

Defect-Tolerant and Long-Range Acoustic Communication in Plugged and Abandoned Wells

Supplementary Note 1: Signal Processing Preliminaries

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1 Signal Processing Preliminaries

To analyze acoustic communication through cement, signal processing tools are required. The fast Fourier transform (FFT) converts discrete time-domain signals into the frequency domain, enabling evaluation of attenuation, dominant frequency components, and harmonic distortion [1]. For a discrete signal, the discrete Fourier transform (DFT), efficiently computed by the FFT, is defined using Eq. 1 [2]:

$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j2\pi \frac{kn}{N}}, \quad k = 0, 1, \dots, N-1 \quad (1)$$

Here, $x[n]$ is the input time-domain signal of length N , and $X[k]$ is the frequency-domain representation at frequency index k .

The frequency response, defined as the output-to-input amplitude ratio versus frequency, indicates how different frequencies are transmitted or attenuated [2]. For communication, identifying bands with low attenuation and minimal distortion is essential for improving signal clarity and channel capacity. The magnitude frequency response between the input and output signals is given by Eq. 2 [2].

$$H(f) = \frac{|Y(f)|}{|X(f)|} \quad (2)$$

Where $X(f)$ and $Y(f)$ are the Fourier transforms of the transmitted and received signals, respectively.

Another important metric is the signal-to-noise ratio (SNR), which quantifies desired signal strength relative to background noise [2, 3]. In cemented environments, material boundaries, microcracks, and inhomogeneities can cause scattering and reflections. Maintaining high SNR at the receiver is therefore essential for reliable communication [2, 3].

The SNR, expressed in dB, is calculated through Eq. 3 [2]:

$$\begin{aligned} \text{SNR} &= \frac{P_{\text{signal}}}{P_{\text{noise}}} = \left(\frac{A_{\text{signal}}}{A_{\text{noise}}} \right)^2 \\ \text{SNR}_{\text{dB}} &= 10 \cdot \log_{10} \left(\frac{P_{\text{signal}}}{P_{\text{noise}}} \right) \\ &= 10 \cdot \log_{10} \left(\left(\frac{A_{\text{signal}}}{A_{\text{noise}}} \right)^2 \right) \\ &= 20 \cdot \log_{10} \left(\frac{A_{\text{signal}}}{A_{\text{noise}}} \right) \end{aligned} \quad (3)$$

In Eq. 3, P_{signal} and P_{noise} denote the signal and noise powers, respectively. The average power of a discrete signal is defined using Eq. 4 [2].

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2 \quad (4)$$

For frequency-domain SNR evaluation, the signal and noise powers are evaluated from the one-sided FFT power spectrum of the received signal. The FFT bin closest to the expected signal frequency was first identified. The signal power, P_{signal} , was obtained by summing the spectral power over the signal-frequency bin and the two neighboring bins on each side. The noise power, P_{noise} , was calculated by summing the remaining spectral power after excluding a guard region extending five FFT bins on either side of the signal-frequency bin. This guard region prevents spectral energy associated with the desired signal from being included in the noise estimate. The resulting powers were then substituted into Eq. 3 to obtain the SNR. A higher SNR indicates a clearer and more detectable signal, which is critical for evaluating transmission performance [2, 3].

References

- [1] Cooley, J.W., Tukey, J.W.: An algorithm for the machine calculation of complex fourier series. *Mathematics of computation* **19**(90), 297–301 (1965)

- [2] Pfister, H.: Discrete-time signal processing. Lecture Note, pfister. ee. duke. edu/courses/ece485/dtsp. pdf (2017)
- [3] Proakis, J.G.: Digital Signal Processing: Principles, Algorithms, and Applications, 4/E. Pearson Education India, ??? (2007)