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Abstract

This article examines fraud against an environmental standard as a multidimensional decision rather than a binary choice between compliance and violation. Motivated by *Dieselgate*, we develop a sequential game in which a regulator chooses the environmental standard, the sophistication of the certification test, and the penalty. The firm then complies or commits fraud. In the latter case, it chooses a circumvention device, the number of vehicles involved, which defines the extensive margin, and excess emissions per vehicle, which defines the intensive margin. We show how the firm balances these margins according to their respective contributions to detection. Without stronger enforcement, a tighter standard can increase fraud-related excess emissions and may therefore undermine environmental protection. We characterize the conditions under which the regulator may forgo complete deterrence in favor of controlled residual fraud. We also show that two fraud configurations with the same aggregate volume can cause different levels of damage when damage depends nonlinearly on extent or intensity. Common enforcement instruments reduce fraud volume without necessarily changing its composition. The results suggest that targeting the two margins differently could provide a more effective strategy. Overall, ambitious standards need to be combined with tests, penalties, and external monitoring that make actual compliance more attractive to the firm than merely apparent compliance.

Keywords: Multidimensional fraud; *Dieselgate*; Environmental regulation; Enforcement; Game theory; Technological circumvention

JEL classification: C72, D82, K32, L51, Q52, Q58

1 Introduction

In September 2015, the U.S. Environmental Protection Agency (EPA) revealed that certain diesel vehicles marketed by Volkswagen in the United States were equipped with defeat devices designed to circumvent official emissions tests. Algorithms and calibrations embedded in the engine control module allowed the emissions control system to operate differently during certification tests and under normal driving conditions. The U.S. settlements covered approximately 590,000 diesel vehicles sold in the United States (EPA, 2026). Worldwide, Volkswagen reported having identified discrepancies between test results and on-road performance in approximately eleven million vehicles equipped with certain diesel engines (Volkswagen AG, 2017).

U.S. court documents describe a particularly revealing mechanism. When the software detected an official test cycle, the vehicle operated in a mode that enabled it to meet U.S. standards for nitrogen oxides (NOx) emissions. In real-world driving, it switched to another mode in which emissions controls were substantially reduced. In January 2017, Volkswagen agreed to plead guilty and to pay \$4.3 billion in criminal and civil penalties, comprising a \$2.8 billion criminal fine and \$1.5 billion in civil settlements (U.S. Department of Justice [DOJ], 2017). The civil settlements also provided for vehicle recalls or modifications, an environmental mitigation fund, and investment in zero-emission vehicles (EPA, 2026).

Volkswagen is an emblematic case, but not an isolated one. In 2022, FCA US was sentenced in the United States to approximately \$300 million in penalties in a case involving more than 100,000 diesel Jeep Grand Cherokee and Ram 1500 vehicles¹. In 2024, Cummins Inc. agreed to a civil penalty of \$1.675 billion in a case involving nearly one million Ram trucks. The EPA distinguishes approximately 630,000 vehicles equipped with illegal defeat devices from a further 330,000 vehicles with auxiliary emissions control devices that had not been disclosed during certification (EPA, 2024). These cases suggest that the problem extends beyond the conduct of any single manufacturer. They reveal a broader vulnerability of certification systems when tests are predictable and firms have the software capabilities to adapt a product's behavior to the conditions under which it is observed.

Dieselgate highlights several features that invite a reconsideration of how environmental fraud is analyzed. It reveals a first dimension of fraud, its extent: the number of vehicles equipped with a defeat device. Fraud did not apply indiscriminately to the group's entire output, but to a substantial, targeted subset of certain diesel vehicles. The case also revealed that emissions exceedances in actual use were not uniform. Tests commissioned by the International Council on Clean Transportation and conducted by West Virginia University showed that certain Volkswagen vehicles substantially exceeded U.S. NOx limits under real-world conditions, with deviations varying across models and testing conditions (Thompson et al., 2014). Fraud thus appears to have a second dimension, its intensity: the magnitude of the exceedance associated with the vehicles concerned.

Finally, the technological device enabled the vehicles to pass routine certification procedures, while independent real-world tests uncovered abnormal emissions discrepancies². Fraud therefore relied on the software's ability to generate apparent compliance in the standardized testing environment, while non-compliance became visible when observation conditions more closely resembled actual use.

¹ Court documents indicate that the emissions control systems had been calibrated to produce lower NOx emissions during federal test cycles than under normal driving conditions, using practices referred to internally as "cycle detection" and "cycle beating" (DOJ, 2022).

² The West Virginia University researchers did not immediately identify the fraudulent software. They first documented an abnormal discrepancy between laboratory and on-road emissions, a signal that subsequently led the EPA and the California Air Resources Board to investigate further.

These features show that *Dieselgate* cannot be reduced to a simple opposition between compliance and violation. They suggest a degree of rationality in the implementation of fraud, which can be organized around several decision margins: limiting or expanding the number of vehicles involved, determining the magnitude of actual exceedances, and investing in a technological device capable of circumventing the public test. These observations raise several questions. When a firm takes these margins into account in designing its fraudulent strategy, how does it choose the form of fraud? How does it balance extent, intensity, and the sophistication of the circumvention device? How should the regulator combine the standard, testing, penalties, and external monitoring when anticipating such a strategy? Finally, under what conditions does a tighter standard actually reduce pollution, and when can it strengthen the incentive to circumvent regulation?

Our analysis focuses on the interaction between the different dimensions of environmental fraud. Its contribution is to examine jointly, for a single firm, the technical circumvention device, the extent of the output involved, and the intensity of non-compliance per unit. This approach identifies how public instruments affect the profitability of fraud, the levels of its two decision margins, and the resulting damage.

This perspective offers a strategic and managerial interpretation of environmental fraud, centered on the strategic, technological, and economic instruments available to the firm in organizing non-compliance. The firm can invest in a device that circumvents the test, determine the extent of fraud through the number of vehicles involved, choose the intensity of emissions exceedances, and account for how visible these choices are to public authorities or external actors. Fraud is thus understood as a multidimensional strategy whose form depends jointly on the expected private gain, the cost of circumvention, the penalty, and the probability of discovery.

Our contribution builds on the economic analysis of deterrence, which relates violations to private gains, detection probabilities, and penalties (Becker, 1968; Polinsky & Shavell, 2000), and on research into avoidance activities (Malik, 1990; Nussim & Tabbach, 2008). We connect these mechanisms to two dimensions of the violation itself: the number of non-compliant units and the exceedance per unit. Technical circumvention conditions entry into fraud, while detection sensitivities determine its composition (see the literature review in Sect. 2, Background).

This connection also informs research on standards, technological investment, and enforcement costs (Arguedas, 2013; Innes, 2004; Stranlund, 2007). Here, technology serves to circumvent certification, and its cost must be considered alongside the profitability of emissions exceedances. The model therefore links the choice of public instruments to the balance between actual compliance and residual fraud. The damage analysis introduces a separate issue: when damage is nonlinear, a given volume of fraud can have different consequences depending on its distribution between extent and intensity.

To analyze this mechanism, we develop a sequential game between a regulator and a firm. The regulator chooses the standard, test sophistication, and penalty. The firm observes these choices and decides whether to comply or commit fraud. When it commits fraud, its decision is multidimensional: it chooses a circumvention device, the number of vehicles involved, and the intensity of the exceedance. The probability of discovery depends on the extent and intensity of fraud and on external monitoring. This structure brings together, within a single framework, standard stringency, the firm's technological and economic strategy, and public deterrence instruments.

The analysis shows that the firm adjusts the extent and intensity of fraud according to their respective contributions to detection. Detection that is more sensitive to large exceedances reduces intensity, whereas detection that is more sensitive to the number of vehicles involved reduces extent. Tightening the standard nevertheless increases the private gain from circumvention. When other instruments remain unchanged, it can increase excess emissions and, under sufficiently weak enforcement, total pollution. The regulator then weighs the cost of complete deterrence against that of controlled residual

fraud. Equilibrium non-compliance may therefore reflect a second-best choice rather than an absence of public intervention.

A further insight concerns the evaluation of enforcement. Stronger external monitoring can reduce fraud without increasing its detection probability, because the firm adjusts both decision margins simultaneously. Moreover, when damage depends nonlinearly on these margins, reducing fraud volume and changing its composition address distinct issues. The common instruments examined here affect fraud volume. Our results support considering whether its composition could be changed through enforcement that targets extent and intensity differently.

The remainder of the article is organized as follows. Section 2, Background, reviews the literature. Section 3 presents the model, and Sect. 4 characterizes the firm's optimal behavior. Section 5 analyzes the effects of standard stringency on fraud and total pollution. Section 6 examines regulatory policy and compares complete deterrence with controlled residual fraud. Section 7 investigates fraud composition and environmental damage. Section 8 discusses the implications of the results, and Sect. 9 concludes.

2 Background

The questions examined here relate to several strands of literature, particularly the economics of enforcement. Since Becker (1968), this literature has shown that the effectiveness of the law depends less on the formal existence of a rule than on the expected cost of violating it. Polinsky and Shavell (2000) clarify the conditions for optimal enforcement, particularly the trade-off between inspection intensity and penalty levels. In environmental economics, Heyes (2000) emphasizes that effective regulation depends on how inspections, penalties, and compliance incentives are combined. We retain this central insight but apply it to a setting in which a violation is not merely chosen: it is technically constructed. The firm can influence its extent, intensity, and detectability.

This issue arises directly in environmental offending. Comparing sanctions imposed in Finland with those implied by a deterrence model, Lindqvist et al. (2024) show that fines can leave a positive expected gain from offending. This finding highlights the importance of the anticipated profitability of non-compliance. Our analysis concerns a different stage: how the firm configures fraud before it is discovered by jointly adjusting its extent, intensity, and the device used to circumvent certification.

The article also relates to the literature on environmental enforcement and regulatory compliance. Harrington (1988) examines how enforcement differentiated according to firms' past behavior can encourage compliance despite infrequent inspections. Nyborg and Telle (2006), however, qualify the existence of a general paradox of high compliance under weak deterrence. Using Norwegian data, they distinguish frequent, lightly sanctioned minor violations from rarer serious violations subject to a credible threat of severe sanctions. Average inspection and penalty levels can therefore give a misleading picture of effective deterrence. Research on inspection targeting (Harrington, 1988; Friesen, 2003), self-reporting and self-auditing (Kaplow & Shavell, 1994; Innes, 1999, 2000, 2001; Pfaff & Sanchirico, 2000), and the interaction between standards and technological investment (Arguedas, 2013) sheds light on different routes to compliance. Our framework builds on this research by explicitly distinguishing, for a single firm, the circumvention device, the share of output involved, and the intensity of non-compliance per unit. It examines their joint choice and their different effects on detection and damage. Other contributions emphasize the strategic relationship between enforcement agencies and polluting firms. Germani et al. (2017) show that partially unpredictable enforcement can strengthen deterrence by creating uncertainty for potential offenders. In a related spirit, our analysis examines how a firm organizes non-compliance when meeting the standard is costly and the certification test can be technically circumvented.

Treating the firm as an offender also connects our analysis to Auriol et al. (2023), who combine the economics of crime with industrial organization to study corporate sanctions. Their contribution highlights, in particular, trade-offs associated with the market consequences of sanctions and the complementarity between self-reporting and independent investigation. We instead isolate the relationship between a regulator and a representative firm: the technical construction of non-compliance is endogenous, but competitive effects are not modeled.

This discussion also relates to debates over the ambition of environmental standards. Research associated with the Porter hypothesis argues that well-designed environmental regulation can stimulate innovation and offset part of the cost of compliance (Porter & van der Linde, 1995). Other approaches emphasize compliance costs and potentially adverse effects when regulation lacks sufficient credibility. Closer to our framework, Arguedas (2013) analyzes the interaction between pollution standards, technological investment, and fines for non-compliance. Our contribution differs, however, by changing the purpose of technological investment. Here, investment is not directed toward emissions abatement or cleaner technology, but toward a device that circumvents the public test. This shift allows us to study a situation in which technology produces apparent compliance rather than improving actual compliance.

A more recent strand of the regulatory auditing literature also emphasizes that public enforcement can change a firm's internal decisions. Dai (2026) studies the interaction between regulatory auditing and firms' self-auditing, showing that enforcement intensity and penalty levels affect the timing of, and incentives for, auditing. Our model shares this sequential regulator–firm logic, but differs in its object of analysis. The firm does not simply choose whether to prevent, disclose, or correct a defect; it chooses a technological device that circumvents the enforcement mechanism. We therefore place not only audits and inspections at the center of the analysis, but also the possibility that the test itself becomes a strategic target.

Ehrmann and Prinz (2025) also identify limitations of auditing arrangements when the private provider is paid by the firm being audited. In their model, transparency or higher fines do not necessarily prevent collusion. Our framework examines a different source of certification failure: even without collusion with the public authority, the firm can exploit the characteristics of the test to generate apparent compliance. The difficulty therefore concerns not only the auditor's incentives, but also the possibility of manipulating what is observed.

The enforcement literature has also extensively analyzed firms' avoidance activities. Malik (1990) shows that offenders can undertake *ex ante* activities that reduce the probability of detection and that such avoidance can create an incentive for screening by enforcement agencies. Kambhu (1989) studies the interaction between regulatory standards, non-compliance, and enforcement when firms can undertake avoidance activities. The effects of regulatory stringency then depend on these response possibilities. Nowell and Shogren (1994) show that higher penalties or a greater probability of inspection do not guarantee a reduction in illegal emissions when firms can invest in challenging enforcement measures. These contributions are particularly relevant because they treat avoidance and circumvention as strategic choices rather than exogenous constraints. Our model builds on this logic by distinguishing two separate margins of fraud, extent and intensity, and incorporating the sophistication of the circumvention device as a third strategic dimension.

Nussim and Tabbach (2008) explicitly study the multiplicity of avoidance responses. When some avoidance activities can be punished and others cannot, sanctioning the former does not necessarily improve deterrence of the underlying offense. Our contribution builds on their analysis of avoidance strategies by connecting them to two margins of the environmental violation itself, its extent and intensity, whose effects on detection and damage may differ.

Innes (2004) analyzes the choice between liability after harm occurs and preventive regulation, accounting for enforcement costs and the structure of sanctions. He shows that preventive regulation can be more efficient even when monitoring precautions is substantially more costly than monitoring accidents. The attractiveness of regulation thus stems from its deterrent effect, not simply from low monitoring costs. Friesen (2003) studies inspection targeting and characterizes a scheme that achieves compliance at minimum inspection cost. In emissions trading programs, Stranlund (2007) shows that the case for tolerating non-compliance depends on the structure of sanctions. Residual non-compliance can be advantageous with an increasing marginal penalty, but a constant-marginal-penalty scheme can achieve the same emissions target at lower cost while ensuring full compliance. This result highlights the role of the available instruments in choosing between compliance and non-compliance. More broadly, Cohen (1999) surveys environmental monitoring and enforcement, focusing in particular on firms' responses to enforcement.

Endogenous detection also matters for penalty policy. Kim (2013) shows that the maximum fine is not necessarily optimal when the apprehension probability depends on the aggregate offense rate. In our framework, endogeneity operates through the extent and intensity choices of a single firm. In the distinct context of piracy, Banerjee and Poddar (2026) show, in an extension where the fine is proportional to the pirate's revenue, that the regulator may benefit from tolerating residual piracy. This possibility depends on the form of sanction available to the regulator. Our analysis instead examines the trade-off between complete deterrence and residual environmental fraud in a certification system, accounting for the costs of testing, penalties, and tightening the standard.

A related strand of literature examines self-reporting and voluntary compliance. Kaplow and Shavell (1994) analyze self-reporting in law enforcement, showing that such arrangements can improve welfare by saving enforcement resources and reducing risk. Innes (1999, 2000, 2001) examines the roles of self-policing, remediation, and avoidance activities in optimal law enforcement. Pfaff and Sanchirico (2000) analyze self-auditing as a two-stage incentive problem: searching for violations and then ensuring compliance. They examine how penalties and reductions granted for violations discovered and reported by the firm affect both decisions. These contributions are relevant because they show how the structure of enforcement instruments can influence firms' compliance and self-reporting decisions, a logic that also informs our analysis of fraud design.

Finally, external discovery of fraud depends on incentives to produce reliable information. Buccirosi et al. (2021) jointly analyze whistleblower rewards, sanctions for false reporting, and judicial errors. Their contribution clarifies the institutional conditions under which private information can support deterrence. We treat external monitoring as exogenous to examine its effect on fraud and public instruments. Endogenizing rewards, the credibility of reports, and protection against retaliation would take this analysis further.

Our contribution is therefore not to contrast strict and moderate standards. Rather, it shows that the effect of a standard depends on how it is combined with enforcement instruments. An ambitious standard can reduce pollution when tests, penalties, and a sufficiently high probability of discovery make it credible. It can become vulnerable when it raises the private gain from circumvention without sufficiently increasing the expected cost of fraud.

3 Model

The model considers two players: the government, or regulator, and a representative firm in the automotive sector. This setting isolates the firm's strategic response to public instruments. Market structure and competition between manufacturers provide directions for further analysis.

The firm produces N vehicles subject to an environmental standard set by the government. The standard defines the legally permitted emissions level per vehicle, denoted by $\bar{q}$. The lower $\bar{q}$, the

stricter the standard. Actual compliance is not directly observable by the government: it is inferred from a public certification test, supplemented by post-market monitoring and signals that may be produced by external actors. This setting reflects the circumstances revealed by *Dieselgate*, where testing includes both official inspections and independent tests undertaken by third parties in response to signals of suspected circumvention.

The government and the firm interact in a two-stage sequential game with complete information. The government moves first and chooses its policy instruments. The firm observes these choices and then decides whether to comply with the environmental standard or implement a fraudulent strategy. If it commits fraud, it simultaneously chooses three instruments that determine the structure of its non-compliance with the standard.

3.1 The firm's constraints and circumvention strategies

The firm's first decision when committing fraud is the sophistication of the circumvention device, denoted by $\gamma \in [0,1]$. Higher values of γ correspond to more elaborate software capable of distinguishing the certification-test environment from actual use. The second decision concerns the extensive margin of fraud, denoted by $n \in [0, N]$: the number of vehicles equipped with the fraudulent device. The third concerns the intensive margin, denoted by $q \geq 0$: excess emissions per fraudulent vehicle above the legal standard $\bar{q}$. The actual emissions level of a fraudulent vehicle is therefore $\bar{q} + q$.

The vehicle's unit selling price is denoted by P , and c_0 denotes its unit production cost excluding the environmental constraint. If the firm complies with the standard, the unit cost of a compliant vehicle is defined as:

$$C^C(\bar{q}) = c_0 + \eta(1 - \bar{q})^2, \quad \eta > 0. \quad (1)$$

The term $\eta(1 - \bar{q})^2$ represents the private cost of environmental compliance. It increases as the standard becomes stricter, that is, as $\bar{q}$ decreases. The case $\bar{q} = 1$ means that the regulator imposes no standard. This cost function captures the idea that abatement becomes more expensive as the permitted emissions level falls.

If the firm commits fraud on a vehicle and exceeds the standard by q , its unit cost is:

$$C^F(q; \bar{q}) = c_0 + \eta(1 - \bar{q})(1 - \bar{q} - q), \quad 0 \leq q \leq 1 - \bar{q}. \quad (2)$$

This function means that a fraudulent vehicle entails less environmental effort than a compliant one. The effort required by the standard is measured by $1 - \bar{q}$, whereas the effort actually exerted for a vehicle emitting q units above the standard is measured by $1 - \bar{q} - q$.

The difference between the two costs yields the gross private gain from exceeding the standard, namely $C^C(\bar{q}) - C^F(q; \bar{q}) = \eta(1 - \bar{q})^2 - \eta(1 - \bar{q})(1 - \bar{q} - q)$, or, more simply:

$$C^C(\bar{q}) - C^F(q; \bar{q}) = \eta(1 - \bar{q})q. \quad (3)$$

Using the notation $B(\bar{q}) = \eta(1 - \bar{q})$, the gross private gain from the exceedance can be written as:

$$C^C(\bar{q}) - C^F(q; \bar{q}) = B(\bar{q})q. \quad (4)$$

The term $B(\bar{q})$ is the marginal cost difference between an actually compliant vehicle and a fraudulent one for one unit of exceedance. Since $B'(\bar{q}) = -\eta < 0$, this difference increases as the standard becomes stricter. The more demanding actual compliance is, the greater the cost gap between actual and apparent compliance.

The firm also incurs a cost of developing the circumvention device. We assume that developing a device of sophistication γ against a public test of sophistication θ costs:

$$C_\gamma(\gamma, \theta) = \frac{c_\gamma}{2} \frac{\gamma^2}{1 - \theta}, \quad c_\gamma > 0. \quad (5)$$

This cost is convex in γ and increasing in θ . A more sophisticated public test therefore raises the cost of any given level of circumvention. To pass certification, the firm's device must be at least as sophisticated as the public test, which we express through the constraint $\gamma \geq \theta$.

3.2 Strategic instruments of public regulation

The government has three instruments. First, it sets the environmental standard $\bar{q} \in [0,1]$. A lower $\bar{q}$ corresponds to a stricter standard. Second, it chooses the sophistication of the public certification test, denoted by $\theta \in [0,1]$. A more sophisticated test makes circumvention more costly for the firm; when it also contributes to ex post detection, it improves the ability to uncover fraud after vehicles enter the market. Finally, the government chooses the monetary penalty $R \geq 0$, applied to the aggregate volume of fraud if it is detected.

We assume that the government minimizes a social loss function with four components:

$$W = \delta(N\bar{q} + nq) + \frac{\lambda}{2}(1 - \bar{q})^2 + \frac{h}{2}\theta^2 + \frac{\rho}{2}R^2, \quad (6)$$

where $\delta > 0$, $\lambda > 0$, $h > 0$, and $\rho > 0$. The first term, $\delta(N\bar{q} + nq)$, represents environmental damage. Total emissions include the legally permitted emissions of the entire fleet, $N\bar{q}$, plus fraud-related excess emissions, nq . The second term, $\frac{\lambda}{2}(1 - \bar{q})^2$, is a convex social cost of tightening the standard. It may reflect, for example, higher vehicle prices, more demanding technical constraints, or reduced performance³. This social cost is specified at the aggregate level and need not coincide with the firm's private compliance cost. The third term, $\frac{h}{2}\theta^2$, is the convex institutional cost of developing and operating a more sophisticated certification procedure. The fourth term, $\frac{\rho}{2}R^2$, is the convex administrative and legal cost of imposing and enforcing penalties.

Note that W does not include the firm's private circumvention cost. The resources devoted to developing concealment software are treated as internal to the firm's fraud decision rather than as a social cost directly internalized by the regulator. In other words, under this specification, the regulator does not internalize the firm's private research and development expenditure on circumvention, except through its consequences for the feasibility of fraud and the resulting damage.

3.3 Identifying fraud and the probability of detection

We consider two independent sources of monitoring that contribute to detecting fraud once vehicles are in use. The first is external monitoring, denoted by $v > 0$, which aggregates the activities of non-governmental organizations, independent laboratories, consumer associations, the media, and whistleblowers. The second is the potential contribution of the public test to post-market detection, measured by $\tau\theta$, with $\tau \geq 0$. When $\tau > 0$, a more sophisticated public test does more than support initial certification: it also improves detection capabilities after vehicles enter the market.

³ In *Dieselgate*, genuinely implementing a more demanding emissions control system could affect vehicle power, fuel consumption, driving comfort, or operating costs. These effects may reduce consumer surplus or the product's commercial appeal. The social cost term associated with the standard aggregates these disadvantages and the broader costs of regulatory tightening.

These two sources are combined in the overall detection environment:

$$\Omega(\theta, v) = v + \tau\theta. \quad (7)$$

The probability of detecting fraud of extent n and intensity q is defined as:

$$\Phi(n, q; \theta, v) = \Omega(\theta, v)(\alpha q + \beta n), \quad \alpha > 0, \quad \beta > 0, \quad (8)$$

with $0 \leq \Phi(n, q; \theta, v) \leq 1$.

The parameter α measures the sensitivity of detection to the intensity of the exceedance: the greater the excess emissions per vehicle, the more visible they are. The parameter β measures the sensitivity of detection to the extent of fraud: the greater the number of vehicles involved, the larger the footprint of fraud. This functional form is linear in n and q for tractability. It is chosen because it explicitly reveals the trade-off between the two margins of fraud.

If fraud is detected, the firm pays a penalty proportional to the aggregate volume of fraud, nq . The expected penalty is therefore:

$$Rnq\Phi(n, q; \theta, v) = Rnq\Omega(\theta, v)(\alpha q + \beta n). \quad (9)$$

If the firm fully complies with the standard, its compliance profit is $\Pi^C(\bar{q}) = N[P - C^C(\bar{q})]$.

Using the compliance cost function gives $\Pi^C(\bar{q}) = N[P - c_0 - \eta(1 - \bar{q})^2]$.

If the firm commits fraud on n vehicles and the fraud remains undiscovered, its profit becomes:

$$\Pi^{F,0} = (N - n)[P - C^C(\bar{q})] + n[P - C^F(q; \bar{q})] - C_\gamma(\gamma, \theta). \quad (10)$$

Comparing this profit with compliance profit and using $C^C(\bar{q}) - C^F(q; \bar{q}) = B(\bar{q})q$, we obtain $\Pi^{F,0} = \Pi^C(\bar{q}) + B(\bar{q})nq - C_\gamma(\gamma, \theta)$.

If fraud is discovered, the firm pays the penalty Rnq . Its profit is then $\Pi^{F,1} = \Pi^{F,0} - Rnq$.

Expected profit from fraud is therefore $\mathbb{E}[\Pi^F] = (1 - \Phi)\Pi^{F,0} + \Phi\Pi^{F,1}$.

Substituting $\Pi^{F,1}$ gives $\mathbb{E}[\Pi^F] = \Pi^C(\bar{q}) + B(\bar{q})nq - C_\gamma(\gamma, \theta) - Rnq\Phi(n, q; \theta, v)$.

The expected net gain from fraud relative to compliance is defined as the difference between expected fraud profit and compliance profit: $\Delta\Pi(\gamma, n, q; \bar{q}, \theta, R) = \mathbb{E}[\Pi^F] - \Pi^C(\bar{q})$.

Thus:

$$\Delta\Pi(\gamma, n, q; \bar{q}, \theta, R) = B(\bar{q})nq - Rnq\Omega(\theta, v)(\alpha q + \beta n) - \frac{c_\gamma}{2} \frac{\gamma^2}{1 - \theta}. \quad (11)$$

The firm commits fraud if and only if the maximum value of this net gain is strictly positive. Otherwise, it complies with the standard. In other words, it commits fraud if:

$$\max_{\gamma \geq \theta, 0 \leq n \leq N, q \geq 0} \Delta\Pi(\gamma, n, q; \bar{q}, \theta, R) > 0. \quad (12)$$

3.4 Timing of the game and equilibrium concept

To formalize the interaction between the government and the firm, we define a two-stage game. At the first stage, the government simultaneously chooses its instruments: the environmental standard $\bar{q}$, test sophistication θ , and penalty R . At the second stage, the firm observes $(\bar{q}, \theta, R)$ and decides whether to comply with the standard or commit fraud. If it commits fraud, it chooses the vector (γ, n, q) , comprising its three decision variables.

The solution concept is subgame-perfect equilibrium. We solve the game by backward induction: first, we solve the firm's problem for each triple of public instruments; we then substitute its best response into the social loss function to obtain the government's reduced problem.

4 The firm's optimal behavior

In this section, the public instruments $\bar{q}$, θ , and R are given. Conditional on committing fraud, the firm chooses its three decision variables. The analysis focuses on interior solutions, in which fraud affects part of the fleet and the probability of detection remains below one. Compliance is determined separately by the condition for entry into fraud.

Since circumvention sophistication γ directly affects neither the gross benefit of fraud, $B(\bar{q})nq$, nor its detection probability, Φ , the firm optimally chooses the minimum level required to pass the public certification test.

Proposition 1. If the firm commits fraud, its optimal sophistication choice is $\gamma^* = \theta$.

Proof. The circumvention cost $C_\gamma(\gamma, \theta) = \frac{c_\gamma}{2} \frac{\gamma^2}{1-\theta}$ is strictly increasing in γ :

$$\frac{\partial C_\gamma}{\partial \gamma} = \frac{c_\gamma \gamma}{1-\theta} > 0 \quad \text{for } \gamma > 0. \quad (13)$$

Since γ has no direct effect on the gain from fraud or its detection probability, the firm chooses the lowest level compatible with passing the public test. The test-passing constraint is $\gamma \geq \theta$, which gives $\gamma^* = \theta$.

The proposition simply states that the firm does not overinvest in circumvention: it chooses exactly the level needed to neutralize the public test. Substituting $\gamma = \theta$ then yields the fixed cost of fraud: $K(\theta) = \frac{c_\gamma}{2} \frac{\theta^2}{1-\theta}$.

Conditional on committing fraud, the firm chooses the number of fraudulent vehicles n and excess emissions per vehicle q to maximize its expected net gain. The fixed circumvention cost $K(\theta)$ does not affect the optimal choices of n and q . The next proposition gives the firm's optimal choices at the second stage of the game.

Proposition 2. Under the assumptions $B(\bar{q}) > 0$, $0 < n^* < N$, and $q^* > 0$, the firm's optimal decisions are given by:

$$n^*(\bar{q}, \theta, R, v) = \frac{B(\bar{q})}{3\beta R \Omega(\theta, v)}, \quad (14)$$

$$q^*(\bar{q}, \theta, R, v) = \frac{B(\bar{q})}{3\alpha R \Omega(\theta, v)}. \quad (15)$$

Proof. See Appendix A.

The firm adjusts the extent and intensity of fraud simultaneously. The optimality condition $\alpha q^* = \beta n^*$, derived from the first-order conditions (see Appendix A), reveals how it balances the two decision margins. If detection is highly sensitive to emissions intensity, that is, if α is large, the firm reduces q^* . If detection is highly sensitive to the number of vehicles, that is, if β is large, it reduces n^* . The parameters R , v , and θ reduce fraud through $R \Omega(\theta, v)$, which captures the combined deterrent effect of penalties, external monitoring, and the public test.

To measure the environmental impact of fraud composition, we consider the following damage function:

$$D(n, q) = \delta nq + \frac{\chi}{2} nq^2 + \frac{\kappa}{2} n^2 q, \quad \delta \geq 0, \chi \geq 0, \text{ and } \kappa \geq 0. \quad (16)$$

Heterogeneity in marginal damage provides an economic motivation for this specification. Muller and Mendelsohn (2009) show that air-pollution damage varies across sources and locations, so identical aggregate emissions need not generate the same environmental costs. In our model, the role of fraud composition is represented by two nonlinear components, one associated with the exceedance per vehicle and the other with the number of vehicles involved. This functional form is a modeling assumption that allows us to compare the consequences of different fraud configurations.

The function $D(n, q)$ illustrates why the choice between extensive fraud, with high n and low q , and intensive fraud, with low n and high q , need not be environmentally neutral. The first term measures damage associated with the aggregate volume of fraud, nq . The other two terms relax the assumption that the extensive and intensive margins are environmentally interchangeable. The parameter χ measures damage convexity with respect to exceedance intensity: when χ is large, fraud concentrated on a small number of vehicles with high exceedances becomes particularly damaging. Symmetrically, κ measures damage convexity with respect to the extent of fraud: when κ is large, fraud spread widely across the fleet becomes particularly costly even if the exceedance per vehicle remains moderate. Thus, if $\kappa = \chi = 0$, damage is linear in nq , and two strategies generating the same aggregate fraud volume are environmentally equivalent. Conversely, whenever $\chi > 0$ or $\kappa > 0$, fraud composition is not neutral: two pairs (n, q) with the same product nq may cause different environmental damage.

At equilibrium, environmental damage can therefore be written as:

$$D(n^*, q^*) = \delta \frac{B(\bar{q})^2}{9\alpha\beta R^2 \Omega(\theta, v)^2} + \frac{B(\bar{q})^3}{54R^3 \Omega(\theta, v)^3} \left(\frac{\chi}{\beta\alpha^2} + \frac{\kappa}{\beta^2\alpha} \right). \quad (17)$$

Equilibrium damage thus depends not only on aggregate fraud volume, but also on how the detection system influences the allocation between n^* and q^* . If detection becomes more sensitive to the intensity of excess emissions, that is, if α increases, the firm reduces q^* . If detection becomes more sensitive to the extent of fraud, that is, if β increases, the firm reduces n^* . When very high emissions intensities are particularly damaging, strengthening detection sensitivity to q becomes more important. When very widespread fraud is particularly damaging, strengthening detection sensitivity to n becomes more decisive.

Proposition 3. The aggregate volume of fraud associated with the firm's optimal choices and the detection probability are respectively:

$$n^* q^* = \frac{B(\bar{q})^2}{9\alpha\beta R^2 \Omega(\theta, v)^2}. \quad (18)$$

$$\Phi^* = \text{Min} \left\{ \frac{2B(\bar{q})}{3R}, 1 \right\} \quad (19)$$

Proof. See Appendix B.

The detection probability does not depend on θ or v . These instruments therefore reduce aggregate fraud volume without affecting the probability that fraud is detected: the firm adjusts its behavior to maintain a constant detection probability.

External monitoring and post-market inspections reduce the aggregate volume of fraud, but the firm adjusts n and q to keep its detection probability at a level compatible with profitability. This result

helps interpret partial-fraud strategies: the firm can limit both the number of vehicles involved and the magnitude of the exceedance to make fraud less visible.

Proposition 4. At the optimum, maximum gross and net fraud profits are respectively:

$$g^* = \frac{B(\bar{q})^3}{27\alpha\beta R^2\Omega(\theta, v)^2}. \quad (20)$$

$$\Delta\Pi^* = \frac{B(\bar{q})^3}{27\alpha\beta R^2\Omega(\theta, v)^2} - \frac{c_\gamma}{2} \frac{\theta^2}{1-\theta}. \quad (21)$$

Proof. See Appendix C.

The expression $\Delta\Pi^*$ connects the firm's conditional behavior, namely its optimal choices of n and q once it has decided to commit fraud, to its entry decision. The firm chooses fraud only if the maximum gain it yields is sufficient to cover the fixed cost of the circumvention device.

Proposition 5. Fraud is deterred if and only if:

$$\frac{B(\bar{q})^3}{27\alpha\beta R^2\Omega(\theta, v)^2} \leq \frac{c_\gamma}{2} \frac{\theta^2}{1-\theta}. \quad (22)$$

Proof. See Appendix D.

The deterrence condition reveals the complementarity between policy instruments. A strict standard increases $B(\bar{q})$ and hence the profitability of fraud. A high penalty reduces that profitability. External monitoring and post-market inspections reduce aggregate fraud volume through $\Omega(\theta, v)$. The public test also operates through the circumvention cost $K(\theta)$.

5 Standard stringency and environmental protection

This section identifies a paradox concerning the effect of tightening the standard on its effectiveness in protecting the environment. To establish it, we examine how a tighter standard affects the firm's fraudulent behavior.

Define $z = 1 - \bar{q}$, which measures standard stringency.

Proposition 6. For given R , θ , and v , a tighter standard increases the aggregate volume of fraud. Formally, $\frac{\partial(n^*q^*)}{\partial z} > 0$.

Proof. See Appendix E.

A tighter standard reduces legally permitted emissions, but it also increases the private gain available from circumvention. Without a simultaneous increase in penalties, monitoring, and enforcement capacity, the aggregate volume of fraud can therefore rise. Tightening environmental standards is consequently insufficient on its own: regulatory ambition must be supported by a sufficiently credible enforcement system.

The net effect of greater stringency on total pollution is obtained from:

$$E^* = N(1 - z) + \frac{\eta^2 z^2}{9\alpha\beta R^2\Omega^2}, \quad \Omega \equiv v + \tau\theta. \quad (23)$$

Hence:

$$\frac{\partial E^*}{\partial z} = -N + \frac{2\eta^2 z}{9\alpha\beta R^2\Omega^2}. \quad (24)$$

The sign of this derivative is ambiguous. When enforcement is sufficiently strong, that is, when $2\eta^2 z / (9\alpha\beta R^2 \Omega^2) < N$, tightening the standard reduces total pollution. When enforcement is weak, that is, when $2\eta^2 z / (9\alpha\beta R^2 \Omega^2) > N$, the induced fraud effect dominates and total pollution increases.

This result reveals a regulatory paradox. When enforcement is sufficiently effective, tighter environmental standards reduce aggregate pollution. Conversely, when monitoring and enforcement are inadequate, stricter standards can have the opposite effect: they strengthen firms' incentives to commit fraud to such an extent that the increase in non-compliance-related emissions more than offsets the expected environmental benefits of tighter regulation.

Thus, stricter standards improve environmental outcomes only when supported by adequate monitoring, effective detection capabilities, and sufficiently deterrent penalties. Otherwise, they can become counterproductive by encouraging regulatory avoidance and strategic non-compliance, thereby undermining the very objectives they are intended to achieve⁴.

6 Optimal regulatory policy

The regulator's problem can be studied under two regimes. In the first, the government allows residual fraud to persist and chooses its instruments to limit the associated social cost. In the second, it seeks to eliminate fraud entirely through a sufficiently deterrent combination of testing and penalties. These regimes do not represent opposing normative assumptions, but two possible configurations of the same policy problem. Complete deterrence may be desirable, yet costly when the standard is highly ambitious or enforcement instruments are difficult to deploy.

6.1 The controlled-fraud regime

The government anticipates the firm's best response and chooses $(\bar{q}, \theta, R)$ to minimize social loss. If it tolerates residual fraud, the reduced-form social loss is:

$$W_F(z, \theta, R) = \delta N(1 - z) + \frac{\lambda}{2} z^2 + \frac{h}{2} \theta^2 + \frac{\rho}{2} R^2 + M \frac{z^2}{R^2 \Omega(\theta, v)^2}, \quad (25)$$

where $z = 1 - \bar{q}$, $M = \frac{\delta \eta^2}{9\alpha\beta}$, and $\Omega(\theta, v) = v + \tau\theta$. Appendix F derives this expression.

Under an interior fraud equilibrium, the optimal public instruments satisfy the following first-order conditions:

$$\begin{aligned} \frac{\partial W_F}{\partial z} &= -\delta N + \lambda z + \frac{2Mz}{R^2 \Omega(\theta, v)^2} = 0, \\ \frac{\partial W_F}{\partial \theta} &= h\theta - \frac{2\tau M z^2}{R^2 \Omega(\theta, v)^3} = 0, \\ \frac{\partial W_F}{\partial R} &= \rho R - \frac{2M z^2}{R^3 \Omega(\theta, v)^2} = 0. \end{aligned} \quad (26)$$

⁴ The European reform illustrates the move toward testing that more closely reflects actual use. From September 1, 2017, new tests became mandatory for new car types: Real Driving Emissions (RDE) are measured using portable systems alongside the harmonized laboratory cycle, the Worldwide Harmonised Light Vehicles Test Procedure (WLTP) (European Commission, 2017). This provides an institutional example of linking certification with real-world monitoring. In our model, the environmental effect of tightening the standard depends on the joint level of detection and penalties, which must sufficiently limit the firm's fraudulent response.

These conditions can be rewritten as:

$$\begin{aligned}\rho R^4 &= \frac{2Mz^2}{\Omega(\theta, v)^2}, \\ h\theta\Omega(\theta, v) &= \tau\rho R^2, \\ \lambda z^2 - \delta Nz + \rho R^2 &= 0.\end{aligned}\tag{27}$$

These first-order conditions characterize the interior optimal values of the standard, test sophistication, and penalty under controlled residual fraud. The system implicitly defines the optimal public strategies. An analytical solution can be obtained in special cases, notably when $\tau = 0$, as detailed in Appendix G.

Second-best policy balances several forces. A tighter standard reduces damage from legally permitted emissions, $\delta N\bar{q}$, but raises the gains from fraud through $B(\bar{q})$. Public tests and penalties reduce fraud but are costly, while external monitoring v reduces the need for costly public instruments without fully replacing public monitoring and enforcement. A higher penalty must be justified by greater fraud-related damage. Public testing is used more intensively when it contributes to ex post detection, that is, when $\tau > 0$. Finally, the standard reflects the trade-off between damage from legally permitted emissions and the cost of increased fraud.

Proposition 7. If $\tau = 0$, then $\Omega(\theta, v) = v$. Under an interior fraud equilibrium, if the government accepts residual fraud, its optimal choice of test sophistication is $\theta_F^* = 0$.

Proof. When $\tau = 0$, reduced-form social loss is:

$$W_F(z, \theta, R) = \delta N(1 - z) + \frac{\lambda}{2}z^2 + \frac{h}{2}\theta^2 + \frac{\rho}{2}R^2 + M\frac{z^2}{R^2v^2}.\tag{28}$$

The variable θ appears only in the public cost term $\frac{h}{2}\theta^2$. Minimization therefore gives $\theta_F^* = 0$.⁵

6.2 The complete-deterrence regime

Complete deterrence requires $\Delta\Pi^* \leq 0$. Using the expression for maximum net fraud profit, this constraint becomes:

$$\frac{B(\bar{q})^3}{27\alpha\beta R^2\Omega(\theta, v)^2} \leq \frac{c_\gamma}{2} \frac{\theta^2}{1 - \theta}.\tag{29}$$

We can therefore derive the penalty that completely deters fraud.

Proposition 8. For a given pair $(\bar{q}, \theta)$, the minimum deterrent penalty is:

$$R_D(\bar{q}, \theta) = \left[\frac{2B(\bar{q})^3(1 - \theta)}{27\alpha\beta c_\gamma \theta^2 \Omega(\theta, v)^2} \right]^{1/2}.\tag{30}$$

Proof. See Appendix H.

⁵ When the government accepts residual fraud, the certification test reduces that fraud if it also improves post-market detection. This channel is represented by $\tau > 0$: a more sophisticated test then strengthens monitoring capabilities after vehicles enter the market. *Dieselgate* illustrates the importance of connecting initial certification with real-world monitoring.

To eliminate all fraud, the government must choose $(\bar{q}, \theta)$ and then apply the minimum penalty $R_D(\bar{q}, \theta)$ that makes fraud unprofitable. It selects this combination to minimize social loss. Its complete-deterrence program is therefore:

$$\min_{\bar{q}, \theta} \left\{ \delta N \bar{q} + \frac{\lambda}{2} (1 - \bar{q})^2 + \frac{h}{2} \theta^2 + \frac{\rho}{2} R_D(\bar{q}, \theta)^2 \right\}. \quad (31)$$

Complete deterrence is not costless. It can be achieved through high penalties, sophisticated tests, strong external monitoring, or less demanding standards. The regulator must therefore compare the cost of complete deterrence with the damage arising from controlled residual fraud. This comparison determines the choice of the optimal regime.

Let W_D^* denote minimum social loss under complete deterrence and W_F^* minimum social loss under an interior fraud equilibrium. The government chooses complete deterrence if $W_D^* \leq W_F^*$, and a second-best policy with controlled residual fraud if $W_F^* < W_D^*$.

By construction, W_D^* and W_F^* are the minimum values of social loss in the only two regimes admissible for the government. As it minimizes social loss, the government necessarily chooses the regime with the lower value.

Fraud at equilibrium may therefore result from a second-best trade-off: complete deterrence is possible but too costly. If the government wishes to maintain strict standards, it must make enforcement credible through monitoring, penalties, and external vigilance.

7 Fraud composition, regulatory policy, and environmental implications

This section examines how key parameters affect equilibrium fraud, optimal regulatory choices, and total environmental damage. The aggregate volume of equilibrium fraud is:

$$n^* q^* = \frac{\eta^2 z^2}{9\alpha\beta R^2 (v + \tau\theta)^2}, \quad z \equiv 1 - \bar{q}. \quad (32)$$

This expression first shows how the penalty R and the detection environment $\Omega, \Omega = v + \tau\theta$, affect fraud: its aggregate volume is inversely proportional to the square of each term. A higher penalty or stronger external monitoring reduces both the extensive and intensive margins of fraud. Conversely, an increase in fraud profitability, captured by η , or in standard stringency, captured by z , increases fraud through a symmetric effect.

The parameters α and β also reduce fraud, but through different channels. An increase in α improves the detection of high emissions exceedances and induces the firm to reduce intensity q^* . An increase in β improves the detection of the extent of fraud and induces the firm to reduce the number of vehicles involved, n^* . Both instruments therefore reduce aggregate fraud volume, but shift its relative composition in opposite directions between the firm's decision margins.

Another result deserves emphasis: under the interiority conditions, the detection probability Φ^* is invariant to the parameters α, β, v, θ , and τ . These instruments reduce fraud volume but not the equilibrium probability that fraud is detected. The firm adjusts its behavior to maintain a detection probability compatible with profitability. The observed detection rate may therefore be a poor indicator of enforcement effectiveness: a more effective policy can result in less fraud without necessarily increasing the observed probability of detection⁶. Finally, the cross-partial derivative $\frac{\partial^2 (n^* q^*)}{\partial z \partial R} =$

⁶ The elasticities confirm this ranking of effects. Aggregate fraud volume has an elasticity of -2 with respect to R and Ω , and an elasticity of $+2$ with respect to η and z . Its elasticity with respect to α and β , however, is -1 . Instruments that affect both margins of fraud therefore have a greater effect than those targeting only one dimension of detection.

$-\frac{4\eta^2 z}{9\alpha\beta R^3 \Omega^2}$ is negative, showing that the marginal effect of tightening the standard on fraud decreases as penalties become more severe. Penalties and standard stringency are thus complementary instruments: a tighter standard entails less fraud risk when accompanied by a sufficiently credible penalty.

7.1 Fraud composition and the design of public enforcement

Environmental fraud can be described along two distinct dimensions: its extent, n^* , the number of vehicles involved, and its intensity, q^* , the magnitude of the exceedance per vehicle. This distinction allows us to examine a point left implicit in the baseline model: two fraud configurations generating the same aggregate volume need not be equivalent in terms of the environmental damage they cause.

7.1.1 The special case of linear damage

As a special case of the general damage function, consider a setting in which fraud-related damage is proportional to the product nq :

$$D(n, q) = \delta nq \quad (\delta > 0) \quad (33)$$

For a given damage level $\bar{D}$, the set of pairs (n, q) compatible with that level forms an iso-damage curve: $nq = \bar{D}/\delta$ or $q = \bar{D}/(\delta n)$.

The curves $q = \bar{D}/(\delta n)$ are decreasing and convex: keeping damage constant requires any increase in extent to be offset by a reduction in intensity. Under this linear specification, fraud composition is neutral. In other words, very extensive but low-intensity fraud and limited but high-intensity fraud are equivalent whenever they generate the same product nq . Figure 1 plots these iso-damage curves for different damage levels.

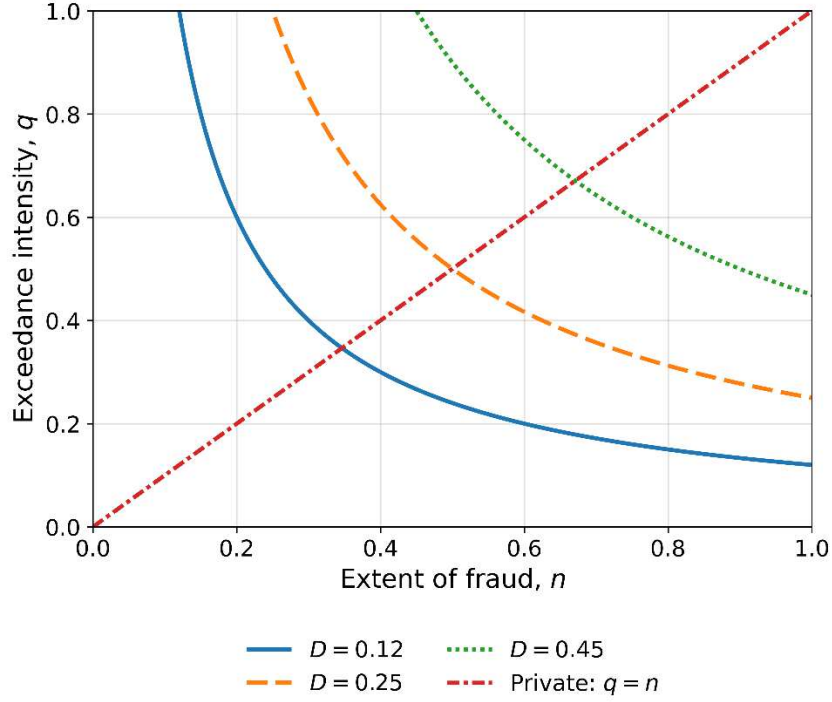


Fig. 1 Iso-damage curves when damage is linear in aggregate fraud volume ($D = \delta nq$); the dashed line represents the private trade-off $q = n$

Figure 1 highlights a difficulty in evaluating enforcement: a fall in the number of fraudulent vehicles may be offset by greater exceedances among the remaining vehicles. The product nq is therefore needed to assess changes in excess emissions and damage, whereas monitoring only the number of vehicles involved says nothing about changes in intensity. The line $q = n$, plotted for $\alpha = \beta$, locates the composition chosen by the firm given the detection technology. In this linear setting, the environmental objective is to reduce total excess emissions, regardless of their initial distribution between the two decision margins.

7.1.2 Convex damage

For vehicle-related air pollution, damage also depends on exposure conditions. Near a busy road, for example, traffic composition, driving conditions, and atmospheric dispersion affect the concentrations to which local residents are exposed (EPA, 2014). We use two assumptions to represent the consequences of these exposures in the model. Convexity in intensity may describe a situation in which very high-emitting vehicles contribute to episodes of elevated concentration with increasing marginal damage. Convexity in extent may capture a different mechanism: a growing number of non-compliant vehicles using the same roads increases the frequency and duration of exposure. When the marginal damage associated with this accumulation rises, each additional vehicle becomes more costly to society. These two configurations distinguish the role of the highest exceedances from that of their spread across the fleet.

The damage function captures these mechanisms through two distinct components: one places increasing weight on the exceedance per vehicle, and the other on the number of vehicles involved. Their relative importance allows us to study how fraud composition changes its environmental cost.

We use the following specification:

$$D(n, q) = \delta nq + \left(\frac{\chi}{2}\right) nq^2 + \left(\frac{\kappa}{2}\right) n^2q \quad (\chi \geq 0, \kappa \geq 0) \quad (34)$$

The parameter χ measures damage convexity with respect to exceedance intensity: when it is large, highly non-compliant vehicles become particularly damaging. The parameter κ measures damage convexity with respect to the extent of fraud: when it is large, fraud spread widely across the fleet becomes particularly costly to society.

Figure 2 isolates convexity in intensity, with $\chi = 3$, $\kappa = 0$, and $\delta = 1$.

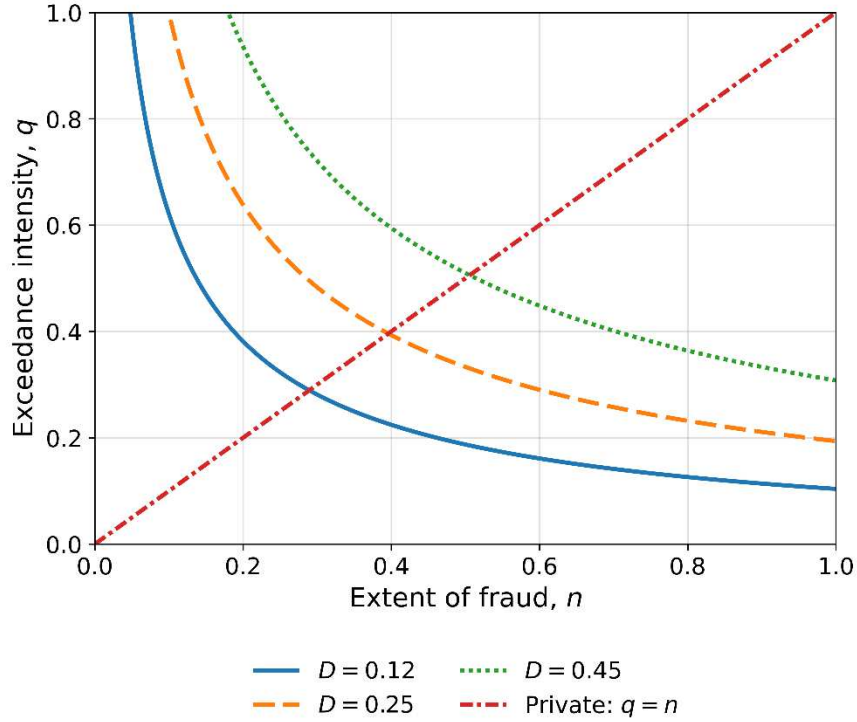


Fig. 2 Iso-damage curves with convexity in exceedance intensity ($\chi = 3$, $\kappa = 0$, $\delta = 1$); the dashed line represents the private trade-off $q = n$

Figure 2 illustrates the environmental cost of concentrating fraud on high-emitting vehicles. For a fixed aggregate volume $V = nq$, damage is $D = \delta V + (\chi/2)Vq$: with identical total excess emissions, it rises with exceedance intensity. A reduction in n accompanied by an increase in q can therefore raise the damage associated with residual fraud. Monitoring volume nq should then be complemented by monitoring the distribution of exceedances across vehicles. For public enforcement, this setting gives particular importance to identifying the most severely non-compliant vehicles, whose contribution to damage exceeds what their share of total emissions alone would suggest.

Figure 3 instead examines convexity in the extent of fraud, with $\kappa = 3$, $\chi = 0$, and $\delta = 1$. Here, the nonlinear damage component is associated with the number of vehicles involved: the spread of the same non-compliance across the fleet becomes progressively more costly. Comparison with Fig. 2 therefore distinguishes two sources of greater damage: concentrating exceedances on a few vehicles and spreading them across many vehicles.

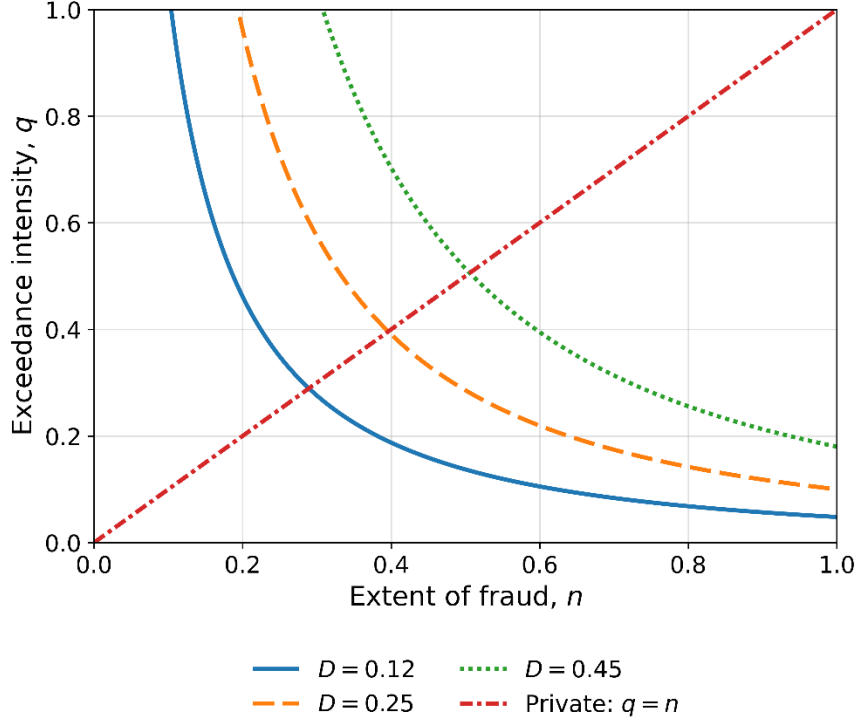


Fig. 3 Iso-damage curves with convexity in the extent of fraud ($\kappa = 3$, $\chi = 0$, $\delta = 1$) and private trade-off curve $q = n$

For a given fraud volume, damage is $D = \delta V + (\kappa/2)Vn$: it increases with the number of vehicles involved. An increase in n offset by a reduction in q leaves total excess emissions unchanged but raises the extent-related damage component. A reduction in the average exceedance per vehicle can thus coexist with higher environmental costs when non-compliance spreads across the fleet.

This setting can be illustrated by a non-compliant calibration shared by several vehicle models or production series. Each vehicle exhibits a moderate exceedance, but repetition of the same calibration extends non-compliance to a substantial share of output. Enforcement must then identify the models, production years, and software versions involved and estimate the corresponding volumes. A sample spread across these categories provides information on the diffusion of fraud, whereas tests focused only on high-emitting vehicles primarily describe its extreme values. The environmental value of these two enforcement approaches therefore depends on which damage component is to be reduced: intensity in Fig. 2 and extent in Fig. 3.

7.1.3 The firm's private trade-off

The model characterizes the firm's behavior through its private trade-off between the two decision margins. At an interior optimum, the firm chooses n and q such that $\alpha q = \beta n$, that is, $q^*/n^* = \beta/\alpha$.

This condition means that the firm allocates its fraudulent activity between extent and intensity according to their respective contributions to the probability of detection. If detection is highly sensitive to intensity (high α), the firm reduces q ; if it is highly sensitive to extent (high β), the firm reduces n . Proposition 2 shows that $n^*(\bar{q}, \theta, R, v)$ and $q^*(\bar{q}, \theta, R, v)$ are both proportional to the same term $B(\bar{q})$, so their ratio $\frac{q^*}{n^*} = \beta/\alpha$ is constant. Thus, whatever the standard $\bar{q}$, penalty R , test sophistication θ , or external monitoring v , the firm's equilibrium moves along a single straight line in the (n, q) plane as long as the solution remains interior.

Figure 4 plots the iso-damage curves $D(n, q) = I$, the private-equilibrium line $q^* = (\beta/\alpha)n^*$, and a boundary separating different damage compositions. When $\chi > 0$ and $\kappa > 0$, this boundary

corresponds to equality between the two nonlinear components: $(\chi/2)nq^2 = (\kappa/2)n^2q$, or $q = (\kappa/\chi)n$ for positive fraud. Above the boundary, the component associated with high exceedances per vehicle is larger; below it, the component associated with the spread of fraud across the fleet is larger. The linear term δnq is added in both regions. The boundary therefore indicates which form of nonlinearity contributes more to damage for a given combination of extent and intensity.

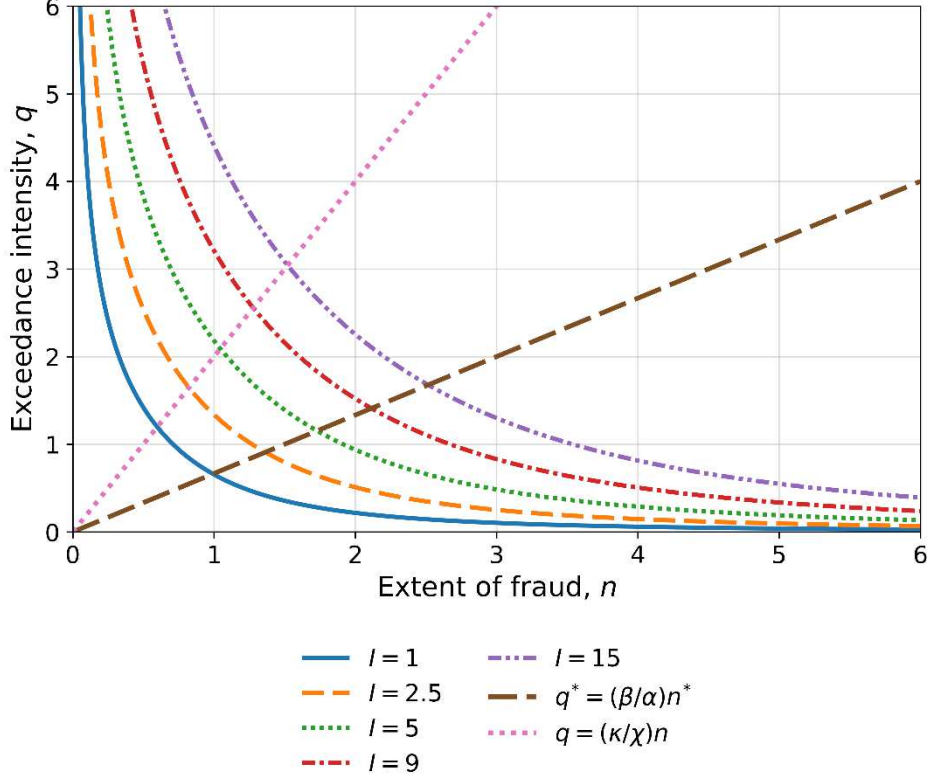


Fig. 4 Iso-damage curves $D(n, q) = I$, the dashed private-equilibrium line $q^* = (\beta/\alpha)n^*$, and the dotted boundary $q = (\kappa/\chi)n$ separating the two nonlinear damage components

Comparing the two lines provides information about the composition induced by enforcement. If $\beta/\alpha > \kappa/\chi$, positive private equilibria lie in the region where the nonlinear intensity-related component is larger. If $\beta/\alpha < \kappa/\chi$, as in the figure, the extent-related component is larger. The slope of the private-equilibrium line depends on the detection technology, whereas the boundary's slope depends on the damage function. There is no reason for the two slopes to coincide.

With detection parameters unchanged, a tighter standard moves the private equilibrium along its line toward higher iso-damage curves. This movement represents the increase in excess emissions established in Proposition 6. The effect on total pollution also depends on the reduction in legally permitted emissions analyzed in Sect. 5. Conversely, an increase in the penalty R or external monitoring v moves the equilibrium toward the origin. Test sophistication θ has the same effect when it improves post-market detection, that is, when $\tau > 0$. It also affects entry into fraud through the circumvention cost. These common instruments leave the ratio q^*/n^* unchanged. Redirecting the composition of residual fraud therefore requires changing the sensitivities α and β differently.

8 Discussion

The model and its results yield several insights, developed below. They first invite a reconsideration of how fraud is understood and illuminate the strategies through which firms organize and adjust the circumvention of public regulations. This perspective then leads us to examine the implications of these behaviors for regulatory design and the effectiveness of environmental protection policies.

8.1 Multidimensional fraud across sectors

The mechanism revealed by *Dieselgate* is not confined to vehicle emissions. In the agri-food sector, China's melamine-contaminated milk scandal provides a particularly telling example (World Health Organization, 2008a, 2008b). Adding melamine to milk artificially increased the apparent protein content measured by certain nitrogen-based tests⁷. As in *Dieselgate*, fraud concerned not only the product's actual quality, but also exploited the way that quality was measured. It can therefore be understood as multidimensional fraud combining a technical adulteration process, an extensive margin associated with the batches and manufacturers involved, and an intensive margin corresponding to the degree of contamination or manipulation of the indicator.

The European horsemeat scandal illustrates fraud concerning product identity: preparations presented as containing beef included undeclared horsemeat. The European control plan of 2013 had two components. The first investigated this substitution in products labeled as containing beef; the second tested for phenylbutazone residues in horsemeat intended for human consumption. The controls therefore addressed both product authenticity and consumer information, on the one hand, and a specific health risk, on the other (Premanandh, 2013; European Commission, 2013). The number of batches involved, the proportion of substituted raw material, and concealment through labeling or traceability provide three dimensions through which to interpret this fraud.⁸

The natural mineral water case examined by the French Senate in 2025 highlights a gap between the processes actually used and the requirements attached to a commercially valued regulatory category. The report documents prohibited treatments and failures of public oversight (Sénat, 2025a, 2025b). A multidimensional interpretation distinguishes the volumes and sites involved, the extent of treatment applied to the water, and the way compliance with the marketed category was presented.⁹

In the pharmaceutical sector, the Ranbaxy case likewise provides an emblematic example of multidimensional fraud¹⁰. Its relevance lies in the fact that pharmaceutical compliance concerns more than the final product. It also relies on manufacturing processes, quality controls, stability tests, technical dossiers, and information supplied to the regulator. Regulatory actions against Ranbaxy included deviations from good manufacturing practice requirements at several production sites, import restrictions, and measures concerning facility compliance (U.S. Food and Drug Administration, 2023). Fraud can therefore be considered multidimensional to some extent: it has an extensive margin associated with the sites, batches, or products involved; an intensive margin associated with the degree of deviation from quality or manufacturing requirements; and an informational dimension associated with how compliance data are produced, retained, or transmitted.

⁷ In September 2008, the World Health Organization reported that more than 54,000 infants and young children had received medical treatment after consuming contaminated dairy products, with three confirmed deaths, after melamine had been found in certain products from at least 22 manufacturers.

⁸ The extensive margin corresponds to the number of batches or products involved, the intensive margin to the proportion of substituted raw material, and the concealment dimension to labeling and traceability practices that hide the substitution. The detection technology identifies the species present in the product. The risk associated with a possible veterinary-drug residue is a distinct health issue, separate from deception about product identity (European Commission, 2013).

⁹ Volume I of the Senate report documents, in particular, the use of activated carbon and ultraviolet lamps at Nestlé Waters sites in the Gard and Vosges departments, as well as questions concerning microfiltration (Sénat, 2025a). Intensity denotes the degree of treatment applied to the water and the deviation between the processes used and those permitted for the marketed category. A treatment can aim to improve microbiological quality while violating the requirements of the "natural mineral water" category.

¹⁰ In 2013, Ranbaxy pleaded guilty in the United States and agreed to pay \$500 million to resolve allegations involving adulterated drugs, violations of good manufacturing practice, and false statements to the Food and Drug Administration (U.S. Department of Justice, 2013).

8.2 Public policy implications

These examples clarify the scope of the model: non-compliance can vary in extent, intensity, and the processes used to conceal it. For the regulator, a first implication is that aggregate fraud volume alone is not always sufficient to assess damage. Under the nonlinearities assumed here, Figs. 2–4 show why an enforcement system should also account for the composition of residual fraud. Detection focused on extreme exceedances and detection focused on the spread of non-compliance do not necessarily address the same environmental concern.

In the model, the standard $\bar{q}$, test θ , penalty R , and external monitoring v move the interior equilibrium along a line with slope β/α . Their effects concern fraud volume, whereas changing its composition requires altering detection sensitivity to n and q differently. A further step would therefore make α and β public choice variables with their own costs. The regulator could then explicitly compare expenditure on identifying large exceedances with expenditure on measuring the spread of non-compliance.

As an illustration, suppose an abnormal signal is observed for a family of diesel engines. One program would expand on-road testing of the models already identified, varying routes and operating conditions to measure exceedances per vehicle more accurately. It would target sensitivity to intensity, represented by α . A second program would extend the sample to other models, production years, and software versions to establish the volumes involved; it would target sensitivity to extent, represented by β . In *Dieselgate*, the first independent tests revealed emissions discrepancies, and the subsequent investigation established the presence of the fraudulent device in the vehicles concerned (Thompson et al., 2014; EPA, 2015). This example shows how two enforcement programs could produce complementary information about the same fraud.

Targeting the two decision margins complements another role of testing: its effect on the cost of circumvention and, through that channel, on entry into fraud. This second mechanism calls for a joint examination of public enforcement design and the role of external actors.

8.3 Public testing and external actors as strategic deterrence instruments

The public test plays a strategic role by determining the technical obstacles the firm must overcome to display apparent compliance. In *Dieselgate*, the software used information including steering-wheel position, vehicle speed, and engine operating time to recognize the test cycle. It then activated a calibration that enabled the vehicle to pass the test, while a different calibration reduced the effectiveness of emissions controls during normal driving (EPA, 2015). Predictability thus allowed measured behavior to diverge from the vehicle's real-world behavior. By making the test more robust or more representative of actual use, the regulator can increase the cost of the device needed to circumvent it. Testing therefore influences the decision to commit fraud before an offense is discovered.

This effect on circumvention costs broadens the instruments of deterrence. Real-world tests, post-approval audits, and less predictable inspections may require the firm to design a device capable of withstanding a wider range of observation conditions. In the sequential game, greater test sophistication directly increases development costs. The penalty and discovery probability, by contrast, reduce the expected gain from fraud. Deterrence policy thus combines two mechanisms: reducing the net benefit of circumvention and increasing the investment needed to make it possible.

The choice of test also accounts for its public cost. A more sophisticated protocol can make circumvention less attractive while requiring more resources for design, expertise, and implementation. This trade-off explains the value of a sufficiently deterrent arrangement rather than maximal sophistication. Research on adaptation to enforcement provides a complementary perspective.

Studying illegal fish sales in Chile, González Lira and Mobarak (2021) show that agents can learn to exploit weaknesses in predictable enforcement. Enforcement design and offenders' capacity to adapt therefore contribute to effectiveness beyond inspection frequency alone.

The cost of monitoring also highlights the complementary role of external actors. In *Dieselgate*, fraud was not uncovered through the routine operation of approval procedures. Independent measurements commissioned by the International Council on Clean Transportation and conducted by West Virginia University produced the initial signal, which was then taken up by the EPA and the California Air Resources Board. This sequence shows that uncovering sophisticated fraud can depend on an institutional chain combining external expertise, evidence production, and public response. In the model, external monitoring complements public enforcement. It increases the probability that actual non-compliance is observed outside the standard test and also reduces the government's cost of deterrence. When non-governmental organizations, researchers, independent laboratories, the media, or whistleblowers produce credible signals, the regulator need not bear the entire monitoring burden alone. Strengthening these actors can therefore reduce the level of public-test sophistication or the penalties needed to make fraud unprofitable.

This insight matters for public policy. Strengthening enforcement does not simply mean increasing official testing or raising fines. The government can also make regulation more credible by facilitating independent testing, access to data, audits of embedded software, whistleblower protection, and authorities' capacity to respond promptly to signals originating outside routine procedures. In sectors where firms possess a substantial informational advantage, external monitoring can introduce the unpredictability that standard public enforcement sometimes lacks.

9 Conclusion

This article analyzes environmental fraud as a multidimensional decision. Motivated by *Dieselgate*, it distinguishes the circumvention device, the extent of fraud, and the intensity of exceedances. The firm adjusts these decision margins according to their profitability and contribution to detection. The model shows that a tighter standard can increase excess emissions when enforcement instruments are not strengthened simultaneously. It also characterizes the regulator's trade-off between the cost of complete deterrence and that of controlled residual fraud.

The contribution connects the usual determinants of offending, private gains, detection risk, and penalties, to the technical choices through which the firm organizes non-compliance. The effectiveness of the standard then depends on the responses it induces in these choices. Its ambition must be combined with testing, penalties, and monitoring that make actual compliance preferable to circumvention.

The damage analysis introduces a further issue of composition. When damage is nonlinear, two fraud configurations with the same aggregate volume can have different environmental costs depending on their extent and intensity. The common instruments examined here reduce fraud volume while leaving the ratio between the two decision margins unchanged at an interior equilibrium. Differential targeting remains a direction for further research: endogenizing detection sensitivities would allow comparison of policies focused on extreme exceedances with policies focused on the spread of fraud.

These results also illuminate the credibility of environmental commitments. When actual performance is difficult to observe, stated commitments benefit from being accompanied by audits, access to information, and internal compliance governance. These arrangements take the analysis of public enforcement further by introducing accountability and incentives within the firm.

The framework isolates the relationship between a regulator and a representative firm, taking external monitoring as given. This structure opens several avenues for further research: the organization of

external enforcement, reputational effects, consumer responses, and competition among producers. Two developments appear particularly useful for connecting compliance decisions to product characteristics and markets.

The first would examine the relationship between the environmental standard and the product's technical performance. Compliance costs may represent industrial abatement efforts as well as the constraints those efforts impose on valued characteristics such as power, fuel consumption, or driving comfort. Circumvention could allow the firm to preserve these characteristics in actual use while still passing the test. Explicitly modeling this trade-off would make it possible to study situations in which fraud serves not only to save production costs but also to avoid modifying the product.

This perspective naturally leads to competition. If some firms genuinely comply with the standard while others circumvent it, the latter may avoid some of the costs or technical constraints of compliance. Fraud could then become an instrument of unfair competition when commercial product characteristics are observable but environmental performance is observed only imperfectly. Examining this advantage, together with consumer responses after its disclosure, would connect the credibility of standards to fair competition and performance certification.

More broadly, the article invites us to view certified-compliance regimes as systems in which firms may seek to manipulate not only actual performance but also the conditions under which it becomes observable. The challenge is therefore not merely to announce ambitious standards or responsible commitments. It is to make actual compliance verifiable and circumvention sufficiently costly for actual compliance to be preferred to its appearance.

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Data availability: This article develops a theoretical model and does not analyze an empirical dataset. The analytical expressions underlying the results and figures are provided in the article and its appendices.

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Appendices

Appendix A: Proof of Proposition 2

The fixed cost $K(\theta)$ does not affect the optimal choices of n and q . The firm maximizes:

$$g(n, q) = B(\bar{q})nq - R\Omega(\theta, v)nq(\alpha q + \beta n). \quad (\text{A.1})$$

The first-order conditions are:

$$\frac{\partial g}{\partial n} = B(\bar{q})q - R\Omega(\theta, v)(\alpha q^2 + 2\beta nq) = 0, \quad (\text{A.2})$$

and

$$\frac{\partial g}{\partial q} = B(\bar{q})n - R\Omega(\theta, v)(2\alpha nq + \beta n^2) = 0. \quad (\text{A.3})$$

For $n > 0$ and $q > 0$, dividing the first equation by q and the second by n gives:

$$B(\bar{q}) = R\Omega(\theta, v)(\alpha q + 2\beta n), \quad (\text{A.4})$$

and

$$B(\bar{q}) = R\Omega(\theta, v)(2\alpha q + \beta n). \quad (\text{A.5})$$

Equating the right-hand sides gives $\alpha q + 2\beta n = 2\alpha q + \beta n$, hence $\beta n = \alpha q$. Substituting $\alpha q = \beta n$ into the first equation yields:

$$B(\bar{q}) = 3R\Omega(\theta, v)\beta n, \quad (\text{A.6})$$

and then:

$$n^* = \frac{B(\bar{q})}{3\beta R\Omega(\theta, v)}. \quad (\text{A.7})$$

Using $\alpha q^* = \beta n^*$, we obtain:

$$q^* = \frac{B(\bar{q})}{3\alpha R\Omega(\theta, v)}. \quad (\text{A.8})$$

To verify that this solution is a global maximum, first maximize $g(n, q)$ with respect to q for a given $n > 0$. This yields a strictly concave quadratic function in q , followed by a reduced function in n whose unique positive maximum is precisely $n^* = B(\bar{q})/[3\beta R\Omega(\theta, v)]$. Under the interiority conditions $0 < n^* < N$, $q^* > 0$, and $\Phi^* \leq 1$, this solution is therefore the global maximum of the firm's problem.

Appendix B: Proof of Proposition 3

Substituting the expressions for n^* and q^* gives:

$$n^* q^* = \frac{B(\bar{q})}{3\beta R\Omega(\theta, v)} \cdot \frac{B(\bar{q})}{3\alpha R\Omega(\theta, v)} = \frac{B(\bar{q})^2}{9\alpha\beta R^2\Omega(\theta, v)^2}. \quad (\text{B.1})$$

Moreover:

$$\Phi^* = \Omega(\theta, v)(\alpha q^* + \beta n^*). \quad (\text{B.2})$$

Since:

$$\alpha q^* = \frac{B(\bar{q})}{3R\Omega(\theta, v)}, \quad \beta n^* = \frac{B(\bar{q})}{3R\Omega(\theta, v)}. \quad (\text{B.3})$$

it follows that:

$$\Phi^* = \Omega(\theta, v) \left(\frac{2B(\bar{q})}{3R\Omega(\theta, v)} \right) = \frac{2B(\bar{q})}{3R}. \quad (\text{B.4})$$

The condition $2B(\bar{q})/(3R) \leq 1$ ensures that the resulting probability remains within its admissible range.

Appendix C: Proof of Proposition 4

At the interior optimum:

$$g^* = B(\bar{q})n^*q^* - R\Omega(\theta, v)n^*q^*(\alpha q^* + \beta n^*). \quad (\text{C.1})$$

We know that:

$$n^*q^* = \frac{B(\bar{q})^2}{9\alpha\beta R^2\Omega(\theta, v)^2}, \quad (\text{C.2})$$

and that:

$$\alpha q^* + \beta n^* = \frac{2B(\bar{q})}{3R\Omega(\theta, v)}. \quad (\text{C.3})$$

Thus:

$$B(\bar{q})n^*q^* = \frac{B(\bar{q})^3}{9\alpha\beta R^2\Omega(\theta, v)^2}, \quad (\text{C.4})$$

and

$$R\Omega(\theta, v)n^*q^*(\alpha q^* + \beta n^*) = \frac{2B(\bar{q})^3}{27\alpha\beta R^2\Omega(\theta, v)^2}. \quad (\text{C.5})$$

Therefore:

$$g^* = \frac{B(\bar{q})^3}{27\alpha\beta R^2\Omega(\theta, v)^2}. \quad (\text{C.6})$$

Maximum net profit is obtained by subtracting the fixed circumvention cost $K(\theta)$:

$$\Delta\Pi^* = g^* - K(\theta) = \frac{B(\bar{q})^3}{27\alpha\beta R^2\Omega(\theta, v)^2} - \frac{c_Y}{2} \frac{\theta^2}{1 - \theta}. \quad (\text{C.7})$$

Appendix D: Proof of Proposition 5

Maximum net profit is:

$$\Delta\Pi^* = \frac{B(\bar{q})^3}{27\alpha\beta R^2\Omega(\theta, v)^2} - \frac{c_Y}{2} \frac{\theta^2}{1 - \theta}. \quad (\text{D.1})$$

Fraud is deterred if and only if $\Delta\Pi^* \leq 0$. This condition immediately gives:

$$\frac{B(\bar{q})^3}{27\alpha\beta R^2\Omega(\theta, v)^2} \leq \frac{c_Y}{2} \frac{\theta^2}{1 - \theta}. \quad (\text{D.2})$$

Appendix E: Proof of Proposition 6

We have:

$$n^* q^* = \frac{\eta^2 (1 - \bar{q})^2}{9\alpha\beta R^2 \Omega(\theta, v)^2}. \quad (\text{E.1})$$

Differentiating with respect to $\bar{q}$ gives:

$$\frac{\partial(n^* q^*)}{\partial \bar{q}} = -\frac{2\eta^2 (1 - \bar{q})}{9\alpha\beta R^2 \Omega(\theta, v)^2} < 0 \quad (\text{E.2})$$

for $\bar{q} < 1$. Setting $z = 1 - \bar{q}$, we obtain:

$$\frac{\partial(n^* q^*)}{\partial z} = \frac{2\eta^2 z}{9\alpha\beta R^2 \Omega(\theta, v)^2} > 0. \quad (\text{E.3})$$

A tighter standard therefore increases the aggregate volume of fraud for given enforcement instruments.

Appendix F: Derivation of reduced-form social loss under controlled residual fraud

Under controlled residual fraud, we substitute equilibrium fraud volume into the social loss function. Since:

$$n^* q^* = \frac{B(\bar{q})^2}{9\alpha\beta R^2 \Omega(\theta, v)^2}, \quad (\text{F.1})$$

and since $B(\bar{q}) = \eta z$, with $z = 1 - \bar{q}$, we obtain:

$$\delta n^* q^* = \frac{\delta \eta^2 z^2}{9\alpha\beta R^2 \Omega(\theta, v)^2}. \quad (\text{F.2})$$

Defining:

$$M = \frac{\delta \eta^2}{9\alpha\beta}, \quad (\text{F.3})$$

reduced-form social loss becomes:

$$W_F(z, \theta, R) = \delta N(1 - z) + \frac{\lambda}{2} z^2 + \frac{h}{2} \theta^2 + \frac{\rho}{2} R^2 + M \frac{z^2}{R^2 \Omega(\theta, v)^2}. \quad (\text{F.4})$$

Appendix G: Closed-form solutions for $\tau = 0$

When $\tau = 0$, we have $\Omega = v$ and, by Proposition 7, $\theta_F^* = 0$. The first-order conditions become:

$$\rho R^4 = \frac{2M z^2}{v^2}, \quad (\text{G.1})$$

$$\lambda z^2 - \delta N z + \rho R^2 = 0. \quad (\text{G.2})$$

The first condition gives:

$$R^2 = \left(\frac{2M}{\rho v^2} \right)^{1/2} z. \quad (\text{G.3})$$

Thus:

$$\rho R^2 = \frac{\sqrt{2M\rho}}{v} z. \quad (\text{G.4})$$

Substituting into the first-order condition for z gives:

$$\lambda z^2 - \delta N z + \frac{\sqrt{2M\rho}}{v} z = 0. \quad (\text{G.5})$$

Discarding the trivial root $z = 0$, the interior solution is:

$$z_F^* = \frac{\delta N - \sqrt{2M\rho}/v}{\lambda}. \quad (\text{G.6})$$

It exists if $0 < z_F^* < 1$. The optimal penalty is:

$$R_F^* = \left(\frac{2M}{\rho v^2}\right)^{1/4} \sqrt{z_F^*}, \quad (\text{G.7})$$

and optimal test sophistication is:

$$\theta_F^* = 0. \quad (\text{G.8})$$

Appendix H: Proof of Proposition 8

Under complete deterrence, aggregate fraud volume is zero, so the social loss function no longer contains the fraud-damage term. To minimize the cost of penalties, the government chooses, for each pair $(\bar{q}, \theta)$, the smallest penalty satisfying the no-fraud constraint. Setting the constraint at equality:

$$\frac{B(\bar{q})^3}{27\alpha\beta R^2\Omega(\theta, v)^2} = \frac{c_\gamma}{2} \frac{\theta^2}{1-\theta}, \quad (\text{H.1})$$

gives:

$$R_D(\bar{q}, \theta) = \left[\frac{2B(\bar{q})^3(1-\theta)}{27\alpha\beta c_\gamma \theta^2 \Omega(\theta, v)^2} \right]^{1/2}. \quad (\text{H.2})$$

Appendix I: Analytical construction of the iso-damage curves

The curves in Figs. 2–4 are obtained by solving $D(n, q) = I$ for q , for each fixed damage level $I > 0$ and given $n > 0$. Substituting the expression for D gives, for fixed n , a quadratic equation in q :

$$\left(\frac{\chi}{2}\right) n q^2 + \left(\delta n + \left(\frac{\kappa}{2}\right) n^2\right) q - I = 0 \quad (\text{I.1})$$

that is, $aq^2 + bq + c = 0$, with $a = \left(\frac{\chi}{2}\right)n$, $b = \delta n + \left(\frac{\kappa}{2}\right)n^2$, and $c = -I$. The discriminant is:

$$\Delta = b^2 + 2\chi n I \quad (\text{I.2})$$

which is always positive. The equation therefore always has two real roots, $q_1 = (-b + \sqrt{\Delta})/(2a)$ and $q_2 = (-b - \sqrt{\Delta})/(2a)$. Since $a > 0$ and $b > 0$, q_2 is always strictly negative and must be discarded because q denotes a nonnegative emissions exceedance. By contrast, q_1 is always nonnegative, since $\Delta = b^2 + 2\chi n I \geq b^2$ and therefore $\sqrt{\Delta} \geq b$. We retain the unique admissible root, denoted by $q(n)$:

$$q(n) = \left(-b + \sqrt{b^2 + 2\chi n I} \right) / (\chi n) \quad (\text{I.3})$$

where $b = \delta n + \left(\frac{\kappa}{2} \right) n^2$

When $\chi = 0$ (Fig. 3), the equation is linear in q and can be solved directly:

$$q(n) = I / \left(\delta n + \left(\frac{\kappa}{2} \right) n^2 \right) \quad (\text{I.4})$$

Applying this closed-form expression point by point on a grid of n values produces each of the iso-damage curves in Figs. 2–4.

References

- Arguedas, C. (2013). Pollution standards, technology investment and fines for non-compliance. *Journal of Regulatory Economics*, 44, 156–176. <https://doi.org/10.1007/s11149-013-9217-8>
- Auriol, E., Hjelmeng, E., & Søreide, T. (2023). Corporate criminals in a market context: enforcement and optimal sanctions. *European Journal of Law and Economics*, 56(2), 225–287. <https://doi.org/10.1007/s10657-023-09773-w>
- Banerjee, D. S., & Poddar, S. (2026). Anti-piracy enforcements and innovation quality. *European Journal of Law and Economics*, 61(3), 297–322. <https://doi.org/10.1007/s10657-026-09868-0>
- Becker, G. S. (1968). Crime and punishment: An economic approach. *Journal of Political Economy*, 76(2), 169–217. <https://doi.org/10.1086/259394>
- Buccirossi, P., Immordino, G., & Spagnolo, G. (2021). Whistleblower rewards, false reports, and corporate fraud. *European Journal of Law and Economics*, 51(3), 411–431. <https://doi.org/10.1007/s10657-021-09699-1>
- Cohen, M. A. (1999). Monitoring and enforcement of environmental policy. In H. Folmer & T. Tietenberg (Eds.), *The International Yearbook of Environmental and Resource Economics 1999/2000: A Survey of Current Issues* (pp. 44–106). Edward Elgar.
- Dai, C. (2026). The interplay of regulatory auditing and self-auditing: Crowd in or out? *Journal of Regulatory Economics*, 69, Article 16. <https://doi.org/10.1007/s11149-026-09514-2>
- Ehrmann, T., & Prinz, A. (2025). The auditing game: the dark side of the private provision of a public good. *European Journal of Law and Economics*, 59(2), 329–376. <https://doi.org/10.1007/s10657-023-09785-6>
- European Commission. (2013). Commission Recommendation 2013/99/EU of 19 February 2013 on a coordinated control plan with a view to establish the prevalence of fraudulent practices in the marketing of certain foods. CELEX: 32013H0099.
- European Commission. (2017). New and improved car emissions tests become mandatory on 1 September. Directorate-General for Internal Market, Industry, Entrepreneurship and SMEs, press release, 4 September 2017.
- Friesen, L. (2003). Targeting enforcement to improve compliance with environmental regulations. *Journal of Environmental Economics and Management*, 46, 72–85. [https://doi.org/10.1016/S0095-0696\(02\)00033-5](https://doi.org/10.1016/S0095-0696(02)00033-5)
- Germani, A. R., Scaramozzino, P., Morone, A., & Morone, P. (2017). Discretionary enforcement and strategic interactions between enforcement agencies and firms: A theoretical and laboratory investigation. *Journal of Regulatory Economics*, 52(3), 255–284. <https://doi.org/10.1007/s11149-017-9341-y>
- González Lira, A., & Mobarak, A. M. (2021). Slippery fish: Enforcing regulation when agents learn and adapt. NBER Working Paper No. 28610. <https://doi.org/10.3386/w28610>
- Harrington, W. (1988). Enforcement leverage when penalties are restricted. *Journal of Public Economics*, 37(1), 29–53. [https://doi.org/10.1016/0047-2727\(88\)90003-5](https://doi.org/10.1016/0047-2727(88)90003-5)
- Heyes, A. (2000). Implementing environmental regulation: Enforcement and compliance. *Journal of Regulatory Economics*, 17(2), 107–129. <https://doi.org/10.1023/A:1008157410380>
- Innes, R. (1999). Remediation and self-reporting in optimal law enforcement. *Journal of Public Economics*, 72, 379–393. [https://doi.org/10.1016/S0047-2727\(98\)00101-7](https://doi.org/10.1016/S0047-2727(98)00101-7)
- Innes, R. (2000). Self-reporting in optimal law enforcement when violators have heterogeneous probabilities of apprehension. *Journal of Legal Studies*, 29, 287–300. <https://doi.org/10.1086/468071>
- Innes, R. (2001). Violator avoidance activities and self-reporting in optimal law enforcement. *Journal of Law, Economics and Organization*, 17, 239–256. <https://doi.org/10.1093/jleo/17.1.239>
- Innes, R. (2004). Enforcement costs, optimal sanctions, and the choice between ex-post liability and ex-ante regulation. *International Review of Law and Economics*, 24(1), 29–48. <https://doi.org/10.1016/j.irle.2004.03.003>
- Kambhu, J. (1989). Regulatory standards, noncompliance and enforcement. *Journal of Regulatory Economics*, 1, 103–114. <https://doi.org/10.1007/BF00140020>

- Kaplow, L., & Shavell, S. (1994). Optimal law enforcement with self-reporting of behavior. *Journal of Political Economy*, 102, 583–606. <https://doi.org/10.1086/261947>
- Kim, J.-Y. (2013). A note on the non-maximality of the optimal fines when the apprehension probability depends on the offense rate. *European Journal of Law and Economics*, 36(1), 131–138. <https://doi.org/10.1007/s10657-012-9341-4>
- Lindqvist, R., Lombardini, C., Suvantola, L., & Ollikainen, M. (2024). The deterrence effect of criminal sanctions against environmental crime in Finland: an application of the rational choice model of crime. *European Journal of Law and Economics*, 58(2), 175–200. <https://doi.org/10.1007/s10657-024-09818-8>
- Malik, A. S. (1990). Avoidance, screening and optimum enforcement. *The RAND Journal of Economics*, 21(3), 341–353. <https://doi.org/10.2307/2555613>
- Muller, N. Z., & Mendelsohn, R. (2009). Efficient pollution regulation: Getting the prices right. *American Economic Review*, 99(5), 1714–1739. <https://doi.org/10.1257/aer.99.5.1714>
- Nowell, C., & Shogren, J. F. (1994). Challenging the enforcement of environmental regulation. *Journal of Regulatory Economics*, 6, 265–282. <https://doi.org/10.1007/BF01064655>
- Nussim, J., & Tabbach, A. D. (2008). (Non)Regulable avoidance and the perils of punishment. *European Journal of Law and Economics*, 25(3), 191–208. <https://doi.org/10.1007/s10657-008-9045-y>
- Nyborg, K., & Telle, K. (2006). Firms' compliance to environmental regulation: Is there really a paradox? *Environmental and Resource Economics*, 35(1), 1–18. <https://doi.org/10.1007/s10640-006-9001-7>
- Pfaff, A. S. P., & Sanchirico, C. W. (2000). Environmental self-auditing: Setting the proper incentives for discovery and correction of environmental harm. *Journal of Law, Economics, and Organization*, 16(1), 189–208. <https://doi.org/10.1093/jleo/16.1.189>
- Polinsky, A. M., & Shavell, S. (2000). The economic theory of public enforcement of law. *Journal of Economic Literature*, 38(1), 45–76. <https://doi.org/10.1257/jel.38.1.45>
- Porter, M. E., & van der Linde, C. (1995). Toward a new conception of the environment-competitiveness relationship. *Journal of Economic Perspectives*, 9(4), 97–118. <https://doi.org/10.1257/jep.9.4.97>
- Premanandh, J. (2013). Horse meat scandal – A wake-up call for regulatory authorities. *Food Control*, 34(2), 568–569. <https://doi.org/10.1016/j.foodcont.2013.05.033>
- Sénat. (2025a). *Eaux minérales naturelles : préserver la pureté* [Natural mineral waters: Preserving purity]. Report No. 628 (2024–2025), Vol. I: Report, submitted May 14, 2025. Committee of inquiry into bottled-water industry practices and public authorities' responsibility for failures in the oversight of these activities. French Senate.
- Sénat. (2025b). *Eaux minérales naturelles : préserver la pureté* [Natural mineral waters: Preserving purity]. Report No. 628 (2024–2025), Vol. II: Proceedings, submitted May 14, 2025. Committee of inquiry into bottled-water industry practices and public authorities' responsibility for failures in the oversight of these activities. French Senate.
- Stranlund, J. K. (2007). The regulatory choice of noncompliance in emissions trading programs. *Environmental and Resource Economics*, 38(1), 99–117. <https://doi.org/10.1007/s10640-006-9058-3>
- Thompson, G. J., Carder, D. K., Besch, M. C., Thiruvengadam, A., & Kappanna, H. K. (2014). In-use emissions testing of light-duty diesel vehicles in the United States. Center for Alternative Fuels, Engines and Emissions, West Virginia University.
- U.S. Department of Justice. (2013). Generic drug manufacturer Ranbaxy pleads guilty and agrees to pay \$500 million to resolve false claims allegations, cGMP violations and false statements to the FDA. Office of Public Affairs.
- U.S. Department of Justice. (2017). Volkswagen AG agrees to plead guilty and pay \$4.3 billion in criminal and civil penalties; six Volkswagen executives and employees are indicted in connection with conspiracy to cheat U.S. emissions tests. Office of Public Affairs.
- U.S. Department of Justice. (2022). FCA US LLC sentenced in connection with conspiracy to cheat U.S. emissions tests. Office of Public Affairs.

- U.S. Environmental Protection Agency. (2014). Near roadway air pollution and health: Frequently asked questions. EPA-420-F-14-044, August 2014. Office of Transportation and Air Quality.
- U.S. Environmental Protection Agency. (2015). Notice of violation: Volkswagen AG, Audi AG and Volkswagen Group of America, Inc. 18 September 2015, pp. 3–4.
- U.S. Environmental Protection Agency. (2024). 2024 Cummins Inc. Vehicle Emission Control Violations Settlement. Retrieved September 8, 2026, from <https://www.epa.gov/enforcement/2024-cummins-inc-vehicle-emission-control-violations-settlement>
- U.S. Environmental Protection Agency. (2026). Volkswagen Clean Air Act civil settlement. Updated February 4, 2026. Retrieved September 8, 2026, from <https://www.epa.gov/enforcement/volkswagen-clean-air-act-civil-settlement>
- U.S. Food and Drug Administration. (2023). Regulatory actions against Ranbaxy and Sun Pharma. Retrieved September 8, 2026, from <https://www.fda.gov/drugs/enforcement-activities-fda/regulatory-actions-against-ranbaxy-and-sun-pharma>
- Volkswagen AG. (2017). Diesel issue. In *Annual Report 2016*. Published March 14, 2017. <https://annualreport2016.volkswagenag.com/group-management-report/diesel-issue.html>
- World Health Organization. (2008a). Melamine-contaminated powdered infant formula in China. Disease Outbreak News, 18 September 2008.
- World Health Organization. (2008b). Melamine-contaminated powdered infant formula in China : Update 2. Disease Outbreak News, 29 September 2008.