

Defect-Tolerant and Long-Range Acoustic Communication in Plugged and Abandoned Wells

Supplementary Note 2: Analog Modulations

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2 Analog Modulation

Analog modulation transmits continuous-valued information by varying the amplitude, frequency, or phase of a carrier signal [1, 2]. Common analog modulation methods include amplitude modulation (AM), frequency modulation (FM), and phase modulation (PM), where FM and PM encode information in instantaneous frequency and phase rather than amplitude.

2.1 Amplitude Modulation

AM conveys analog information by varying the carrier amplitude according to a baseband message signal. Let the carrier be $c(t) = A_c \cos(2\pi f_c t)$ and the normalized message signal be $m(t)$, where $|m(t)| \leq 1$. The AM signal is expressed using Eq. 1 [1].

$$s(t) = A_c [1 + \mu m(t)] \cos(2\pi f_c t) \quad (1)$$

where μ denotes the modulation index.

From Eq. 1, the spectrum consists of a carrier at f_c and two sidebands at $f_c \pm f_m$. Therefore, the required transmission bandwidth is given by Eq. 2 [1].

$$B_{AM} = 2B_m \quad (2)$$

where B_m is the message bandwidth. This makes AM spectrally inefficient compared to digital schemes. For an available channel bandwidth of $B_{\text{ch}} = 8\text{kHz}$, Eq. 2 gives a maximum message bandwidth of $B_m = B_{\text{ch}}/2 = 4\text{kHz}$.

Demodulation is typically performed using an envelope detector, and the recovered signal is approximated using Eq. 3 [1].

$$\hat{m}(t) \approx \frac{|s(t)|}{A_c} - 1 \quad (3)$$

AM is simple and does not require phase synchronization, but it is highly sensitive to amplitude noise and channel fading [1, 2].

2.2 Frequency Modulation

FM encodes information by varying the carrier instantaneous frequency according to the message signal. The FM signal is expressed using Eq. 4 [1].

$$s(t) = A_c \cos \left(2\pi f_c t + 2\pi k_f \int m(\tau) d\tau \right) \quad (4)$$

where k_f is the frequency sensitivity and $m(t)$ is the message signal. The instantaneous frequency is therefore:

$$f_i(t) = f_c + k_f m(t) \quad (5)$$

The bandwidth of an FM signal is approximated using Carson's rule, as expressed by Eq. 6 [1].

$$B_{FM} \approx 2(\Delta f + B_m) = 2(\beta + 1)B_m \quad (6)$$

where $\Delta f = k_f A_m$ is the peak frequency deviation and $\beta = \Delta f / B_m$ is the modulation index. A larger β improves noise immunity but increases bandwidth consumption. For an available channel bandwidth of $B_{\text{ch}} = 8\text{kHz}$ and $\beta = 4$, Eq. 6 gives a maximum message bandwidth of $B_m = B_{\text{ch}}/[2(\beta + 1)] = 0.8\text{kHz}$.

FM demodulation is commonly performed using a frequency discriminator or phase-locked loop (PLL). The demodulator estimates $f_i(t)$ and recovers the message signal as:

$$\hat{m}(t) = \frac{f_i(t) - f_c}{k_f} \quad (7)$$

FM is resistant to amplitude noise because information is carried in frequency variations [1, 2].

2.3 Phase Modulation

PM encodes information in the carrier instantaneous phase. The PM signal is given by Eq. 8 [1].

$$s(t) = A_c \cos (2\pi f_c t + k_p m(t)) \quad (8)$$

where k_p is the phase sensitivity. The instantaneous phase is therefore:

$$\phi(t) = 2\pi f_c t + k_p m(t) \quad (9)$$

The bandwidth of PM is similar to FM, where increasing β improves noise robustness at the cost of increased bandwidth [1]. For an available channel bandwidth of $B_{\text{ch}} = 8\text{kHz}$ and $\beta = 4$, the corresponding maximum message bandwidth is $B_m = B_{\text{ch}}/[2(\beta + 1)] = 0.8\text{kHz}$.

PM demodulation is typically performed using a phase detector or PLL-based approach. The instantaneous phase is extracted and the message signal is recovered as:

$$\hat{m}(t) = \frac{\phi(t) - 2\pi f_c t}{k_p} \quad (10)$$

PM is less sensitive to amplitude variations but requires accurate phase tracking, making it more complex than AM [1, 2].

References

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- [2] Tomasi, W.: Electronic Communications System: Fundamentals Through Advanced, 5/e. Pearson Education India, ??? (2009)