

Defect-Tolerant and Long-Range Acoustic Communication in Plugged and Abandoned Wells

Supplementary Note 3: Digital Modulations

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3 Digital Modulation

Digital modulation encodes discrete information by modifying carrier amplitude, phase, or frequency [1]. Common digital modulation methods include On-Off Keying (OOK), 4-level frequency shift keying (4-FSK), and quadrature phase-shift keying (QPSK), each offering different trade-offs among complexity, bandwidth, and robustness.

3.1 On-Off Keying

OOK is the simplest digital modulation scheme, where the carrier is switched on or off according to the transmitted bit. The transmitted signal is defined using Eq. 1 [1, 2].

$$s(t) = b(t)A_c \cos(2\pi f_c t), \quad b(t) \in \{0, 1\} \quad (1)$$

From Eq. 1, no signal is transmitted when $b(t) = 0$, enabling energy-efficient idle periods. However, this makes OOK highly sensitive to amplitude fluctuations and noise [1, 2].

Demodulation is performed using envelope detection based on the magnitude of the analytic signal obtained from the Hilbert transform. The decision threshold between the ON and OFF states is selected experimentally from the received envelope. The detected bit sequence is then obtained by comparing the recovered envelope with

this threshold. Although OOK is simple to implement, its performance degrades significantly in noisy or fading environments [1].

For an available channel bandwidth of $B_{\text{ch}} = 8\text{kHz}$, the OOK bandwidth relation gives a maximum bit rate of $R_b = B_{\text{ch}}/2 = 4\text{kb/s}$. The approximate required bandwidth for OOK is $B_{\text{OOK}} \approx 2R_b$.

3.2 4-Frequency Shift Keying

4-FSK uses four distinct frequencies to encode two bits per symbol. The transmitted signal is given by Eq. 2 [1].

$$s(t) = A_c \cos(2\pi f_i t), \quad i \in \{1, 2, 3, 4\} \quad (2)$$

Each frequency corresponds to a unique two-bit symbol, improving spectral efficiency at the cost of increased bandwidth [1, 2]. The approximate bandwidth of 4-FSK is $B_{\text{4FSK}} \approx (M - 1)\Delta f + 2R_s$, where M is the number of frequency states, Δf is the tone spacing, and R_s is the symbol rate. For an available channel bandwidth of $B_{\text{ch}} = 8\text{kHz}$, assuming orthogonal tone spacing such that $\Delta f = R_s$, the 4-FSK bandwidth relation becomes $B_{\text{4FSK}} \approx 5R_s$ for $M = 4$. Since each 4-FSK symbol carries two bits, $R_b = 2R_s$. Therefore, $R_s = B_{\text{ch}}/5 = 1.6\text{ksymbol/s}$, giving a maximum bit rate of $R_b = 3.2\text{kb/s}$.

Detection is performed using correlators or matched filters, where the received signal is compared against all possible tones and the maximum-correlation symbol is selected [1].

3.3 Quadrature Phase Shift Keying

QPSK improves spectral efficiency by transmitting two bits per symbol using orthogonal carriers. The transmitted signal is expressed using Eq. 3 [1].

$$s(t) = A_c [I(t) \cos(2\pi f_c t) - Q(t) \sin(2\pi f_c t)] \quad (3)$$

where $I(t), Q(t) \in \{-1, +1\}$.

From Eq. 3, the signal represents two independent modulations on orthogonal carriers, allowing QPSK to double the data rate without increasing bandwidth [1].

QPSK demodulation uses coherent detection by separating the received signal into in-phase (I) and quadrature (Q) components. The received signal $r(t)$ is multiplied by $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$, yielding [1]:

$$z_I(t) = r(t) \cos(2\pi f_c t), \quad z_Q(t) = -r(t) \sin(2\pi f_c t) \quad (4)$$

After low-pass filtering, the baseband signals $I(t)$ and $Q(t)$ are obtained, and the transmitted bit pair is recovered from the sign of each component. This approach requires accurate carrier phase and quadrature synchronization for reliable detection [1, 2]. For an available channel bandwidth of $B_{\text{ch}} = 8\text{ kHz}$, using the bandwidth

approximation $B_{\text{QPSK}} \approx R_b$, the maximum bit rate is

$$R_b \approx B_{\text{ch}} = 8\text{ kbit/s}.$$

Thus, QPSK provides a maximum bandwidth-limited bit rate of 8kbit/s under this bandwidth assumption.

For digital modulation, the bit-error rate (BER) is evaluated by direct bit-by-bit comparison of the transmitted and detected binary sequences. The BER is calculated as $\text{BER} = N_{\text{error}}/N_{\text{total}}$, where N_{error} is the number of incorrectly detected bits and N_{total} is the total number of transmitted bits. When no bit errors are observed over a finite sequence, the result is reported as zero observed errors, with the experimental BER resolution limited by the sequence length. Any theoretical BER estimate is reported separately and is not interpreted as a directly measured BER.

References

- [1] Sklar, B.: Digital Communications: Fundamentals and Applications. Pearson, ??? (2021)
- [2] Simon, M.K., Alouini, M.-S.: Digital Communication over Fading Channels. John Wiley & Sons, ??? (2004)