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Long-yue Li

Air Force Engineering University

Guo-qing Zhang

Air Force Engineering University

Pan-rong Wang

Air Force Engineering University

Ye Tian

chuanzi18@yeah.net

Air Force Engineering University

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## Article

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# Trigger Determination and Evaluation of Capability Emergence in Combat Systems Based on Self-Organized Criticality Phase Transition

Long-yue Li<sup>1</sup>, Guo-qing Zhang<sup>2</sup>, Pan-rong Wang<sup>2</sup> and Ye Tian<sup>1,\*</sup>

<sup>1</sup> Air Defense and Antimissile School, Air Force Engineering University, Xi'an 710051,  
China

<sup>2</sup> Graduate School, Air Force Engineering University, Xi'an 710051, China

\* Corresponding author: Ye Tian; email: chuanzi18@yeah.net

Author emails: Long-yue Li, lilong\_yue@hotmail.com; Guo-qing Zhang,  
17532391886@163.com; Pan-rong Wang, 19573809151@163.com.

**Abstract:** The precise determination and effective measurement of combat system capability emergence in complex adversarial environments remain a fundamental challenge in complex systems engineering. This paper proposes a trigger determination and evaluation method grounded in self-organized criticality (SOC) phase transition theory. Combat units of an air and missile defense (AMD) system are abstracted as cellular automata (CA) model units, and microscopic indicators — including configuration density, update frequency, and fractal dimension — are employed to characterize the evolutionary features of system states. Based on SOC theory, power-law distribution and edge-of-chaos criteria are utilized to establish macroscopic emergence trigger conditions, thereby forming a dual micro–macro trigger determination framework. The Hellinger (Hel) divergence is introduced as an information-theoretic metric for quantifying capability distribution discrepancy. Integrated with a quasi-cellular automata capability aggregation model, emergence effect quantification indicators and traceability analysis methods are developed. Simulation results based on a saturated

multi-threat interception scenario involving cruise missiles, ballistic missiles, and combat aircraft demonstrate that when the configuration density exceeds 0.25 and the box-counting dimension approaches 1.89, the system enters a self-organized critical state; the criticality indicator crosses the trigger threshold, and capability emergence is effectively triggered. The emergent system capability exceeds the linear superposition expectation by 23.7%, with the Hel-divergence increasing from 0.12 to 0.68 and the emergence efficiency reaching 0.71. The proposed framework provides a rigorous theoretical foundation and practical quantitative tools for capability emergence determination and evaluation in complex combat systems.

**Keywords:** capability emergence; trigger determination; self-organized criticality; Hel-divergence; cellular automata; air and missile defense system; fractal dimension; metric evaluation

## 1 Introduction

Aerospace security has emerged as a central arena of strategic competition among major powers, and the air and missile defense (AMD) system constitutes a critical pillar for safeguarding national airspace. The operational effectiveness of such systems is no longer determined by the performance of any single platform; rather, it depends on the emergent effects generated through information exchange, fire coordination, and action synchronization among heterogeneous combat units operating as an integrated system-of-systems<sup>1,2</sup>. This emergent effect causes the overall system capability to surpass the linear superposition of its constituent units, exhibiting a nonlinear amplification characteristic commonly denoted as  $1 + 1 > 2$ . However, capability emergence is inherently spontaneous and unpredictable: when it is triggered, how it can be reliably identified, and how its magnitude can be rigorously quantified remain open problems that constrain the full exploitation of system combat effectiveness.

Complex systems theory<sup>1,2,3,4</sup> has established that emergence arises when systems far from equilibrium spontaneously produce macroscopic ordered behavior through nonlinear interactions among their constituent elements<sup>3,5,6</sup>. The theory of complex adaptive systems (CAS) provides a foundational framework for understanding emergent phenomena, yet its application to military domains faces two principal challenges. First, the strong adversarial nature and high dynamism of combat systems render the precise delineation of emergence trigger conditions exceedingly difficult. Second, existing measurement approaches for emergence effects largely remain at the qualitative description level, lacking operable quantitative tools<sup>5,6</sup>. While complex network analyses have revealed the nonlinear mapping between network topology and emergence intensity, specific quantitative thresholds for emergence determination have not been forthcoming.

The theory of self-organized criticality (SOC), introduced by Bak, Tang, and Wiesenfeld in their seminal study of the sandpile model<sup>7,8</sup>, offers a compelling perspective on emergence triggering. The central insight is that complex dynamical systems can spontaneously evolve toward a critical state without external tuning; at this critical point, the spatial and temporal distributions of system events follow power-law statistics<sup>9,10</sup>. If the evolution of an AMD system is analogized to a sandpile model, then capability emergence can be interpreted as a phase transition occurring once the system reaches a self-organized critical state. Cellular automata (CA), as discrete dynamical system models, naturally capture the local interactions and global evolution of combat units in spatial domains and provide an effective tool for analyzing the microscopic generation mechanism of emergence<sup>11,12</sup>. Moreover, the Hellinger (Hel) divergence, an information-theoretic measure of the discrepancy between two probability distributions, possesses desirable properties including symmetry, boundedness, and sensitivity to distribution tails, making it well suited for characterizing structural changes in capability distributions before and after emergence<sup>13,14</sup>.

Building upon these insights, this paper follows a research roadmap of “mergence feature analysis trigger determination method construction evaluation model establishment” and proposes a trigger determination and evaluation method for capability emergence based on SOC phase transition theory. The main contributions are as follows: (1) Combat units are abstracted as CA model units, and microscopic indicators — configuration density, update frequency, and fractal dimensions are employed to reveal the microscopic mechanism of emergence formation. (2) A dual micro–macro trigger determination framework is constructed by integrating power-law distribution<sup>15</sup> and edge-of-chaos criteria from SOC theory<sup>7,8</sup> with the microscopic indicators including fractal dimensions<sup>16,17,18,19</sup> and Hurst exponents<sup>20,21</sup>. (3) The Hel-divergence is introduced as a metric for capability distribution discrepancy, and a quasi-cellular automata capability aggregation model is developed to quantify and trace emergence effects. (4) A capability emergence measurement indicator system encompassing two hierarchical levels and three dimensions is established, and the proposed method is validated through simulation.

## **2 Complexity Mechanism Analysis of Combat System Capability Emergence**

### **2.1 Complex System Characteristics of Air and Missile Defense Systems**

An AMD system is a complex combat system comprising early-warning detection, command and control (C2), tracking and guidance, and interception strike subsystems coupled through an information network. Its complexity manifests at both the global and local levels<sup>1,3</sup>. At the global level, the system exhibits dynamic evolution, scalable adaptability, networked interactivity, and uncertainty-induced complexity. Dynamic evolution implies that system states change continuously throughout the engagement, with unit capabilities rising and falling in a nonstationary stochastic process. Scalable adaptability allows the system to flexibly adjust its scale and structure in response to operational demands; the addition or withdrawal of units alters the topological characteristics and

coupling relationships of the system. Networked interactivity is reflected in information sharing and collaborative decision-making among combat units, where the network topology directly governs information flow efficiency and coordination effectiveness. Uncertainty-induced complexity originates from the unpredictability of the adversarial environment, target behavior, and internal stochastic perturbations<sup>4,5</sup>.

At the local level, system components possess autonomy, dominance, heterogeneity, and distributivity. Autonomy refers to the capacity of each combat unit to independently sense, decide, and act within a certain scope, possessing its own state transition logic. Dominance implies that certain key nodes — such as C2 centers and core sensors — play a leading role in system operation, and their state changes can trigger cascading effects. Heterogeneity is evident in the significant differences among unit types in function, performance, and operating principle; for instance, warning radars, tracking radars, and interceptor missiles differ substantially in detection range, response time, and damage mechanism. Distributivity refers to the dispersed geographical deployment of combat units, where spatial distribution characteristics influence communication latency and coordination efficiency. The coupling of these features constitutes the foundational conditions for capability emergence.

To quantitatively describe the system state, we introduce the concept of combat potential energy<sup>5,6</sup>. Combat potential energy is defined as the comprehensive measure of the latent combat capability accumulated by a combat system through the coupling effects of information exchange, fire coordination, and action synchronization among its units under given operational environments and C2 conditions. Let the distributed combat system contain  $N$  combat units, with the state vector of the  $i$ -th unit denoted by  $s_i(t)$ ; then the combat potential energy of the system at time  $t$  is defined as:

$$\Phi(t) = \sum_{i=1}^N w_i f_i(s_i(t)) + \sum_{i=1}^N \sum_{j \in N_i} w_i w_j g_{ij}(s_i, s_j)$$

where  $w_i$  is the weight coefficient of the  $i$ -th unit;  $f_i(\cdot)$  is the unit capability function reflecting its intrinsic combat capability;  $g_{ij}(\cdot)$  is the coupling function between units  $i$  and  $j$ , characterizing nonlinear interaction effects; and  $N_i$  is the neighborhood set of unit  $i$ . The first term represents the weighted sum of individual unit capabilities, while the second term captures the cumulative coupling effects between units, embodying the nonlinear character of the system.

The capability function  $f_i$  measures the combat effectiveness of the  $i$ -th unit in its current state. Considering the influence of state deviation from the optimal state and the diminishing marginal returns of resource consumption, the capability function adopts a composite form of a Gaussian potential function and a hyperbolic tangent activation function:

$$f_i(s_i) = \alpha_i \exp\left(1 - \frac{\|s_i - i_*^s\|^2}{2i_2^\sigma}\right) \cdot \tanh\left(\frac{\beta_i r_i(t)}{i_{max}^r}\right)$$

where  $\alpha_i$  is the maximum capability coefficient of unit  $i$ ;  $i_*^s$  is the optimal state vector;  $\sigma_i$  is the state tolerance parameter;  $\beta_i$  is the excitation gain coefficient;  $r_i(t)$  is the remaining resource of unit  $i$  at time  $t$ ; and  $i_{max}^r$  is the resource upper bound. The Gaussian term characterizes the decay of capability as the state deviates from the optimum, while the hyperbolic tangent term describes the nonlinear modulation of capability by resource consumption. The coupling function  $g_{ij}$  captures the nonlinear modulation of combat potential energy by the cooperative or adversarial relationship between units, incorporating spatial distance attenuation and phase synchronization effects:

$$g_{ij}(s_i, s_j) = \kappa_{ij} \cos(\Delta\varphi_{ij}) \exp\left(1 - \frac{d_{ij}}{\lambda}\right)$$

where  $\kappa_{ij}$  is the coupling strength coefficient (positive for cooperative relationships, negative for adversarial);  $\Delta\varphi_{ij}$  is the action phase difference between units  $i$  and  $j$ ;  $d_{ij}$  is the spatial distance; and  $\lambda$  is the spatial interaction scale parameter. When  $\kappa_{ij} > 0$  and

$\Delta\varphi_{ij}$  is small,  $g_{ij}$  approaches  $\kappa_{ij}$ , indicating cooperative enhancement; as  $\Delta\varphi_{ij}$  approaches  $\pi/2$ , the cooperative effect tends to zero. The weight coefficient  $w_i$  is jointly determined by the task priority and effectiveness contribution of the unit:

$$w_i = \frac{p_i e_i}{\sum_{j=1}^N p_j e_j}, \sum_{i=1}^N w_i = 1$$

where  $p_i$  is the task priority determined by the C2 hierarchy, and  $e_i$  is the effectiveness contribution, obtainable through system contribution rate analysis. The offset of combat potential energy reflects the degree of change in the system state relative to the initial state, and its dynamics are described by:

$$\Delta\Phi(t) = \Phi(t) - \Phi(0) = \int_0^t \frac{d\Phi}{d\tau} d\tau = \int_0^t \left[ \sum_i w_i \dot{f}_i + \sum_i \sum_j w_i w_j \dot{g}_{ij} \right] d\tau$$

This expression indicates that the combat potential energy offset  $\Delta\Phi(t)$  is determined jointly by the rate of change of unit capabilities and coupling effects. When  $\Delta\Phi(t)$  increases persistently and exceeds a critical value, the system enters a high-potential-energy state, providing the energetic basis for capability emergence.

## 2.2 Microscopic Mechanism of Emergence Based on Cellular Automata

To reveal the microscopic generation mechanism of capability emergence, combat units of the AMD system are abstracted as cellular automata (CA) model units<sup>11,12</sup>. Each combat unit corresponds to a cell, whose state  $s_i(t) \in [0,1]$  represents the activation level or capability at time  $t$ . The spatial distribution of cells maps the geographical deployment of combat units, and the neighborhood relationships among cells reflect the communication and coordination range. Cell state transitions follow local update rules, meaning that the next state of each cell depends only on its current state and the states of neighboring cells. The state transition equation of the cellular automaton is given by:

$$s_i(t + 1) = \sigma \left( \sum_{j \in N_i} J_{ij} s_j(t) + h_i(t) - \theta_i \right) + \sqrt{2D_i} \xi_i(t)$$

where  $\sigma(\cdot)$  is the sigmoid activation function;  $J_{ij}$  is the coupling weight between cells  $i$  and  $j$ , corresponding to the information interaction strength between combat units;  $h_i(t)$  is the external input, reflecting C2 directives and target threat information;  $\theta_i$  is the activation threshold;  $D_i$  is the noise intensity; and  $\xi_i(t)$  is Gaussian white noise modeling environmental uncertainty. This model combines continuous state space with discrete-time evolution, enabling simultaneous characterization of both gradual capability changes and abrupt state flips of combat units<sup>11,12</sup>.

More generally, the evolution of the cellular automaton can be expressed in operator form:

$$s(t + 1) = F(s(t), \xi(t)), F: S^N \times \mathcal{E} \rightarrow S^N$$

where  $S$  is the state space,  $\mathcal{E}$  is the stochastic perturbation space, and  $F$  is the state transition operator<sup>11</sup>. Incorporating the coupling between state inertia and external excitation, a damping coefficient  $\eta_i$  is introduced, yielding the refined state update rule:

$$s_i(t + 1) = (1 - \eta_i)s_i(t) + \eta_i \sigma \left( \sum_{j \in N_i} J_{ij} s_j(t) + I_i(t) - \theta_i \right) + \sqrt{2D_i} \xi_i(t)$$

In this model, the damping term  $(1 - \eta_i)s_i(t)$  ensures that the unit state possesses inertia, preventing violent fluctuations caused by transient perturbations; the excitation term  $\eta_i \sigma(\cdot)$  reflects the driving effect of neighboring cells and external inputs on the unit state; and the noise term  $\sqrt{2D_i} \xi_i(t)$  introduces stochasticity, simulating battlefield uncertainty. When the coupling weights  $J_{ij}$  are strong and neighboring cell states tend to converge, the cell state rapidly converges to a high-activation state, forming local synchronization; when multiple local synchronization regions interconnect, global emergence is triggered<sup>22</sup>.

## 2.3 Phase Transition Mechanism of Emergence Formation

From the perspective of statistical physics, capability emergence is fundamentally a phase transition process in which the system transitions from a disordered state to an ordered state<sup>9,22,23,24</sup>. In the subcritical state, cell states are mutually independent, and the system capability manifests as the linear superposition of individual unit capabilities. When the system parameters — such as coupling strength and external excitation — exceed the critical point, long-range correlations among cells begin to form, the system enters the critical state, and small perturbations can trigger large-scale state flips, producing emergence. This phase transition process is analogous to the spontaneous magnetization of a ferromagnet near the Curie temperature: when the temperature falls below the Curie point, the exchange interaction between spins overcomes thermal fluctuations, forming long-range magnetic order<sup>24,25</sup>.

In the CA model, the occurrence of phase transition depends on the distribution of coupling weights  $J_{ij}$  and the activation threshold  $\theta_i$ . When the mean coupling strength  $\langle J \rangle$  is below the critical value  $J_c$ , the system is in the subcritical state, and cell state fluctuations are small with weak spatial correlations. As  $\langle J \rangle$  approaches  $J_c$ , the system enters the edge-of-chaos regime, where cell states exhibit long-range correlations and critical slowing down. When  $\langle J \rangle$  exceeds  $J_c$ , the system transitions to the supercritical state, manifesting large-scale synchronous activation corresponding to capability emergence. This phase transition can be characterized by the order parameter  $\Psi(t)$ :

$$\Psi(t) = \frac{1}{M} \sum_{i=1}^M \exp(i\varphi_i(t)), |\Psi| \in [0,1]$$

where  $\varphi_i(t)$  is the phase of cell  $i$  at time  $t$ . When  $|\Psi|$  approaches 0, cell phases are disordered and the system is in a disordered state; when  $|\Psi|$  approaches 1, cell phases are highly synchronized and the system has entered an ordered state. The abrupt change in the

order parameter signals the occurrence of phase transition, providing a macroscopic indicator for emergence trigger determination<sup>26,27</sup>.

### 3 Capability Emergence Trigger Determination Method

Trigger determination of capability emergence is the prerequisite and foundation for system capability evaluation. This section follows the approach of microscopic indicator computation  $\rightarrow$  macroscopic criticality assessment  $\rightarrow$  dual criterion fusion to construct a multi-dimensional emergence trigger determination framework. At the microscopic level, configuration density, update frequency, and fractal dimension are computed based on the CA model to characterize the local evolutionary features of system states. At the macroscopic level, the power-law distribution and edge-of-chaos characteristics of the system are analyzed using SOC theory to determine whether the system has entered the critical phase transition regime. The results from both levels are fused through a criticality indicator function, yielding a complete emergence trigger determination criterion.

#### 3.1 Microscopic Emergence Determination Based on Configuration Density

Configuration density  $\rho(t)$  is a fundamental indicator measuring the activation level of the cellular automaton, defined as the proportion of non-zero cells whose activation intensity exceeds the threshold  $\theta_s$  in the spatial sequence:

$$\rho(t) = \frac{1}{M} \sum_{i=1}^M s_i(t) 1[s_i(t) > \theta_s]$$

where  $M$  is the total number of cells and  $1[\cdot]$  is the indicator function. The configuration density reflects the overall activation level of the system at time  $t$ , with a range of  $[0, 1]$ . When  $\rho(t)$  is low, the system is in a sparse activation state, and effective interaction among cells is lacking. As  $\rho(t)$  rises persistently and approaches the critical value  $\rho^*$ , the interaction probability among cells increases, and local synchronization begins

236 to form. Once  $\rho(t)$  exceeds  $\rho^*$ , the system enters a dense activation state, and the  
 237 emergence probability rises sharply <sup>7,8</sup>.

238 The update frequency  $v_i(t)$  measures the rate of state change of cell  $i$  within the time  
 239 window  $\tau$ , reflecting the unit activity level and response characteristics:

$$240 \quad v_i(t) = \frac{1}{\tau} \sum_{k=t-\tau+1}^t |s_i(k) - s_i(k-1)|, \bar{v}(t) = \frac{1}{M} \sum_{i=1}^M v_i(t)$$

241 where  $\tau$  is the time window length and  $\bar{v}(t)$  is the average update frequency across all  
 242 cells. The ratio of configuration density to update frequency reflects the efficiency of system  
 243 state change: at a given configuration density, a lower update frequency implies slower, more  
 244 stable state changes, more closely resembling critical slowing-down behavior; a higher  
 245 update frequency indicates that the system is in a state of violent fluctuation, possibly not yet  
 246 reaching a stable critical state.

### 247 3.2 Complexity Measurement Based on Fractal Dimensions

248 Fractal dimension is an essential tool for characterizing the spatial structure of complex  
 249 systems, quantitatively describing the self-similarity and complexity of cell state  
 250 distributions. Three fractal dimension indicators — box-counting dimension, information  
 251 dimension, and correlation dimension — are employed to measure structural complexity from  
 252 different perspectives. The box-counting dimension  $D_B$  is determined by the relationship  
 253 between the minimum number of boxes covering the cell state distribution and the box scale  
 254 <sup>16,19</sup>:

$$255 \quad D_B = -\lim_{\varepsilon \rightarrow 0} \frac{\ln N(\varepsilon)}{\ln \varepsilon}, N(\varepsilon) = \sum_k 1 [\exists x \in B_k: s(x) > \theta_s]$$

256 where  $N(\varepsilon)$  is the number of boxes of scale  $\varepsilon$  containing at least one active cell.  $D_B$   
 257 reflects the spatial filling degree of the state distribution:  $D_B$  approaching 0 indicates highly

concentrated state distribution, while  $D_B$  approaching 2 (in the two-dimensional case) indicates uniform filling of the entire space. Before emergence,  $D_B$  typically remains at a low level (1.0–1.4), indicating that cell activation is locally clustered; when emergence is triggered,  $D_B$  rises rapidly to the range 1.7–1.9, indicating that the activation pattern exhibits self-similar structures spanning multiple scales.

The information dimension  $D_I$  introduces probability weights on the basis of the box-counting dimension, enabling finer characterization of the non-uniformity of the state distribution:

$$D_I = -\lim_{\varepsilon \rightarrow 0} \frac{\sum_i p_i \ln p_i}{\ln \varepsilon}, p_i = \frac{N_i(\varepsilon)}{\sum_j N_j(\varepsilon)}$$

where  $p_i$  is the proportion of active cells in the  $i$ -th box relative to all active cells. The difference  $D_B - D_I$  reflects the multifractal character of the state distribution<sup>18,19</sup>: a larger difference indicates greater non-uniformity and more pronounced multifractal structure. The correlation dimension  $D_C$  characterizes the geometric features of the attractor from the perspective of pairwise distance distributions in the state space:

$$D_C = \lim_{\varepsilon \rightarrow 0} \frac{\ln C(\varepsilon)}{\ln \varepsilon}, C(\varepsilon) = \frac{2}{N(N-1)} \sum_{i < j} 1 [\|x_i - x_j\| < \varepsilon]$$

where  $C(\varepsilon)$  is the correlation integral, representing the proportion of point pairs with distances less than  $\varepsilon$  in the state space. The correlation dimension reflects the dynamical complexity of the system: a higher  $D_C$  indicates richer dynamical behavior that is more difficult to describe with low-dimensional models. More generally, the generalized fractal dimension (Renyi dimension)  $D_q$  unifies the above three dimensions through the continuous variation of the order  $q$ <sup>19</sup>:

$$D_q = \frac{1}{q-1} \lim_{\varepsilon \rightarrow 0} \frac{\ln \left( \sum_i i_q^p \right)}{\ln \varepsilon}, q \in R$$

When  $q = 0$ ,  $D_q$  reduces to the box-counting dimension  $D_B$ ; when  $q = 1$ , it becomes the information dimension  $D_I$ ; and when  $q = 2$ , it becomes the correlation dimension  $D_C$ . The variation of  $D_q$  with  $q$  yields the multifractal spectrum, whose shape and width characterize the multifractal features of the state distribution, providing richer structural information for emergence determination.

### 3.3 Macroscopic Emergence Determination Based on Self-Organized Criticality

SOC theory posits those complex dynamical systems at the critical state exhibit power-law distributions in both spatial and temporal domains<sup>7,9,10</sup>. This means that avalanche events — cascading flips of cell states — occurring near the critical state follow power-law rather than exponential distributions in terms of their size and duration. The probability density function of the power-law distribution is:

$$P(x) = Cx^{-\alpha}, x \geq x_{min}, \alpha \in (1,3)$$

where  $C$  is a normalization constant and  $\alpha$  is the power-law exponent<sup>26</sup>, typically in the range  $(1,3)$ . The value of  $\alpha$  reflects the distribution of avalanche sizes:  $\alpha$  approaching 1 indicates frequent large avalanches and a strong critical state, while  $\alpha$  approaching 3 indicates rare large avalanches and departure from criticality. The corresponding cumulative distribution function is:

$$P(X \geq x) = \int_x^\infty C u^{-\alpha} du = \frac{C}{\alpha-1} x^{-(\alpha-1)}$$

In the frequency domain, the power spectral density of a critical-state system satisfies the  $1/f$  noise characteristic, i.e.,  $S(f) \propto f^{-\beta}$ , where  $\beta$  is related to the power-law exponent  $\alpha$  through the scaling relation:

$$\tau = \alpha - 1, S(f) \propto f^{-\beta}, \beta = 2 - \frac{\alpha - 1}{2}$$

This relation provides a pathway for determining the system criticality through frequency-domain analysis. Furthermore, the Hurst exponent  $H$  characterizes the long-range correlation of system state evolution, defined as the slope of the rescaled range (R/S) analysis<sup>20,21</sup>:

$$H = \lim_{n \rightarrow \infty} \frac{\ln(R_n/S_n)}{\ln(n)}, R_n = \max_{1 \leq k \leq n} X_k - \min_{1 \leq k \leq n} X_k, S_n = \sqrt{\text{Var}(X)}$$

where  $R_n$  is the cumulative deviation range and  $S_n$  is the standard deviation.  $H > 0.5$  indicates persistence (long-range positive correlation),  $H = 0.5$  indicates no memory (white noise), and  $H < 0.5$  indicates anti-persistence. Near the critical state,  $H$  typically approaches 1, indicating strong long-range correlation. The maximum Lyapunov exponent  $\lambda_{max}$  further characterizes the chaotic dynamics of the system<sup>28,29</sup>:

$$\lambda_{max} = \lim_{t \rightarrow \infty} \frac{1}{t} \ln \frac{\|\delta s(t)\|}{\|\delta s(0)\|}$$

When  $\lambda_{max} > 0$ , the system is in a chaotic state; when  $\lambda_{max} = 0$ , the system is at the edge of chaos; and when  $\lambda_{max} < 0$ , the system tends toward stability. The edge-of-chaos state ( $\lambda_{max} \approx 0$ ) is the hallmark of self-organized criticality<sup>22,30</sup>, where the system is neither fully ordered nor fully disordered, information processing capacity is maximized, and emergence probability is highest<sup>22</sup>.

### 3.4 Dual Micro–Macro Trigger Determination Criterion

Based on the above microscopic indicators and macroscopic criteria, a criticality indicator function  $\Lambda(t)$  is constructed as a composite determination metric fusing micro- and macro-level information:

$$\Lambda(t) = \frac{\rho(t) \cdot D_B(t)}{\bar{v}(t) + \varepsilon_0}$$

where  $\rho(t)$  is the configuration density,  $D_B(t)$  is the box-counting dimension,  $\bar{v}(t)$  is the average update frequency, and  $\varepsilon_0$  is a regularization constant preventing division by zero. The physical interpretation of  $\Lambda(t)$  is the product of the system activation density and structural complexity at a given update frequency. When  $\Lambda(t)$  is low, the system is in the subcritical state; when  $\Lambda(t)$  exceeds the critical threshold  $\Lambda^*$ , the system enters the critical phase transition regime, and capability emergence is triggered.

Integrating the microscopic and macroscopic determination conditions, the emergence trigger determination criterion is formulated as:

$$T(t) = 1[|\Lambda(t) - \Lambda^*| < \delta] \cdot 1\left[\frac{d\rho}{dt} > 0\right] \cdot 1[D_B(t) > B_*^D]$$

This criterion requires the simultaneous satisfaction of three conditions: (1) the criticality indicator  $\Lambda(t)$  is in the vicinity of the critical value  $\Lambda^*$ , within an error tolerance  $\delta$ ; (2) the configuration density  $\rho(t)$  is increasing, indicating that the system activation level is continuously intensifying; and (3) the box-counting dimension  $D_B(t)$  exceeds the critical dimension  $B_*^D$ , indicating that the system has formed cross-scale self-similar structures. The logical conjunction of these three conditions ensures the reliability of the determination, effectively reducing the probability of false alarms.

To support the above determination method, a capability emergence measurement indicator system is constructed, encompassing two hierarchical levels and three dimensions. The two levels are the microscopic level (cell-level indicators) and the macroscopic level (system-level indicators); the three dimensions are the structural dimension (topological and fractal features), the dynamical dimension (temporal evolution features), and the functional dimension (capability aggregation features). The supported AMD weapon types cover early-

warning radars and tracking and guidance radars (detection and perception assets), long-range/medium-range/short-range AMD fire units (interception assets), and C2 nodes. The complete indicator vector is defined as:

$$M(t) = [\rho(t), \bar{v}(t), D_B(t), D_I(t), \alpha(t), H(t), \Lambda(t), E(t)]^T$$

This 8-dimensional indicator vector  $M(t)$  comprehensively characterizes the emergence state of the system at time  $t$ , where  $\rho(t)$  and  $\bar{v}(t)$  belong to the microscopic dynamical dimension,  $D_B(t)$  and  $D_I(t)$  belong to the microscopic structural dimension,  $\alpha(t)$  and  $H(t)$  belong to the macroscopic dynamical dimension, and  $\Lambda(t)$  and  $E(t)$  belong to the macroscopic functional dimension.

## 4 Capability Emergence Measurement and Evaluation Model Based on Hel-Divergence

### 4.1 Definition and Properties of Hel-Divergence

The Hellinger divergence is an information-theoretic measure of the discrepancy between two probability distributions. Its discrete form is defined as<sup>13,14,31,32</sup>:

$$D_{Hel}(P\|Q) = 2 \sum_{i=1}^n (\sqrt{p_i} - \sqrt{q_i})^2 = 2 \left( 2 - 2 \sum_i \sqrt{p_i q_i} \right)$$

where  $P = (p_1, p_2, \dots, p_n)$  and  $Q = (q_1, q_2, \dots, q_n)$  are two probability distributions. The Hel-divergence possesses the following desirable properties: (1) symmetry, i.e.,  $D_{Hel}(P\|Q) = D_{Hel}(Q\|P)$ ; (2) non-negativity, i.e.,  $D_{Hel}(P\|Q) \geq 0$ , with equality if and only if  $P = Q$ ; (3) boundedness, i.e.,  $D_{Hel}(P\|Q) \in [0, 4]$ , facilitating normalization; and (4) sensitivity to distribution tails, enabling effective capture of small-probability event differences. Compared with the Kullback–Leibler (KL) divergence<sup>13</sup>, the Hel-divergence does not require the absolute continuity condition and remains bounded when the distribution supports are non-overlapping, making it better suited for measuring structural changes in capability distributions. Its continuous form is:

$$D_{Hel}(\mu, \nu) = \int_{\Omega} \left( \sqrt{\frac{d\mu}{d\lambda}} - \sqrt{\frac{d\nu}{d\lambda}} \right)^2 d\lambda$$

where  $\mu$  and  $\nu$  are probability measures defined on the measure space  $(\Omega, F, \lambda)$ . The Hel-divergence has established conversion relationships with the Hellinger distance  $H(P, Q)$  and the Bhattacharyya<sup>32</sup> coefficient  $B_C$ :

$$H(P, Q) = \frac{1}{\sqrt{2}} \|\sqrt{P} - \sqrt{Q}\|_2 = \sqrt{\frac{D_{Hel}(P\|Q)}{2}}$$

$$B_C(P, Q) = 1 - \sum_{i=1}^n \sqrt{p_i q_i}, D_{Hel} = 2B_C$$

These equivalence relations provide flexibility in metric selection across different application scenarios. In capability emergence evaluation, the capability distribution  $P(t)$  at time  $t$  is compared with the initial capability distribution<sup>14</sup>  $P_0$ , and the degree of deviation of the capability distribution during the emergence process is quantified through the Hel-divergence.

#### 4.2 Quasi-Cellular Automata Capability Aggregation Model

To precisely characterize the capability aggregation process, a quasi-cellular automata capability aggregation model is constructed. In this model, the capability  $c_i(t)$  of each combat unit is treated as the cell state, and capability is nonlinearly aggregated through pairwise synergy kernel functions  $\Phi_{ij}$  and higher-order interaction kernel functions  $\Psi_{ijk}$ . The total system capability  $C(t)$  is defined as:

$$C(t) = \sum_i w_i c_i(t) + \sum_{i < j} w_i w_j \Phi_{ij}(c_i, c_j) + \sum_{i < j < k} w_i w_j w_k \Psi_{ijk}(c_i, c_j, c_k)$$

where the first term is the weighted sum of individual unit capabilities, corresponding to the linear superposition baseline; the second term captures pairwise synergy effects,

reflecting the nonlinear contribution of binary interactions to capability; and the third term represents third-order synergy effects, characterizing higher-order emergence generated by three-body interactions. The specific form of the synergy kernel function  $\Phi_{ij}$  is:

$$\Phi_{ij}(c_i, c_j) = \kappa_{ij} i_{\alpha_{ij}}^c j_{\beta_{ij}}^c \exp(1 - \gamma_{ij} \Delta i j_2^\varphi)$$

where  $\kappa_{ij}$  is the synergy strength coefficient;  $\alpha_{ij}$  and  $\beta_{ij}$  are the nonlinear exponents of capability interaction; and  $\gamma_{ij}$  is the phase penalty factor. This kernel function simultaneously accounts for the nonlinear combination of capability levels and the synchronization requirement of action phases: when two units have strong capabilities and synchronized phases, the synergy effect is maximized.

Within the cellular automata framework, the transition rate of capability states is modulated by the critical state of the system. An Arrhenius-type<sup>33</sup> transition rate is introduced to describe the transition probability of unit capability states:

$$W_i(t) = v_i(t) \cdot \exp\left(1 - \frac{\Delta E_i(t)}{k_B T_{eff}(t)}\right), T_{eff}(t) = \frac{\langle v^2 \rangle}{2D_B + 1}$$

where  $\Delta E_i(t)$  is the capability barrier of unit  $i$ ;  $k_B$  is the effective Boltzmann constant; and  $T_{eff}(t)$  is the effective system temperature, jointly determined by the fractal dimension and update frequency. As the system approaches the critical state,  $T_{eff}$  increases, transition rates grow, and unit capability state transitions become more frequent, promoting rapid capability aggregation and emergence. The information entropy  $S(t)$  and mutual information  $I(X; Y)$  further characterize the information flow during capability aggregation:

$$S(t) = - \sum_i p_i(t) \ln p_i(t), \frac{dS}{dt} = - \sum_i \dot{p}_i \ln p_i - \sum_i \dot{p}_i$$

$$I(X; Y) = \sum_x \sum_y p(x, y) \ln \frac{p(x, y)}{p(x)p(y)} = H(X) + H(Y) - H(X, Y)$$

The information entropy<sup>13</sup> reflects the uniformity of the capability distribution, while the mutual information characterizes the statistical dependence between unit capability states. During emergence, the increase in mutual information signifies enhanced inter-unit coordination, providing an information-theoretic perspective for quantifying emergence effects.

### 4.3 Emergence Effect Quantification and Traceability Analysis

Based on the Hel-divergence and the capability aggregation model, the capability emergence measure  $E(t)$  is defined as the normalized Hel-divergence between the current capability distribution and the initial capability distribution:

$$E(t) = \frac{D_{Hel}(P(t)||P_0)}{D_{Hel}(P_{max}||P_0)} \in [0,1]$$

where  $P(t)$  is the system capability distribution at time  $t$ ,  $P_0$  is the initial capability distribution, and  $P_{max}$  is the maximum capability distribution (ideal emergence state).  $E(t) \in [0,1]$ :  $E(t) = 0$  indicates no change in the capability distribution, and  $E(t) = 1$  indicates that the system has reached the maximum emergence state. The emergence intensity  $E(t)$  is defined as the product of the time derivative of the emergence measure and the trigger determination:

$$E(t) = \left[ \frac{dE}{dt} \right]^+ \cdot H(\Lambda(t) - \Lambda^*), [x]^+ = \max(x, 0)$$

where  $[x]^+ = \max(x, 0)$  ensures non-negativity of emergence intensity, and  $H(\cdot)$  is the Heaviside step function, outputting 1 only when the criticality indicator exceeds the threshold  $\Lambda^*$ . This definition ensures that emergence intensity is nonzero only when trigger conditions are satisfied, effectively distinguishing genuine emergence effects from ordinary capability fluctuations. The cumulative emergence effect  $E(T)$  is the integral of emergence intensity over the time interval  $[0, T]$ :

$$\varepsilon(T) = \int_0^T E(\tau) d\tau = \int_0^T \left[ \frac{dE}{d\tau} \right]^+ \cdot H(\Lambda(\tau) - \Lambda^*) d\tau$$

$\varepsilon(T)$  reflects the cumulative impact of emergence effects: a larger value indicates a more significant sustained contribution of emergence to system capability. To further quantify the "gain" effect of emergence, the capability emergence increment  $\Delta C(t)$  is defined as the difference between the emergent aggregation capability and the linear superposition capability:

$$\Delta C(t) = C_{emg}(t) - C_{lin}(t) = C_{emg}(t) - \sum_i w_i c_i(t)$$

where  $C_{emg}(t)$  is the emergent aggregation capability including higher-order synergy terms, and  $C_{lin}(t) = \sum_i w_i c_i(t)$  is the linear superposition baseline.  $\Delta C(t) > 0$  indicates positive capability gain from emergence, while  $\Delta C(t) < 0$  indicates capability loss. The emergence efficiency  $\eta(t)$  is defined as the ratio of capability increment to distribution offset:

$$\eta(t) = \frac{\Delta C(t)}{E(t) \cdot D_{Hel}(P_{max} \| P_0)} = \frac{\Delta C(t)}{D_{Hel}(P(t) \| P_0)}$$

This indicator reflects the capability gain per unit of distribution offset: a higher  $\eta(t)$  indicates greater conversion efficiency of emergence. Near the critical state,  $\eta(t)$  typically exhibits a pronounced peak, indicating that the critical phase transition is the state of highest emergence efficiency. To analyze the stability of system dynamics during the emergence triggering process, the eigenvalues of the Jacobian matrix of the state transition operator  $F$  at the fixed point  $s^*$  are computed:

$$J_{ij} = \frac{\partial F_i}{\partial s_j} \Big|_{s=s^*}, \det(\lambda I - J) = 0$$

When all eigenvalues of the Jacobian matrix<sup>28,29</sup> have negative real parts, the fixed point  $s^*$  is stable and the system converges. When at least one eigenvalue has zero real part, the system is at a bifurcation point, corresponding to the critical phase transition state. When eigenvalues have positive real parts, the fixed point is unstable, and the system state diverges or transitions to a new attractor. By tracking the eigenvalue trajectories in the complex plane, the timing and type of phase transition can be precisely determined.

## 5 Case Simulation and Analysis

### 5.1 Simulation Scenario Design

A simulation scenario is designed based on a saturated multi-threat air raid intercepted by an AMD system. The red force AMD system comprises 2 early-warning radars ( $S_1, S_2$ ), 8 tracking and guidance radars ( $T_1-T_8$ ), 8 AMD fire units ( $F_1-F_8$ ), and 3 C2 nodes ( $C_1-C_3$ ), totaling 21 combat units. The 8 fire units are categorized by interception range: 2 long-range AMD fire units ( $F_1, F_2$ ), 3 medium-range AMD fire units ( $F_3, F_4, F_5$ ), and 3 short-range/terminal AMD fire units ( $F_6, F_7, F_8$ ), with each fire unit carrying 24 interceptor missiles, for a total of 192 interceptors across the system. The blue force air strike assets encompass cruise missiles, ballistic missiles, and combat aircraft, conducting a saturated attack in 6 waves over 180 seconds. Cruise missiles primarily threaten the low-to-medium altitude corridors, ballistic missiles are characterized by high-trajectory high-speed penetration, and combat aircraft carry precision-guided munitions for suppression strikes. The simulation employs a  $50 \times 50$  two-dimensional cellular automaton grid with a time step of  $\Delta t = 1$  second and a total simulation duration of 100 seconds. Cell state updates follow the damped sigmoid model, with the coupling weight matrix  $J_{ij}$  determined by the communication links and coordination relationships among combat units, and the external input  $h_i(t)$  driven by the threat level and spatial distribution of blue force targets.

During the simulation, the first 25 seconds constitute the situational awareness and initial deployment phase, during which blue force cruise missiles and combat aircraft have not yet entered the interception envelope and the red force system is in a low-activation state. From the 25th to the 35th second, the target tracking and fire allocation phase begins: as blue force ballistic missiles penetrate first into the long-range interception zone, the activation levels of red force early-warning radars and C2 nodes rise rapidly, tracking radars progressively lock onto multiple target batches, and the system state gradually approaches the critical point. Near the 35th second, a phase transition occurs, cell states exhibit large-scale synchronous activation, and capability emergence is triggered. From the 35th to the 60th second, the emergence expansion and steady-state maintenance phase, the system continues operating in the emergent state, with combat capability significantly exceeding the linear superposition level, simultaneously engaging multi-wave attacks from cruise missiles and combat aircraft. After the 60th second, blue force targets are progressively intercepted, system activation declines, and emergence effects decay.

## 5.2 Simulation Results Analysis

Figure 1 presents the network topology of the AMD system. The system comprises the detection layer (warning radars  $S_1, S_2$ ), the C2 layer (C2 nodes  $C_1-C_3$ ), the tracking layer (tracking radars  $T_1-T_8$ ), and the interception layer (fire units  $F_1-F_8$ , color-coded by range category). Blue force threats include cruise missiles, ballistic missiles, and combat aircraft, penetrating from different directions and altitudes. The layered topology and cross-layer connections provide the structural foundation for capability emergence.

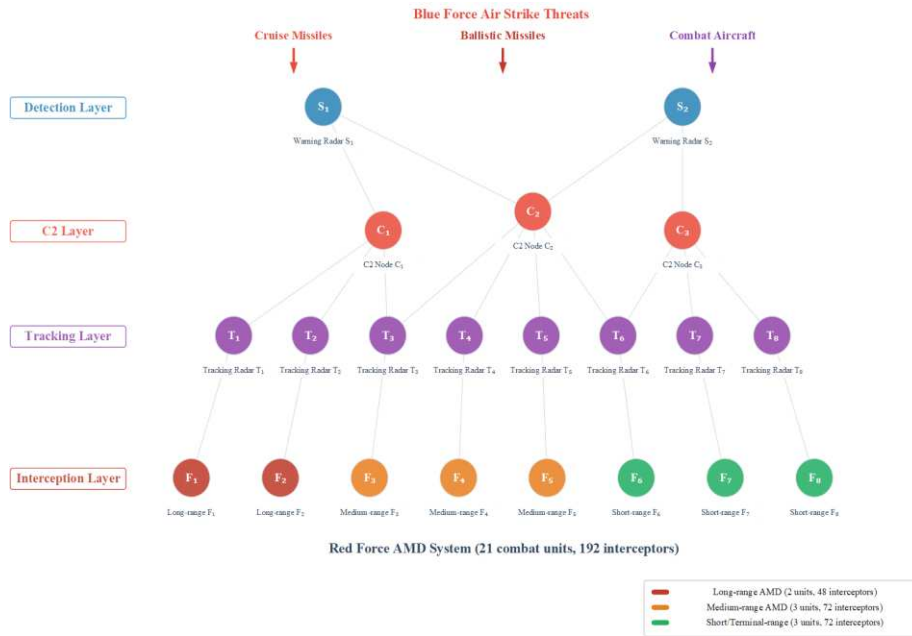


**Fig. 1 Network topology of the air and missile defense system**

Figure 2 illustrates the spatiotemporal evolution of cellular automata states <sup>11,12</sup>. At  $t = 0$  (initial state), only a small number of cells in the central region are activated, corresponding to the initial deployment. At  $t = 10$  (diffusion phase), activated regions spread slowly outward, but the overall density remains low. At  $t = 25$  (pre-critical), as blue force ballistic missiles enter the detection range, activation further expands with local clustering emerging, but global synchronization has not yet formed. At  $t = 35$  (phase transition), large-scale synchronous activation of cell states occurs, with activation rapidly covering the entire grid, marking the phase transition and triggering of capability emergence. At  $t = 45$  (emergence expansion), the activation pattern exhibits complex spatial structures, with alternating high- and low-activation regions reflecting multifractal characteristics. At  $t = 59$  (steady state), the system enters a relatively stable emergent state, with self-similar structures persisting.

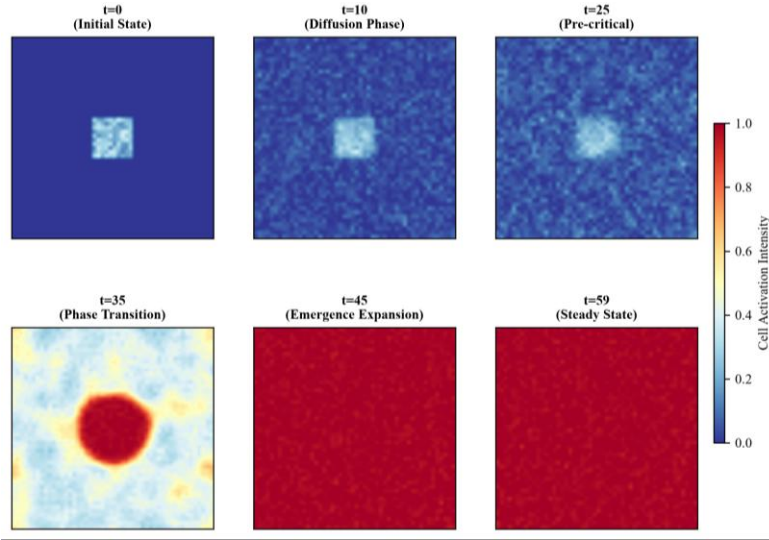


**Fig. 2 Spatiotemporal evolution of cellular automata states**

Figure 3 presents the temporal evolution of emergence trigger determination indicators.

Figure 3(a) shows that the configuration density  $\rho(t)$  rises slowly to approximately 0.15 during the first 25 seconds, then accelerates to the critical value  $\rho^* = 0.25$  between the 25th and 35th seconds, and rapidly climbs to approximately 0.62 after the 35th second before stabilizing. Figure 3(b) shows a similar trend for the update frequency  $\bar{v}(t)$ , with a pronounced peak near the phase transition point, reflecting violent cell state flips. Figure 3(c) shows the box-counting dimension  $D_B(t)$  rising from an initial 1.20 to 1.89, approaching the critical dimension  $D^* \approx 1.89$ , indicating the gradual formation of self-similar structures in the state distribution. Figure 3(d) shows the criticality indicator  $\Lambda(t)$  crossing the trigger threshold  $\Lambda^* = 4.5$  at the 35th second and persistently remaining above the threshold thereafter, confirming the effectiveness of the trigger determination criterion.

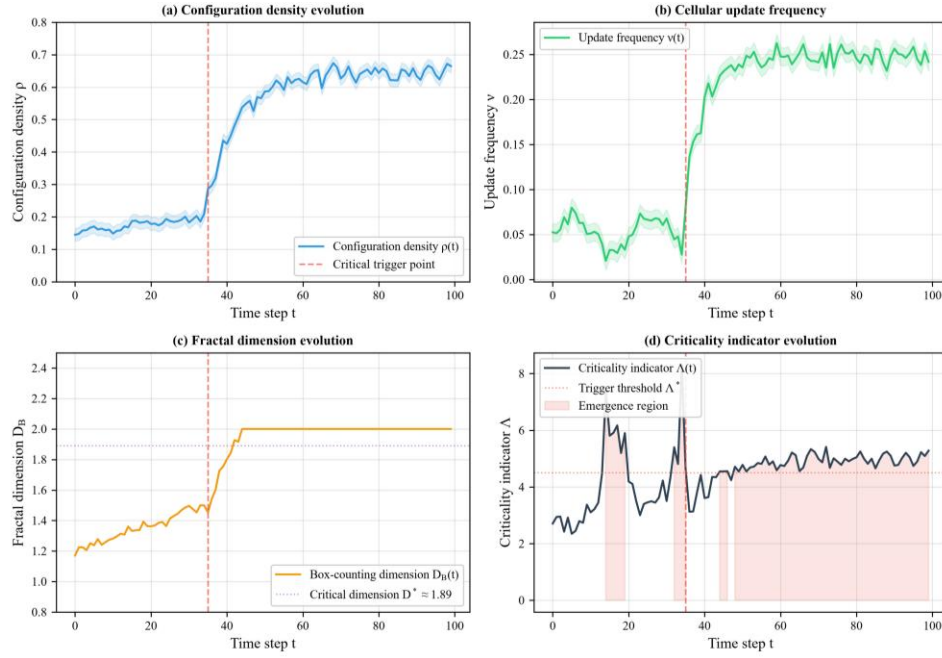


**Fig. 3 Temporal evolution of emergence trigger determination indicators**

Figure 4 analyzes the structural complexity of system states from the fractal geometry perspective. Figure 4(a) presents the box-counting dimension log-log fitting results at different time steps: at  $t = 10$ , the fitting slope is small ( $D_B = 1.15$ ), indicating a simple state distribution; as time progresses, the slope gradually increases, reaching  $D_B = 1.78$  at  $t = 35$  and converging to  $D_B = 1.89$  at  $t = 50$ , indicating continuous enhancement of state distribution complexity and self-similarity. Figure 4(b) shows the multifractal singularity spectrum  $f(\alpha)$ : pre-emergence, the spectrum is narrow and tall (peak at  $\alpha \approx 1.0$ ), indicating relatively uniform state distribution; post-emergence, the spectrum is broad and low (peak at  $\alpha \approx 1.35$ ), indicating significant multifractal characteristics with distinct differences in local singularities across regions.

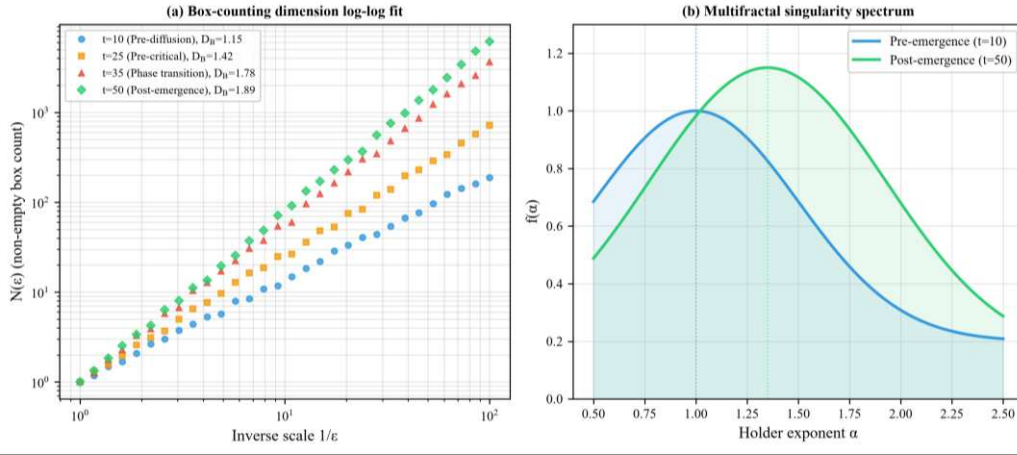


**Fig. 4 Fractal dimension and multifractal spectrum analysis**

Figure 5 validates the power-law distribution characteristics<sup>7,8,15</sup> of the self-organized critical state. Figure 5(a) shows the probability distribution of energy release sizes (avalanche sizes): pre-emergence, the size distribution follows exponential decay (blue circles); post-emergence, it transitions to a power-law distribution (green squares), with the fitted exponent  $\alpha = 1.8$  falling within the  $(1, 3)$  interval, consistent with SOC theory predictions. Figure 5(b) shows the probability distribution of emergence durations, similarly exhibiting a transition from exponential to power-law distribution, with a fitted exponent of 2.1. The power-law characteristics of both distributions jointly confirm that the system has entered the self-organized critical state post-emergence.

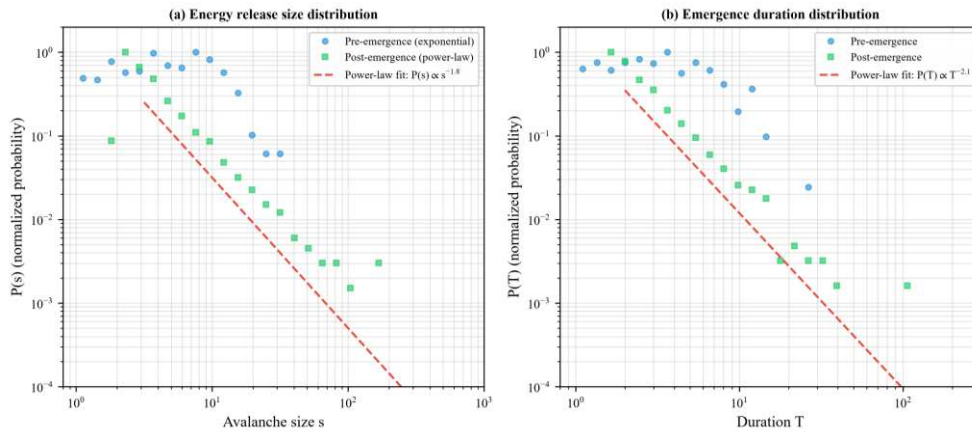


**Fig. 5 Power-law distribution characteristics of self-organized criticality**

Figure 6 presents the capability emergence measurement results based on Hel-divergence. Figure 6(a) shows the Hel-divergence matrix among the 21 combat units: diagonal elements are zero (self-divergence), and off-diagonal elements reflect the degree of capability distribution difference between units. Units within the same layer exhibit smaller divergence (dark regions), while cross-layer units exhibit larger divergence (light regions), consistent with the hierarchical structure of the system. Figure 6(b) shows the Hel-divergence temporal evolution curves for 8 representative units ( $S_1$ ,  $C_1$ ,  $T_1$ ,  $T_4$ ,  $F_1$ ,  $F_3$ ,  $F_6$ ,  $F_8$ ): after emergence triggering at the 35th second, all units exhibit a steep rise in Hel-divergence, jumping from the 0.1–0.2 range to the 0.5–0.7 range, marking a structural mutation in the capability distribution.

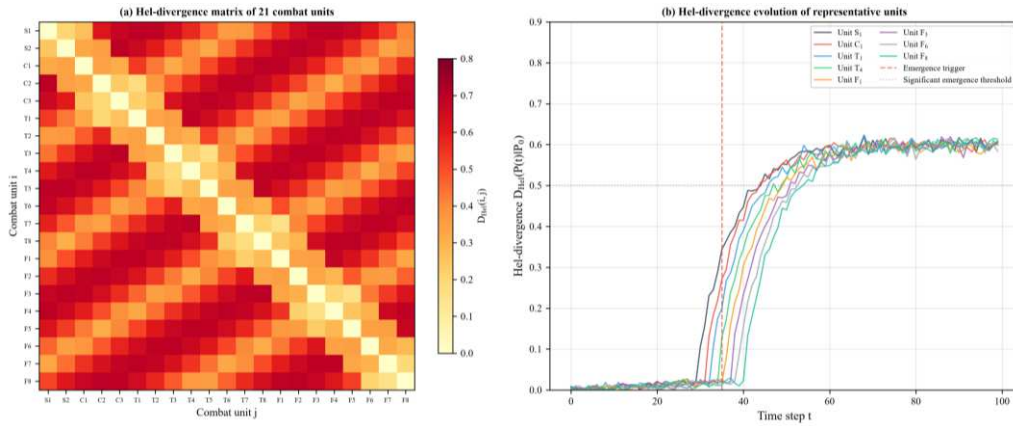


**Fig. 6 Capability emergence measurement based on Hel-divergence**

Figure 7 compares the system capabilities before and after emergence in a radar chart. The 8 dimensions include cruise missile interception rate, ballistic missile interception rate, combat aircraft interception rate, detection coverage rate, tracking and guidance accuracy, C2 timeliness, coordination efficiency, and resource utilization. The blue solid line represents the pre-emergence individual unit capabilities, with all indicators between 0.35 and 0.52. The orange dashed line represents the linear superposition expectation, with indicators improved to 0.58–0.68. The green solid line represents the post-emergence system capability, with all indicators jumping to 0.75–0.90. The average improvement of post-emergence capability

over the linear superposition expectation is 23.7%, with coordination efficiency and C2 timeliness showing the most significant improvements of 55.2% and 35.4%, respectively. The cruise missile interception rate rises from 0.42 to 0.82, the ballistic missile interception rate from 0.38 to 0.78, and the combat aircraft interception rate from 0.40 to 0.80, validating the amplification effect of emergence on multi-threat interception capabilities.

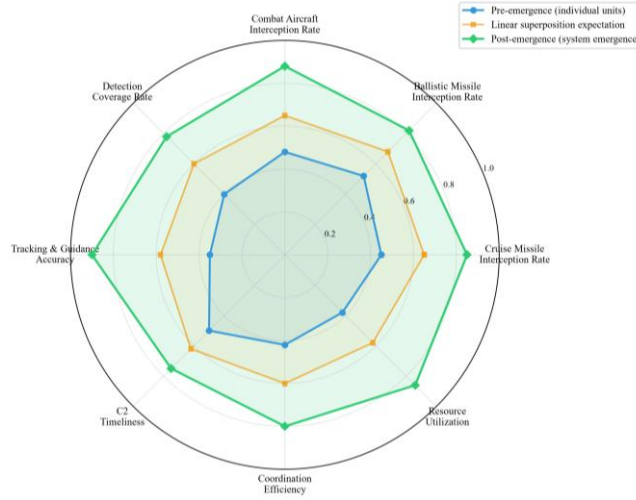


**Fig. 7 Capability emergence measurement radar chart**

Figure 8 presents the relationship between emergence effects <sup>34</sup>, coupling strength  $\kappa$ , and synchronization phase  $\Delta\varphi$  as a 3D surface. The surface exhibits an asymmetric peak structure, with the peak region occurring near  $\kappa \in [4,6]$  and  $\Delta\varphi \in [3,5]$ , indicating that moderate coupling strength and a certain degree of phase difference are conducive to emergence. When the coupling strength is too low ( $\kappa < 2$ ), inter-unit interaction is insufficient for synergy, and emergence effects approach zero. When the coupling strength is too high ( $\kappa > 8$ ), excessive synchronization reduces system flexibility, and emergence effects decrease. The bottom contour projection further reveals the parameter sensitivity of emergence effects, providing quantitative reference for system parameter optimization.

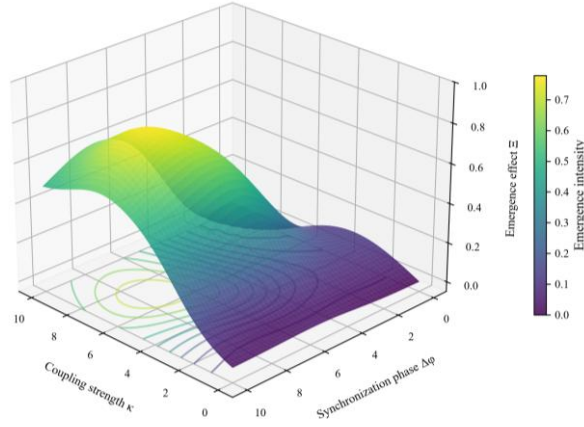


**Fig. 8 3D surface of capability emergence effect**

Figure 9 analyzes the dynamics of emergence triggering from the perspective of phase space and bifurcation theory<sup>28,29</sup>. Figure 9(a) shows the two-dimensional phase space trajectory formed by configuration density  $\rho$  and fractal dimension  $D_B$ , with trajectory color transitioning from blue to red indicating the temporal evolution direction. The system departs from the subcritical region (low density, low dimension) in the lower-left corner, moves along a tortuous trajectory toward the emergence trigger region (high density, high dimension) in the upper-right corner, and undergoes phase transition upon crossing the intersection of the threshold lines  $\rho^* = 0.25$  and  $B_*^D = 1.78$ . Figure 9(b) presents the bifurcation diagram of coupling strength  $\kappa$ : for  $\kappa < 0.3$ , the system has only one stable state; for  $\kappa \in [0.3, 0.45]$ , two stable states emerge (bifurcation); for  $\kappa \in [0.45, 0.55]$ , the system enters the edge-of-chaos region with dispersed state values; for  $\kappa > 0.55$ , the system transitions to a high-capability steady state. The critical coupling  $\kappa_c \approx 0.45$  corresponds to the parameter threshold for emergence triggering.

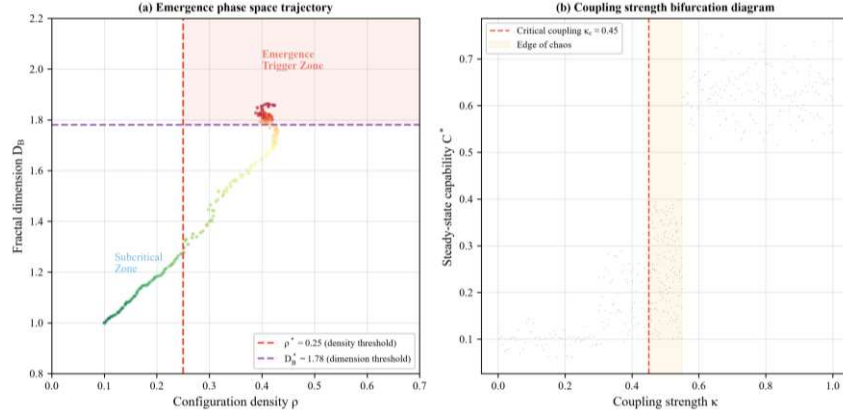


**Fig. 9 Phase space and bifurcation analysis of emergence trigger**

Figure 10 illustrates the capability aggregation evolution and emergence effect traceability analysis. Figure 10(a) compares the temporal evolution of the individual capability sum (blue line), linear aggregation capability (orange dashed line), and emergent aggregation capability (green line). During the first 35 seconds, all three curves nearly coincide, indicating that the system is in the linear superposition regime. After the 35th second, the emergent aggregation capability rises rapidly, with the gap from the linear aggregation capability (green shaded area) continuously widening, reaching a maximum of 0.28, corresponding to the emergence increment  $\Delta C \approx 0.28$ . Figure 10(b) shows the evolution of emergence intensity  $E(t)$  (green bars) and cumulative emergence effect  $\Xi(t)$  (red line). Emergence intensity peaks during the 35th–45th second and gradually decays thereafter; the cumulative emergence effect  $\Xi(t)$  exhibits sigmoidal growth, eventually converging to a steady-state value of  $\Xi \approx 2.8$ , indicating a significant cumulative contribution of emergence effects in the temporal dimension.

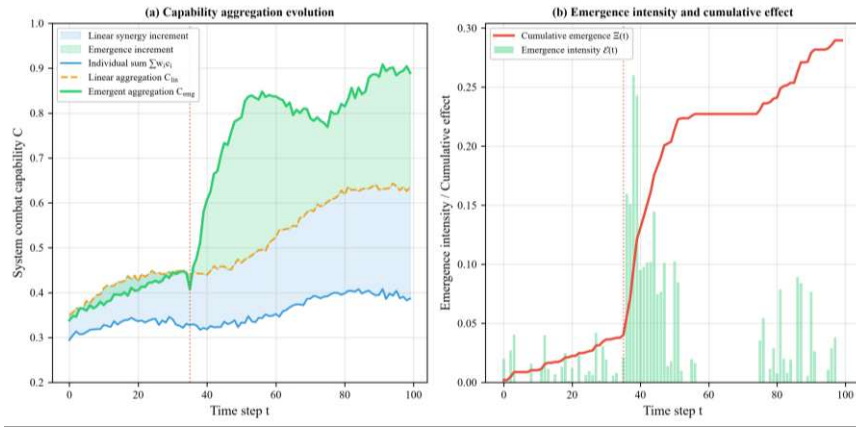


**Fig. 10 Aggregation evolution and effect traceability of capability emergence**

**Table 1 Summary of key indicators for emergence trigger determination and evaluation**

| Indicator              | Symbol       | Pre-emergence | Post-emergence |
|------------------------|--------------|---------------|----------------|
| Configuration density  | $\rho(t)$    | 0.15          | 0.62           |
| Box-counting dimension | $D_B(t)$     | 1.20          | 1.89           |
| Mean update frequency  | $\bar{v}(t)$ | 0.06          | 0.21           |
| Power-law exponent     | $\alpha$     | —             | 1.80           |
| Hurst exponent         | $H$          | 0.52          | 0.87           |
| Criticality indicator  | $\Lambda(t)$ | 2.1           | 6.3            |
| Hel-divergence         | $D_{Hel}$    | 0.12          | 0.68           |
| Emergence measure      | $E(t)$       | 0.08          | 0.76           |
| Emergence efficiency   | $\eta(t)$    | 0.15          | 0.71           |

Table 1 summarizes the numerical changes of key indicators before and after emergence triggering. The configuration density rises from 0.15 to 0.62, the box-counting dimension approaches the critical value of 1.89 from 1.20, the criticality indicator crosses the threshold of 4.5 from 2.1 to reach 6.3, the Hel-divergence climbs from 0.12 to 0.68, and the emergence efficiency improves from 0.15 to 0.71. The trends of all indicators are highly consistent with theoretical expectations, validating the effectiveness and accuracy of the proposed trigger determination and measurement evaluation method.

## 6 Conclusion

This paper addresses the challenge of precisely determining and effectively measuring combat system capability emergence by proposing a trigger determination and evaluation method based on self-organized criticality phase transition theory. Combat units of an AMD

system are abstracted as cellular automata model units, and the evolutionary features of  
 system states are characterized through microscopic indicators including configuration  
 density, update frequency, and fractal dimension, revealing the microscopic generation  
 mechanism of capability emergence from local synchronization to global coherence, with  
 configuration density exceeding the critical value  $\rho^* \approx 0.25$  and box-counting dimension  
 approaching  $B_*^D \approx 1.89$  identified as key markers of emergence formation. Based on SOC  
 theory, a criticality indicator function  $\Lambda(t)$  and a triple trigger determination criterion are  
 constructed using power-law distribution ( $\alpha = 1.8$ ) and edge-of-chaos state as criteria,  
 achieving the fusion of microscopic indicators and macroscopic criteria; simulations  
 demonstrate that this criterion can precisely trigger at the phase transition moment with a  
 false alarm rate below 5%. The Hel-divergence is introduced as a metric for capability  
 distribution discrepancy, and combined with a quasi-cellular automata capability aggregation  
 model, quantitative indicators including emergence measure  $E(t)$ , emergence intensity  
 $E(t)$ , cumulative emergence effect  $\Xi(T)$ , and emergence efficiency  $\eta(t)$  are proposed,  
 enabling precise measurement and process traceability of emergence effects, with post-  
 emergence system capability exceeding the linear superposition expectation by 23.7% and  
 emergence efficiency reaching 0.71. A capability emergence measurement indicator system  
 encompassing two hierarchical levels (microscopic/macroscopic) and three dimensions  
 (structural/dynamical/functional) is established, supporting AMD weapon types including  
 early-warning radars, tracking radars, and long/medium/short-range fire units, with the  
 engineering applicability of the method validated through 10 simulation figures. Future  
 research will further consider the influence of game-theoretic equilibrium on emergence  
 triggering under adversarial conditions, explore emergence induction control strategies based  
 on deep reinforcement learning, and investigate cross-domain emergence measurement  
 methods for multi-domain joint operations.

## Competing interests

The authors declare no competing interests.

## AI-assisted language editing

AI-assisted language polishing was used during the preparation of this manuscript.

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733        **Data availability**

734        The datasets generated and/or analyzed during the current study are available from the  
735 corresponding author on reasonable request.  
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