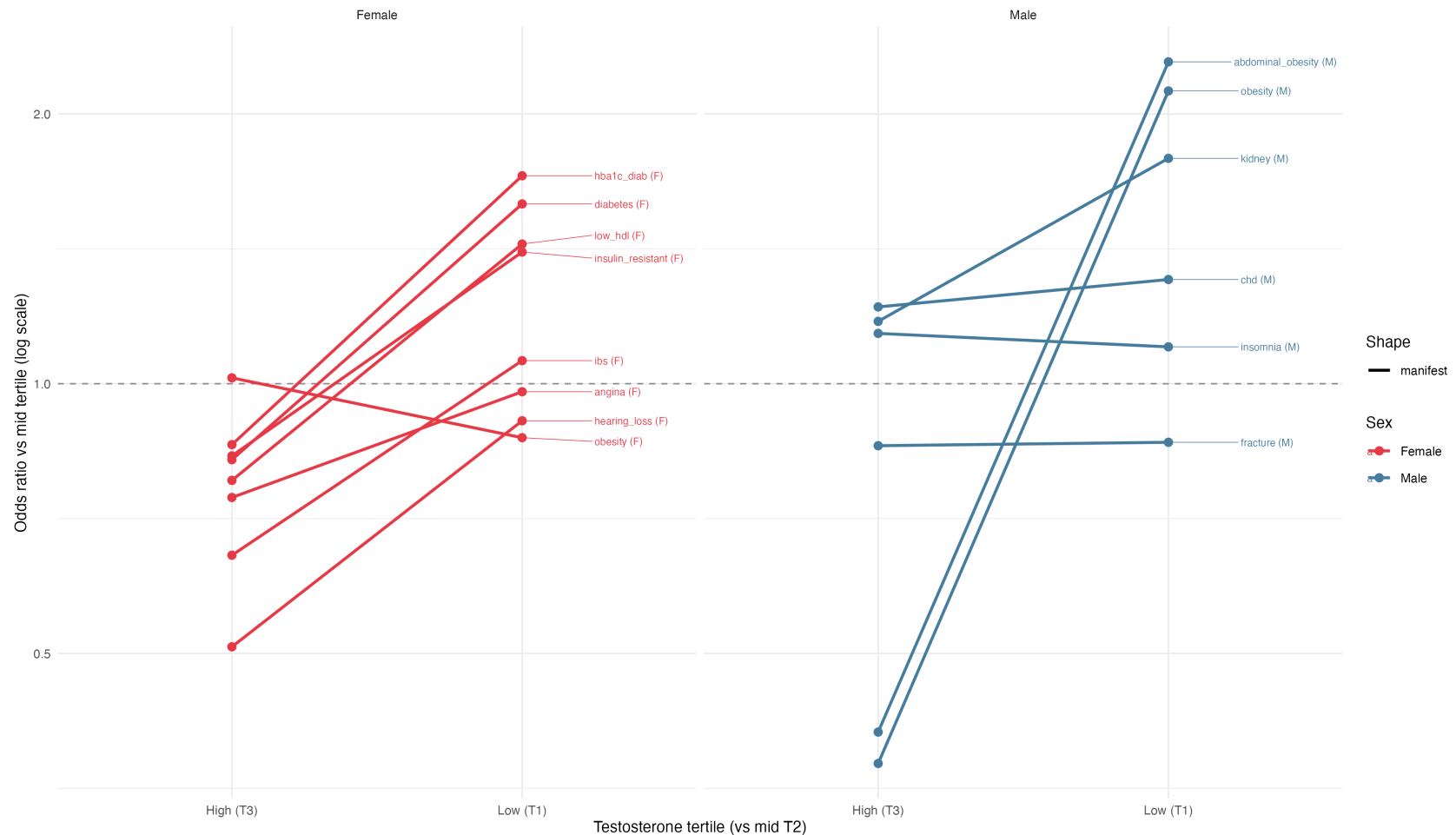


NHANES 2011-2023 | Total testosterone | bend-model characterisation

Supplementary Figure S7. Curvature map — young-stratum versus old-stratum quadratic coefficient per disease, with U/inverted-U quadrants and the curvature-masking pair highlighted.

Figure 5 | Non-linear associations — total testosterone

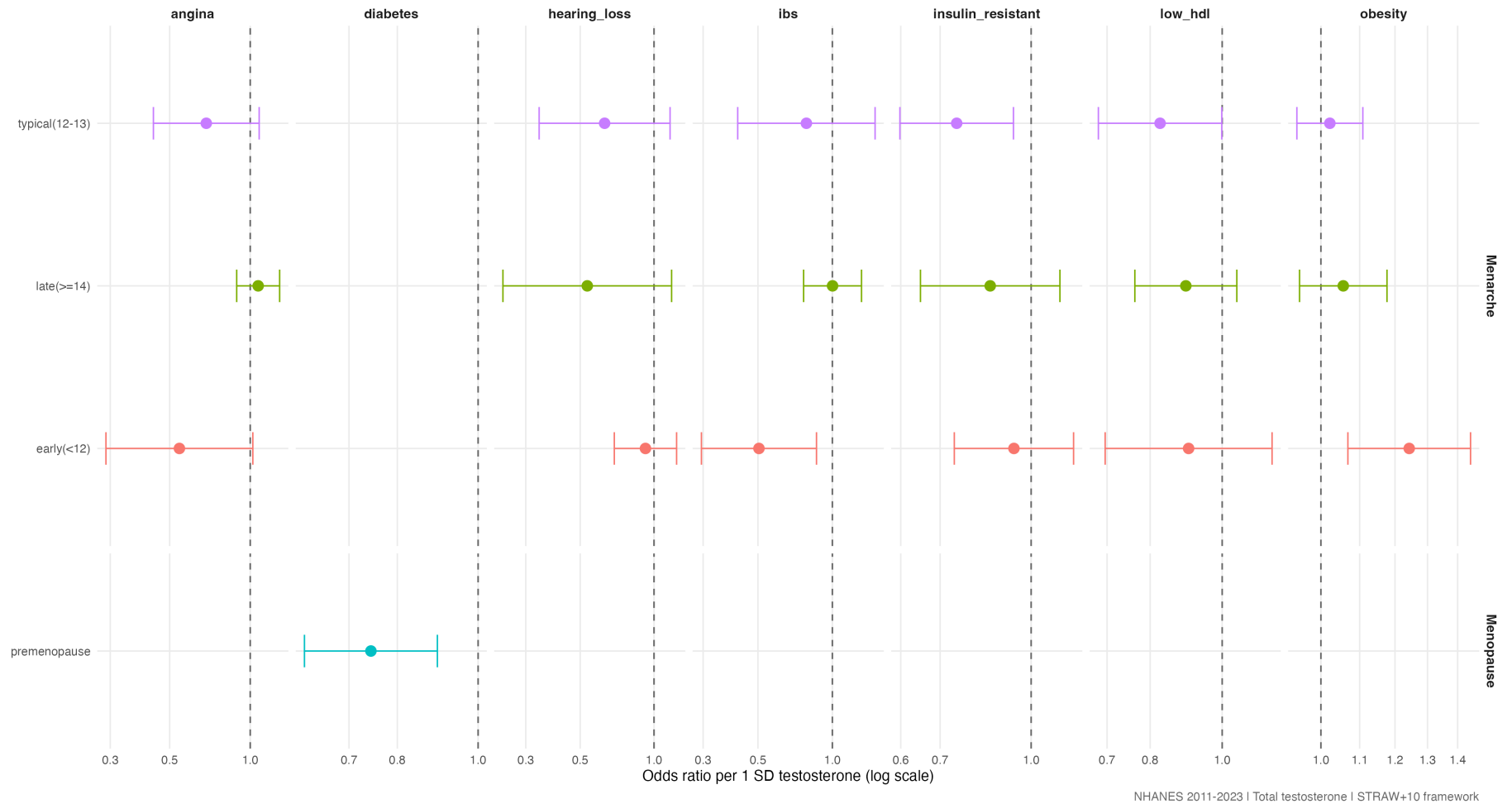
14 disease-sex pairs shown | both ends vs mid tertile (T2). Both ends >1 = U; both <1 = inverted-U.



Supplementary Figure 6. Non-linear associations by testosterone tertile. Odds ratio at the low (T1) and high (T3) testosterone tertiles relative to the middle tertile (T2) reference, drawn as a line for each disease and faceted by sex. Both endpoints above unity describe a U-shape and both below an inverted-U. The female metabolic outcomes (HbA1c-defined diabetes, low HDL cholesterol, and insulin resistance) show the U-shaped testosterone relationship where both low and high tertiles carry higher disease odds than the middle tertile, indicating a non-linear dose-response that a purely linear regression on total testosterone would attenuate. In men, coronary heart disease and insomnia show approximately linear tertile gradients, while abdominal obesity and obesity carry both endpoints below unity in the same direction, consistent with monotone dose-response rather than curvature

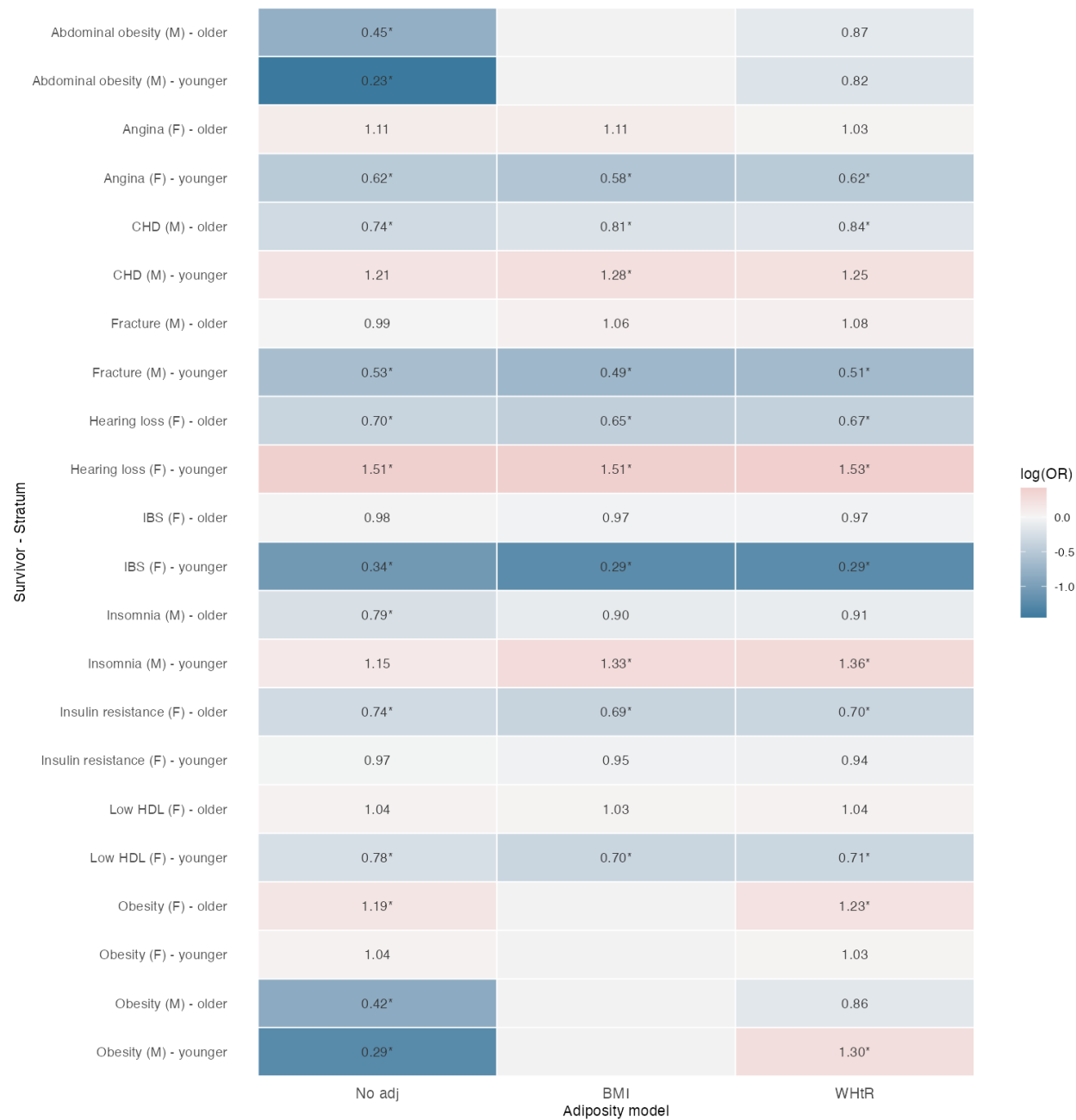
Supplementary Figure S7 | Menarche and menopause stratified odds ratios

Female reversals stratified by reproductive stage. Bar = 95% CI.



Supplementary Figure S7. Menarche- and menopause-stratified odds ratios for the female reversals.

Supplementary Figure S9. Adiposity odds-ratio grid
OR per stratum under three adiposity models. * = $p < 0.05$. Colour consistency across a row = robust.

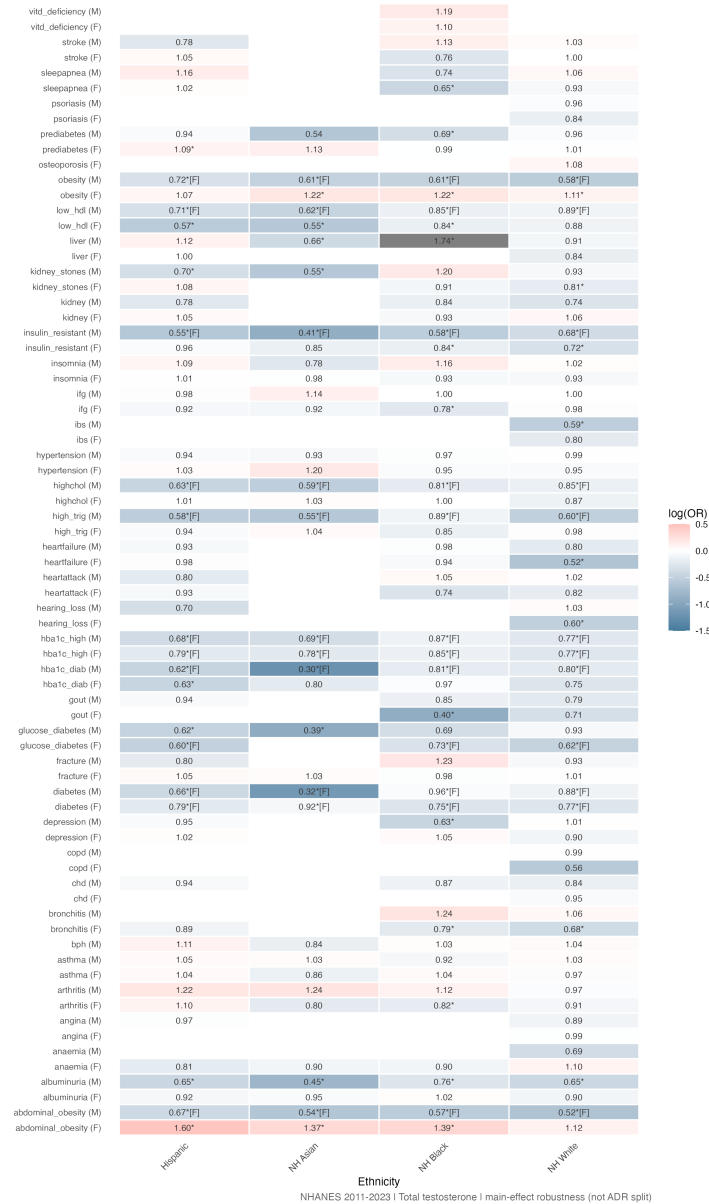


NHANES 2011-2023 | Total testosterone | complementary to Figure S8

Supplementary Figure S8. Adiposity odds-ratio grid — per surviving reversal and stratum, the testosterone odds ratio under each adiposity model, coloured by log odds ratio with significance marked; a consistent colour across a row indicates robustness to the adiposity measure.

Supplementary Figure S10 | Ethnicity main-effect heatmap

Testosterone OR per disease/sex within each NHANES ethnicity. * = $p < 0.05$, [F] = FDR-sig in main PheWAS.



NHANES 2011-2023 | Total testosterone | main-effect robustness (not ADR split)

Supplementary Figure S9. Ethnicity main-effect heatmap — testosterone odds ratio per disease and sex within each NHANES ethnicity group, with nominal-significance and main-screen-FDR annotations (main-effect robustness, not an age-boundary split).