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Robust Optimization of Multiple Tuned Mass Dampers for Multimodal Seismic Response Mitigation

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Abstract

This study investigates the robust optimization of multiple tuned mass dampers (MTMDs) for multimodal seismic response mitigation of a multi degree of freedom shear structure under structural parameter and earthquake record uncertainties. A 20-story shear frame subjected to 44 individual seismic records is considered, and the performance of an MTMD system is compared with that of a single tuned mass damper (STMD). Bayesian optimization is employed to simultaneously optimize the mass, stiffness, and damping of each absorber, while uncertainty-aware optimization accounts for variations in structural stiffness, mass, and damping, as well as record to record variability. A Sobol-based sampling strategy is used to generate the uncertain realizations, with the number of samples selected through a convergence assessment based on the stabilization of the coefficients of variation of the considered response metrics. Deterministic and robust designs are subsequently evaluated using an independent set of structural and seismic realizations. The results show that the relative advantage of MTMD over STMD is strongly dependent on the modal composition of the seismic response: STMD is marginally superior in Mode 1 dominant cases, whereas MTMD provides substantial improvements when higher modes contribute significantly, with the robustly optimized MTMD achieving up to 50.7% and 72.1% lower mean and standard deviation of time integrated modal energy compared to STMD. This is supported by strong correlations between MTMD improvement over STMD and higher mode contribution, with Pearson correlation coefficients of 0.951 and 0.808 for the mean and standard deviation, respectively. Furthermore, MTMD exhibits a greater improvement from deterministic to robust optimization than STMD, highlighting its greater potential to benefit from uncertainty-aware parameter optimization.

Keywords: Structural Control; Multiple Tuned Mass Dampers; Seismic Vibration Mitigation; Robust Optimization.

1. Introduction

Reducing unwanted structural vibrations through passive control devices is an effective means of enhancing structural reliability and safety. Without the need for external power sources, sensors,

or complex algorithms, passive control devices mitigate harmful oscillations induced by environmental and operational excitations, such as earthquakes, wind, and traffic loads. Ongoing research continues to advance the understanding and design of passive solutions for structural control.

The tuned mass damper (TMD), owing to its cost-effectiveness, simplicity in design, and ease of implementation, has been widely adopted as a vibration control strategy in both research and practice since its initial proposal by Frahm ([1911](#)). Building on this foundational concept, subsequent efforts have focused on refining the design of TMDs, comprising mass, spring, and damping elements, to facilitate their practical implementation in real-world engineering applications ([Brock](#), 1946; Den Hartog, 1956; Ormondroyd and Den Hartog, 1928; [Warburton](#) and Ayorinde, 1980).

Uncertainties inherent in the estimation of structural properties and external excitations can compromise TMD effectiveness, leading to detuning, particularly when the excitation exhibits significant high-frequency content. Consequently, accounting for these uncertainties in the TMD design process is imperative to ensure robust absorber performance. In addition to robust optimization, the limitations of a single TMD (STMD) can be further addressed by employing multiple TMDs (MTMD) distributed at critical locations throughout the structure. This spatial distribution of damping not only enhances the overall robustness of the TMD system but also improves its effectiveness in suppressing vibrations across a broader range of excitation frequencies (Igusa and Xu, 1994; Qu et al., 2026; Xu and Igusa, 1992). Furthermore, numerous TMD variants have been developed to improve performance, including pendulum TMD (Gerges and Vickery, 2005; Roffel et al., 2013), shape memory alloy spring TMD (Mishra et al., 2013; Zuo et al., 2022), TMD-inerter systems (Chan, 2026; Marian and Giaralis, 2014), friction TMD (Etedali et al., 2019; Gewei and Basu, 2011), Magnetic TMD (Pourjafar, 2026; Pourjafar and Razzaghi, 2026), and hybrid solutions, such as base isolation-active TMD hybrid (Akbari et al., 2024; Akbari et al., 2026), active tendons-magnetorheological dampers (Akbari et al., 2024), active tendons-double TMD inerters ([Akbari](#) et al., 2025).

A TMD mitigates structural vibrations by being tuned to a target natural frequency (typically the fundamental mode) and resonating in counter phase to dissipate energy. However, its

effectiveness is sensitive to frequency mismatch. Inaccuracies in estimating structural properties or changes in the system over time can detune the TMD, particularly when all damper mass is concentrated at a single location, as in a conventional STMD. Moreover, when the excitation contains substantial high-frequency content, higher modes may contribute significantly to the overall response, yet the STMD remains tuned exclusively to the first mode. To address these limitations, distributing the damping mass across multiple locations has been proposed as a means of targeting multimode response and enhancing robustness to such uncertainties (Abé M, Fujino, 1994; Mohebbi et al., 2013).

Given the large number of parameters involved in MTMD design, the optimization process often struggles to locate the global minimum. To reduce the computational burden, several studies have opted to fix a subset of MTMD parameters (Elias et al., 2019; Hussan et al., 2018; Mohebbi et al., 2013; Zuo et al., 2017), sacrificing some design flexibility for a more manageable optimization problem. However, fixing certain parameters may inadvertently exclude promising solutions, and accordingly, Li and Ni ([2007](#)) proposed a gradient-based method for non-uniform MTMDs; their method explicitly accounted for potential estimation errors to achieve a robust design. Bandivadekar et al. ([2013](#)) investigated the optimization of MTMDs under harmonic external excitation and derived an explicit formula for optimal MTMD parameters via curve fitting. Cetin et al. ([2025](#)) employed a differential evolution algorithm to optimize MTMD parameters attached to a cantilever beam under both deterministic and stochastic excitations, reporting multimodal response reduction and robust performance. Zakaria et al. ([2026](#)) used particle swarm optimization to identify optimal MTMD parameters for linear and nonlinear Bernoulli-Euler beams, demonstrating improved performance and robustness in both regimes. Xiang and Nishitani ([2014](#)) proposed a floor vibration control system that integrates floor isolation with TMDs, effectively treating the floors as a distributed MTMD system. Their optimized configuration showed that the proposed concept retains the advantages of isolation systems while substantially reducing the seismic response of the main structure. Vellar et al. ([2019](#)) employed a search group algorithm to perform robust optimization of MTMDs for a 10-storey structure subjected to random excitation.

Chan ([2025](#)) proposed inverse element exchange method combined with multi-level programming for the optimal placement of MTMDs, later extending the approach to offshore wind

turbine towers ([Chan, 2025](#)). The same methodology has also been applied to the optimal placement of viscous dampers in structures under seismic loading ([Chan et al., 2022](#); [Chan and Ma, 2025](#)). Araz ([2021](#)) employed a simulated annealing approach to optimize parallel and series TMD configurations under harmonic loading, subsequently deriving an explicit design expression through curve fitting. The study was later extended by adopting particle swarm optimization to determine optimal parameters for parallel MTMD systems under seismic excitation, with particular attention to the peak ground velocity to peak ground acceleration ratio and soil-structure interaction (SSI) effects ([Araz, 2024](#)). The framework was further developed to investigate the influence of SSI on the optimization of double TMDs using the Jaya algorithm ([Araz et al., 2023](#)). Series MTMD configurations have also been explored for mitigating train-induced vibrations in railway bridges ([Kahya and Araz, 2015](#)).

This study investigates the robust optimization of MTMDs for a benchmark 20-storey shear frame subjected to a suite of 22 far-field earthquake records, comprising 44 individual horizontal components. In addition to uncertainty in the structural parameters, namely mass, damping, and stiffness, the selection of the seismic record is explicitly treated as an uncertain variable. This differs from most uncertainty-aware approaches in which the seismic input is commonly represented either by a prescribed deterministic record or by a stochastic ground motion model. Following normalization and spectral acceleration based scaling, the earthquake records are divided into separate optimization and testing groups, allowing the robust designs to be evaluated against previously unseen records. Structural uncertainty realizations are generated using a Sobol sequence, selected following a comparison of parameter space coverage with random and Latin hypercube sampling, most commonly used in Monte Carlo simulations. The number of realizations is determined through a convergence assessment of the coefficient of variation of the response metrics adopted for optimization and evaluation, namely the time-integrated total modal energy, root mean square (RMS) top floor displacement, and maximum inter-storey drift. Separate uncertainty ensembles are then used for optimization and independent evaluation. Bayesian optimization is employed to simultaneously determine the mass, stiffness, and damping parameters of the MTMD, with analogous optimization and evaluation performed for STMD to provide a consistent benchmark. The resulting framework therefore integrates structural parameter uncertainty and record selection uncertainty into a unified robust optimization procedure, while explicitly assessing the generalization of the resulting absorber designs to unseen seismic records.

2. Methodology

2.1. Structural model

The multiple degree of freedom (MDOF) structure considered in this study is based on a benchmark 20-storey frame (Spencer et al., 1999), whose first five normalized mode shapes and corresponding natural frequencies are presented in Fig. 1. The number of TMDs employed in the MTMD design is determined based on the modal energy contribution of the structure when subjected to the 44-record suite, as illustrated in Fig. 2. It is evident that, for all records, the majority of the modal energy is concentrated in the first three modes, which account for an average of 96.8% of the total modal energy across the entire record suite, while higher modes contribute negligibly to the overall structural response. Accordingly, the first three modes are selected for control using MTMDs. The placement of each TMD is determined based on the maximum mode shape amplitude of the corresponding mode. Based on the computed eigenmodes, the TMDs are positioned at the top, seventh, and fourth floors, as shown in Fig. 3. For the STMD case, the damper is placed at the top floor.

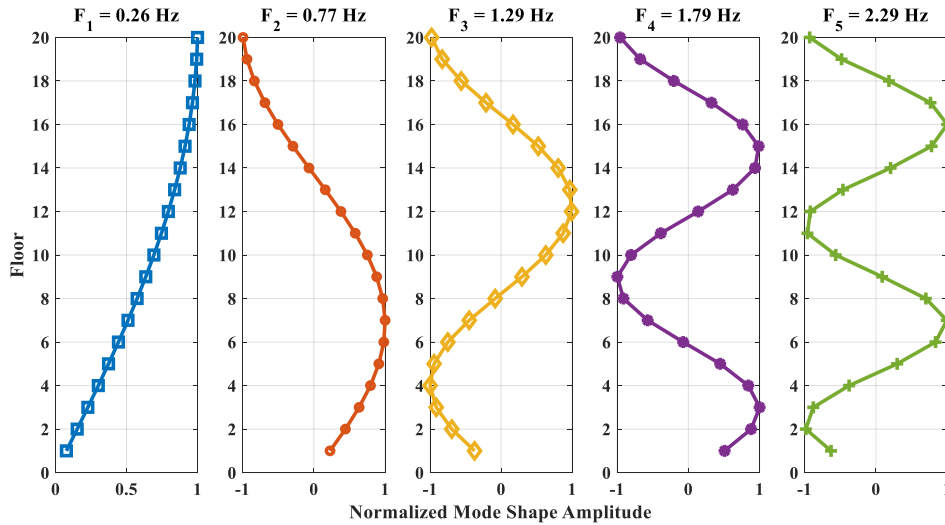


Fig. 1. First five normalized mode shapes and corresponding natural frequencies of the benchmark structure.

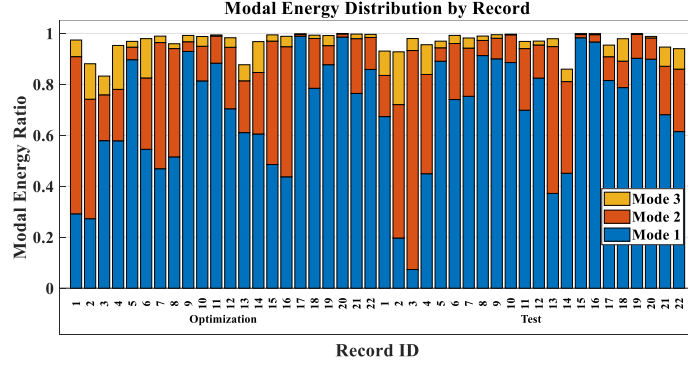


Fig. 2. Modal energy distribution of the structure across the earthquake record suite.

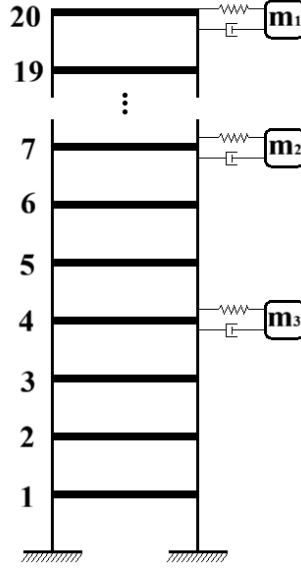


Figure 3. Schematic of the 20-storey benchmark structure with MTMD placement locations.

The mass, stiffness, and damping matrices of the augmented system are defined as follows:

$$M = \begin{bmatrix} [M_s]_{20 \times 20} & [0]_{20 \times 3} \\ [0]_{3 \times 20} & [M_d]_{3 \times 3} \end{bmatrix} \quad (2.1)$$

$$K = \begin{bmatrix} [K_s]_{20 \times 20} + [K_d]_{20 \times 20} & -[K_d]_{20 \times 3} \\ -[K_d]_{3 \times 20} & [K_d]_{3 \times 3} \end{bmatrix} \quad (2.2)$$

$$C = \begin{bmatrix} [C_s]_{20 \times 20} + [C_d]_{20 \times 20} & -[C_d]_{20 \times 3} \\ -[C_d]_{3 \times 20} & [C_d]_{3 \times 3} \end{bmatrix} \quad (2.3)$$

where the subscripts s and d denote the structural and TMD components, respectively. The damping matrix C_s is constructed using Rayleigh damping, with modal damping ratios ranging from 2% to 10%, consistent with the values recommended in (Spencer et al., 1999).

The dynamic equations of motion for the augmented system are given by:

$$[M]\ddot{\mathbf{X}}(t) + [C]\dot{\mathbf{X}}(t) + [K]\mathbf{X}(t) = -[M]\mathbf{r}\ddot{x}_g(t) \quad (2.4)$$

where $\ddot{\mathbf{X}}(t)$, $\dot{\mathbf{X}}(t)$ and $\mathbf{X}(t)$ denote the acceleration, velocity, and displacement response vectors of the augmented system, respectively; $\mathbf{r}$ is the influence and $\ddot{x}_g(t)$ is the ground acceleration. The dynamic equations of motion were numerically integrated in time domain using a Runge-Kutta-Nyström scheme.

2.2. Ground motion

A suite of 22 far-field earthquake records (44 horizontal components), recommended by FEMA-P695 (FEMA, 2009) and sourced from the Pacific Earthquake Engineering Research Center database (PEER), was selected as the seismic input for this study. The records are listed in Table 1 and are partitioned into two independent sets: one for optimization (Optimization in Fig. 2) and the other for validation (Test in Fig. 2). Each record pair was first normalized using the FEMA-P695 normalization factors, then scaled to match a target acceleration response spectrum, defined as the median of the spectra across the entire suite (Fig. 4). For each record, a linear amplitude scaling factor was determined by minimizing the root mean square error (RMSE) between its response spectrum and the target spectrum over the period range $0.75T_1$ to $1.25T_1$, where T_1 is the fundamental natural period of the structure. The scaling factor for the i -th record, SF_i , is therefore obtained as follows:

$$SF_i = \arg \min_s \left\{ \sqrt{\frac{1}{N} \sum_{k=1}^N \left[\frac{s \cdot S_{a,i}(T_k) - S_{a,target}(T_k)}{S_{a,target}(T_k)} \right]^2} \right\}, T_k \in [0.75T_1, 1.25T_1] \quad (2.5)$$

where s is a positive scaling factor, $S_{a,i}(T_k)$ denotes the response spectrum of the i -th record evaluated at period T_k , and N is the number of discrete period points considered within the specified range.

This period range, also indicated in Fig. 4, was selected to minimize the deviation in the maximum top-floor displacement response across the suite, thereby reducing the optimizer's tendency to favor records that induce larger amplitudes when identifying optimal damper

parameters. The coefficient of variation (CV) of the maximum top floor displacement decreased from 0.5762 for the unscaled records to 0.1984 after scaling, a substantial reduction in dispersion, as summarized in [Table 2](#) along with the corresponding mean and standard deviation (Std) values.

Table 1. Earthquake record suite used for optimization and validation; peak ground acceleration (PGA) values are reported following FEMA P695 normalization.

Optimization				Test			
Record	PGA (m/s ²)		Duration (s)	Record	PGA (m/s ²)		Duration (s)
	Comp1	Comp2			Comp1	Comp2	
Northridge	2.65	3.29	30	Landers	3.18	4.69	28
Northridge	3.34	3.93	20	Loma Prieta	5.64	4.73	40
Duzce, Turkey	4.48	5.07	56	Loma Prieta	4.79	3.17	40
Hector Mine	2.84	3.6	45	Manjil, Iran	3.97	3.83	53
Imperial Valley	3.05	4.51	100	Superstition Hills	3.05	2.2	40
Imperial Valley	3.61	3.77	40	Superstition Hills	5.14	3.45	22
Kobe, Japan	5.16	5.09	41	Cape Mendocino	3.09	4.41	36
Kobe, Japan	2.62	2.28	41	Chi-Chi, Taiwan	1.42	1.77	90
Kocaeli, Turkey	2.1	2.41	27	Chi-Chi, Taiwan	4.46	4.81	90
Kocaeli, Turkey	2.91	1.99	30	San Fernando	4.31	3.58	28
Landers	2.36	1.46	44	Friuli, Italy	4.96	4.44	36

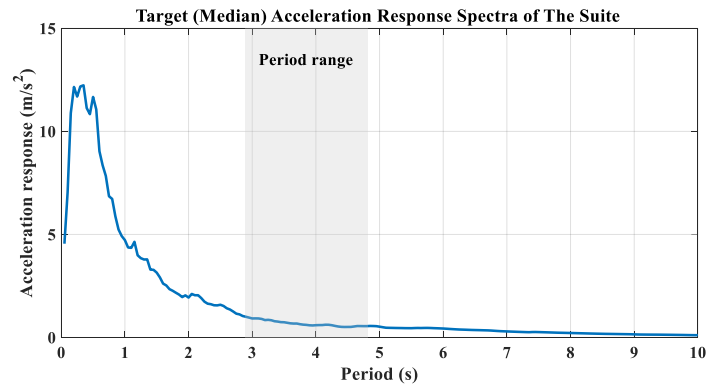


Fig. 4. Median acceleration response spectrum of the earthquake record suite.

Table 2. Statistical comparison of the maximum top floor displacement for the unscaled and scaled earthquake record suites.

Records case	Mean	Std	CV
Unscaled	0.3715	0.214	0.5762
Scaled	0.2933	0.0582	0.1984

2.3. Uncertainty Analysis

The uncertainty in Structural parameters and earthquake incidence in the optimization is propagated through variations in structural mass (α_M), damping (α_C), stiffness (α_K) and record ID selection, where continuous points in structural parameters, with $\pm 15\%$ variation in nominal values and integer points in record ID selection are combined to account for the uncertainty. This

is typically done using random/quasi-random sampling methods to reduce computational cost of the design. Most studies with Monte Carlo simulations use random or Latin hypercube sampling (LHS) to select uncertainty combination points in parameter space instead of costly structured grid sampling. In this study, however, a Sobol sampling approach was adopted, on the basis that it provides more uniform coverage of the parameter space. This is evident from [Fig. 5](#) for two-dimensional combinations of 200 uncertainty points in structural parameters, where Sobol shows noticeably better coverage. Therefore, it was opted for the uncertainty sampling for both design and evaluation here.

To ensure robustness and generalizability, two independent batches of 200 uncertainty points were generated - one for optimization and the other for subsequent testing and evaluation. While a larger sample size would enhance the reliability of the analysis, it would also increase computational cost. A convergence study was therefore conducted to balance reliability and efficiency. The uncontrolled structure was analyzed using varying numbers of uncertainty points, and the CV of the response metrics was recorded for each population size, as shown in [Fig. 6](#). The results indicate that the CV stabilizes with negligible variation beyond a sample size of 200. Accordingly, 200 uncertainty points were selected for the subsequent optimization and validation phases.

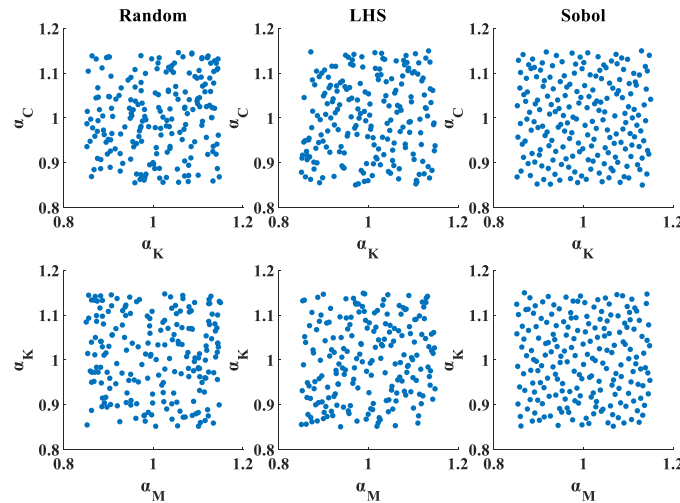


Fig. 5. Comparative assessment of space-filling capabilities of random, Latin hypercube, and Sobol sampling approaches for propagating uncertainty in structural parameters.

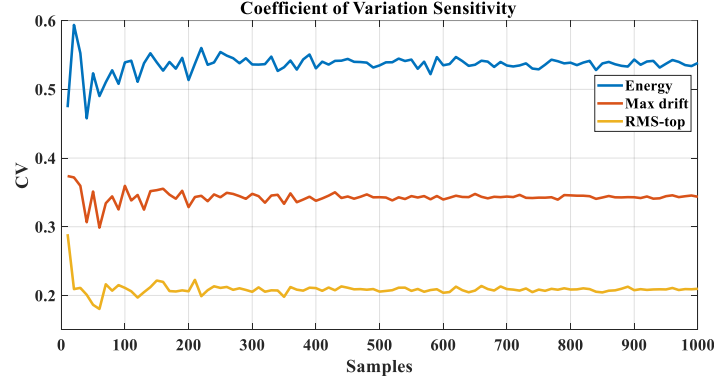


Fig. 6. Convergence of the CV of key response metrics for the uncontrolled structure with respect to sample size.

2.4. Bayesian Optimization

Bayesian optimization (BO) provides an efficient framework for minimizing expensive black-box objective functions of the form

$$\min_{p \in \mathcal{X}} J(p)$$

where each evaluation of $J(p)$ is computationally demanding and gradient information is unavailable ([Garnett, 2023](#)). BO constructs a probabilistic surrogate model of the objective function, typically a Gaussian process, and iteratively selects the next sampling point by balancing exploration of uncertain regions with exploitation of areas expected to yield low objective values.

In the present study, the objective function $J(p)$ is defined as a weighted average of three normalized performance metrics: the time-integrated modal energy E , the peak inter-storey drift $\max(\text{MD})$, and the RMS of the top-floor displacement $\text{RMS}(u_{\text{top}})$. These metrics were selected to capture complementary aspects of the structural response: total modal energy reflects the overall vibration intensity and velocity content; peak inter-story drift quantifies the maximum relative deformation, which is directly linked to potential damage; and the RMS of top-floor displacement provides a measure of the sustained absolute displacement at the location where the largest motion is expected. Each metric is normalized by its counterpart for the uncontrolled system. Two optimization strategies are pursued: (i) a deterministic approach, in which the structural parameters are held at their nominal values and the structure is subjected to all records in the optimization batch; and (ii) a robust approach, in which both the structural parameters and

the earthquake record (from the optimization batch) are treated as uncertain and sampled quasi-randomly. The optimal damper parameters are those that minimize the average response across all realizations, specifically, across the 22 records in the deterministic case, and across the combined uncertainty samples in the robust case.

The optimization problem is expressed as:

$$J(\mathbf{p}) = w_1 J_1 + w_2 J_2 + w_3 J_3 \quad (2.6)$$

with equal weights $w_1 = w_2 = w_3 = 1/3$. The individual objective components are defined as:

$$J_1 = \frac{E_{cnt}}{E_{unc}} \quad (2.7)$$

$$J_2 = \frac{Max(MD)_{cnt}}{Max(MD)_{unc}} \quad (2.8)$$

$$J_3 = \frac{RMS(u_{top})_{cnt}}{RMS(u_{top})_{unc}} \quad (2.9)$$

where the subscripts cnt and unc denote the controlled and uncontrolled systems, respectively. The total modal energy E is computed as the time integral of the sum of kinetic and potential energies:

$$E = \int_0^T (E_k(t) + E_p(t)) dt \quad (2.10)$$

where the instantaneous kinetic and potential energies are evaluated from the physical degrees of freedom as:

$$E_k(t) = \frac{1}{2} \dot{\mathbf{X}}_s(t)^\top \mathbf{M}_s \dot{\mathbf{X}}_s(t) \quad (2.11)$$

$$E_p(t) = \frac{1}{2} \mathbf{X}_s(t)^\top \mathbf{K}_s \mathbf{X}_s(t) \quad (2.12)$$

with $\mathbf{X}_s$ and $\dot{\mathbf{X}}_s$ denoting the displacement and velocity vectors of the structural degrees of freedom, and $\mathbf{M}_s$ and $\mathbf{K}_s$ the corresponding mass and stiffness matrices.

Bayesian optimization seeks the optimal parameter vector $\mathbf{p} = [m_1, m_2, m_3, c_1, c_2, c_3, k_1, k_2, k_3]$ that minimizes the objective function $J(\mathbf{p})$, subject to a total mass constraint:

$$\min_p J(p) \text{ s.t. } m_1 + m_2 + m_3 \leq m_{\max} \quad (13)$$

where $m_{\max} = 0.03 M_{\text{total}}$, corresponding to a 3% mass ratio relative to the total structural mass. The same mass constraint is imposed for the STMD case.

The optimization was performed using MATLAB's bayesopt function, employing the expected improvement acquisition function under the assumption of a deterministic objective. Parallel evaluations were enabled via the clipped model prediction parallelization strategy to accelerate convergence. The maximum number of function evaluations was set to 200. The mass constraint was enforced directly through the optimizer's built-in constraint handling.

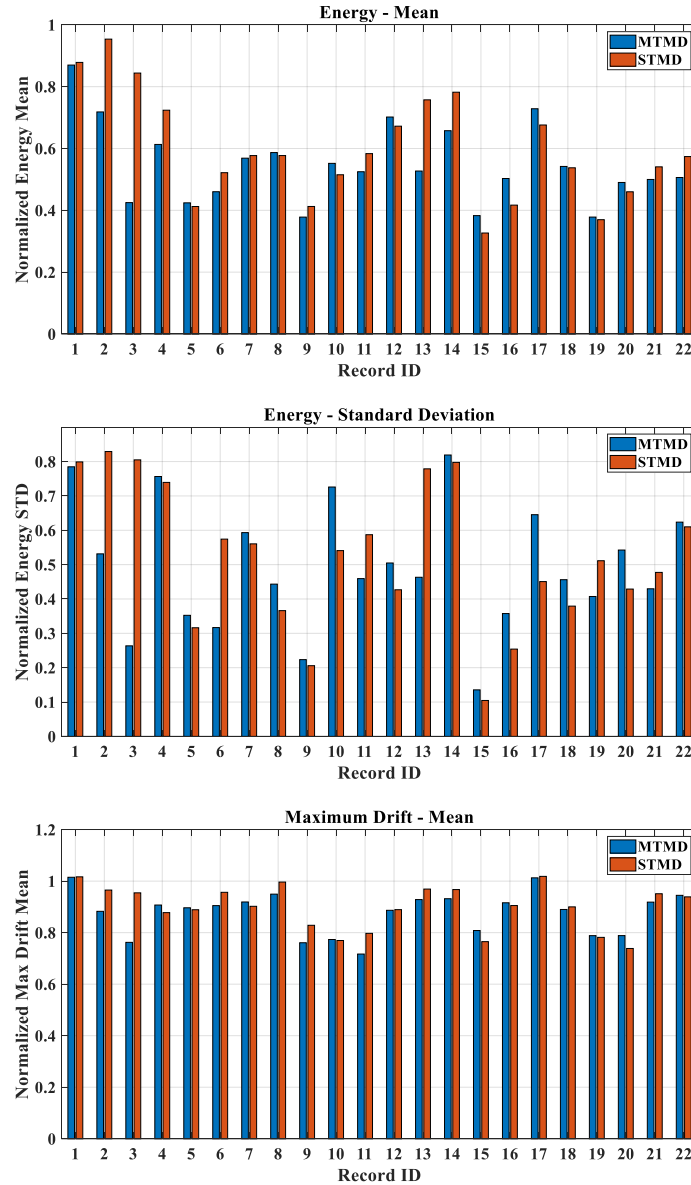
3. Results and Discussion

3.1. Deterministic optimization

The performance of the deterministically optimized dampers is evaluated under uncertainty-aware simulations. The evaluation keeps the record ID fixed for each run while varying only the structural parameters using an independent batch of 200 uncertainty samples. This allows for a record-by-record assessment of MTMD and STMD performance on unseen excitations.

[Fig. 7](#) presents the mean and Std of the response metrics, normalized by their uncontrolled counterparts, for the deterministically optimized MTMD and STMD across the test-set uncertainty samples. The mean and Std are used as indicators of performance and robustness, respectively. In line with the modal energy distributions shown in [Fig. 2](#), MTMD generally outperforms STMD, particularly when the first mode is less dominant in the overall response. For instance, in record 3 of the test record set, where Mode 2 is overwhelmingly dominant, MTMD achieves 49.7% and 67.3% reductions in the mean and Std of structural energy compared to STMD. Similar improvements are observed for maximum drift and RMS top-floor displacement, with MTMD yielding 20.1% and 3.3% lower mean and Std of maximum drift, and 23.5% and 74% lower mean and Std of RMS top-floor displacement, respectively. However, this trend is less pronounced for the Std of maximum drift, suggesting a weaker correlation between robustness improvement in drift and higher-mode contribution. Additionally, MTMD exhibits generally lower effectiveness

in improving both performance and robustness of RMS top-floor displacement relative to STMD, an expected outcome. Since STMD concentrates the entire damper mass at the top floor, where the first mode (to which it is tuned) has its maximum modal amplitude, it is inherently more effective in reducing top-floor displacements. The optimal parameters for both deterministically optimized MTMD and STMD are reported in [Table 3](#).



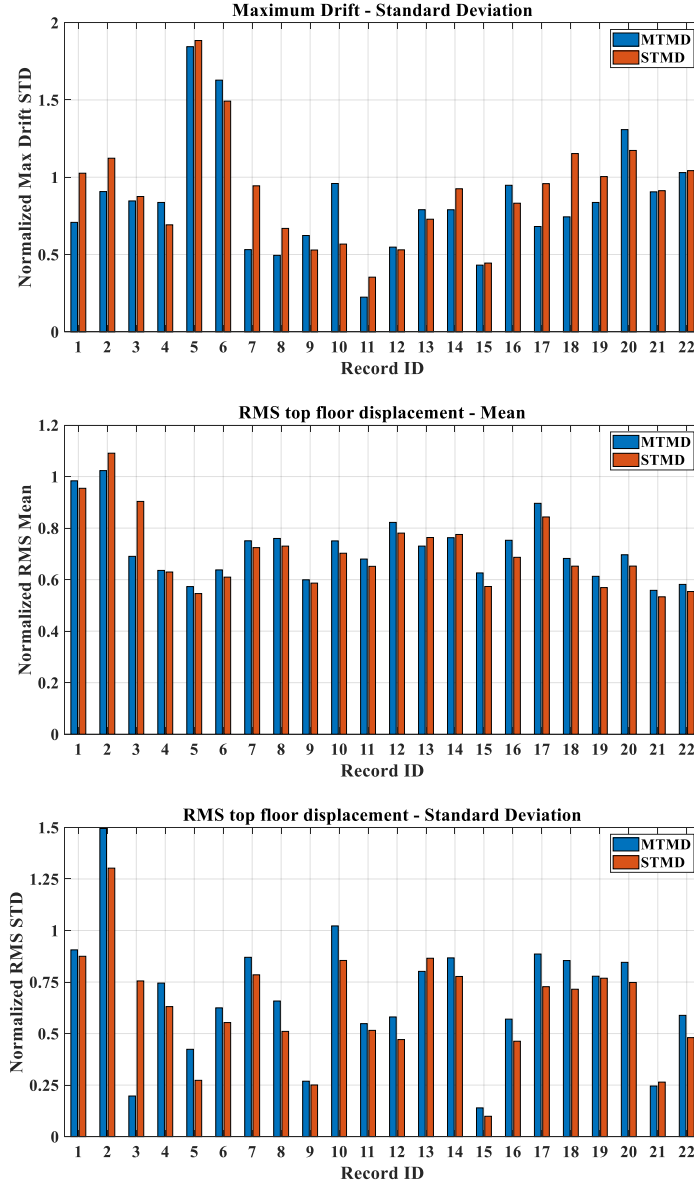


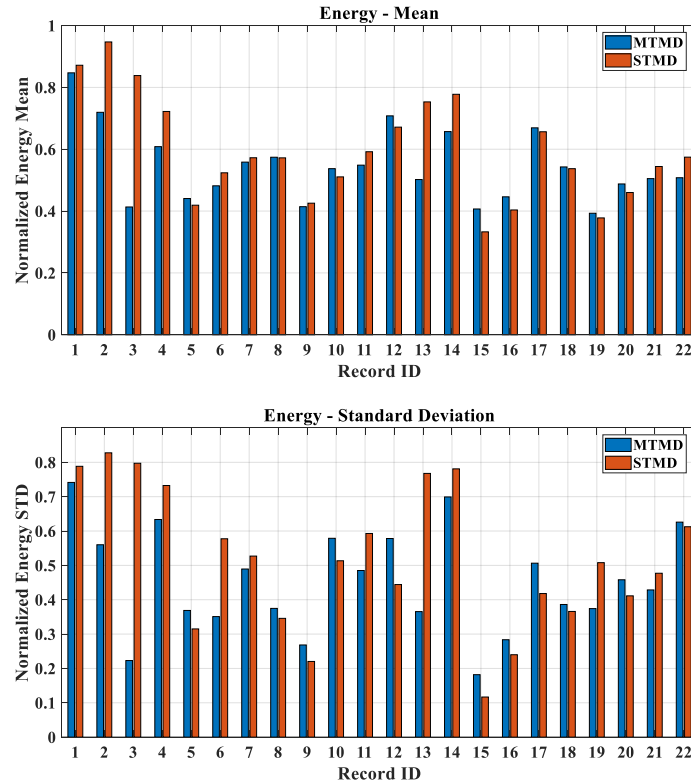
Fig. 7. Mean and Std of response metrics for deterministically optimized MTMD and STMD under uncertainty-aware simulations, normalized by uncontrolled response.

Table 3. Deterministically optimized MTMD and STMD parameters.

Case\Parameter	M_d (Kg)			C_d (N. s/m)			K_d (N/m)		
	m_1	m_2	m_3	c_1	c_2	c_3	k_1	k_2	k_3
MTMD	1.15e5	4.83e4	1.80e3	3.67e4	2.46e4	6.10e3	2.76e5	1.15e6	5.67e4
STMD	1.65e5			7.11e4			3.78e5		

3.2. Robust optimization

[Fig. 8](#) presents the mean and Std of the response metrics obtained using the robust optimization framework described in the previous section. The trends are broadly consistent with those observed for the deterministically optimized parameters in terms of modal energy distribution; however, the improvements are generally more pronounced for MTMD under robust optimization. Specifically, MTMD outperforms STMD in energy and maximum drift for the majority of records, with larger improvements observed for records dominated by higher-frequency content. The reductions in the Std of energy and maximum drift are particularly notable at the individual record level. Across the test record set, MTMD achieves up to 50.7% and 72.1% reductions in the mean and Std of energy, 20.2% and 45.1% reductions in maximum drift, and 24.5% and 79.7% reductions in RMS top-floor displacement compared to STMD. While STMD exhibits only marginal superiority in certain cases, MTMD shows substantial improvements in several records. Moreover, the advantage of MTMD is significantly more pronounced in the Std of the response metrics, indicating that MTMD primarily enhances robustness. The robustly optimized parameters are reported in [Table 4](#).



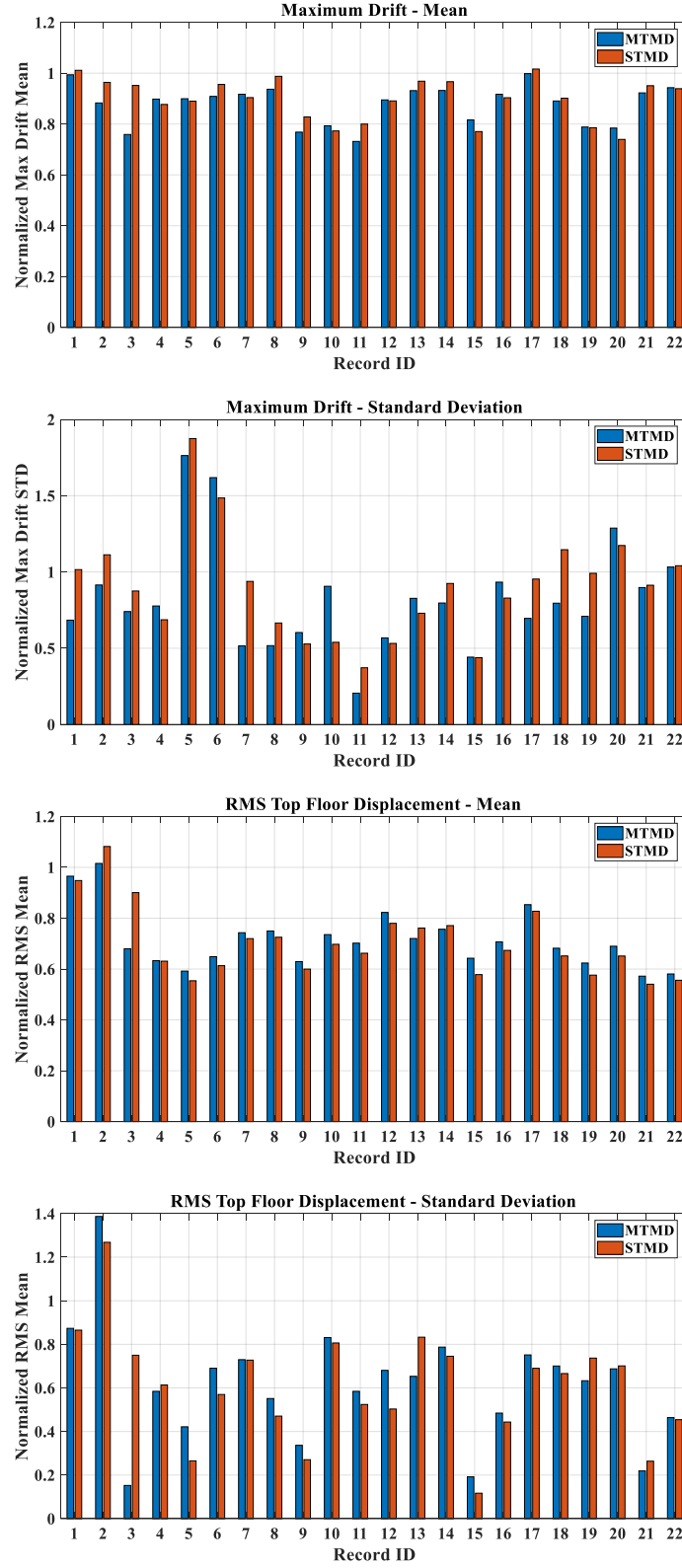


Fig. 8. Mean and Std of response metrics for robustly optimized MTMD and STMD under uncertainty-aware simulations, normalized by uncontrolled response.

Table 4. Robustly optimized MTMD and STMD parameters

Case\Parameter	M_d (Kg)			C_d (Kg/s)			K_d (Kg/s ²)		
	m_1	m_2	m_3	c_1	c_2	c_3	k_1	k_2	k_3
MTMD	1.16e5	4.53e4	2.32e3	4.03e4	2.5e4	1.18e4	2.62e5	1.03e6	1.65e5
STMD	1.65e5			7.53e4			3.68e5		

To further illustrate the advantage of MTMD over STMD for records with significant higher-mode contributions, the improvement of MTMD relative to STMD is defined as:

$$I_{MTMD} = \frac{(\bar{j}_n)_{STMD} - (\bar{j}_n)_{MTMD}}{(\bar{j}_n)_{STMD}} \quad (3.1)$$

where $\bar{j}_n$ denotes the mean or Std of a given response metric. This improvement is then plotted against the higher-mode contribution factor E_h , defined as:

$$E_h = E_{2r} + E_{3r} \quad (3.2)$$

in which E_{2r} and E_{3r} are the time-integrated energies of the second and third modes, normalized by the total energy as per Eq. (2.10). The relationship between I_{MTMD} and E_h is presented in [Fig. 9](#), along with regression line and the Pearson correlation coefficient, calculated as ([Berman, 2016](#)):

$$R = \frac{\sum_{i=1}^N (E_{h,i} - \bar{E}_h)(I_{MTMD,i} - \bar{I}_{MTMD})}{\sqrt{\sum_{i=1}^N (E_{h,i} - \bar{E}_h)^2} \sqrt{\sum_{i=1}^N (I_{MTMD,i} - \bar{I}_{MTMD})^2}} \quad (3.3)$$

where N is the number of records, and $\bar{E}_h$ and $\bar{I}_{MTMD}$ denote the sample means of E_h and I_{MTMD} , respectively.

The regression line and correlation coefficients indicate a strong positive correlation between MTMD improvement over STMD and higher-mode contribution. This trend is most evident in the performance metrics (mean responses); the Std metrics exhibit a similar but somewhat weaker correlation, particularly for the maximum drift, where the relationship remains positive yet less pronounced. Overall, these findings provide compelling evidence that MTMDs are particularly advantageous for excitations that induce significant higher-mode participation in the structural response.

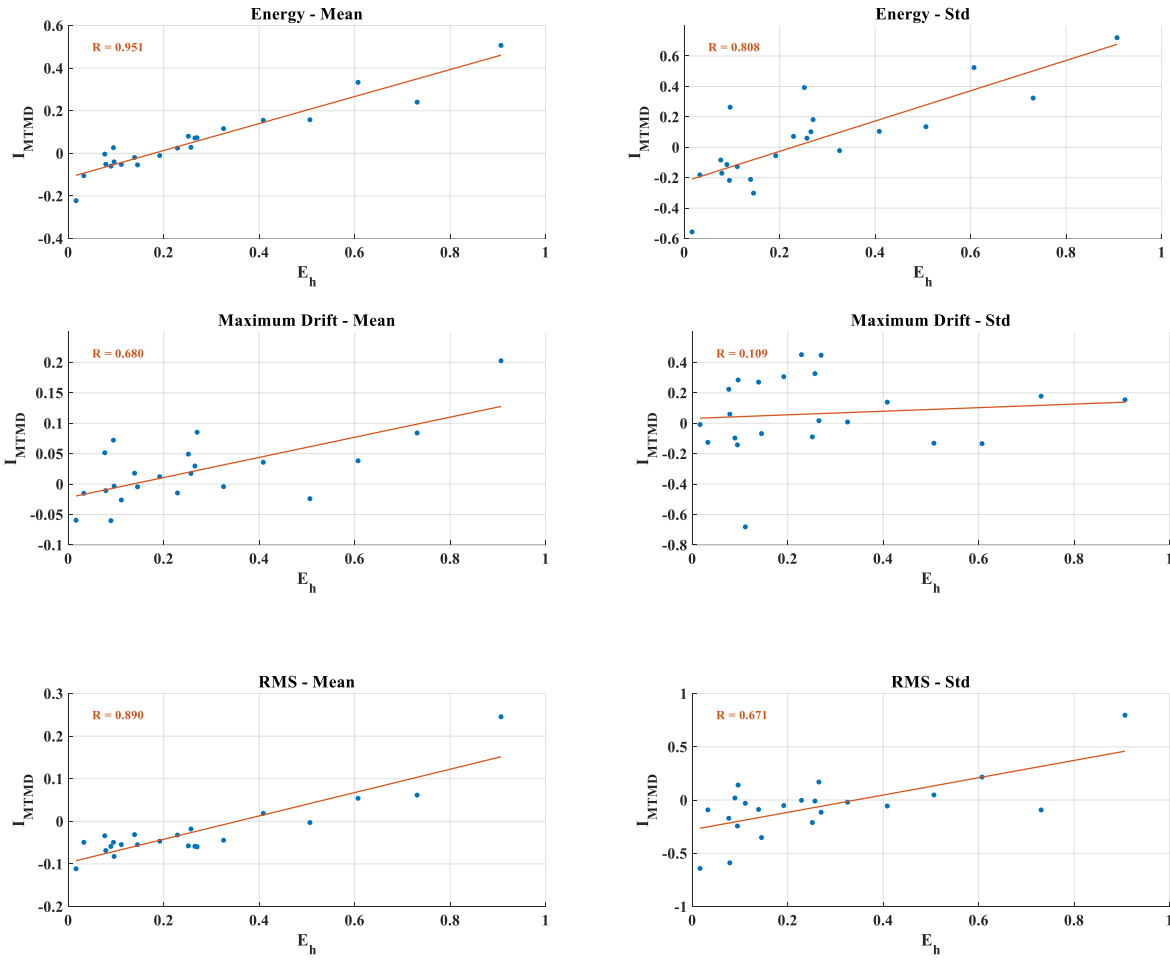


Fig. 9. Correlation between MTMD improvement over STMD and higher-mode contribution factor.

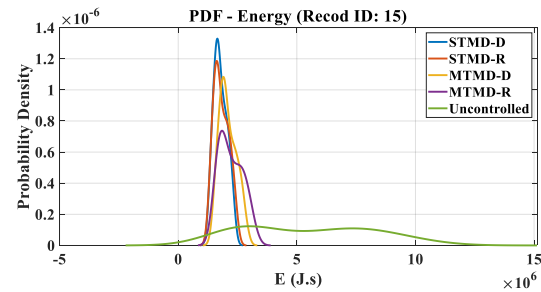
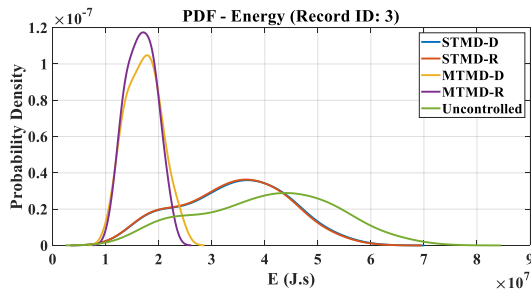
[Fig. 10](#) presents the probability density functions (PDFs) of the uncertainty-realized response metrics for four selected records: record 15 (highest mode 1 contribution), record 3 (highest mode 2 contribution), and records 4 and 14 (moderate contributions from both modes). Results are shown for the deterministically and robustly optimized STMD and MTMD, alongside the uncontrolled structure.

For record 3, where mode 2 is overwhelmingly dominant, the MTMD distributions are substantially narrower (indicating greater robustness) and shifted toward lower response values (indicating better performance) compared to both STMD and the uncontrolled case. In contrast, STMD shows only marginal improvement over the uncontrolled structure for this record. For record 15, dominated by mode 1, the STMD distributions are slightly narrower and shifted to lower

values than MTMD, though the difference is modest. This suggests that while STMD holds a slight advantage in mode 1 dominant responses, MTMD's gain over STMD in mode 2 dominant cases is far more substantial. For Records 4 and 14, the two configurations yield closer results; however, MTMD still shows improvements over STMD in energy and maximum drift, albeit with less effectiveness in RMS top floor displacement.

The cumulative distribution functions (CDFs) presented in [Fig. 11](#) further support these observations. For record 3, the MTMD CDF curves exhibit smaller spread (greater robustness) and are shifted leftward (better performance) relative to both STMD and the uncontrolled case. For record 15, STMD demonstrates superior performance, while records 4 and 14 show comparable behavior between the two configurations.

A key finding is that when the response is dominated by mode 2, the STMD controlled structure performs only marginally better than the uncontrolled case, whereas MTMD provides substantial improvement. Conversely, in mode 1 dominant cases where STMD is superior, MTMD lags only slightly behind, with results much closer to STMD than to the uncontrolled structure. Another important observation is that robustly optimized MTMD exhibits a more pronounced difference from its deterministically optimized counterpart compared to STMD, whose results remain largely aligned across both optimization strategies. This indicates that MTMD responds more favorably to robust design. Nevertheless, even for MTMD, the improvement is moderate, largely because the deterministic optimization, despite assuming nominal structural parameters, still accounts for load variability through the suite of optimization records, which differ in frequency content, duration, and amplitude. Consequently, the deterministic optimization framework adopted in this study inherently yields a notable level of robustness due to the richness of the seismic suite.



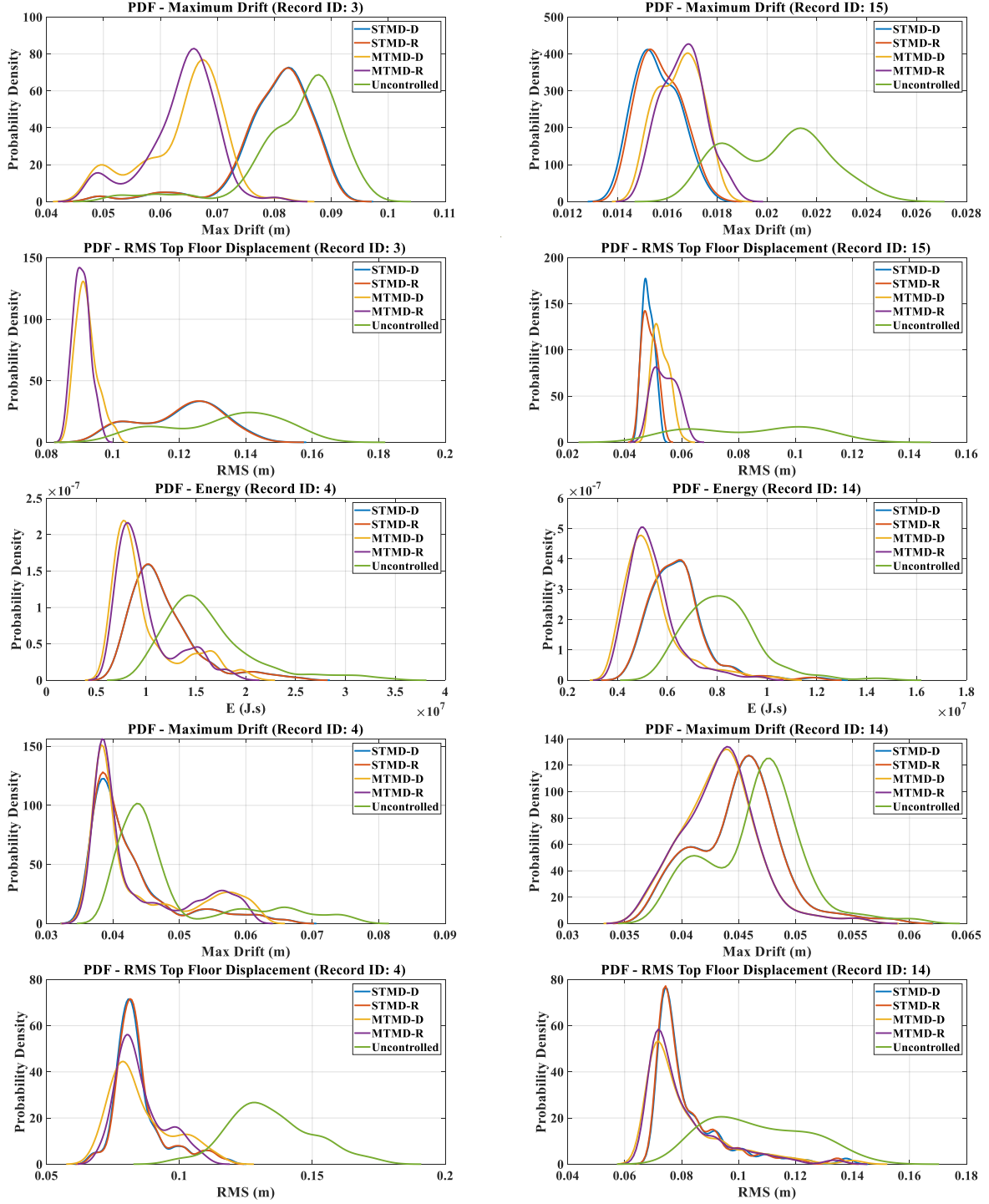
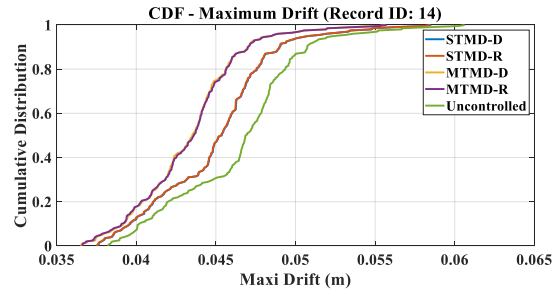
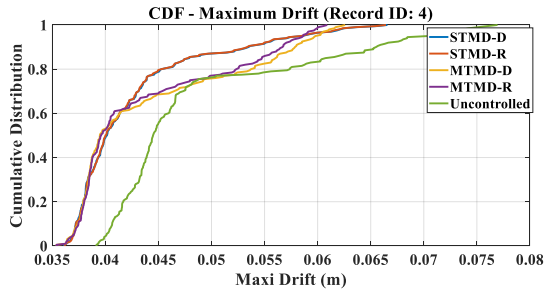
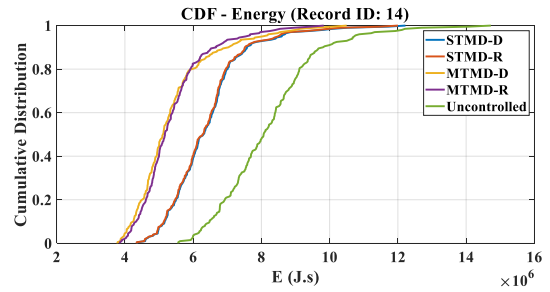
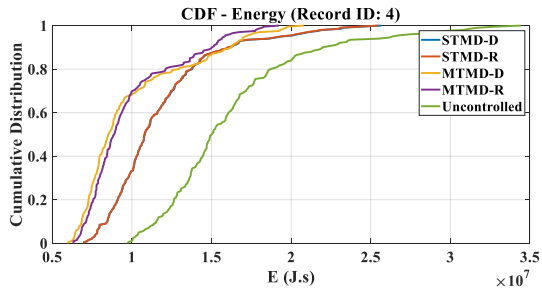
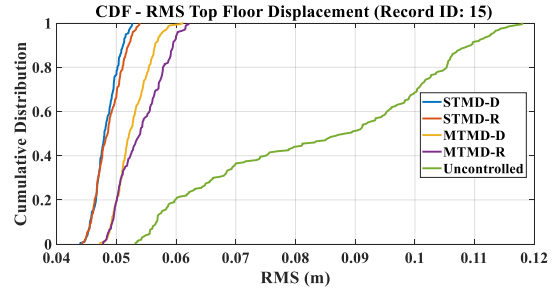
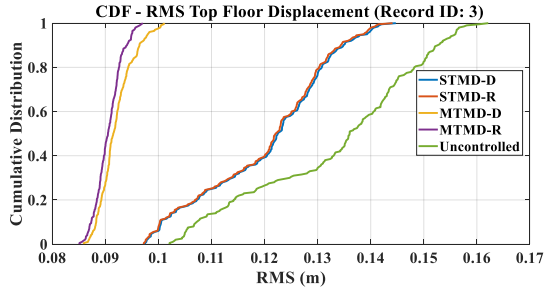
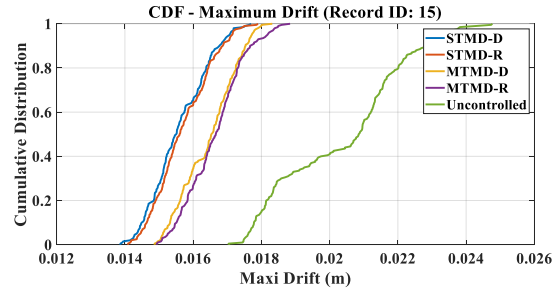
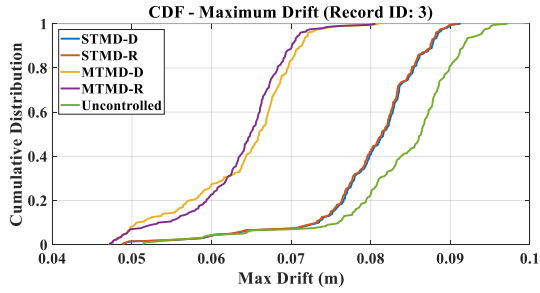
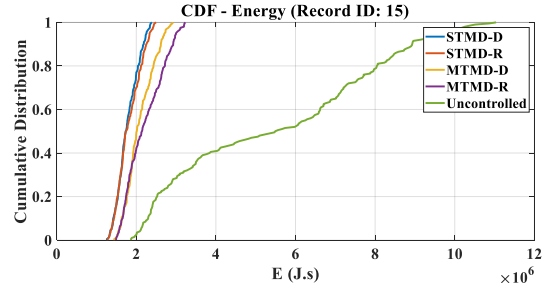
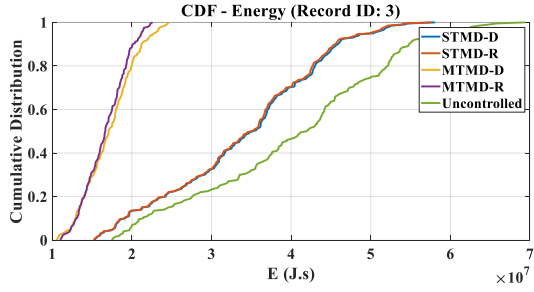


Fig. 10. Probability density functions (PDFs) of response metrics for the uncontrolled structure and the structure controlled with deterministically (d) and robustly (r) optimized STMD and MTMD.



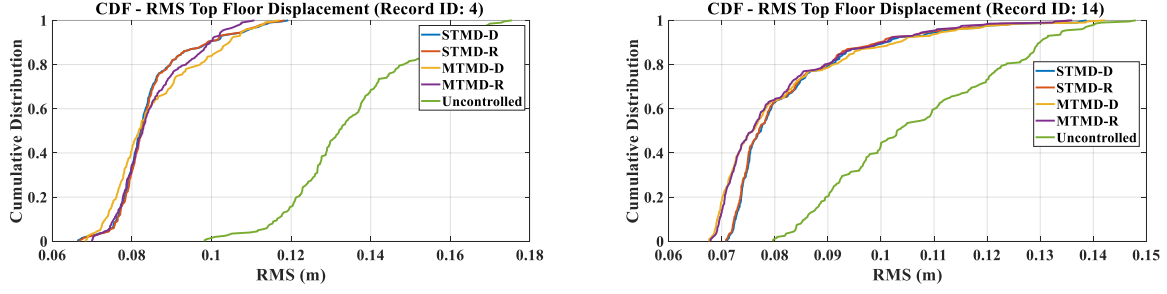
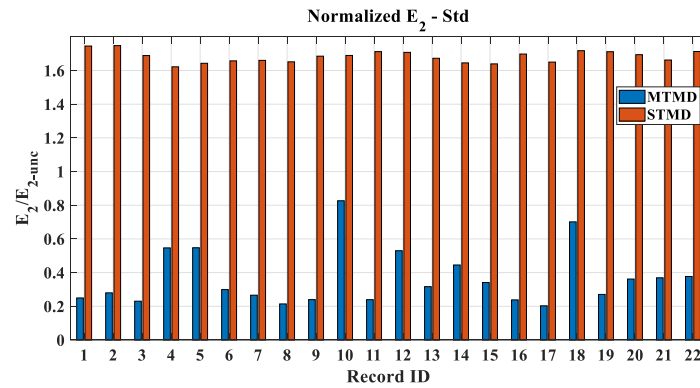
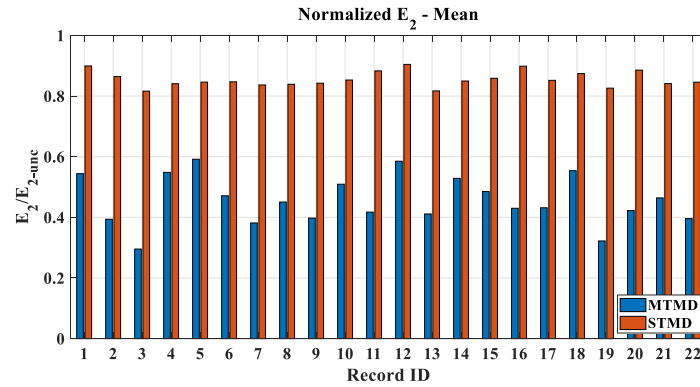
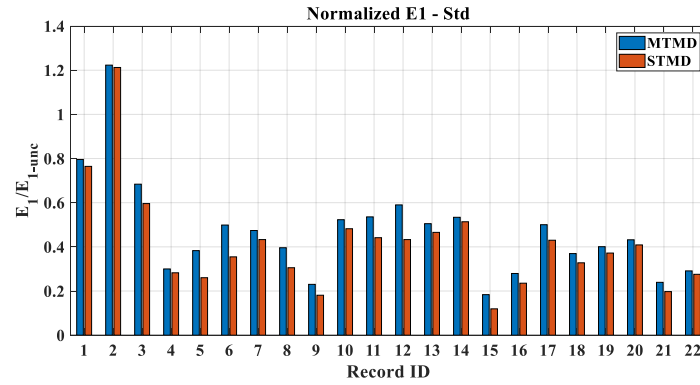
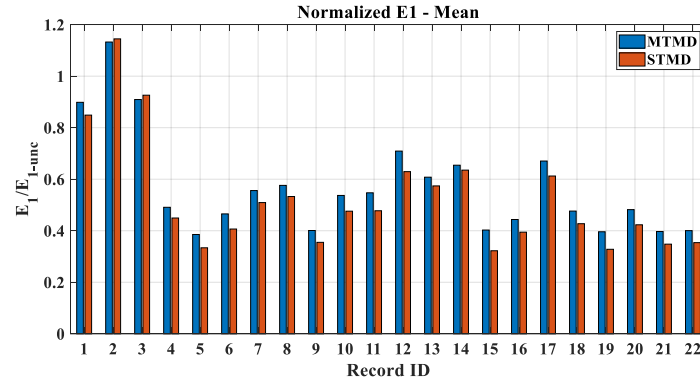


Fig. 11. Cumulative distribution functions (CDFs) of response metrics for the uncontrolled structure and the structure controlled with deterministically (d) and robustly (r) optimized STMD and MTMD.

3.3. Multimodal response reduction

[Fig. 12](#) presents the mean and Std of the time-integrated modal energy of the first three structural modes for the robustly optimized MTMD and STMD, normalized by the corresponding uncontrolled values for each mode, across the test record set. MTMD is considerably more effective and robust than STMD in reducing mode 2 energy compared to mode 1. These findings further support the earlier observations: MTMD's improvement in reducing mode 2 response is substantially greater than STMD's advantage over MTMD in mode 1 response reduction. This is particularly evident in the Std, where MTMD achieves an average reduction of 78.1% in mode 2 energy Std across all records relative to STMD.

Both STMD and MTMD exhibit similarly marginal improvements in reducing the mean and Std of mode 3 energy, with no clear superiority of MTMD over STMD. This is attributed to the relatively small contribution of mode 3 across the entire suite, averaging only 4.86% of the total energy over the 44 records. Consequently, the optimizer allocates only a very small mass to the TMD located at the mode 3 maximum amplitude location, as reflected in [Tables 3 and 4](#), yielding minimal mode 3 response reduction. It is worth noting that the marginal reductions observed for mode 3, particularly in the STMD case, are largely due to the large mode shape amplitude at the top floor (0.97), where the STMD attachment effectively acts as a viscous damper, slightly reducing the amplitude and overall modal energy response.



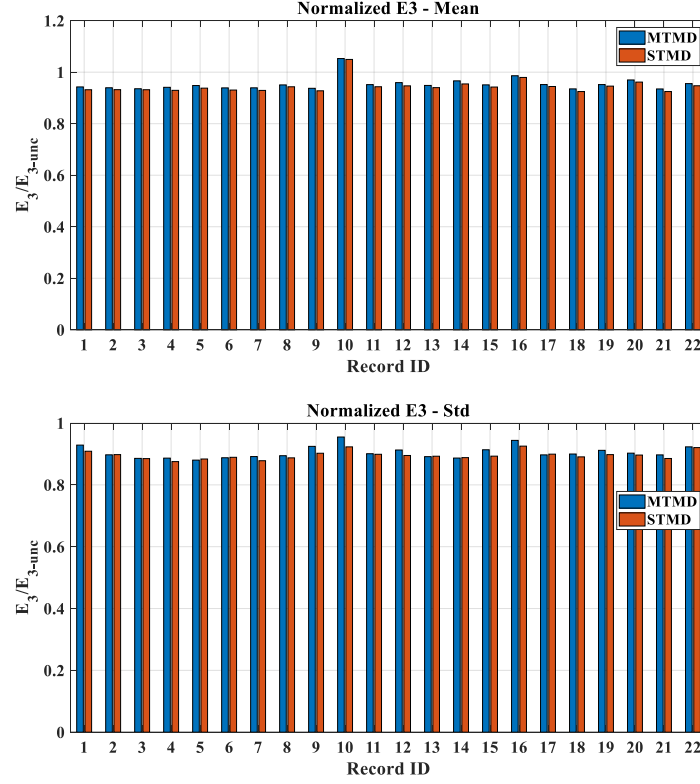


Fig. 12. Normalized mean and standard deviation of modal energy response for the first three structural modes.

4. Conclusions

This study investigated robust, multimodal response mitigation of a 20-storey shear frame under seismic excitation, treating structural mass, damping, and stiffness, as well as the earthquake record ID, as sources of uncertainty. The mean and Std of three response metrics, time-integrated modal energy, maximum inter-story drift, and RMS top-floor displacement, were evaluated for both deterministically and robustly optimized STMD and MTMD configurations, using a set of records not employed in the optimization process.

The results demonstrate that MTMD consistently outperforms STMD in terms of both performance (mean) and robustness (Std) when higher modes contribute significantly to the structural response. Under robust optimization, MTMD achieved reductions of up to 50.7% and 72.1% in the mean and Std of energy, 20.2% and 45.1% in maximum drift, and 24.5% and 79.7% in RMS top-floor displacement compared to STMD. Importantly, MTMD's improvement over

STMD was more pronounced in reducing response variability, highlighting its effectiveness in enhancing robustness.

A strong correlation was observed between MTMD's relative improvement and higher-mode contribution, with correlation coefficients of 0.981, 0.680, and 0.890 for the mean of energy, maximum drift, and RMS top-floor displacement, respectively. Conversely, STMD exhibited only marginal superiority over MTMD in mode 1 dominant cases, while showing limited effectiveness in mode 2 dominant scenarios. These findings underscore the suitability of MTMDs for multimodal vibration control, particularly for structures susceptible to higher-mode excitation.

The robustly optimized MTMD showed greater improvement over its deterministic counterpart compared to STMD, although the gains were moderate. This is partly attributed to the inherent robustness imparted by the deterministic optimization, which, despite assuming nominal structural parameters, still accounts for significant load variability through the use of a diverse suite of earthquake records.

Some limitations of this study should be acknowledged. The analyses were performed on a fixed-base benchmark structure, neglecting soil-structure interaction effects. Additionally, the record suite, while comprising 44 far-field horizontal components, excludes near-field pulse type records. Including a broader range of ground motions would enhance the generalizability of the framework. Addressing these limitations is a promising direction for future work.

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Competing Interests

The authors have no relevant financial or non-financial interests to disclose.