

Arm Redesign and Endurance Maximization of Hexarotor using Stochastic Optimization and Machine Learning

Oguz KOSE

oguz.kose@erzincan.edu.tr

Erzincan Binali Yıldırım University

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Abstract

In this paper, it is aimed to maximise the endurance in simultaneous hexarotor longitudinal and lateral flight using arm redesign, optimization method and machine learning. Arm redesign rate, longitudinal and lateral flight controller coefficients were estimated by stochastic optimization. Battery weight and moments of inertia of hexarotor were estimated by two different machine learning algorithms. Drawings were made in Solidworks for the model parameters required for the optimization and machine learning. The hexarotor modelling was performed in the state space model with linear equations of motion using the Newton-Euler method. Simulations were performed in MATLAB/Simulink environment. The results obtained from two machine learning algorithms are given comparatively. The total cost was reduced by 16% with the optimization algorithm. With the parameters estimated by optimization and machine learning, the hexarotor successfully followed the given trajectory in longitudinal and lateral flight. In endurance optimization, the minimum cost was obtained with 40 min.

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Paper type - Research paper.

1 Introduction

Today, unmanned aerial vehicles (UAVs) are among the technologies that increase productivity by offering revolutionary innovations in many sectors. Among these vehicles, hexarotor UAVs are unmanned aerial vehicles with six motors and propellers, and stand out with their high stability and carrying capacity. Thanks to these features, it offers a wide range of uses in military, civilian and industrial areas. They are especially preferred in applications such as aerial photography, mapping, search and rescue operations and agricultural spraying. Advanced sensor systems and autonomous flight capabilities enable these vehicles to be used reliably in sensitive missions. Today, hexarotor UAVs play a critical role in disaster management and security. In emergencies such as earthquakes, floods or fires, they can provide instant data to the authorities by conducting rapid reconnaissance. It minimises human intervention in risky areas by being used for surveillance and reconnaissance by police and military units. It also stands out as an alternative that accelerates transport in the logistics sector. The importance of these unmanned aerial vehicles is increasing day by day with technology. Thanks to artificial intelligence-supported analysis systems and advanced communication technologies, they are used efficiently in more complex and sensitive missions. In particular, autonomous navigation capability and high manoeuvrability make hexarotor UAVs an indispensable tool in various industries.

In addition to these features, with today's developing technology, both the endurance optimisation of UAVs and the ability to adapt to different missions come to the fore. In hexarotor UAVs, endurance is of great importance in terms of the ability to provide uninterrupted operation in long-term missions. Especially in missions such as military reconnaissance, search and rescue, agricultural analysis and border security, the endurance of a UAV directly affects the success of the operation. Higher battery capacities, the use of lightweight materials and energy-efficient engine systems can increase the endurance level of hexarotor UAVs. This feature is critical for long-range operations and wide-area scans, increasing operational efficiency and saving manpower and costs. Arm redesign

refers to the ability of hexarotor UAVs to adapt to different missions and environmental conditions. The ability to make hardware and software changes according to changing weather conditions, difficult terrain conditions or different mission requirements enables the versatile use of these vehicles. Thanks to their modular design and flexible software integration, UAVs with enhanced metamorphosis capability can be customised according to user needs, addressing a wider area of use and offering a more effective solution for multi-purpose missions.

In recent years, there have been many studies on endurance and morphing characteristics of UAVs. Gatti et al. [1] present an analytical framework to analyse the endurance of a battery-powered multi-rotor UAV. With a particular focus on hovering time, the balance between power requirement and battery capacity, which is critical for aircraft design, is analysed. Mathematical models for endurance are derived based on aircraft airframe characteristics, rotor parameters and battery capacity. In addition, two different approximate solutions are presented to determine the optimum battery capacity and the proposed method is validated by numerical simulations and flight tests. The developed model helps to determine the optimum endurance conditions by considering factors such as aircraft weight, battery capacity and rotor efficiency during the preliminary design phase. In particular, it is emphasised that randomly increasing the battery capacity during the design process is not effective, instead, the endurance time should be improved by optimising the aircraft components. In addition, it is predicted that endurance time can be calculated more efficiently by using artificial intelligence supported optimisation algorithms. Ming et al. [2] aimed to develop a method for testing and evaluating the endurance performance (EP) of electric multi-rotor spray drones (EMSD). The effectiveness of EMSDs in agricultural aviation applications is directly related to their operation time and mission completion capacity. In this context, a test system was designed and an evaluation method was proposed to determine the endurance performance. Firstly, a test model based on the performance test platform was created to model the energy consumption of EMSDs. In the study, an EP evaluation method is established by considering key factors such as power consumption, thermal efficiency ratio and power consumption ratio per unit load. Using an integrated index method and weighting technique based on expert opinion, EMSDs are evaluated under multiple criteria. The validity of the proposed method is verified through experimental tests on three different drone models widely used in the market. The results show that the proposed system is able to accurately measure the endurance performance of EMSDs and provides useful data for their optimisation. The findings of this study contribute to the establishment of drone performance standards in the field of agricultural aviation and provide a new test methodology to guide the design and development processes of EMSDs. Panigrahi et al. [3] discuss the design, analysis and testing of a hybrid vertical take-off and landing (VTOL) tilt-rotor UAV with increased endurance. Although the use of UAVs in various fields has become widespread, the limitations of rotor-based systems in terms of flight time and range, and the need for take-off and landing runway for fixed-wing aircraft create significant disadvantages. In this study, a hybrid VTOL tilt-rotor UAV combining the advantages of both structures is designed. During the design process, existing hybrid UAV configurations were analysed and a bi-copter model with thrust steering capability using minimum actuators was created. The mechanical design was developed to ensure aerodynamic efficiency and weight optimisation, and the airfoil and engine placement were carefully determined. The dynamic model of the UAV was created, aerodynamic coefficients were calculated by numerical simulations and the overall flight dynamics of the system were analysed. Simulations have shown that the UAV can smoothly switch between VTOL, transition and fixed-wing flight modes and flight stability is ensured. Experimental tests have confirmed that the system can glide stably in the air and that the specified control mechanisms work effectively. Future work

will focus on aerodynamic optimisations for longer range and development of automatic flight transition algorithms. El-Atab et al., [4], investigate the technologies developed to increase the endurance of solar-powered small unmanned aerial vehicles (SUAVs). Conventional battery-powered UAVs have limited flight time. To overcome this problem, UAVs equipped with solar panels have been developed and it has been shown that flight time can be significantly increased. The study addresses different classifications of solar-powered UAVs and the electrical design challenges of these systems. In particular, the evaluation of the solar panels used in terms of material and mechanical flexibility, energy storage solutions and control algorithms are analysed in detail. In order to provide continuous flight, the solar energy collected during the daytime must be capable of both charging the batteries and meeting the instantaneous energy needs of the UAV. However, due to variables such as weather conditions, optimising energy storage strategies is of great importance. The paper also attempts to determine the most suitable energy sources for SUAVs by comparing different battery types and solar panels. Furthermore, limitations of current solar-powered UAVs and suggestions on how these technologies can be developed in the future are presented. In conclusion, new system designs and energy management strategies to increase the flight time of solar-powered UAVs form the basis of advances in this field. Many studies have been carried out in recent years on the control of rotary wing UAVs with arm redesign or morphing. Parancheerivilakkathil et al. [5] investigate the control strategies used in aircraft with morphing characteristics. In the study, the performances of different control approaches are evaluated by experimental tests and numerical simulations. In particular, closed-loop control systems developed for wing shaping control are tested in real time to optimise the actuators. Among the key findings, a closed-loop control method applied to the airfoil morphing profile in a wind tunnel experiment was shown to increase the lift-to-drag ratio (L/D) by 16%. Furthermore, the proportional integral (PI) controller used for actuator temperature control was found to reduce the heating power by 20%. Comparison of open-loop and closed-loop control showed that the closed-loop control method provides higher stability. In addition, in experiments with shape memory alloys (SMA) used for deformation of flexible wing surfaces, it was observed that the actuator motion accuracy remained within the error limit of 0.05 mm. Within the scope of hybrid control systems, the combination of fuzzy logic PID control and on/off control increased the effectiveness of the metamorphosis mechanism by minimising the delays in the cooling and warm-up processes. In wind tunnel tests, it was also shown that the morphing process provides aerodynamic improvements that can directly affect fuel consumption. Xing et al. [6] provide a comprehensive review of morphing quadrotors developed to provide UAVs with greater manoeuvrability and flexibility. It is emphasised that conventional fixed-frame quadrotor systems are limited in tight spaces or obstacle-filled environments, while morphing systems can overcome these limitations thanks to their ability to structurally change during flight. The article focuses on the basic design principles of morphing mechanisms, actuator systems and flight control strategies. Frame systems that can be configured in real time with the integration of smart materials and advanced actuators are discussed, and the effects of morphing on stability and energy consumption are analysed in detail. However, it is noted that morphing quad-rotors face challenges such as more complex mechanical structures, increased energy demand and the need for advanced control systems. The study summarises the potential future uses of morphing technology, particularly by evaluating biomimetic design approaches, artificial intelligence-based control strategies and adaptive flight algorithms. In the future, it is suggested that the integration of smart materials and machine learning techniques can improve the performance of these systems and enable them to be extended to wider application areas. Vundela and Muralidharan [7] present a new design of a shape-shifting drone called QBQ-Copter. The drone has two main

arms, with the upper arm pivoting axially relative to the lower arm, and is equipped with tilting propellers. In this way, the QBQ-Copter is able to change from the traditional four-rotor mode to the Bi-Quadcopter mode, which is a two-rotor assisted structure thanks to its tilt propellers, in order to better manoeuvre in tight spaces. The paper details the control allocation mechanism developed for this morphing drone and describes the dynamic control algorithms. A two-stage control system is implemented to ensure stable flight of the UAV in different modes: one is a nonlinear position controller for position tracking and the other is a two-stage cascade control mechanism for attitude control. Simulation results show that the QBQ-Copter can successfully switch between different flight modes and maintain stability. Kose and Oktay [8] use deep neural networks (DNN) and Simultaneous Perturbation Stochastic Approximation (SPSA) algorithm to optimise both arm morphing and autopilot system for hexarotor UAV. The research aims to improve the autonomous flight performance by varying the arm length of the hexarotor. In the study, the appropriate PID controller coefficients and the optimal morphing rate are optimised by the SPSA algorithm. Since the moments of inertia of the hexarotor change as the arm length changes, deep neural networks are used to calculate these parameters. Both longitudinal and lateral flight simulations were performed with the best parameters obtained. The results show that the use of SPSA and DNN provides a significant improvement in the flight performance of the hexarotor. In particular, design performance criteria such as rise time, settling time and overshoot amount are optimised and the hexarotor is more stable to follow the specified route. This study contributes to the literature by providing an innovative approach to the adaptive control of unmanned aerial vehicles and optimising their dynamic structures. Kose [9] deals with the optimisation of flight control system for an octorotor UAV under active morphing effect. The research aims to improve autonomous flight performance by improving both longitudinal and lateral flight control. In the study, octorotor models with different arm lengths were created in Solidworks and these arm lengths were optimised using the SPSA algorithm. In order to predict the changing moments of inertia, the Stochastic Gradient Descent (SGD) algorithm, which is frequently used in machine learning, is preferred. For flight control, PID controller was used due to its simplicity in structure. The SPSA algorithm optimised a total of seven parameters: three longitudinal flight PID gains, three lateral flight PID gains and one morphing ratio. The PID gains remained within the specified limits and successful results were obtained. At the same time, the generated cost index showed a success rate of 93%. The SGD algorithm optimised the moment of inertia estimates in 1,570 iterations. The results obtained show that SPSA and SGD methods are effective in octorotor flight control. The simulations show that the octorotor successfully follows the specified routes and significant improvements are achieved in the design performance criteria. The study presents an innovative approach for octorotor UAVs using a combination of active morphing and optimisation methods. Kose [10], the design of a simultaneous longitudinal and lateral autonomous flight system under differential morphing effect for an octorotor type UAV is discussed. The dynamic model is obtained by Newton-Euler approximation and linearised in MATLAB/Simulink environment and longitudinal and lateral flight simulations are performed. Differential morphing was applied with X-type and plus (+) type configurations of the octorotor, and the moments of inertia changing according to randomly determined arm lengths in each iteration were estimated by deep neural network (DNN). In addition, the PID control coefficients for both longitudinal and lateral flight were estimated separately using the stochastic optimisation method SPSA and the design performance criteria were minimised. As a result, a 49.45% improvement in the total cost index was achieved and the octorotor was successfully controlled under randomly determined arm lengths and metamorphosis effects. The findings of the study show

that optimisation techniques can be successfully applied to octorotor type UAVs and both longitudinal and lateral flight can be successfully controlled under differential morphing effect.

In this study, it is aimed to determine the battery mass according to the arm redesign for the hexarotor and accordingly, the endurance of the hexarotor is maximised and the longitudinal and lateral flight costs are minimised and flight control is ensured. For this purpose, optimisation algorithms and different machine learning algorithms were used comparatively. Seven parameters including arm redesign rate, longitudinal flight controller PID gains and lateral flight controller PID gains were determined by SPSA. Battery mass and moments of inertia were estimated separately by both artificial neural network (ANN) and SGD. A data set was created for the machine learning algorithms and the data were obtained from full models drawn in Solidworks programme. R^2 , Mean Absolute Percentage Error (MAPE), Mean Squared Error (MSE) were used for the performance metrics of the machine learning algorithms and a comparison was made between each other. Newton-Euler approach was used for the hexarotor linear mathematical model and the model was transformed into a state space model. In addition, longitudinal and lateral flight simulations were carried out in MATLAB/Simulink environment. Flight controller costs are presented in percentages and machine learning predictions are presented with performance metrics for comparison. In this context, the endurance for the hexarotor UAV is maximised. As a result, with arm redesign rate maximum endurance, this study will be an innovative method for the endurance and control of hexarotor UAVs in terms of the use of both arm redesign rate and optimization and machine learning and its contribution to the literature and will eliminate the gap in this subject.

2 Hexarotor Mathematical Model, Arm Redesign and Endurance

2.1 Hexarotor Mathematical Model

The hexarotor is a six-rotor UAV model. This type of UAV consists of six arms, each with a rotor and propeller. Six rotors provide more lift and better manoeuvrability. The six-rotor structure allows the hexarotor to continue flying even in the event of an engine failure. This increases reliability. Hexarotors offer more lift and better stability compared to quadrotors with four rotors. This makes them particularly suitable for carrying heavy loads, long duration flight and more complex missions.

Hexarotor modelling is the process of mathematically expressing and simulating the dynamic behaviour of a UAV. For hexarotor modelling, the hover mode is considered [11]. This modelling is the basis for the design, performance analysis and simulations of the hexarotor's control systems. Hexarotor modelling is usually based on physical principles such as Newton-Euler or Lagrange dynamics. In this study, hexarotor modelling is divided into longitudinal flight and lateral flight and Newton-Euler is used. Longitudinal flight is concerned with the hexarotor forward-backward (x-direction) and pitch (θ) motion. The longitudinal flight equations of motion are shown in equation 1.

$$x_{lon} = [x \quad \theta \quad \dot{x} \quad q]^T \quad (1)$$

Where x_{lon} is the longitudinal flight, x is the position in the forward direction, θ is the pitch angle, $\dot{x}$ is the forward velocity and q is the pitch angular velocity.

To model longitudinal motion, inputs must be applied to the system. Longitudinal flight inputs are shown in equation 2.

$$u_{lon} = [F_T \quad \tau_y]^T \quad (2)$$

Where u_{lon} is the longitudinal flight inputs, F_T is the total thrust, τ_y is the pitch moment.

Lateral motion is lateral motion (y-direction) and is related to roll (ϕ) motion. The equations for lateral motion are shown in equation 3.

$$x_{lat} = [y \quad \phi \quad \dot{y} \quad p]^T \quad (3)$$

x_{lat} is the lateral flight, y is the position in the lateral direction, ϕ is the roll angle, $\dot{y}$ is the lateral velocity and p is the roll angular velocity. To model lateral motion, inputs must be applied to the system. The lateral flight inputs are shown in equation 4.

$$u_{lat} = [F_T \quad \tau_x]^T \quad (4)$$

Where u_{lat} is the lateral flight inputs, F_T is the total thrust, τ_x is the roll moment.

The state space model is a mathematical representation that allows dynamic systems to be expressed by linear or nonlinear differential equations. In this study, the hexarotor longitudinal and lateral equations of motion are considered as linear differential equations. The state space model represents the relationship between input, output and state variables using matrices. The general form of the state space model is shown as follows.

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

Where;

x , state vector,

u , input vector (control inputs),

y , output vector (measured variables),

A , system matrix (interaction of state variables within itself),

B , input matrix (how input variables affect the system),

C , output matrix (which state variables are measured),

D , direct transmission matrix (how inputs directly affect outputs),

Accordingly, the longitudinal flight state space model is shown as follows.

$$\begin{aligned} A_{lon} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & B_{lon} &= \begin{bmatrix} 0 & 0 \\ 0 & 1/I_y \\ 1/m & 0 \\ 0 & 1/I_y \end{bmatrix} \\ C_{lon} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} & D_{lon} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

The lateral flight state space model is shown as follows.

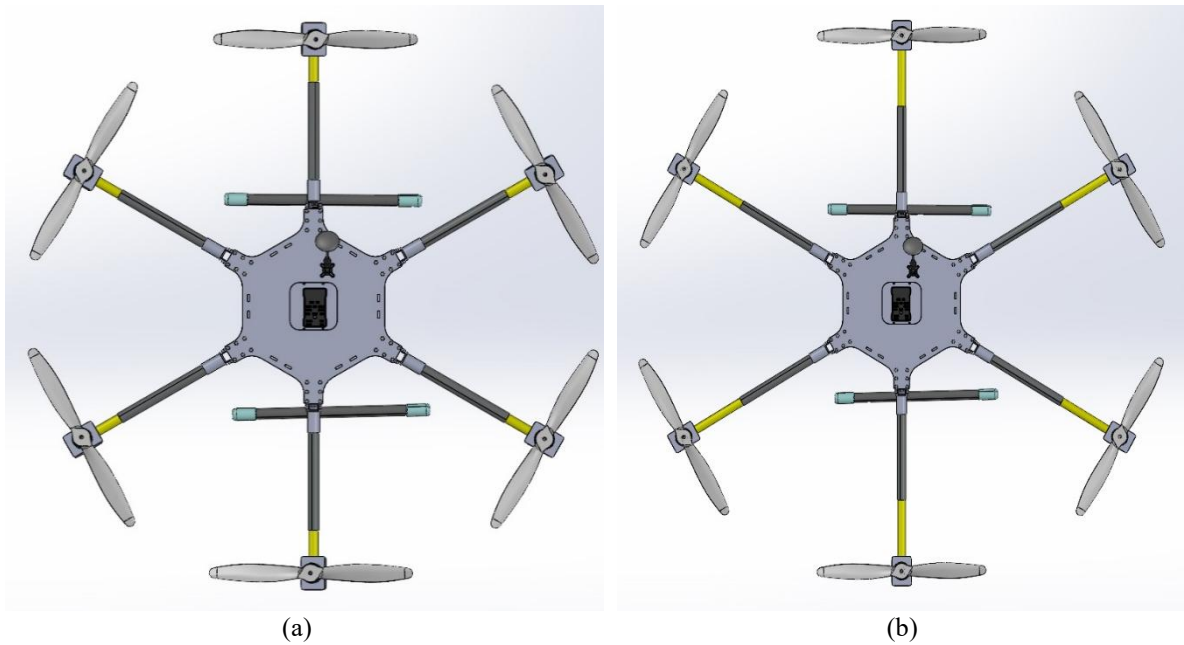
$$\begin{aligned} A_{lat} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & B_{lat} &= \begin{bmatrix} 0 & 0 \\ 0 & 1/I_x \\ 1/m & 0 \\ 0 & 1/I_x \end{bmatrix} \\ C_{lat} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} & D_{lat} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

For longitudinal flight, the given state space model is used for the control of forward- backward motion, i.e. speed and pitch angle. The state space model given for lateral flight is used for the control of the lateral motion, i.e. the lateral speed and roll angle.

2.2 Hexarotor Arm Redesign

In hexarotor UAV, arm redesign refers to the active change of the aerodynamic or structural characteristics of the vehicle to improve its flight performance and mission effectiveness [12]. With arm redesign, the hexarotor can dynamically adapt to different flight conditions and mission requirements. In the hexarotor, which is a rotary wing UAV, they can increase fuel efficiency, improve manoeuvrability or minimize wind resistance by changing the arm length with arm redesign. Especially in missions such as reconnaissance, surveillance and search and rescue, arm redesign capability makes UAVs more flexible and robust [13]. In the future, with advanced materials and intelligent control systems, this technology is expected to become even more widespread.

In the hexarotor UAV, arm redesign is performed before the start of flight. Arm redesign is performed by lengthening and shortening the arms. The arm length can be between 37 cm and 65 cm. Figure 1 shows examples of arm redesign.



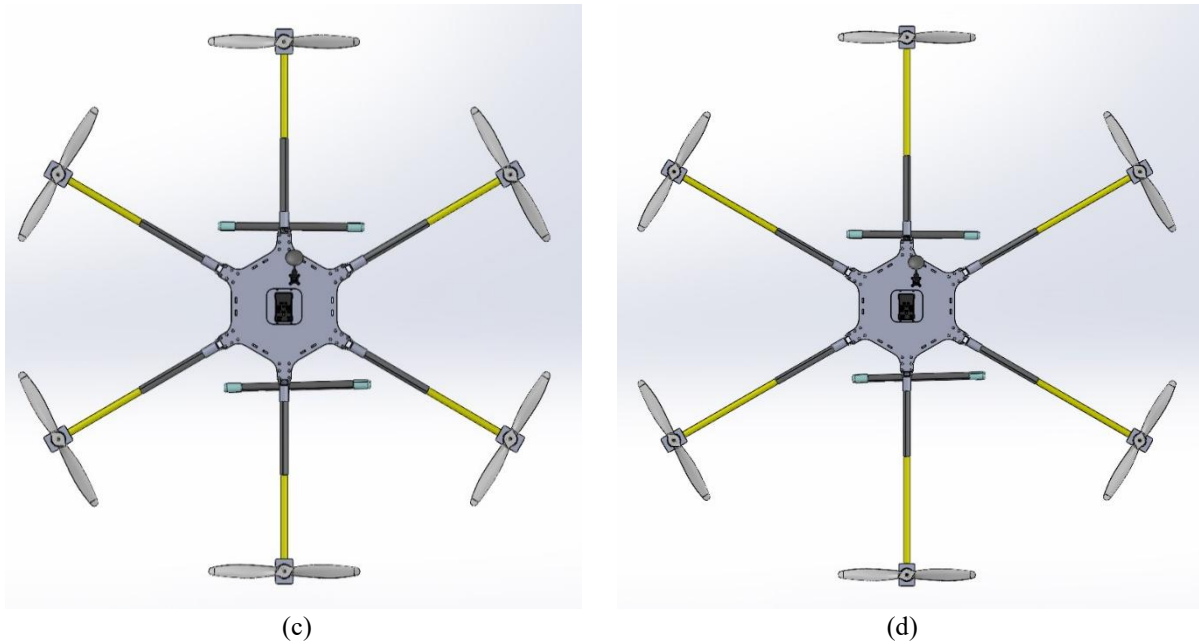


Figure 1. Hexarotor arm redesign (a) 37 cm arm length, (b) 45 cm arm length, (c) 55 cm arm length and (d) 65 cm arm length

With Arm redesign, the lengthening and shortening of the hexarotor arms is realized by means of a DC motor placed inside the arm. In an arm, there are two arms placed inside each other, the outer arm and the inner arm. The outer arm moves the inner arm back and forth with the DC motor, causing an increase and decrease in the total length of the arm, thus performing arm redesign. The cross section of the arm where the arm redesign takes place is given in Figure 2.

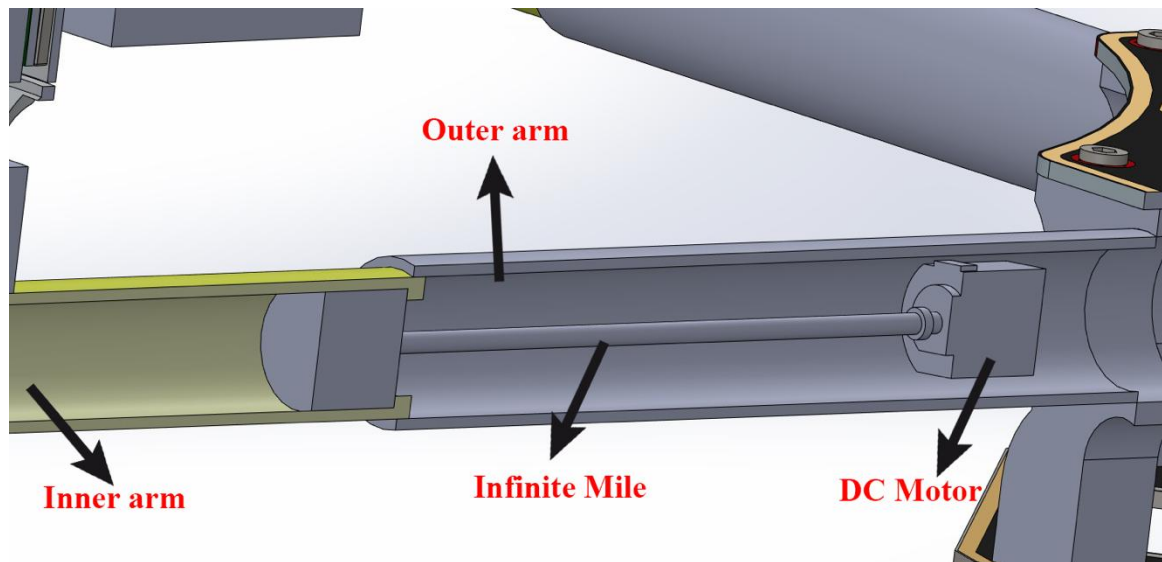


Figure 2. Arm redesign system

Arm redesign does not change the mass of the hexarotor. However, since the geometric shape of the hexarotor changes with the arm redesign, changes will occur on the moments of inertia. Hexarotor full models are drawn in Solidworks drawing program. The moments of inertia in each axis are taken from there.

2.3 Hexarotor Endurance

In the hexarotor, maximizing the flight time or providing the required flight time for a chosen arm length is a fundamental endurance-related problem. The endurance will vary according to the hexarotor arm length or the payload it carries. Hexarotors are UAVs with six propellers that provide more lift and stability. Hexarotors are specifically designed for long-duration missions, and endurance depends on factors such as battery technology, aerodynamic design and weight optimization. Lithium-polymer (LiPo) batteries are often used to increase the endurance of hexarotors, but these batteries have limited energy density and charge-discharge cycles. Therefore, lightweight materials are used, and the aerodynamic performance of propellers is optimized to improve energy efficiency [3]. Furthermore, autonomous flight systems and energy management algorithms minimize the energy consumption of the hexarotor and extend the flight time. The endurance of hexarotors is critical in many fields such as military reconnaissance, agriculture, search and rescue, and environmental monitoring [14]. To calculate the endurance of a hexarotor, the hover mode is considered as in the modelling and accordingly, the endurance of an electric UAV can be calculated with the following formula [15].

$$T = \sqrt{\frac{\pi\rho}{2g^3}} \zeta\eta_0 \frac{WD}{m_{takeoff}} \sqrt{\frac{n}{m_{takeoff}}} \quad (5)$$

In Equation 5, ρ is the air density at the relevant altitude, g is the gravitational acceleration, and these values cannot be changed by the designer. $\zeta\eta_0$ is the power efficiency coefficient [15]. The rest of the formula consists of the values that should be chosen as the main goal of the UAV design. The energy storage capacity of the LiPo battery affects the flight time in direct proportion. The flight time is directly proportional to the square root of the diameter and number of propellers, but inversely proportional to the mass of the device. Where W is the battery energy storage capacity (mAh), D is the propeller diameter (cm), M is the hexarotor mass and n is the number of propellers. These values cannot be changed independently of each other. Increasing or decreasing one of these values will have an inversely proportional effect on the total mass. In this study, a 16x55 inch propeller was used and the power efficiency coefficient of the propeller was taken as 0.695 [16]. Since the energy storage capacity and battery mass vary depending on the arm length, these values were estimated separately with ANN, a machine learning algorithm, and stochastic gradient descent (SGD), a gradient-based optimization algorithm in machine learning.

3 Optimization Algorithm

3.1 Artificial Neural Network and Stochastic Gradient Descent

Both battery mass and energy storage capacity change according to the hexarotor arm length. The total mass of the hexarotor changes with changing battery mass. The changing parameters change the hexarotor moments of inertia. All changing parameters are estimated by both ANN and SGD.

ANNs are mathematical models inspired by biological neural networks. They form the basis of machine learning and deep learning. These models consist of neurons in interconnected layers. Each neuron produces an output by applying certain weights to the inputs. During the training process, algorithms such as backpropagation are used to optimize the weights and allow the model to learn complex situations in the data. ANN has been successfully applied in many fields such as image and voice recognition, natural language processing, prediction

models and autonomous systems. However, they have some limitations such as the need for large amounts of data for training and their computational complexity.

The ANN model is designed in such a way that the output parameter is calculated from the input parameters by organizing the basic connections. Here, feed-forward neural network-based models are simulated using the input parameters battery weight, moment of inertia on x -axis I_x , moment of inertia on y -axis I_y , moment of inertia on z -axis I_z , and hexarotor arm redesign as output parameters. The neural network has one hidden layer and 15 neurons. The values of the input and output parameters for the data set were obtained from full models drawn in Solidworks. The data set is shown in table 1. 70% of the dataset was used for training and 30% for testing. Levenberg-Marquardt was used as the training algorithm and the details were analyzed in MATLAB. For training, the number of iterations, i.e. epochs, was set to 1000 and the error threshold, goal, was set to 1e-6.

Among the common techniques for solving the optimization problem, the SGD method has been a common application in machine learning. They aim to minimize empirical risk by iteratively taking the gradient calculated from randomly sampled data. SGD is a way to update and minimize an objective function by changing parameters in the opposite direction of the gradient.

Stochastic gradient descent performs a parameter update for each training sample $x(i)$ and label $y(i)$. This is expressed in equation 6 [17].

$$\theta_{t+1} = \theta_t - \eta \nabla L(\theta_t; x_i, y_i) \quad (6)$$

Where;

θ_t, t . Model parameters in iteration,

η , learning rate,

$L(\theta_t; x_i, y_i)$, the loss function with input x_i and output y_i ,

∇L is the derivative (gradient) of the missing function with respect to the parameters.

SGD offers a faster and cost-effective learning process by updating the gradient using a single random sample at each iteration. SGD aims to find the global minimum by changing the network structure after each training phase. This approach reduces error by approximating the gradient for a randomly selected group instead of finding the gradient for the entire dataset [18]. SGD usually uses a dataset to reach the global minimum. It performs training and validation by randomly selecting data from the dataset at each iteration. In this study, arm length is used as input, battery weight as output, moment of inertia I_x on the x -axis, moment of inertia I_y on the y -axis, and moment of inertia I_z on the z -axis. Accordingly, SGD outputs the predicted values at the end of training according to the arm length. Initial weights are chosen randomly in the first iteration between 0-1. The learning rate is set to 0.01 and the number of epochs is set to 1000. The data set is shown in table 1. All values in the dataset are normalized between 0-1.

Table 1. Both ANN and SGD dataset

INPUT		OUTPUT		
Arm Length (m)	Battery Mass (gr)	$I_x(kg * m^2)$	$I_y(kg * m^2)$	$I_z(kg * m^2)$
0,37	436	0,19	0,34	0,18
0,37	605	0,19	0,34	0,19
0,37	935	0,19	0,34	0,19
0,37	1057	0,19	0,34	0,19
0,37	1351	0,19	0,34	0,19
0,37	1677	0,19	0,34	0,19
0,45	436	2,75	2,94	0,23

0,45	605	2,90	3,09	0,23
0,45	935	3,19	3,38	0,24
0,45	1057	3,30	3,49	0,24
0,45	1351	3,56	3,75	0,24
0,45	1677	3,85	4,05	0,24
0,55	436	2,87	3,14	0,3
0,55	605	3,02	3,29	0,31
0,55	935	3,32	3,59	0,31
0,55	1057	3,42	3,7	0,31
0,55	1351	3,69	3,96	0,31
0,55	1677	3,98	4,25	0,31
0,65	436	3,01	3,36	0,39
0,65	605	3,16	3,51	0,39
0,65	935	3,46	3,8	0,39
0,65	1057	3,57	3,91	0,39
0,65	1351	3,83	4,18	0,39
0,65	1677	4,12	4,47	0,39

ANN and SGD application was run on MATLAB 2017 version. In addition, the computer on which ANN and SGD is run has i7-12700H processor and 32 GB RAM memory.

R^2 (Determination Coefficient), Mean Absolute Percentage Error (MAPE) and Mean Squared Error (MSE) are used as performance metrics for ANN and SGD models. R^2 is used as a statistical metric that measures the explanatory power of a regression model's independent variables on the dependent variable. An R^2 value between 0 and 1 indicates the explanatory power of the model; values close to 1 indicate that the independent variables explain the dependent variable to a large extent, while values close to 0 indicate that the model is insufficient. Negative R^2 values mean that the model's predictions are even worse than predictions using the mean value. However, a high R^2 does not always mean a good model; situations such as overfitting or linear dependence between variables can give the model a misleading improvement.

MAPE is an error metric used to measure the prediction accuracy of a model. It expresses the average absolute error rate of the predicted values relative to the true values in percentage terms. Mathematically, it is the absolute value of the difference between actual values (y_i) and predicted values ($\hat{y}_i$), normalized as a percentage and averaged over all observations. A lower MAPE indicates better model prediction; generally, values below 10% indicate high accuracy, while values above 50% indicate poor model performance.

MSE is a statistical metric commonly used to measure the prediction error of a model and is calculated as the mean of the squares of the differences between predicted values and actual values. Mathematically, the MSE is shown in equation 7.

$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2 \quad (7)$$

Where y_i represents the true values and $\hat{y}_i$ represents the model predicted values. The MSE gives more weight to large errors than small errors, allowing a more precise assessment of the model's large deviations.

3.2 Control Algorithm and Simultaneous Perturbation Stochastic Approximation

The PID control algorithm is a basic feedback control mechanism widely used in industrial control systems. The PID controller tries to minimize the error signal of the system, i.e. the difference between the desired value and the actual output, using three basic components: proportional (P), integral (I) and derivative (D). The

proportional term considers the current error, the integral term the accumulation of past errors and the derivative term the rate of change of the error. These three terms allow the controller to respond quickly, eliminate steady-state error and reduce overshoot in dynamic systems [19].

The popularity of PID controllers is due to their simple structure, wide range of applications and the adaptability of their tuneable parameters (k_p , k_i and k_d) to the system dynamics [20]. However, the performance of PID controllers depends on accurate tuning of the parameters. This tuning process can be performed by experimental approaches such as the Ziegler-Nichols method or by modern optimization techniques [21]. The separate PIDs for longitudinal and lateral flight are shown in equation 8 and 9.

$$u(t)_{lon} = k_{p_{lon}}e(t) + k_{i_{lon}} \int_0^t e(t)d(t) + k_{d_{lon}} \frac{de(t)}{d(t)} \quad (8)$$

$$u(t)_{lat} = k_{p_{lat}}e(t) + k_{i_{lat}} \int_0^t e(t)d(t) + k_{d_{lat}} \frac{de(t)}{d(t)} \quad (9)$$

PID controllers have been successfully applied in many fields such as industrial automation, robotics, process control and energy systems. For example, they are widely used in applications such as temperature control, pressure regulation, and motor speed control [22]. However, the performance of PID controllers may be limited in nonlinear systems or high-order systems. In these cases, advanced control techniques or PID-based hybrid control methods may be preferred [23].

SPSA is a stochastic approximation method used especially for solving high-dimensional and noisy optimization problems. Unlike classical gradient-based optimization techniques, SPSA obtains an approximate gradient estimate with only two random perturbed evaluations instead of computing the derivative of all parameters individually at each iteration. This significantly reduces the computational cost, especially for large-scale problems, and can give good results for noisy functions [24]. The SPSA algorithm is often applied in areas such as machine learning, signal processing, control systems and engineering optimization. It is especially effective when the derivative information cannot be obtained directly or is costly to compute [25].

In this study, longitudinal flight PID coefficients ($k_{p_{lon}}$, $k_{i_{lon}}$ and $k_{d_{lon}}$) and lateral flight PID coefficients ($k_{p_{lat}}$, $k_{i_{lat}}$ and $k_{d_{lat}}$) are estimated for arm redesign with SPSA. Optimal results in SPSA depend on both longitudinal and lateral flight performance. The cost of longitudinal and lateral flight performance is shown in equation 10.

$$J = T_{rt} + T_{st} + OS \quad (10)$$

The cost function consists of the design performance criteria of rise time, settling time and overshoot. The separate cost indices for longitudinal and lateral flight are shown in equations 11 and 12.

$$J_{lon} = a * T_{rt_{lon}} + b * T_{st_{lon}} + c * OS \quad (11)$$

$$J_{lat} = d * T_{rt_{lat}} + e * T_{st_{lat}} + f * OS \quad (12)$$

The coefficients a,b,c,d,e and f are determined in the first iteration and try to minimize the cost function. In the first iteration, rise time, settling time and overshoot values are taken and whichever value is the smallest is approximated, and the cost index is minimized. The coefficients obtained in the first iteration are given in table 2.

Table 2. Longitudinal and lateral cost function coefficients

Coefficients	Value
a	0.080
b	1
c	0.020
d	0.082
e	1
f	0.021

Total cost is an index that shows the effect of optimization parameters on cost in percentage terms and provides information for cost improvement [26]. The total cost is obtained by combining both longitudinal and lateral cost [26].

$$\%COST_{tot_i} = \%COST_{lon_i} + \frac{\%COST_{lon_0}}{\%COST_{lat_0}} + \%COST_{lat_i} \quad (13)$$

4 Results and Discussions

According to the hexarotor arm redesign rate, it is difficult to obtain battery weight and moments of inertia values with conventional methods. Because it is difficult to establish a connection between these values. Optimization techniques and machine learning are used to obtain battery weight and moments of inertia according to redesign rate. Machine learning algorithms are discussed separately for longitudinal and lateral flight. Here, to compare machine learning methods, not the same arm redesign rate but different arm redesign rates determined by SPSA are used for prediction in ANN and SGD. Figure 3 shows the prediction by SPSA for both ANN and SGD according to the arm redesign rate.

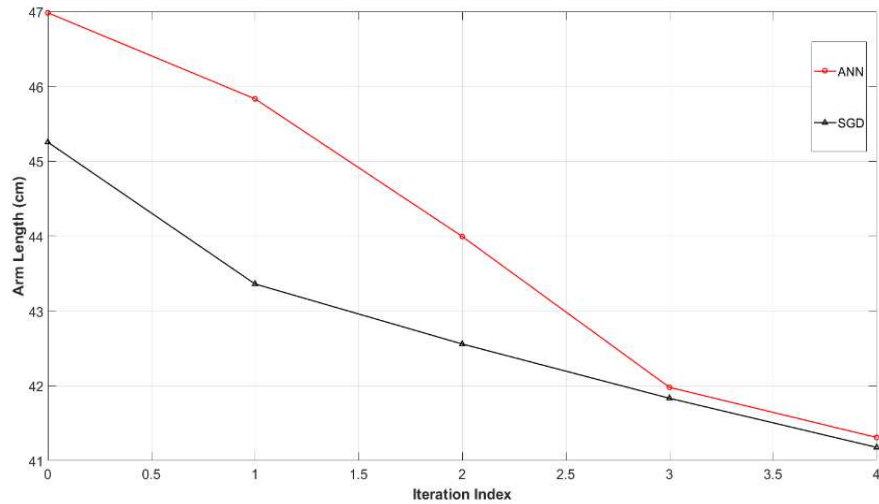


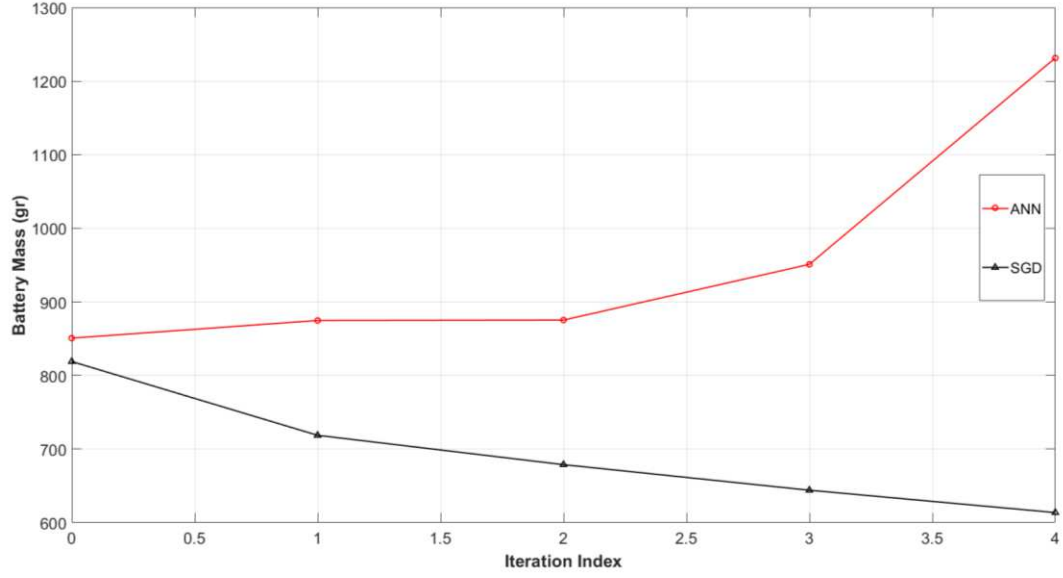
Figure 3. Arm redesign rate estimated by SPSA

After the arm redesign rate is estimated, it is sent as a parameter to the previously trained ANN and SGD. The ANN and SGD output the battery weight and moments of inertia. The accuracy of the estimated values is measured by performance metrics. The performance metrics for ANN and SGD are shown in table 3.

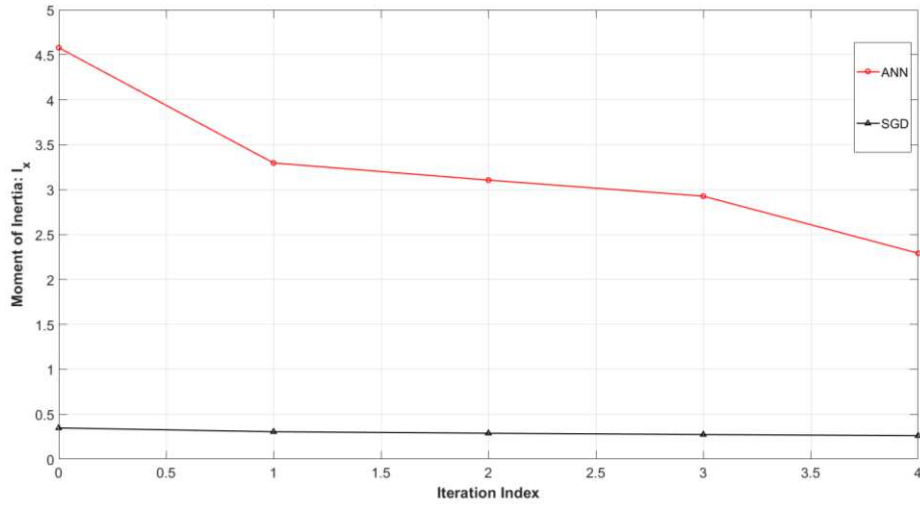
Table 3. ANN and SGD performance metrics

Performance metric	ANN	SGD
$R^2 [m_{bat} \ I_x \ I_y \ I_z]$	[0.7627 0.9448 0.9508 0.9612]	[0.1850 0.9585 0.9597 0.9763]
MAPE	14.6684	12.6761
MSE	0.0668	0.0500

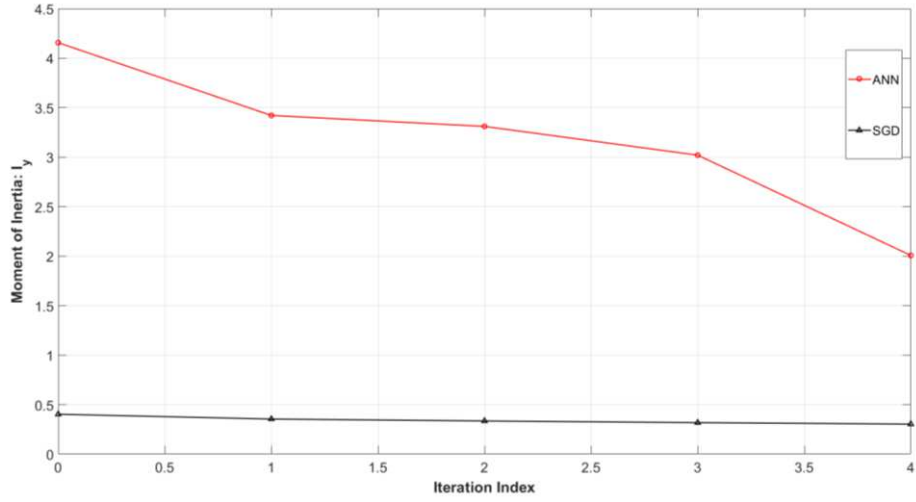
Accordingly, the battery weights and moments of inertia estimated by ANN and SGD are shown in Figure 4.



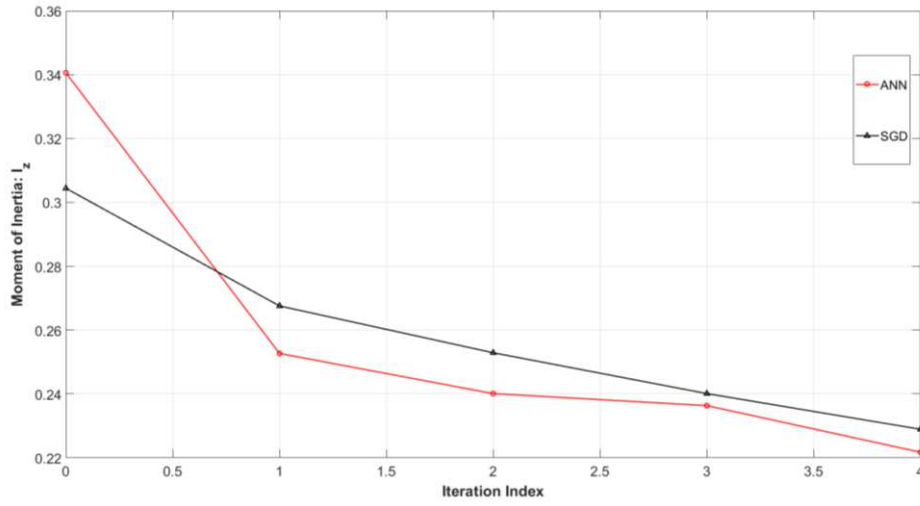
(a)



(b)



(c)



(d)

Figure 4. ANN and SGD prediction (a) Battery mass, (b) Moment of inertia: I_x , (b) Moment of inertia: I_y and (c) Moment of inertia: I_z

After the battery mass is obtained separately by both ANN and SGD, this weight is summed with the hexarotor mass to obtain the take-off weight. The hexarotor mass is obtained as 2350 g in Solidworks programme drawings without battery. Take-off mass is shown in equation 14.

$$m_{takeoff} = m_{hexa} + m_{battery} \quad (14)$$

The hexarotor take off mass obtained in each iteration separately in ANN and SGD according to equation 14 is shown in Figure 5.

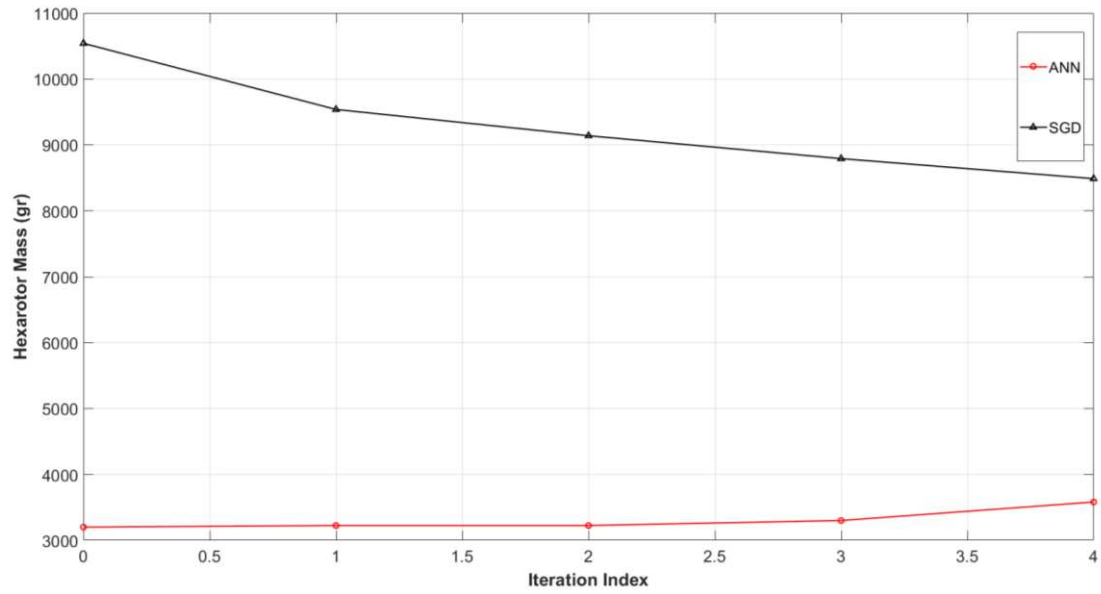


Figure 5. Hexarotor mass

According to the battery masses estimated by ANN and SGD, the closest battery energy storage capacity in the data set is considered as the energy storage capacity of the relevant battery. Figure 6 shows the battery energy storage capacity for ANN and SGD.

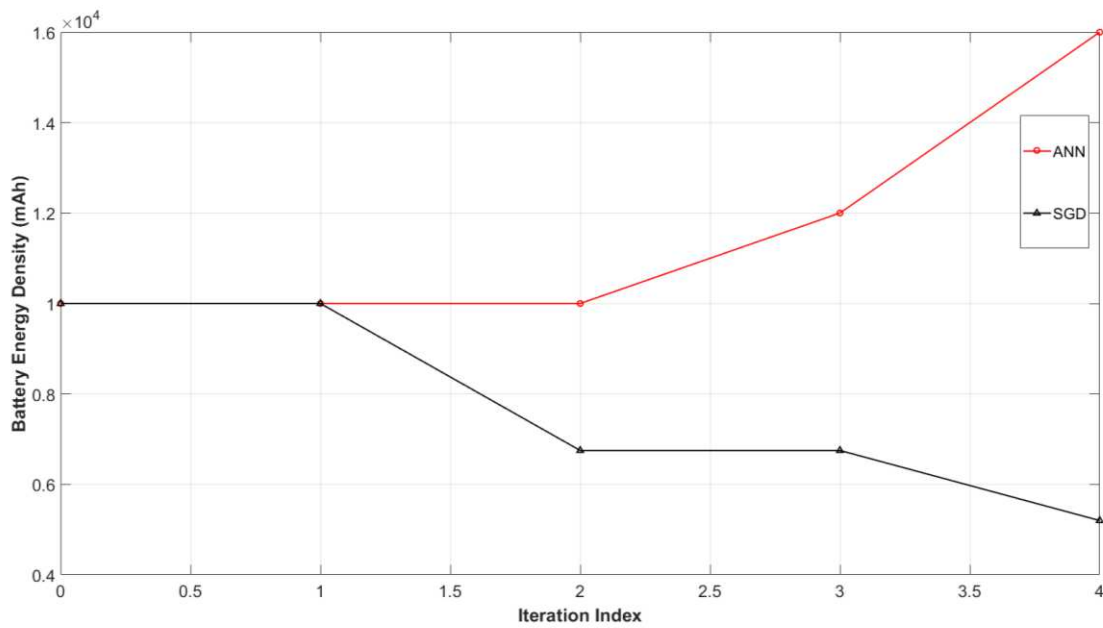


Figure 6. Energy density of battery for ANN and SGD

With battery energy densities, hexarotor endurance is calculated according to equation 5. The calculated endurance for ANN and SGD is shown in Figure 7.

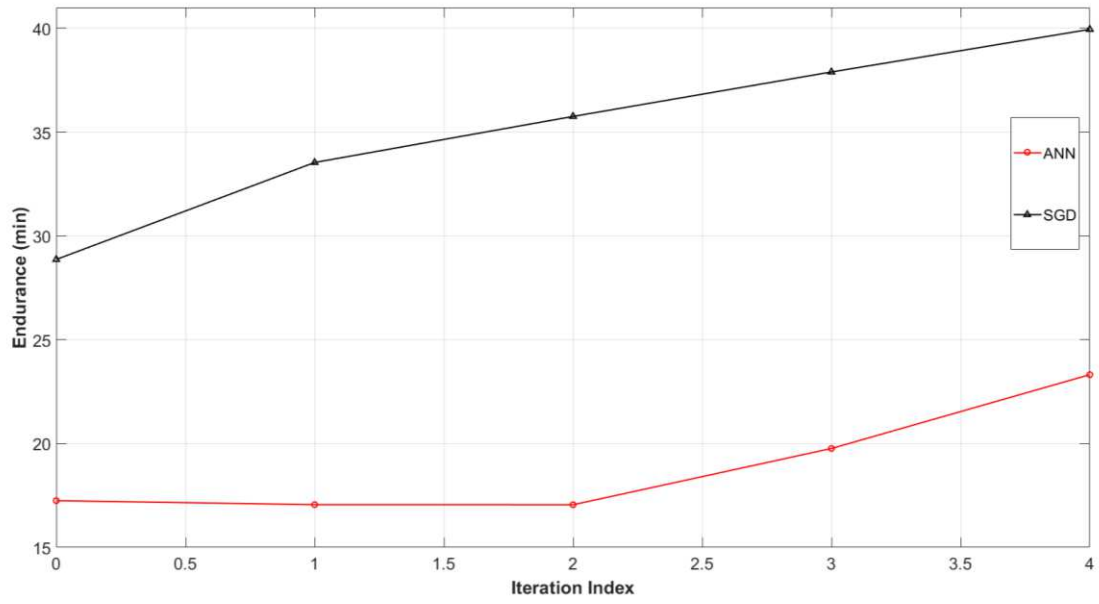
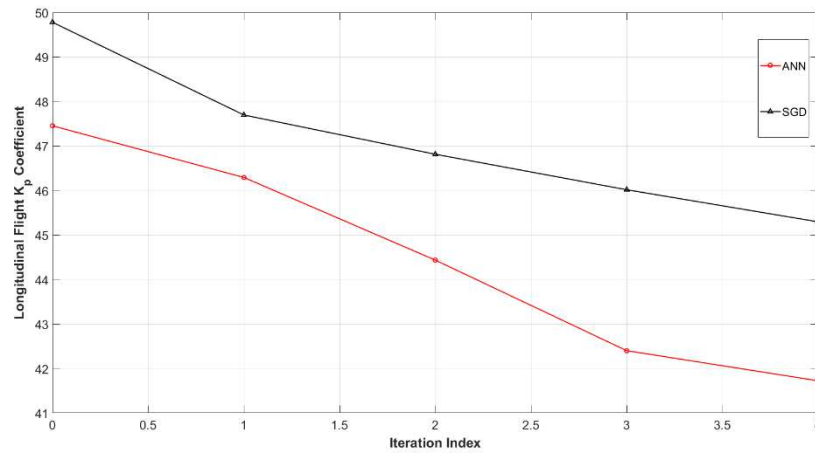
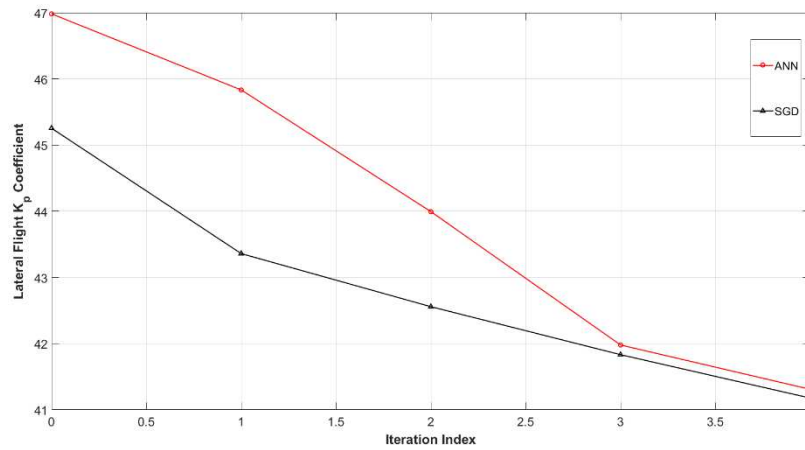


Figure 7. Endurance for ANN and SGD

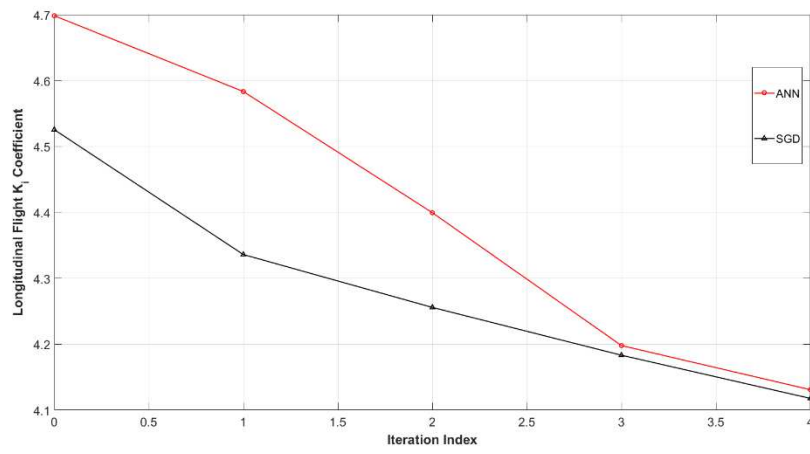
After obtaining the hexarotor mass, the PID coefficients for longitudinal and lateral flight are estimated separately by SPSA. The PID coefficients k_p , k_i and k_d for longitudinal and lateral flight are shown in Figure 8'de respectively.



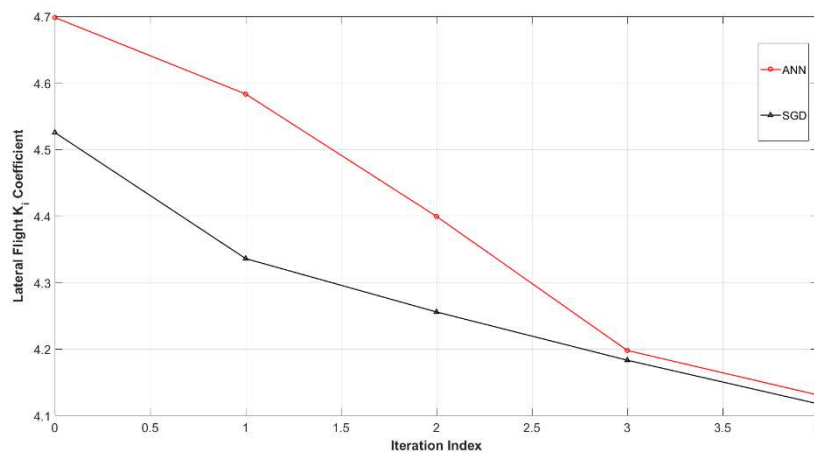
(a)



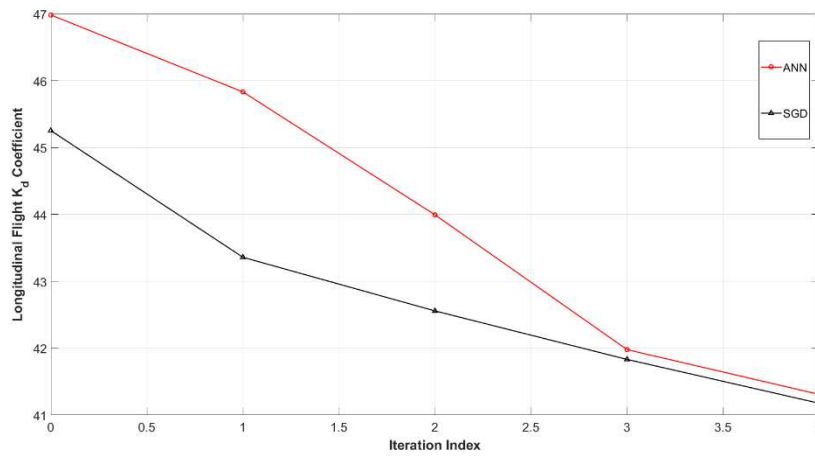
(b)



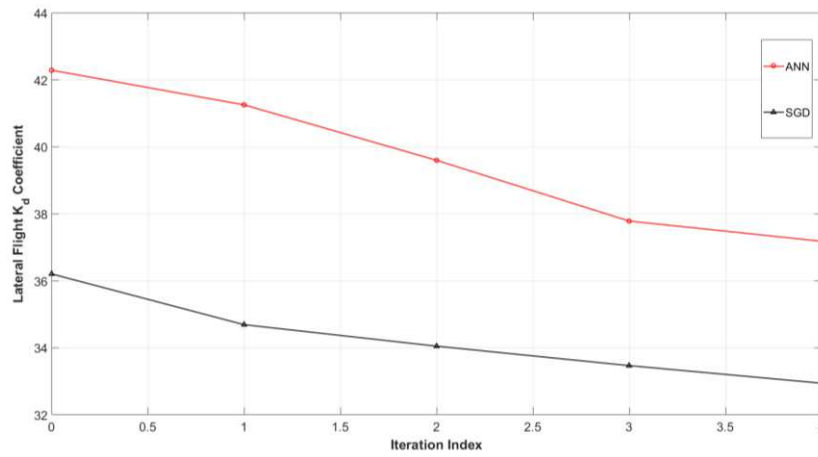
(c)



(d)



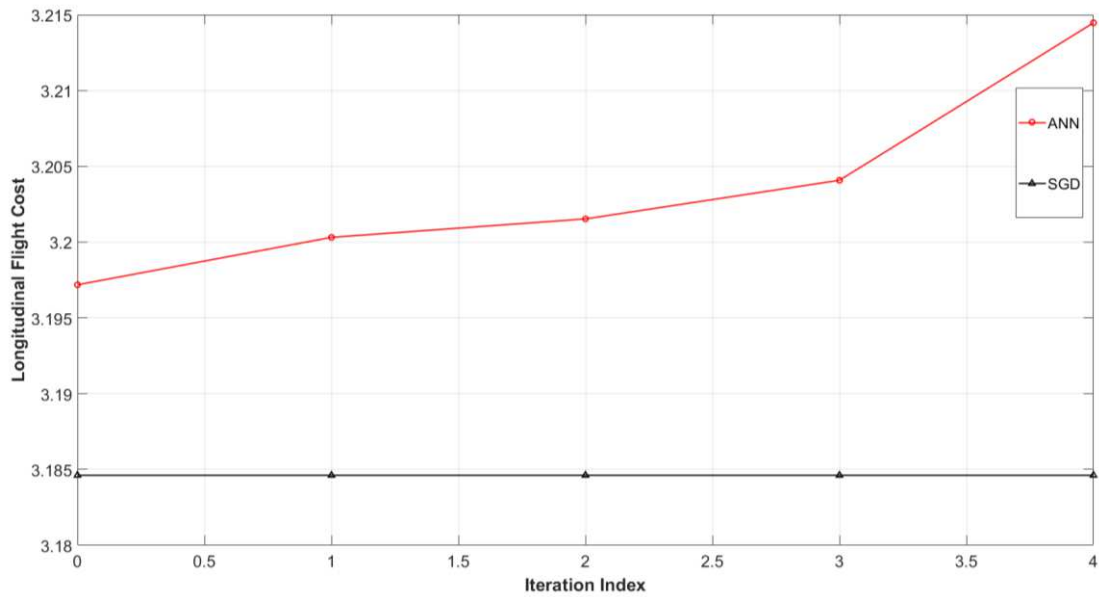
(e)



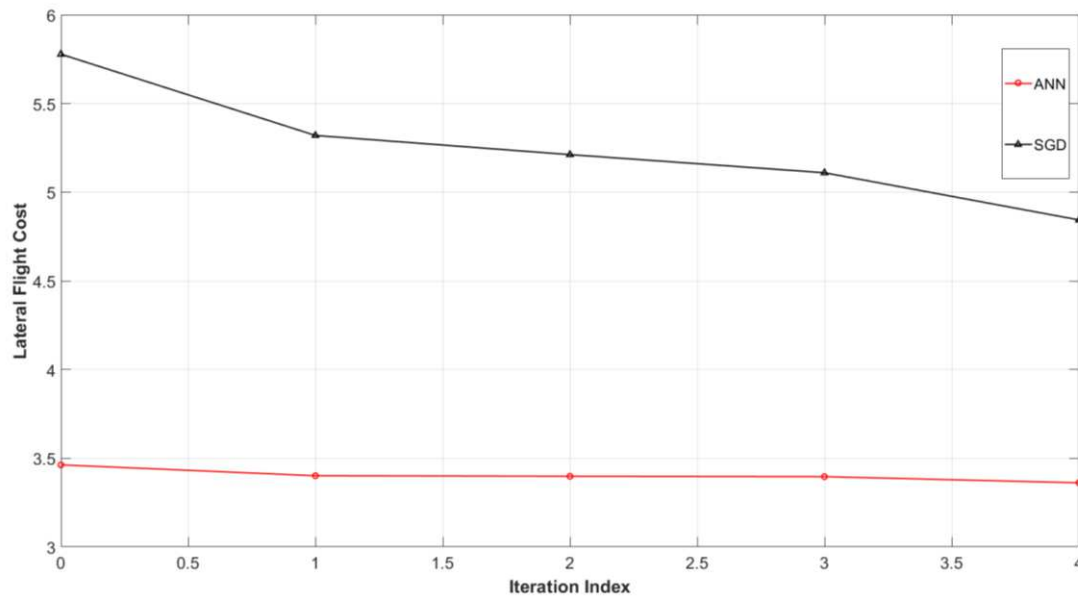
(f)

Figure 8. Longitudinal and lateral flight PID coefficients

After the hexarotor mass, moments of inertia and PID coefficients are obtained, longitudinal and lateral state space models are used to obtain the longitudinal and lateral flight cost functions. With the coefficients obtained from the models, the cost functions for longitudinal and lateral flight are obtained separately using equations 11 and 12. Accordingly, the cost functions are obtained separately with the parameters from ANN and SGD. The cost functions are shown in Figure 9.



(a)



(b)
Figure 9. (a) Longitudinal flight cost and (b) Lateral flight cost

After obtaining the longitudinal and lateral cost function, the total cost is obtained according to equation 10. The total cost for ANN and SGD is shown in Figure 10.

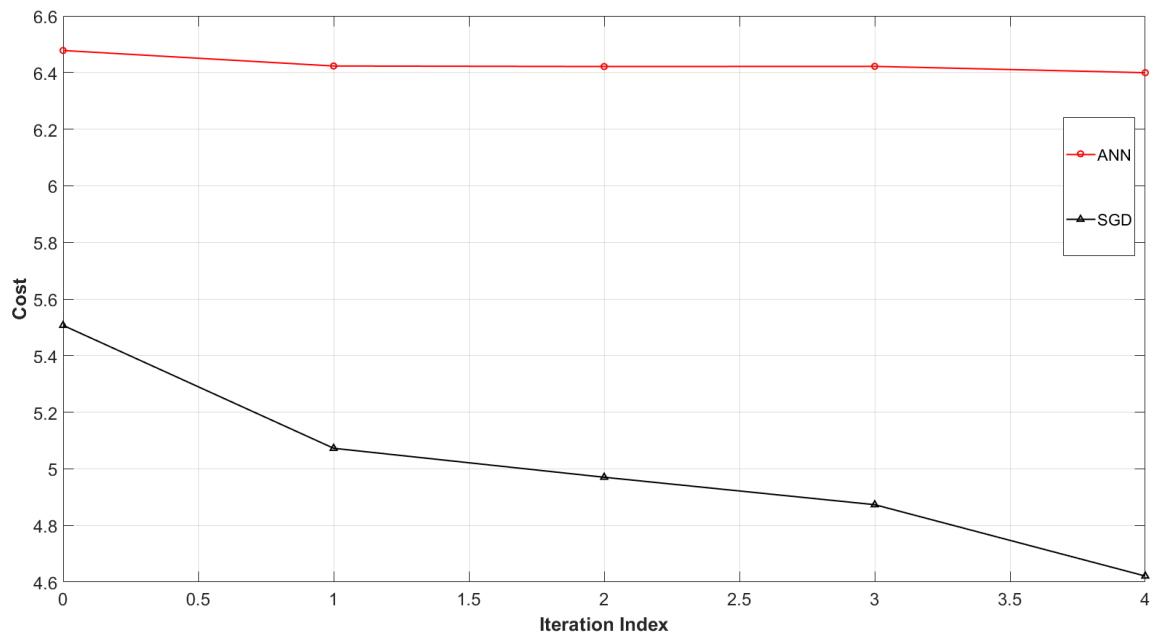


Figure 10. Total cost for ANN and SGD

According to equation 13, the improvement in cost is expressed as a percentage. Percentage improvement in cost is shown in Figure 11 for ANN and SGD.

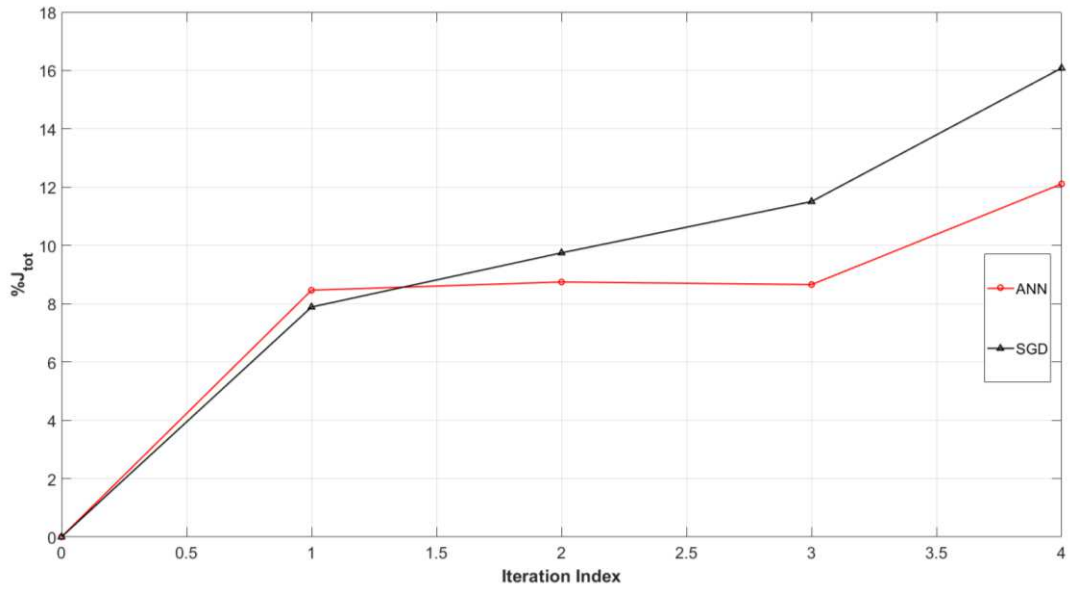


Figure 11. Percentage improvement in total cost for ANN and SGD

After all parameters are obtained, simulation of each iteration is performed in Matlab/Simulink environment. Simulation is performed using the state space model in Simulink environment. Figure 12 shows the longitudinal and lateral model in MATLAB/Simulink environment.

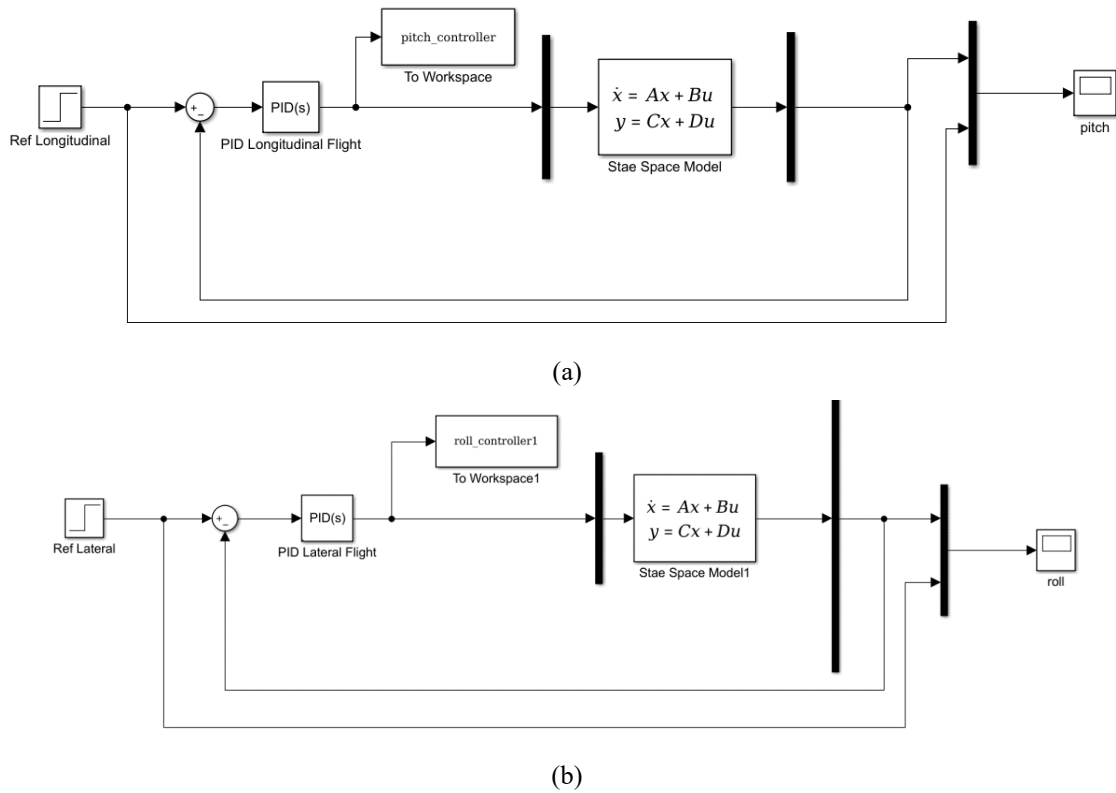
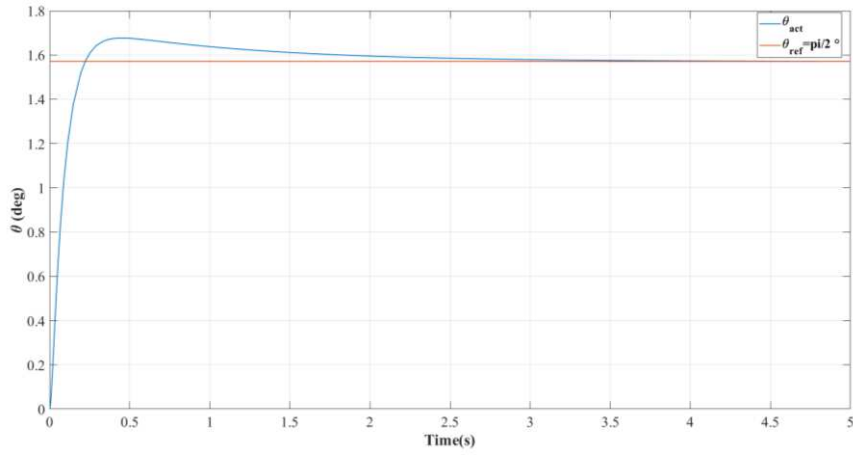


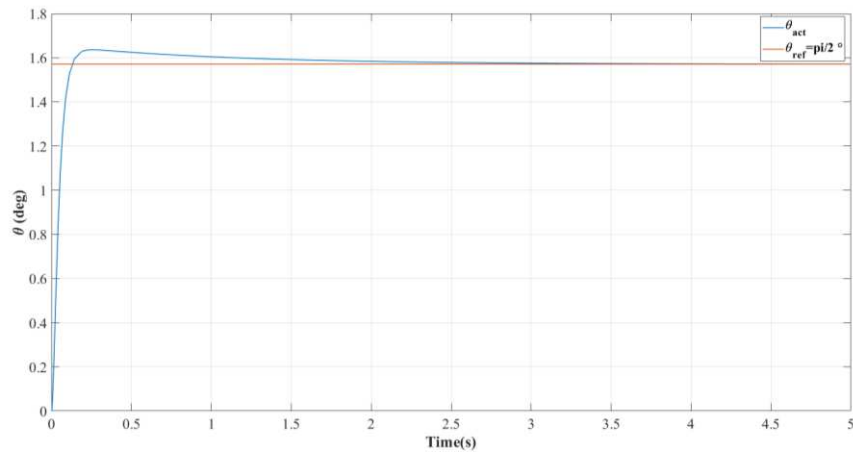
Figure 12. Matlab/Simulink Model (a) Longitudinal Flight and (b) Lateral Flight

It is seen from the iteration index value that SPSA reached the optimum result in 5 iterations in total. The first iteration is the initial iteration and the last iteration is the iteration where the optimum value is reached. In this study, in addition to obtaining maximum endurance, longitudinal and lateral flight control was also performed using the model created in MATLAB/Simulink environment with the values obtained from SPSA and machine

learning algorithms. Longitudinal and lateral flight control was performed by taking the values obtained from ANN and SGD separately for the model. For longitudinal flight, the hexarotor was requested to follow a $\pi/2^\circ$ trajectory. For ANN, Figure 13 a shows the first iteration of longitudinal flight and b shows the final iteration simulation result.



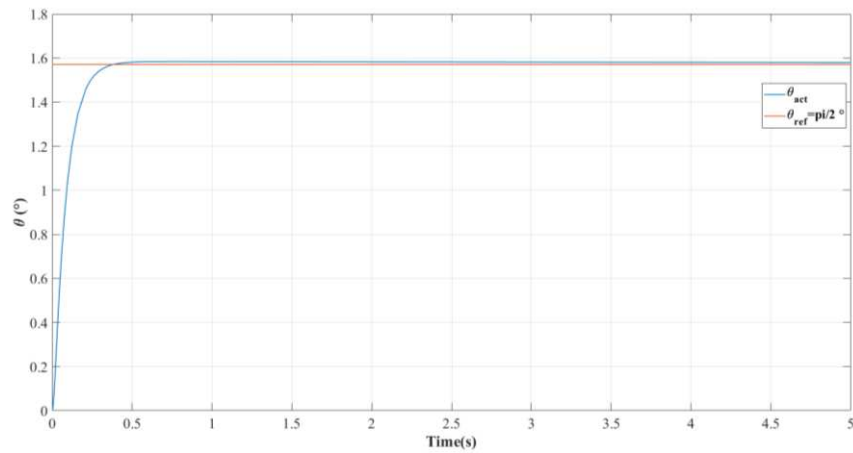
(a)



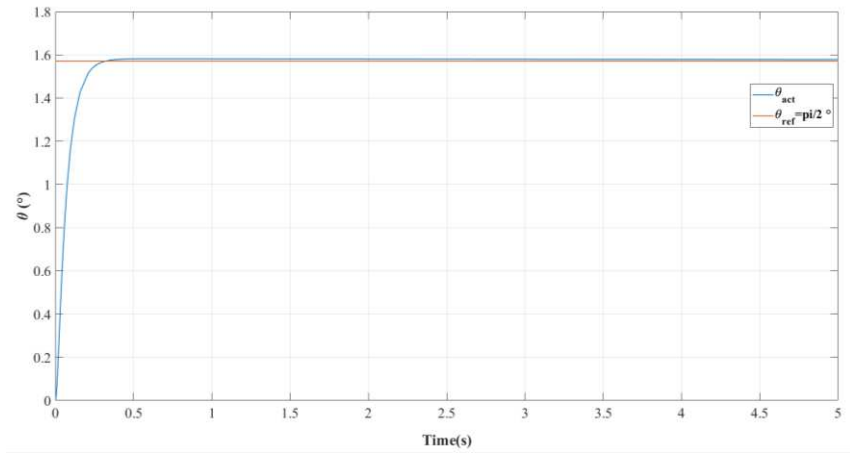
(b)

Figure 13. ANN longitudinal flight simulation results (a) First iteration and (b) Final iteration

For SGD, Figure 14 a shows the first iteration of longitudinal flight and b shows the last iteration simulation result.



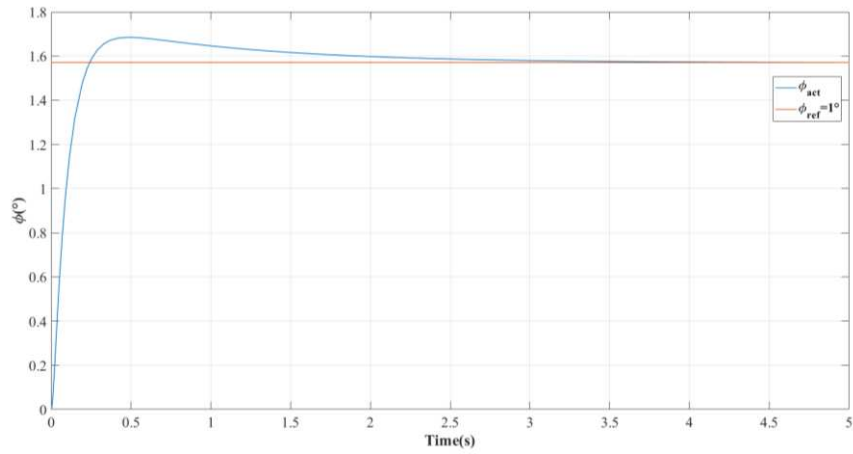
(a)



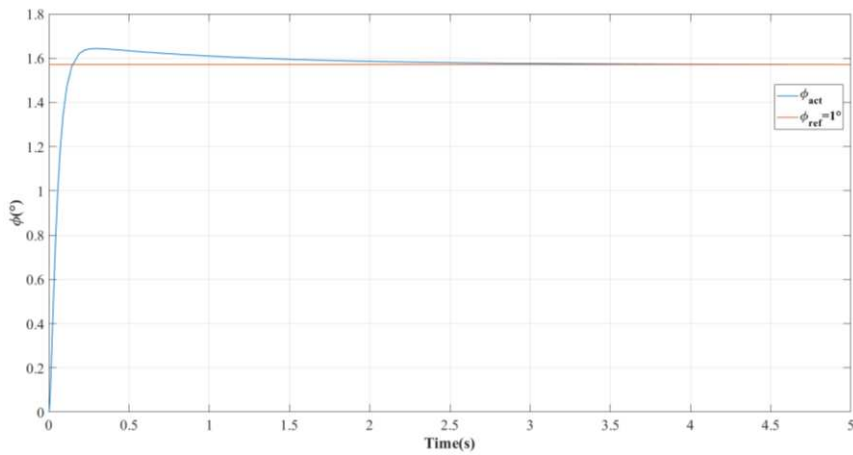
(b)

Figure 14. SGD longitudinal flight simulation results (a) First iteration and (b) Final iteration

For lateral flight, the hexarotor was desired to follow the 1° trajectory. For the ANN, Figure 15 a shows the first iteration of the lateral flight and b shows the final iteration simulation result.



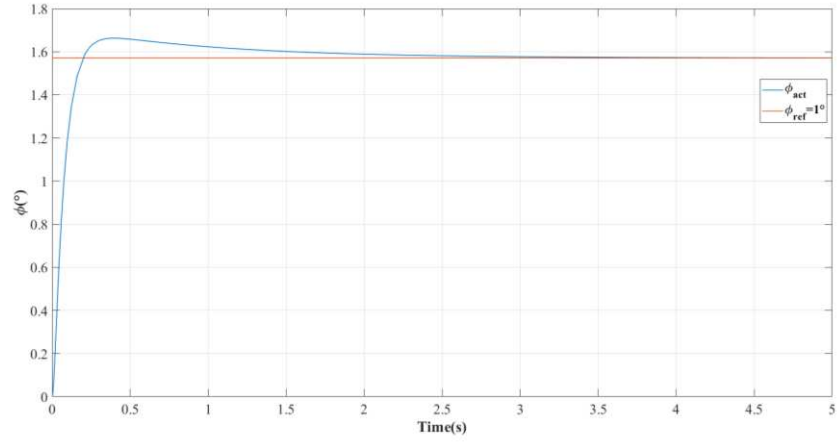
(a)



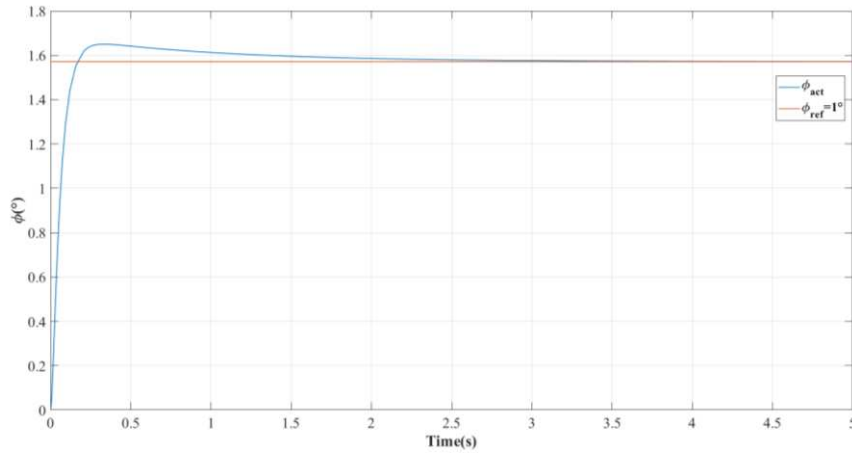
(b)

Figure 15. ANN lateral flight simulation results (a) First iteration and (b) Final iteration

For SGD, Figure 16 a shows the first iteration of lateral flight and b shows the final iteration simulation result.



(a)



(b)

Figure 16. SGD lateral flight simulation results (a) First iteration and (b) Final iteration

4. Conclusions

In this paper, it is aimed to maximise the endurance in simultaneous hexarotor longitudinal and lateral flight using arm redesign, optimisation method and machine learning. For this purpose, battery mass, which is one of the most important factors affecting endurance, is taken into account. The mathematical modelling of the hexarotor was performed by Newton-Euler method. After the mathematical modelling, the model was transferred to MATLAB/Simulink environment by using the state space model approach separately for both longitudinal and lateral flight. Using the stochastic optimisation method SPSA, the total cost function is minimised by using the design performance criteria of rise time, settling time and overshoot. Longitudinal and lateral flight cost functions are considered separately in minimising the total cost function. Arm redesign rate, 3 PID gains for longitudinal flight ($k_{p_{lon}}$, $k_{i_{lon}}$ and $k_{d_{lon}}$) and 3 PID gains for lateral flight ($k_{p_{lat}}$, $k_{i_{lat}}$ and $k_{d_{lat}}$) were estimated with SPSA. Thus, a total of 7 parameters were estimated with SPSA. The estimation of the battery mass ($m_{battery}$) and the moments of inertia in three axes (I_x , I_y and I_z) according to the arm redesign rate was performed with ANN and SGD. The data set for ANN and SGD was obtained from the models drawn in Solidworks drawing programme. The performance comparison of ANN and SGD was carried out with the performance metrics of R^2 , MSE and MAPE. Using SPSA and the parameters obtained by ANN and SGD separately, it was requested to follow a

trajectory of $\pi/2^\circ$ for 5 seconds for longitudinal flight and a trajectory of 1° for 5 seconds for lateral flight. Simulation results were evaluated based on the design performance criteria.

When the arm redesign rate was analysed, SPSA predicted a value between 37 cm and 65 cm. The prediction was made for both ANN and SGD. When Figure 3 is analysed, it is seen that arm redesign rate decreases for both ANN and SGD. Arm redesign rate reduction is important for the hexarotor to fly faster.

The data set given in table 1 is used to estimate the battery mass according to the arm redesign rate estimated in Figure 3. 70% of the data set is used for training and 30% for testing. The same data set is used for ANN and SGD. The performance of ANN and SGD is analysed according to the performance metrics in table 3. When the metrics are analysed, the R^2 value is examined separately for each output. Here, it is seen that the first output, battery mass, gives better results in ANN. The other outputs, moments of inertia, gave better results for SGD with very small differences. However, the inputs of the battery mass for SGD require better values to be selected in the data set. If the MAPE value between 5-10% is considered acceptable, it gives better results for SGD, while MSE is desired to be close to zero, here again SGD gave better results.

Figure 4-a shows the battery mass estimated by ANN and SGD. In ANN, it is seen that the battery mass increases while the arm redesign rate shortens. In the prediction made with SGD, it is seen that the battery mass decreases.

Figure 4-b, c and d show the moments of inertia. It is seen that the moments of inertia are within satisfactory limits compared to the hexarotor weight and the data in the dataset.

Figure 6 shows the energy density according to the battery mass. No estimation was made for the energy density parameter and the closest density was obtained by comparing the data in the data set according to the battery mass.

In Figure 7, the maximum endurance, which is the main topic of the article, is calculated according to the battery weight estimated by ANN and SGD. According to the estimated data, SGD started the optimisation with a higher endurance estimate and continued by increasing the endurance. ANN, on the other hand, started with a lower endurance and increased the endurance after the 2nd iteration. The reason for this is that the decrease in battery mass in SGD estimations decreases the $m_{takeoff}$ value in equation 14. Since this value is included in the equation as a divisor in equation 5, it increases endurance. It is seen that endurance increases with SGD. In this respect, it is seen that endurance increases numerically with the values predicted by SGD.

In Figure 8, the PID coefficients for both longitudinal and lateral flight are estimated by SPSA according to the arm redesign rate. The estimated PID coefficients did not show excessive oscillations and remained within satisfactory limits. According to the estimated PID coefficients, simultaneous design was performed for longitudinal and lateral flight. The improvement and performance improvement in the simultaneous design were measured by the total cost index. Longitudinal and lateral flight costs are a function of total cost. When Figure 9-a is analysed, it can be seen that for longitudinal flight, there is a cost increase in the values predicted with ANN, while there is no change in the cost with SGD. This situation shows that the predictions for longitudinal flight should be improved. When Figure 9-b is analysed, it is seen that the cost value decreases in the last iteration in the values predicted with ANN for lateral flight, while the cost value decreases from the first iteration in the values predicted with SGD. This situation affects the total cost. When the total cost is analysed in Figure 10, it is seen that the total cost is more minimised in the values estimated with SGD. In addition, Figure 11 shows the

improvement of the total cost as a percentage. When the graphs are analysed, it is seen that there is an improvement of 12% in total cost with ANN and 16% with SGD.

With the values obtained from SPSA, ANN and SGD in Figure 12, longitudinal and lateral flight simulations were performed for 5 seconds for the first iteration and the last iteration with the model created in Figure 15 in MATLAB/Simulink environment. When Figures 13, 14, 15, and 16 are analysed, longitudinal and lateral flight simulations gave satisfactory values. Simulations were performed for all iterations in the model. The simulations were compared with the design performance criteria of rise time, settling time and overshoot values. The design performance values for longitudinal and lateral flight are given in Tables 4 and 5.

Table 4. Longitudinal flight design performance criteria

	1st iteration		2nd iteration		3rd iteration		4th iteration		5th iteration	
	ANN	SGD	ANN	SGD	ANN	SGD	ANN	SGD	ANN	SGD
Rise time (sec)	0.141	0.169	0.119	0.153	0.120	0.147	0.115	0.141	0.075	0.137
Settling time (sec)	1.750	0.290	1.560	0.265	1.570	0.254	1.510	0.245	1.06	0.237
Overshoot (%)	6.730	0.784	5.880	0.626	5.910	0.626	5.700	0.600	4.150	0.655

Table 5. Lateral flight design performance criteria

	1st iteration		2nd iteration		3rd iteration		4th iteration		5th iteration	
	ANN	SGD	ANN	SGD	ANN	SGD	ANN	SGD	ANN	SGD
Rise time (sec)	0.154	0.123	0.115	0.112	0.112	0.108	0.111	0.104	0.087	0.101
Settling time (sec)	1.860	1.480	1.510	1.380	1.490	1.340	1.480	1.300	1.220	1.270
Overshoot (%)	7.260	5.860	5.700	5.480	5.620	5.320	5.560	5.180	4.620	5.050

When the design performance criteria were analysed, SGD gave better results than ANN except SGD rise time for longitudinal flight. Although the values for lateral flight are approximately the same, it is seen that ANN overshoot and settling time initial values are higher than SGD. In general, it is seen that the predictions made with SGD are optimum for longitudinal and lateral flight.

In this paper, an attempt has been made to maximise endurance and reduce the cost of longitudinal and lateral flight with arm redesign rate for hexarotor. The battery mass and moments of inertia changing with arm redesign rate are estimated by ANN and SGD. Arm redesign rate, longitudinal and lateral flight PID coefficients were estimated with SPSA. In the estimated values, it was observed that endurance was increased to maximum duration in the values estimated with SGD, and longitudinal and lateral flight gave better results in design performance criteria in the values estimated with SGD. It was also found that the total cost showed a 16% better improvement in the values predicted with SGD. It is observed that SGD predictions will give better results if the data set is selected and determined more appropriately. The real-time applications of this study will be realized within the scope of The Scientific and Technological Research Council of Turkey (TÜBİTAK) projects according to the data obtained from the simulation results. This study is an initial infrastructure for real-time application.

CRediT authorship contribution statement

Oguz KÖSE: Writing – review & editing, Writing – original draft, Visualization, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

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Data availability

Data will be made available on request

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