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Valley topological acoustic waveguides with novel diamond-geometry for ultrasonic sensing

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Abstract

In his work, we introduce a novel design for a two-dimensional topological acoustic waveguide based on three rhombic prisms (diamond-shaped) silicon phononic crystal within a conservative Hamiltonian framework. The unit cell structure with C_{3v} symmetric shows topological phase transition, in which the non-trivial bandgap shifts from the first to the second bandgap. Besides, the topological phase transition between unit cells oriented at $\alpha = 30^\circ$ and $\alpha = -30^\circ$ was quantitatively determined through Berry curvature calculations. The supercell interface is formed by arranging elements in the unit cell according to C_{3v} symmetry. Numerical analysis of the waveguide interface demonstrates robust transmission across the frequency interval corresponding to the bulk bandgap extends from 306 to 332 kHz. This mechanism enables novel pathways for acoustic wave propagation and lays a robust foundation for next-generation acoustic sensor technologies.

Keywords: Phononic crystal, Unit cell, Berry curvature, Supercell interface, Waveguide design, Transmission.

1. Introduction

Phononic crystal based topological waveguides represent essential devices for the efficient control of wave propagation in modern photonics, elastics, and phononics, as they provide reliable performance in applications such as communication, sensing, and energy-efficient integrated switchable acoustic devices. Phononic crystals (PnCs), structures exhibiting topologically conserved robustness are typically designed using unit cells with C_{3v} symmetry. These unit cells reveal topological phase alterations in their divergence properties [1]. The functionality of a topological acoustic waveguide depends critically on the band properties of a supercell interface,

which enables active control of efficient and robust propagation [2-3]. Several studies have already demonstrated that topological phase transitions can transfer energy and/or information not only in quantum systems but also in classical matters. This has triggered comprehensive studies across diverse platforms, ranging from photonics to acoustics, and from mechanics to electrics [4-8]. Basically, a similar phenomenon has been discovered in quasi two-dimensional phononic crystals, commonly known to as phononic-crystal plates [9-10]. For real world application, photons are presented by optical waves [11-12], while phonons are diagnosed by acoustic or elastic waves [13]. A tunable PnC plate incorporating Y-shaped prisms has been depicted to demonstrate valley transportation of resilient waves. In this system, pseudospin states of flexible waves show reverse chirality, analogous to spins of electrons, and are built on the principles of the quantum valley Hall effect [14]. By contrast, quantum spin Hall (QSH) systems require the presence of a double-Dirac cone, which is typically realized through the zone folding method at the high-symmetrical points. However, the non-trivial phase transitions for Lamb waves have successfully been reported realizing the low-frequency Hall effect in thin phononic crystal plates [15]. A waveguide for the propagation of sound has already been proposed by Z. Zhang et al. by revolving a unit cell with three legs without changing their lattice architecture [16].

In addition, a different type of elastic-wave insulator with reconfigurability has been modeled and experimentally demonstrated by some groups [17]. These works endorsed the mechanically spinning of the unit cell arrangement and loading/draining fluid into holes. Subsequently, a GaAs PnC has been modeled and unraveled to control over the chiral behaviors of mass valley modes by Kim *et al.* [18]. This is achieved by revolving triangular holes within the unit cell, thereby inducing phase transitions. Recently, W. Yuan et al. [19] and J. Sun et al. [20] have addressed phase changes in sonic crystals that in turn guide the development of topological sound waveguides. Topological protected waveguide has emerged as a novel class of architected structures that are inherently immune to defects and imperfections [21]. Their unique topological characteristics protect them against scattering losses, ensuring robust wave transmission. This immunity to defects in both photonic and phononic systems is of significant attraction due to their remarkable capability to confine and guide waves. Such properties make them highly desirable for a wide range of utilizations, including wave filters [22-24], waveguides [25-29], and sensors [30-31].

Although numerous studies have explored acoustic wave transmission, prior research has revealed that waveguides are complex for edge-mode propagation, thereby restricting the advancement of integrated switchable acoustic devices. In fact, these investigations highlighted the necessity for specialized mechanisms to restructure waveguide configurations while preserving their inherent robustness. In this study, we have introduced an innovative framework for a robust interface that presents a straightforward yet effective approach toward the advancement of highly integrated tunable acoustic devices. We performed the modeling and computational evaluations on diamond-shaped silicon topological acoustic waveguides embedded in a silicone rubber system using the finite element method (FEM). The structure outlines the pressure acoustic model emphasizing the diamond-shaped unit cell within the Brillouin Zone as well as the supercell interface to illustrate the theoretical methodology for phononic band characteristics. Then novel straight and Z-shaped valley topological acoustic waveguides have been designed and simulated enabling active regulation of robust transmission through the supercell interface.

2. Crystal design and band computation

2.1. Phononic crystals (PnCs) design

Phononic crystals (PnCs) can be realized in one, two, or three-dimensional structures. The periodic arrangement of phononic crystals (PnCs) induces constructive and destructive interference of acoustic waves at the interfaces of their constituent materials. Such interference can generate a stop band, referred to as a band gap, within a specific frequency range, where destructive interference inhibits wave propagation. The unit cell was designed using three rhombic silicon prisms positioned in a hexagonal lattice framework and enclosed within silicone rubber as shown in Fig. 1(a). This configuration, originally introduced by Okuno et al. and Ali et al. [3–4, 34–35], was selected for its straightforward fabrication. The geometric parameters of the structure were specified as follows: the lattice constant was set to $a = 2.2$ mm, while the prism rods possessed a diameter of $d = 1.1$ mm, yielding a corresponding radius of $r = d/2 = 0.55$ mm. Fig. 1(b) illustrates the corresponding Brillouin Zone, which retains the hexagonal symmetry characteristic of valley phononic crystals. The high-symmetry points, particularly K^+ and K^- , play a pivotal role in governing valley-dependent dynamics. Rotation of the unit cell modifies the band dispersion near these points, resulting in band inversion and the emergence of pseudo-spin states. Subsequently, a supercell consisting of valley phononic crystals (PnCs) was constructed. Each unit cell comprised three rhombic prism rods arranged in a configuration exhibiting C_{3v} symmetry, with $\alpha =$

30° and $\alpha = -30^\circ$, as elucidated in Fig. 1(c). The interface of the supercell, formed by two distinct PnC orientations, serves as the region through which acoustic waves within the band gap can propagate.

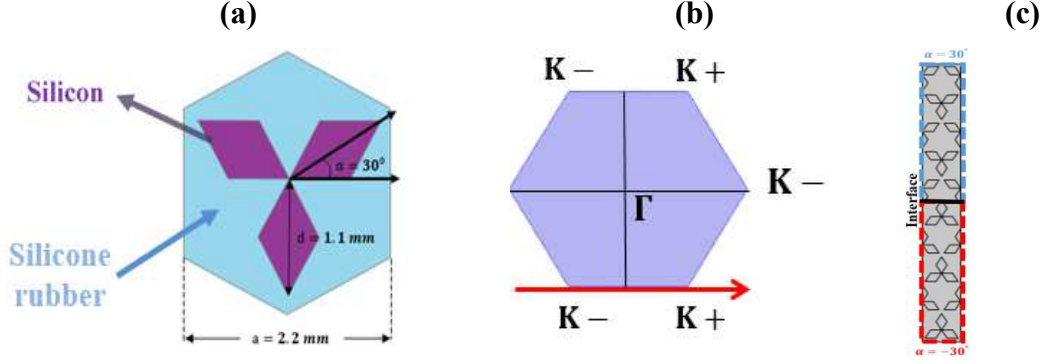


Fig. 1: Schematic diagram of a diamond-shaped silicon PnC: (a) hexagonal unit cell, (b) hexagonal Brillouin zone, and (c) supercell.

2.2 Computation of PnC waveguides

The diamond-shaped silicon phononic crystal (PnC) waveguide embedded in a silicone rubber matrix was modeled in a two-dimensional geometry using eigenfrequency analysis in COMSOL Multi-physics (version 6.3), utilizing the Acoustic Pressure Domain module. After constructing the unit cell, material properties such as speed of sound and density for silicon and silicone rubber were assigned from the built-in material library as Table 1. Floquet periodic boundary conditions were imposed, with the wave vectors designated as k_x and k_y . Following mesh generation, a parametric sweep over the wave vector range (0 to 3) was conducted. The results were visualized using 1D global plots and 2D surface and arrow plots of acoustic pressure (acpr). The same simulation procedure was repeated for structures with rods rotated by $+30^\circ$ and -30° . From these analyses, the band gaps were identified, and the band inversion property of the crystals was observed. For the supercell analysis, a similar approach was followed to evaluate band structures. The supercell was designed such that regions with $+30^\circ$ and -30° rotated crystals formed an interface between the structures. For each calculation, wave vectors (k-points) were set as global variables as the structure required. Finally, a waveguide of $p \times q$ dimensions were designed. Perfectly Matched Layers (PML) along with other appropriate boundary conditions were implemented to accurately compute transmission using an averaging operator (aveop). Wave propagation characteristics were also examined through 2D field distribution plots.

Table 1: Properties of materials used for the design of diamond-shaped silicon PnC.

Materials	Speed of sound (m/s)	Density (kg/m ³)
Silicone Rubber	1100	1100
Silicon	8433	2300

3. Results and analysis

3.1 Orientation-dependent band structure of diamond-shaped silicon PnCs

3.1.1 Dispersion relation of the silicon phononic crystal unit cell

Fig. 2 illustrates the band structures of the diamond-shaped silicon PnC unit cell embedded in silicone rubber. The angle (α) is the respective orientation of the silicon pole array within the hexagonal lattice governs the symmetrical configuration. When $\alpha = 30^\circ$, the band structures shown in Fig. 2(a), display valley-type divergence at the k point, accompanied by the emergence of a bandgap in the frequency range of 306–332 kHz. In contrast, at $\alpha = 0^\circ$, the presence of bulk mode is observed, as depicted in Fig. 2(b). Alternatively, when the orientation angle is reversed to $\alpha = -30^\circ$, a band inversion emerges as unveiled in Fig. 2(c). Rotating the unit cell from $+30^\circ$ to -30° induces band inversion and generates topologically protected edge states, enabling robust phononic waveguides.

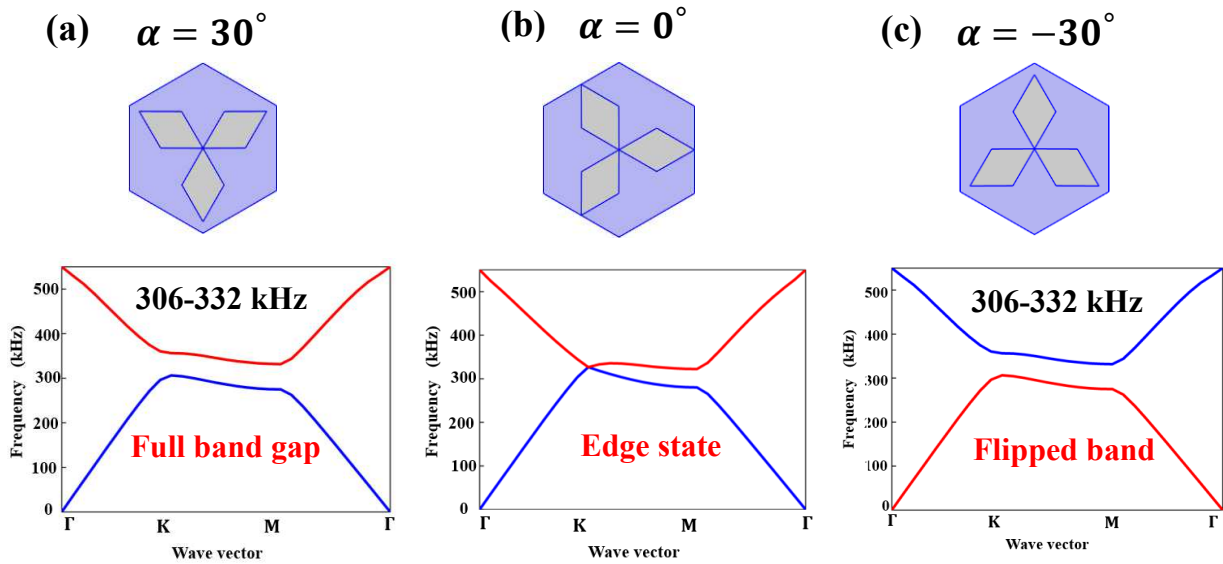


Fig. 2: Band structure of a C_{3v} symmetry hexagonal diamond-shape silicon PnC unit cell: (a) $\alpha = 30^\circ$, (b) $\alpha = 0^\circ$, and (c) $\alpha = -30^\circ$.

3.1.2 Topological phase in terms of Berry curvature

We employed the Berry curvature framework by examining the unit cell contributions from both the upper and lower bands near the band gap to determine the topological phase of each phononic band. The Berry phase of the n th band along a closed contour C is defined in terms of the Berry connection, while its curl yields the Berry curvature.

$$\gamma_n = \oint_C \vec{\Lambda}_n(\vec{k}) \cdot d\vec{k} = \iint_S \Omega_n(\vec{k}) d^2k$$

The quantity $\Lambda_n(\vec{k})$ is the Berry connection, where $\Lambda_n(\vec{k}) \equiv \langle u_n(\vec{k}) | i \nabla_{\vec{k}} | u_n(\vec{k}) \rangle$, and $\Omega_n(\vec{k}) = \nabla_{\vec{k}} \times \Lambda_n(\vec{k})$ is the Berry curvature [36].

In our formulation, the Berry curvature at a specific point (k_x, k_y) in reciprocal space is evaluated using four surrounding points [37-38] that form a counterclockwise rectangular loop.

$$\Omega(\vec{k}) = - \frac{I_m \left[\langle u(\vec{k}_1) | u(\vec{k}_2) \rangle + \langle u(\vec{k}_2) | u(\vec{k}_3) \rangle + \langle u(\vec{k}_3) | u(\vec{k}_4) \rangle + \langle u(\vec{k}_4) | u(\vec{k}_1) \rangle \right]}{\delta k_x \times \delta k_y}$$

where $\vec{k}_j$ ($j = 1, 2, 3, 4$), and

$$\begin{aligned} k_1 &= k_1 \left(k_x - \frac{\delta k_x}{2}, k_y - \frac{\delta k_y}{2} \right) \\ k_2 &= k_2 \left(k_x - \frac{\delta k_x}{2}, k_y + \frac{\delta k_y}{2} \right) \\ k_3 &= k_3 \left(k_x + \frac{\delta k_x}{2}, k_y + \frac{\delta k_y}{2} \right) \\ k_4 &= k_4 \left(k_x + \frac{\delta k_x}{2}, k_y - \frac{\delta k_y}{2} \right) \end{aligned}$$

The displacement vector field $u_n(\vec{k})$ corresponds to the two-dimensional eigenmode, and in our numerical analysis all eigenmodes $\langle u_n(\vec{k}) | u_n(\vec{k}) \rangle$ were normalized through integration over the unit cell surface.

Fig. 3(a), and Fig. 3(b) illustrate the topological characteristics of the upper and lower bands for unit cell configurations with $\alpha = 30^\circ$, and $\alpha = -30^\circ$, respectively. These results reveal that the sign of the Berry curvature around the K^+ and K^- valleys in both bands undergoes a continuous reversal during the translational shift, highlighting the phase transition behavior.

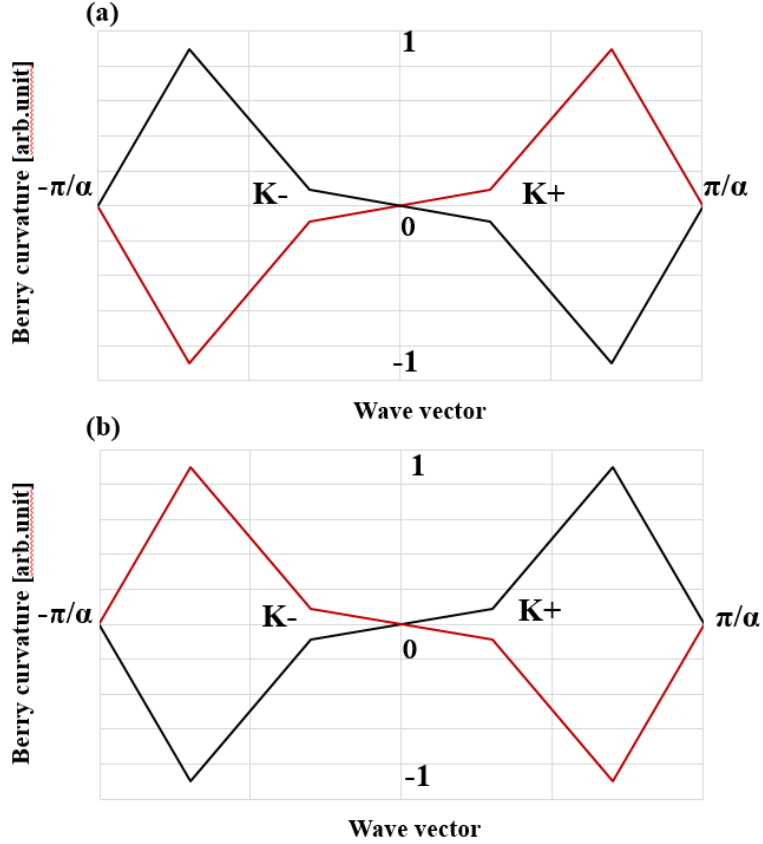


Fig. 3: Berry curvature distributions of the lower and upper phononic bands in reciprocal space around the K^+ and K^- valleys: (a) unit cell with $\alpha = 30^\circ$, and (b) unit cell with $\alpha = -30^\circ$.

As illustrated in Fig. 4(a) and Fig. 4(b), the acoustic intensity distribution of the unit cell is displayed for orientation of $\alpha = 30^\circ$, and $\alpha = -30^\circ$. The upper and lower bands are represented as inverted prisms within the hexagonal unit cell, highlighting the reversal in direction. The distribution of the pressure field at 356 kHz (upper mode) of in an $\alpha = 30^\circ$ oriented unit cell is shown to be like the distribution of pressure field at 306 kHz (lower mode) in an $\alpha = -30^\circ$ oriented unit cell. Likewise, the distribution of the pressure field at 306 kHz (lower mode) in an $\alpha = 30^\circ$ oriented unit cell is observed to be similar to the distribution of pressure field at 356 kHz

(upper mode) in an $\alpha = -30^\circ$ oriented unit cell. The acoustic pressure distribution at the K^+ and K^- points demonstrates that the chiral states of the upper and lower bands undergo continuous flipping. This behavior signifies the occurrence of a band inversion process, which serves as a hallmark of a topological phase transition. Such a transition enables valley-selective propagation of acoustic waves, thereby confirming the system's nontrivial topological characteristics.

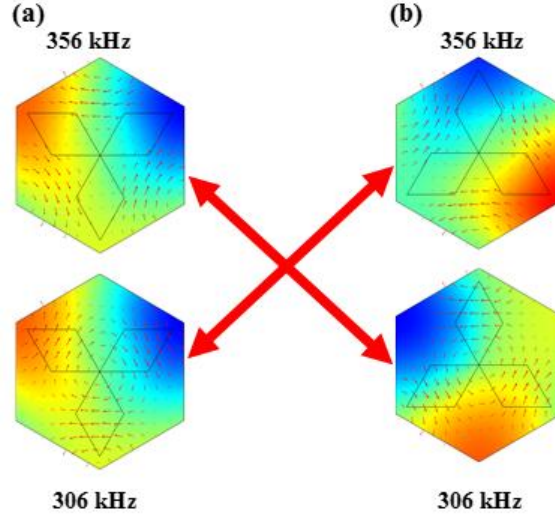


Fig. 4: Acoustic intensity of the diamond-shaped silicon PnC unit cell for (a) $\alpha = 30^\circ$, and (b) $\alpha = -30^\circ$.

3.1.3. Band structure of diamond-shaped silicon PnC supercell

A silicon phononic crystal (PnC) supercell composed of valley phononic crystals has been designed, in which each unit cell contains three rhombic prism rods arranged with C_{3v} symmetry at an orientation angle of $\alpha = 30^\circ$, as shown in Fig. 5(a). This supercell has a width of a , and a height of $\frac{5a}{\sqrt{3}}$, respectively. Within this arrangement, a clear phononic bandgap appears in the frequency range of 306–332 kHz. Fig. 5(b) further demonstrates localized modes near this frequency window. In a subsequent configuration, another silicon PnC supercell is constructed wherein every unit cell consists of three rhombic prism poles arranged in a C_{3v} symmetry with orientations of $\alpha = 30^\circ$ and $\alpha = -30^\circ$ as illustrated in Fig. 5(d). The supercell consists of upper and lower layers of unit cells oriented in opposite directions, with a topological interface separating them. An edge mode is identified within the same frequency range of 306–332 kHz, with two pseudo-spin modes emerge at the K^+ and K^- points (nearby the K points in the supercell Brillouin

zone), as depicted in Fig. 5(c). The acoustic pressure distribution field confirms the presence of bulk modes. This finding not only substantiates the topological characteristics of the system but also highlights its influence on waveguide transmission along the supercell interface.

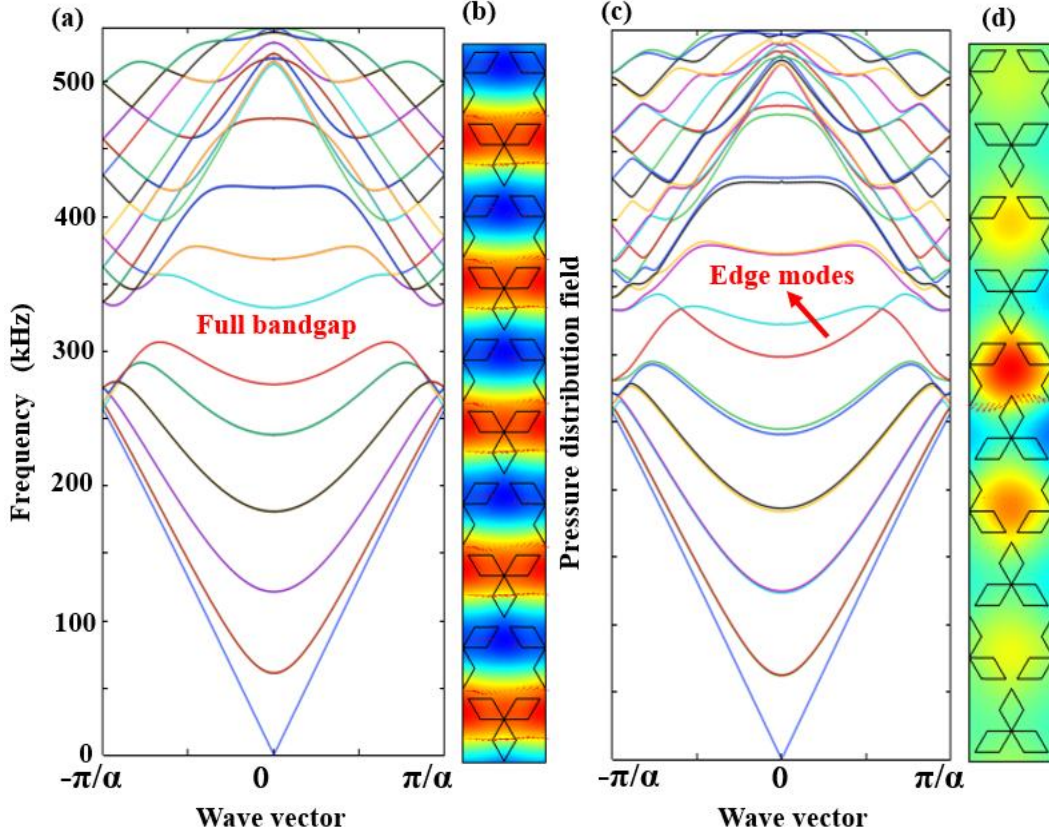


Fig. 5: Supercell phononic band and normalized pressure field (a-b) C_{3v} symmetric uniform structure with orientation $\alpha = 30^\circ$, and (c-d) C_{3v} symmetric structure with different oriented layers separated by topological interface.

3.2 Waveguide design and Transmission analysis

3.2.1. Characteristics of straight waveguide

A valley topological uniform waveguide is initially constructed, consisting of 40×53 rectangular unit cell arrays arranged in C_{3v} symmetry with $\alpha = 30^\circ$, and embedded in silicone rubber. The configuration and corresponding pressure field distribution are shown in Fig. 6(a), wherein an incoming acoustic wave is introduced from the left port and transmission was measured at the right output. No transmission is observed along the bulk band gap frequency 306-332 kHz, as shown in

Fig. 6(b). Subsequently, a valley straight waveguide is designed using the same 40×53 unit cell arrays arranged in two C_{3v} symmetrical orientations, $\alpha = 30^\circ$ and $\alpha = -30^\circ$, in turn, and embedded in silicone rubber. Fig. 6(c) presents the waveguide layout and pressure field distribution under the same excitation conditions. The yellow and blue regions in Fig. 6(c) correspond to unit cells oriented at $\alpha = 30^\circ$, and $\alpha = -30^\circ$, respectively. The yellow, and blue color (highlighted in the upper and lower portion) of Fig. 5(c) represents different layers of unit cells $\alpha = 30^\circ$, and $\alpha = -30^\circ$, respectively. As highlighted by the red arrow in Fig. 5(d), effective wave propagation is observed within the bandgap of 306–332 kHz, consistent with the analysis discussed earlier.

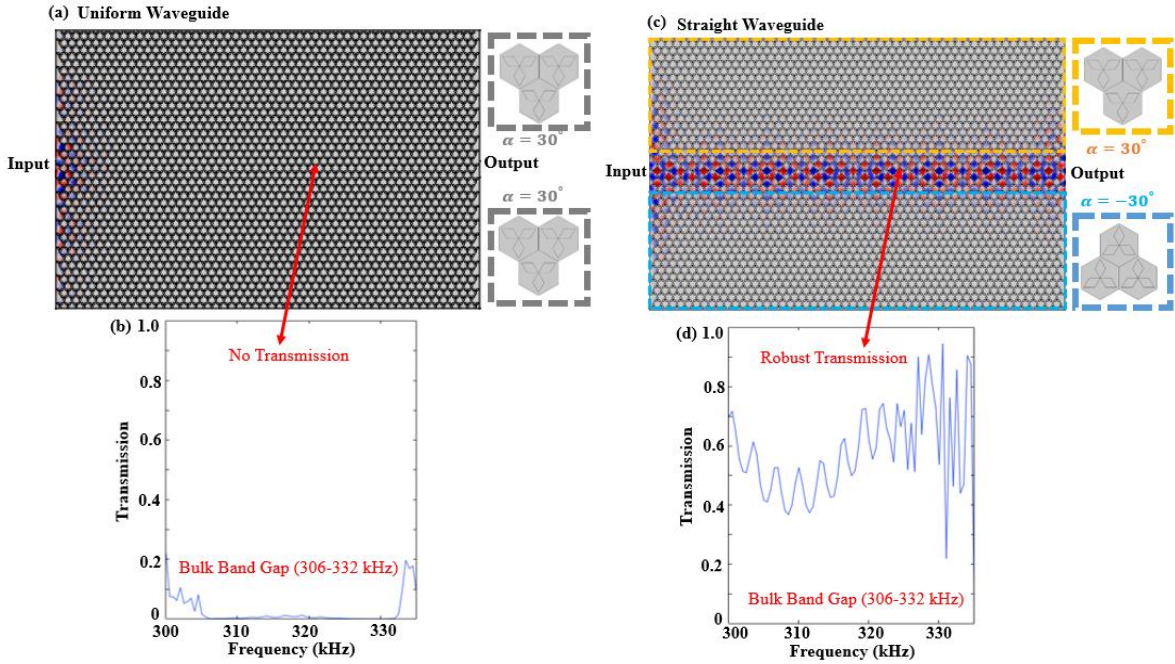


Fig. 6: Normalized pressure and transmission spectrum of valley phononic (a-b) uniform waveguide, and (c-d) straight waveguide.

3.2.2. Characteristics of Z-shaped waveguide

A Z-shaped acoustic waveguide was also designed to verify the robustness of topological edge states when subjected to structural bends and imperfections. While straight waveguides allow acoustic waves to propagate along the interface, they do not fully demonstrate resilience. To address this, a 40×53 rectangular array of unit cells is constructed in a uniform C_{3v} symmetric arrangement with an orientation of $\alpha = 30^\circ$, immersed in silicone rubber. Subsequently, unit cells

with C_{3v} symmetry oriented at $\alpha = -30^\circ$ are introduced, separated from the $\alpha = 30^\circ$ configuration in the half layer of the right-side by a slanted interface. The positions of the top ten layers are fixed, while the bottom ten layers were fixed with unit cells oriented at $\alpha = 30^\circ$. Fig. 7(a) illustrates the Z-shaped waveguide layout with the pressure distribution under excitation from the left port, with transmission measured at the right output. Fig. 7(b) and Fig. 7(c) highlight red arrow indicators that clearly demonstrate effective wave propagation within the bulk band-gap frequency range of 306–332 kHz, consistent with the analysis presented earlier.

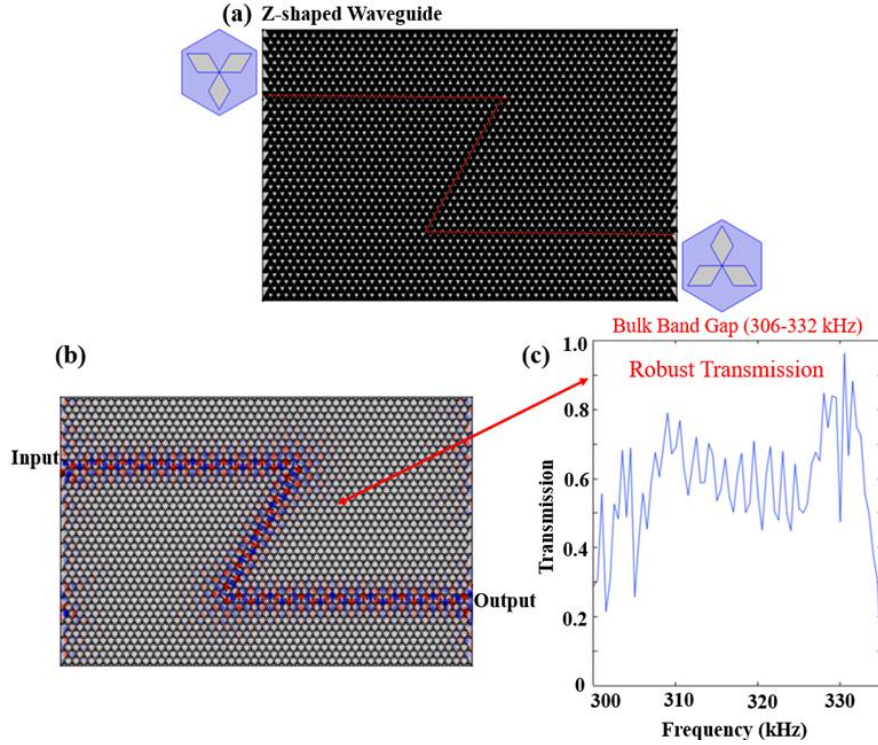


Fig. 7: (a) Design of a Z-shaped (Mesh) waveguide, (b) normalized pressure field of the proposed Z-shaped waveguide, and (c) the transmission spectrum.

4. Conclusions

We have introduced a novel design of an acoustic waveguide that exhibits a topological phase transition, quantitatively characterized through Berry curvature analysis, and enables dynamic regulation of robust transmission along the supercell boundary/interface. The proposed design demonstrates effective applicability to vibration control in engineering structures and noise reduction within complex systems. Furthermore, numerical simulations confirm the potential of

such systems for developing energy-conserving surface acoustic and elastic wave devices based on valley-topological phononic crystals. The results reveal that the present work paves a new avenue toward a deeper comprehension of topological elastic and acoustic phenomena, highlighting the central role of Berry curvature in bridging local geometric properties with global topological invariants.

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Ethics, Consent to Participate, and Consent to Publish declarations

Not applicable

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