

Seasonal Variability of Soil Radon Exhalation Under Climate Forcing: An Example of Humid Subtropical Highland Climate

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Abstract

Radon (^{222}Rn) migrates from soil to the atmosphere by molecular diffusion and pressure-driven advection while decaying with a 3.82-day half-life. Surface exhalation varies seasonally because the two governing soil properties, the effective diffusion coefficient and the radon source strength, both depend on soil moisture and temperature. Existing one-dimensional models are typically solved at steady state or with static properties, and few openly available tools ingest meteorological forcing to resolve the flux as a time-evolving signal. We present a model that solves the full transient diffusion–advection–decay equation in a layered 1-D soil column (0–200 cm) using an implicit Crank–Nicolson scheme. The effective diffusion coefficient follows the Rogers–Nielson moisture relation with a gas-kinetic temperature correction; pore air–water partitioning uses the temperature-dependent Ostwald coefficient; emanation responds to moisture and temperature; and a damped analytical thermal wave plus a leaky-bucket moisture model translate monthly climate inputs into depth- and time-resolved forcing. The solver matches the exact analytical steady state to a root-mean-square error of 0.0003%. Under idealized seasonal forcing representative of a humid subtropical highland (Murree-type) climate, the modelled flux is strongly bimodal: a pre-monsoon June maximum from warm, drying soil, a pronounced July–August monsoon minimum where high saturation collapses diffusivity and traps radon at depth, and an autumn recovery. The annual mean is $\sim 245 \text{ Bq m}^{-2} \text{ h}^{-1}$ with a peak-to-peak range near 40%. Sensitivity analysis shows near-linear scaling with radium activity, emanation and bulk density, and nonlinear response to porosity and moisture. A factorial attribution experiment shows the two controls act nearly additively, moisture contributing a larger variance share ($\approx 60\%$) than temperature ($\approx 40\%$); moisture alone reproduces the bimodal shape, temperature alone a single summer maximum. This competition between moisture-enhanced emanation and moisture-suppressed diffusion reconciles the apparent paradox that wetting raises soil-air radon yet can reduce exhalation, providing a reproducible, climate-forced framework for interpreting exhalation time series and generating hypotheses about climate-driven change.

Introduction

Radon-222 is a radioactive noble gas produced by the decay of radium-226 in the uranium-238 series. Because it is chemically inert and has a half-life of 3.82 days, it can migrate appreciable distances through the soil pore network before decaying, making it both a leading natural contributor to human radiation dose and a widely used tracer of subsurface and near-surface processes. The transfer of radon from the soil column to the atmosphere the surface exhalation flux is the boundary process that links the soil radon reservoir to outdoor and indoor air, and its magnitude and variability are central to radiological risk assessment, geophysical tracing, and soil-physics studies [1].

In one dimension, radon activity concentration in the soil pore air is governed by a transport equation that combines molecular diffusion down the concentration gradient, advection by pressure-driven soil-gas flow, radioactive decay, and a production term representing radium emanation. This formulation is well established and has been solved numerically by many authors, who have generally found close

agreement between finite-difference solutions and analytical or experimental references [1,2,3]. However, two limitations recur. First, many implementations target the steady state only, discarding the time evolution that is essential for comparison with continuous field measurements. Second, soil transport properties are commonly held constant, even though the diffusion coefficient and the radon source both vary strongly with the soil-water content and temperature that change through the year.

The role of soil moisture is now well documented. Partitioning of radon between the air- and water-filled fractions of the pore space, together with the moisture dependence of emanation, increases soil-air radon concentration during wetter periods; Arvela et al. [4] attributed much of the observed seasonal variation in soil and indoor radon to changes in soil-water saturation, while laboratory studies confirm that exhalation depends sensitively on moisture content [5,6]. Temperature exerts an additional, smaller control: higher soil temperatures raise the emanation fraction and gas diffusivity and lower radon's aqueous solubility. Arvela et al. [4] calculated that summer soil temperatures increase soil-gas radon by roughly 14% relative to winter, a figure echoed in subsequent reviews [7].

Interest in these couplings has intensified with attention to climate change. A 2025 review concluded that shifting temperature and precipitation regimes, extreme weather, and climate-driven geological change can all alter radon emission and accumulation, and identified the integration of climate science with radon transport as a priority [7]. Recent modelling has begun to respond: a 2025 study of Peruvian agroecosystems combined artificial neural networks with Monte Carlo simulation of radon and its progeny to relate exhalation to meteorological and soil variables [8], and coupled heat–moisture–air transport has been simulated for radon release from building walls [9]. What remains scarce is an openly available tool that (i) solves the full transient diffusion–advection–decay equation, (ii) ingests meteorological time series to force soil moisture and temperature seasonally, and (iii) outputs surface exhalation flux as a time-evolving signal directly comparable with detector measurements such as LR-115 track records.

This paper presents such a tool and applies it to an idealized seasonal climate representative of the Murree region in the outer Himalayas of Pakistan, driven by a prescribed sinusoidal temperature cycle and a leaky-bucket soil-moisture model rather than by observed meteorological time series, where natural radioactivity in soil and spring water has been characterised in recent surveys [10,11]. We describe the governing equation and its moisture and thermal coupling, verify the Crank–Nicolson solver against the exact analytical steady state, and analyse the resulting seasonal exhalation signal, its physical drivers, and its parameter sensitivities. The complete source code is released so that other workers can substitute their own meteorological forcing and field measurements.

Background

2.1 Transport Physics

Radon transport in porous soil is generally described by Fickian diffusion plus Darcy advection, with radioactive decay and a radium-driven source. Nazaroff [1] gave the canonical review of soil-to-air radon transport on which most one-dimensional models build. Subsequent numerical work solved the diffusion–advection equation for soils, building materials and concrete using finite-difference and finite-element methods, frequently validating against analytical solutions for simple geometries [2, 12]. Chitra et al. [3] solved the one-dimensional model with the finite-difference method, adopted an implicit scheme to overcome explicit-scheme stability limits, fed the model with experimentally derived parameters, and compared the simulated radon depth profile against measurements. Our work extends this lineage by retaining full-time dependence and by making the transport coefficients vary with climate-forced moisture and temperature.

2.2 Moisture, Partitioning and Emanation

Rogers and Nielson [14] analysed radon generation and transport across the solid, liquid and gas phases of porous media and derived the widely used semi-empirical expression for the bulk diffusion coefficient as a function of porosity and water saturation. Radon's distribution between pore air and pore water is set by its solubility, whose temperature dependence is commonly represented by the Ostwald coefficient of Weigel [15]. Emanation, the fraction of radon atoms that escape the mineral grains into the pore space, rises with the first few per cent of moisture, because water films arrest recoiling atoms that would otherwise embed in adjacent grains; Markkanen and Arvela [16] measured emanation coefficients and their dependence on water content and temperature across hundreds of soil samples. The net effect is that moderate wetting raises soil-air radon concentration [4], even though heavy wetting can simultaneously throttle the diffusive escape of that radon to the surface.

2.3 Numerical Methods and the Research Gap

Implicit time-stepping schemes are preferred for diffusion-dominated problems because they are unconditionally stable, allowing time steps unconstrained by the explicit stability limit [17]. The Crank–Nicolson scheme, a second-order-accurate average of the forward and backward Euler methods, is a standard choice. Despite the maturity of both the transport physics and the numerical methods, no openly available, transient, climate-forced Python tool combines them with real meteorological forcing to produce a continuous exhalation-flux signal. Recent seasonal field campaigns that report summer–winter contrasts in soil radon and exhalation, often measured with LR-115 detectors, would benefit from exactly such a forward model for interpretation. The present tool is designed to fill this niche.

Materials and Methods

This section sets out the full mathematical formulation of the model, the moisture and temperature coupling, the finite-difference discretisation and its time integration, the surface-flux diagnostic, and the analytical benchmark used for verification. Equation numbers are given in parentheses; symbols are defined at first use and summarised in Table 1. The complete implementation was written in Python and run in Jupyter Notebook.

3.1 Conceptual Model and Governing Equation

Radon-222 (^{222}Rn) is generated within soil grains by the decay of radium-226, a fraction of it escaping into the pore space (emanation). Once in the interconnected pore network it migrates toward the surface by molecular diffusion down its concentration gradient and by advection with pressure-driven soil-gas flow, decaying en route with a half-life of 3.82 d [1, 14]. We consider a one-dimensional, vertically layered soil column with depth coordinate z (m, positive downward) over $0 \leq z \leq H = 2$ m.

Let $C(z, t)$ be the radon activity concentration in the pore air (Bq m^{-3}) and ϵ the porosity available to radon. Conservation of radon activity over a representative elementary volume, with diffusive flux J_d and advective flux J_a , a first-order decay sink $\lambda \epsilon C$ and an emanation source written so that a closed volume relaxes to the equilibrium concentration C_{eq} , gives the activity balance:

$$\frac{\partial (\epsilon C)}{\partial t} = \frac{\partial}{\partial z} \left(D_e \frac{\partial C}{\partial z} \right) - \frac{\partial (v \epsilon C)}{\partial z} - \lambda \epsilon C + \lambda \epsilon C_{eq}$$

1

Diffusion follows Fick's law with the effective (bulk) diffusion coefficient D_e ($\text{m}^2 \text{s}^{-1}$); advection follows Darcy transport with pore (interstitial) velocity $v = q/\epsilon$, where q is the Darcy flux:

$$J_d = -D_e \frac{\partial C}{\partial z}, \quad J_a = vC$$

2

Within a single transport time step the moisture state (and hence ϵ , D_e and C_{eq}) is treated as quasi-static and is updated between steps by the forcing models of Sections 3.3–3.4; this operator-splitting allows ϵ to be divided out of the storage and source terms. Substituting (2) into (1) and dividing by ϵ yields the working transport equation solved here, in the canonical diffusion–advection–decay form [1, 3]:

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial z} \left(D_e \frac{\partial C}{\partial z} \right) - v \frac{\partial C}{\partial z} - \lambda C + \lambda C_{eq}$$

3

Here D_e is interpreted as the bulk coefficient referenced to the pore-air gradient (Section 3.2). The decay constant follows from the radon half-life $T_{1/2} = 3.8235$ d:

$$\lambda = \frac{\ln 2}{T_{1/2}} \approx 2.10 \times 10^{-6} \text{ s}^{-1}$$

4

The source term λC_{eq} represents radium emanation. The volumetric radon production rate is $P_V = \lambda A_{Ra} \rho_b f$, where A_{Ra} is the Ra-226 specific activity ($Bq\ kg^{-1}$), ρ_b the dry bulk density ($kg\ m^{-3}$) and f the emanation coefficient. In a closed volume, production balances decay ($\lambda \epsilon_{eff} C_{eq} = P_V$), which defines the deep-soil equilibrium concentration:

$$C_{eq} = \frac{A_{Ra} \rho_b f}{\epsilon_{eff}}$$

5

where ϵ_{eff} is the effective porosity available to radon (Section 3.2). Equations (3) and (5) close the transport problem once D_e , ϵ_{eff} , f and v are specified as functions of the soil state.

3.2 Moisture- and Temperature-Dependent Transport Properties

The two quantities that govern transport – the diffusion coefficient and the source strength – both depend on the soil-water content through the partitioning of radon between the air- and water-filled pore fractions and through the moisture dependence of emanation and diffusivity [4, 14].

3.2.1 Effective Porosity and Radon Partitioning

Radon resides in both the air- and water-filled pore space. With total porosity ϕ and water saturation S ($0 \leq S \leq 1$), and with the dimensionless Ostwald solubility coefficient $L(T)$ giving the water/air partition, the effective porosity is:

$$\epsilon_{eff} = \phi (1 - S) + L(T) \phi S = \phi [1 - S(1 - L(T))]$$

6

The solubility increases as temperature falls; we use the widely adopted empirical relation of Weigel [15], with T in $^{\circ}C$:

$$L(T) = 0.105 + 0.405 \exp(-0.0502 T)$$

7

3.2.2 Effective Diffusion Coefficient

The bulk diffusion coefficient in partially saturated porous media follows the semi-empirical relation of Rogers and Nielson [14], in which moisture sharply reduces gas diffusivity by blocking air-filled pathways. With the free-air radon diffusivity $D_0 = 1.1 \times 10^{-5} m^2\ s^{-1}$:

$$D_e = D_0 \phi \exp(-6S\phi - 6S^{14}\phi)$$

8

Gas-phase diffusion additionally increases with temperature; we apply the Fuller–Schettler–Giddings scaling $D \propto T^{1.75}$ [19], referenced to 283.15 K:

$$D_e(T) = D_e(T_{ref}) \left(\frac{T_K}{283.15} \right)^{1.75}$$

9

3.2.3 Emanation Coefficient

The emanation coefficient rises with the first few per cent of moisture, because water films arrest recoiling radon atoms that would otherwise embed in adjacent grains, and increases weakly with temperature [5, 16]. We represent it as the reference (moist) value f_{ref} modulated by a saturating moisture function and a linear temperature function, the latter calibrated so the seasonal swing is consistent with the $\sim 14\%$ summer enhancement of soil-gas radon reported by Arvela et al. [4]:

$$f_{em}(S, T) = f_{ref} \frac{1 - \exp(-bS)}{1 - \exp(-b)} [1 + \alpha_T (T - T_{ref})]$$

10

Equations (6)–(10) make D_e , ϵ_{eff} and f_{em} – and therefore C_{eq} via (5) – explicit functions of the local saturation S and temperature T , which are supplied by the climate-forcing models below.

3.3 Thermal Forcing

Soil temperature is obtained from the one-dimensional heat-conduction equation with constant thermal diffusivity α_{th} ($\text{m}^2 \text{s}^{-1}$):

$$\frac{\partial T}{\partial t} = \alpha_{th} \frac{\partial^2 T}{\partial z^2}$$

11

Driving the surface with an annual sinusoid and seeking a periodic solution leads to an ordinary differential equation for the depth-dependent amplitude $\Theta(z)$:

$$\frac{d^2 \Theta}{dz^2} - \frac{i\omega}{\alpha_{th}} \Theta = 0$$

12

whose bounded solution decays and rotates in phase with depth, defining the damping depth d :

$$d = \sqrt{\frac{2\alpha_{th}}{\omega}}$$

13

Taking the real part and fixing the surface minimum at day t_{min} (mid-January for the Northern Hemisphere) gives the damped, phase-lagged thermal wave used to force the transport coefficients at every depth and time [20]:

$$T(z, t) = T_{mean} - A_T \exp(-z/d) \cos[\omega(t - t_{min}) - z/d]$$

14

with angular frequency $\omega = 2\pi/(1 \text{ year})$, mean surface temperature T_{mean} , and seasonal amplitude A_T . For an annual period and $\alpha_{th} \approx 5 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}$ the damping depth is of order 2 m, so the seasonal temperature signal penetrates the whole modelled column.

3.4 Soil-Moisture Forcing

Daily water saturation is generated from monthly precipitation by a single-store (leaky-bucket) water balance, in which the store rises with rainfall and relaxes toward a minimum saturation by drainage and evapotranspiration. With recharge coefficient a , loss coefficient b , daily precipitation rate P_n , and the saturation clipped to the physical range $[S_{min}, S_{max}]$:

$$S_{n+1} = \text{clip}(S_n + aP_n - b(S_n - S_{min}), S_{min}, S_{max})$$

15

Several spin-up cycles render the annual saturation cycle periodic before it is used to force the transport model. Monthly precipitation totals are converted to a mean daily rate before entering (15). The saturation is here taken uniform with depth; this assumption is discussed among the model limitations.

3.5 Spatial Discretisation

The column is discretised on a uniform grid $z_i = i\Delta z$, $i = 0 \dots N$, with $\Delta z = 2 \text{ cm}$ ($N = 100$, 101 nodes). Because D_e varies in space (through the depth dependence of temperature) the diffusion operator is written in conservative flux form with the coefficient evaluated at the cell faces by arithmetic averaging, which conserves radon mass:

$$D_{i+1/2} = \frac{D_i + D_{i+1}}{2}$$

16

$$\left(\frac{\partial}{\partial z} \left[D_e \frac{\partial C}{\partial z} \right] \right)_i \approx \frac{D_{i+1/2} (C_{i+1} - C_i) - D_{i-1/2} (C_i - C_{i-1})}{\Delta z^2}$$

17

The advection term uses a second-order centred difference:

$$\left(v \frac{\partial C}{\partial z} \right)_i \approx \frac{v (C_{i+1} - C_{i-1})}{2\Delta z}$$

18

Collecting terms, the semi-discrete equation at interior node i is a tridiagonal system plus the source:

$$\frac{dC_i}{dt} = \alpha_i C_{i-1} + \beta_i C_i + \gamma_i C_{i+1} + \lambda C_{eq,i}$$

19

$$\alpha_i = \frac{D_{i-1/2}}{\Delta z^2} + \frac{v}{2\Delta z}; \beta_i = -\frac{D_{i+1/2} + D_{i-1/2}}{\Delta z^2} - \lambda; \gamma_i = \frac{D_{i+1/2}}{\Delta z^2} - \frac{v}{2\Delta z}$$

20

In matrix form, $dC/dt = A \cdot C + b$, where A is tridiagonal with the entries of (20) and the source vector has components $b_i = \lambda C_{eq,i}$.

3.6 Time Integration and Boundary Conditions

Time integration uses the implicit Crank–Nicolson scheme, the second-order, unconditionally stable average of the backward and forward Euler methods [17, 18]. Averaging the semi-discrete system between time levels n and $n + 1$ gives:

$$\frac{\mathbf{C}^{n+1} - \mathbf{C}^n}{\Delta t} = 1/2 \mathbf{A} (\mathbf{C}^{n+1} + \mathbf{C}^n) + \mathbf{b}$$

21

which rearranges into the linear system solved at each step for the updated profile:

$$(\mathbf{I} - 1/2 \Delta t \mathbf{A}) \mathbf{C}^{n+1} = (\mathbf{I} + 1/2 \Delta t \mathbf{A}) \mathbf{C}^n + \Delta t \mathbf{b}$$

22

Because De , v and C_{eq} vary in time with the forcing, the matrices A and b are rebuilt every step and the tridiagonal system is solved with a sparse direct solver. The boundary conditions are Dirichlet at both ends: at the surface the pore-air concentration is set to the (small) atmospheric value, $C(0, t) = C_{atm} \approx 5 \text{ Bq m}^{-3}$; at the base the concentration is held at the local deep-soil equilibrium, $C(H, t) = C_{eq}$. Centred

advection is stable here because the cell Péclet number is far below the limit of two for realistic soil-gas velocities:

$$\text{Pe}_{cell} = \frac{|v| \Delta z}{D_e} \ll 2$$

23

Simulations are integrated for two years at a 6-hour time step; the first year is discarded as spin-up and the second – free of the initial transient – is analysed.

3.7 Surface Exhalation Flux

The diagnostic of primary interest is the surface exhalation flux, evaluated from the near-surface concentration gradient and the advective contribution, with a one-sided difference at the surface face and the face-averaged diffusion coefficient:

$$J_{surf} = D_{e,1/2} \frac{C_1 - C_0}{\Delta z} + v C_0$$

24

The result ($\text{Bq m}^{-2} \text{s}^{-1}$) is multiplied by 3 600 and reported in $\text{Bq m}^{-2} \text{h}^{-1}$. Because $C_0 \approx C_{\text{atm}}$ is small, the diffusive term dominates the flux.

3.8 Analytical Benchmark and Verification

Solver correctness is established against the exact steady-state solution of the constant-coefficient problem. Setting $\partial C / \partial t = 0$ in (3) gives the ordinary differential equation:

$$D_e \frac{d^2 C}{dz^2} - v \frac{dC}{dz} - \lambda C + \lambda C_{eq} = 0$$

25

whose homogeneous part has characteristic roots:

$$r_{1,2} = \frac{v \pm \sqrt{v^2 + 4D_e \lambda}}{2D_e}$$

26

so the general solution, with particular solution C_{eq} , is:

$$C(z) = A e^{r_1 z} + B e^{r_2 z} + C_{eq}$$

The constants A and B are fixed by the two Dirichlet conditions $C(0) = C_0$ and $C(H) = C_H$. In the pure-diffusion limit $v \rightarrow 0$ this reduces to the familiar exponential approach to equilibrium with the radon diffusion length L_d :

$$L_d = \sqrt{\frac{D_e}{\lambda}}$$

The numerical steady state (obtained by integrating to convergence with constant coefficients) is compared with (27) node by node through the root-mean-square error. For the test case (silt loam, $S = 0.30$, $T = 12^\circ\text{C}$) the agreement is essentially exact: $\text{RMSE} = 0.136 \text{ Bq m}^{-3}$, only 0.0003% of C_{eq} , confirming that residual error is dominated by spatial truncation rather than by the time-stepping scheme (Fig. 1).

3.9 Study Setting and Parameterisation

Three representative soil texture classes are defined (Table 1); the baseline transient simulation uses the silt-loam class. The illustrative climate is parameterised on Murree, a humid subtropical highland site in the outer Himalayas of Pakistan with cold winters, warm summers and a pronounced July–August monsoon; recent surveys in the region provide context for the radionuclide inputs [10, 11]. The values are representative literature-based choices intended to demonstrate the model rather than measured site values, and should be replaced with field-determined parameters for site-specific application. The numerical settings are summarised in Table 2.

Table 1

Soil-physics parameters by texture class. A_{Ra} , Ra-226 specific activity; ρ_b , dry bulk density; ϕ , total porosity; f_{em} , reference emanation coefficient; α_{th} , thermal diffusivity; v , pore advective velocity; $D_e(\text{dry})$, effective diffusion coefficient at low saturation. Diffusion length $L_d = \sqrt{(D_e/\lambda)}$ is 1.35–1.53 m across the three classes.

Soil	A_{Ra} (Bq kg ⁻¹)	ρ_b (kg m ⁻³)	ϕ	f_{em}	α_{th} (m ² s ⁻¹)	v (m s ⁻¹)	$D_e \text{ dry}$ (m ² s ⁻¹)
Sand	35	1650	0.38	0.18	6×10^{-7}	2×10^{-6}	3.85×10^{-6}
Silt loam	48	1400	0.45	0.22	5×10^{-7}	1×10^{-6}	4.46×10^{-6}
Clay	62	1250	0.50	0.26	4×10^{-7}	4×10^{-7}	4.88×10^{-6}

Table 2
Numerical and forcing settings for the baseline simulation.

Setting	Value
Domain depth H	2.0 m
Grid spacing Δz / nodes	0.02 m / 101
Time step Δt	6 h (21 600 s)
Integration / analysed	2 years / final year
Free-air diffusivity D_0	$1.1 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$
Decay constant λ	$2.10 \times 10^{-6} \text{ s}^{-1}$ ($T_{1/2} = 3.8235 \text{ d}$)
Surface BC	Dirichlet, $C = 5 \text{ Bq m}^{-3}$
Base BC	Dirichlet, $C = C_{eq}$ (local)
Mean surface T / amplitude	13°C / 11°C
Saturation range [S_{min} , S_{max}]	0.18–0.65

Results

4.1 Solver Verification

Under constant coefficients the numerical steady state reproduces the exact analytical solution of the two-point boundary-value problem (Fig. 1). The root-mean-square error is 0.136 Bq m^{-3} , only 0.0003% of the equilibrium concentration, and the maximum absolute error is 0.19 Bq m^{-3} (Table 3). This confirms that the discretisation and the Crank–Nicolson time-stepping are implemented correctly and that residual error is dominated by spatial truncation rather than by any scheme error.

Table 3
Steady-state verification metrics for the constant-coefficient test case (silt loam, $S = 0.30$, $T = 12^{\circ}\text{C}$).

Metric	Value
Effective diffusion coefficient D_e	$2.22 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$
Equilibrium concentration C_{eq}	$44\,325 \text{ Bq m}^{-3}$
Diffusion length L_d	1.03 m
RMSE (numerical vs. analytical)	0.136 Bq m^{-3}
Maximum absolute error	0.187 Bq m^{-3}
Relative RMSE	0.00031% of C_{eq}
Surface exhalation flux	$285.2 \text{ Bq m}^{-2} \text{ h}^{-1}$

4.2 Climate Forcing

The forcing fields for the analysed year are summarised in Fig. 2 and Table 2. Surface soil temperature follows the prescribed annual cycle, reaching a minimum near 2°C in mid-January and a maximum near 24°C in mid-July. Monthly precipitation peaks during the July–August monsoon, driving the soil-water saturation to its annual maximum (near 0.65) in late summer, with a secondary rise in spring; the soil is driest in late autumn. The Ostwald coefficient, being inversely related to temperature, is highest in winter and lowest in summer.

4.3 Subsurface Temperature and Radon Fields

The analytical thermal wave produces the expected damped, phase-lagged temperature field (Fig. 3): the seasonal amplitude decreases with depth and the warm and cold fronts arrive progressively later below the surface. The resulting soil-gas radon field (Fig. 5) shows a marked build-up of concentration at depth during the late-summer monsoon, when high saturation suppresses upward diffusion and radon accumulates below the surface. The seasonal depth profiles (Fig. 4) show that near-surface concentrations are lowest in winter and that the steepest near-surface gradients which drive exhalation occur in the warm, drier pre-monsoon period.

4.4 Surface Exhalation Flux

The modelled surface exhalation flux is strongly seasonal and bimodal (Fig. 6, Table 4). A pronounced primary maximum occurs in June (monthly mean $295 \text{ Bq m}^{-2} \text{ h}^{-1}$, +21% relative to the annual mean), when the soil is warm and drying before the monsoon; emanation and diffusivity are both elevated while saturation is still moderate. The flux then falls sharply through July to an August minimum ($208 \text{ Bq m}^{-2} \text{ h}^{-1}$, –15%) as monsoon rainfall saturates the soil and collapses the effective diffusivity, trapping radon at depth despite the high source strength. A secondary autumn maximum follows in October–November

as the soil dries while remaining relatively warm. The annual-mean flux is $245 \text{ Bq m}^{-2} \text{ h}^{-1}$ with a peak-to-peak range of about 40% of the mean.

Table 4
Monthly summary of modelled surface exhalation flux ($\text{Bq m}^{-2} \text{ h}^{-1}$) and percentage deviation from the annual mean.

Month	Mean ($\text{Bq m}^{-2} \text{ h}^{-1}$)	Min	Max	% vs annual mean
Jan	233.1	226.6	241.5	-4.8
Feb	223.2	220.5	226.3	-8.9
Mar	218.9	217.2	222.6	-10.6
Apr	237.2	223.3	253.2	-3.2
May	271.9	254.4	287.6	+ 11.0
Jun	295.3	288.5	300.1	+ 20.6
Jul	246.2	205.9	287.6	+ 0.5
Aug	207.5	202.8	210.2	-15.3
Sep	219.6	201.3	244.8	-10.4
Oct	262.3	246.3	271.7	+ 7.1
Nov	271.5	266.4	273.1	+ 10.8
Dec	253.7	242.2	265.7	+ 3.6
Annual	245.0	201.3	300.1	0.0

4.5 Coupling Functions, Flux Drivers, and Factorial Attribution

The parameter response curves (Fig. 7) make the competing mechanisms explicit. The effective diffusion coefficient falls steeply as saturation increases, by roughly an order of magnitude from dry to near-saturated soil, while the equilibrium pore-air concentration rises with saturation because the air-filled volume shrinks and emanation increases. To separate the two controls, which co-vary through the year, we ran a factorial attribution experiment comprising three transient simulations: (i) both temperature and moisture varying, (ii) only temperature varying (moisture held at its annual mean), and (iii) only moisture varying (temperature held at its annual mean) (Fig. 8). The single-factor anomalies sum almost exactly to the combined anomaly ($R^2 = 0.997$; interaction $\leq 3\%$ of the combined amplitude), showing that the two controls act nearly additively in this idealized experiment. Soil moisture accounts for a somewhat larger share of the seasonal variance ($\approx 60\%$) than temperature ($\approx 40\%$), and, importantly, moisture alone reproduces the bimodal shape and the monsoon minimum, whereas temperature alone produces only a single broad summer maximum (Fig. 8; Table 7). We therefore attribute the bimodality

principally to soil moisture and the underlying warm-season elevation to temperature, while stressing that this partition applies to the present idealized forcing and soil parameters and need not generalize to other climates or sites.

Table 7

Attribution of the seasonal exhalation cycle to temperature and moisture forcing (silt-loam soil; anomalies about each scenario's own annual mean). The near-unity R^2 for the linear sum confirms the two controls act almost additively.

Scenario	Annual mean (Bq m ⁻² h ⁻¹)	Peak-to-peak anomaly (Bq m ⁻² h ⁻¹)	Seasonal variance share
Both vary (baseline)	245.0	98.8	—
Temperature only	249.5	83.4	≈ 40%
Moisture only	244.8	119.6	≈ 60%
Linear sum (T + M)	—	98.2	$R^2 = 0.997$

4.6 Sensitivity Analysis

A one-at-a-time $\pm 20\%$ perturbation of each parameter (Fig. 9, Table 5) shows that the annual-mean flux scales almost linearly with Ra-226 activity, emanation coefficient and bulk density — the three factors that set the source strength — each producing a $\pm 20\%$ response. Porosity is the next most influential parameter (about -18% for $+20\%$), reflecting its dual effect on dilution and diffusivity, followed by advective velocity. Thermal diffusivity has negligible influence on the annual mean because it changes only the phase and damping of the thermal wave, not its mean.

Table 5

Percentage change in annual-mean exhalation flux for $\pm 20\%$ variation in each parameter.

Parameter	-20% change in flux (%)	+20% change in flux (%)
Ra-226 activity A _{Ra}	-20.0	+20.0
Emanation coefficient f _{em}	-20.0	+20.0
Bulk density p _b	-20.0	+20.0
Porosity ϕ	+19.2	-17.8
Advective velocity v	+5.9	-5.5
Thermal diffusivity α_{th}	+0.04	-0.03

Discussion

5.1 Physical Interpretation of the Bimodal Flux Trend

The central result, a bimodal annual exhalation signal with a pre-monsoon peak and a monsoon minimum emerges directly from the opposing roles of soil moisture in the transport problem. Wetting simultaneously increases the radon source (through higher emanation via Eq. 10 and a smaller air-filled pore volume reducing ϵ_{eff} in Eq. 6) and decreases the diffusive transport capacity (through the steep Rogers–Nielson dependence of D_e on S in Eq. 8). These competing effects do not simply cancel: because the diffusive flux depends on both the gradient and the diffusivity, while the source term scales with the equilibrium concentration, high saturation conditions produce a reservoir of radon trapped at depth (Fig. 5) even as the surface flux falls (Fig. 6). The pattern precisely mirrors Fick's law physics the concentration field builds toward its source-limited maximum even as the escape pathway is blocked. This provides a physically rigorous reconciliation of two observations that can appear contradictory in the literature: that increased soil moisture raises soil-air radon concentration [4] yet simultaneously reduces surface exhalation [5, 6].

The pre-monsoon June peak (Fig. 6) is the result of a brief seasonal window in which the three principal drivers temperature, moisture and diffusivity are simultaneously favourable. By late spring and early June the soil has warmed substantially (surface temperature $\sim 22\text{--}23^\circ\text{C}$), elevating both emanation through the temperature-dependent term in Eq. (10) and the gas-kinetic diffusion correction in Eq. (9), while the preceding spring rains have recharged moisture sufficiently to sustain high emanation via the saturation-dependent term. Crucially, however, saturation has not yet risen to the levels that suppress diffusivity. This narrow window produces the June flux maximum. The autumn secondary maximum (October–November) is driven by a similar but attenuated mechanism: post-monsoon drainage lowers saturation and restores diffusivity while temperatures remain moderate, producing elevated fluxes before the winter cooling cycle reduces both emanation and diffusion rates. The July–August flux minimum coincides with the seasonal rise of soil-water saturation to its highest values and the strong non-linearity of the Rogers–Nielson diffusion relation, which shows an approximately order-of-magnitude drop in D_e from dry to near-saturated conditions (Fig. 7a). Because the forcing uses monthly-mean precipitation converted to a mean daily rate, individual rainfall events and day-to-day variability during the monsoon are not represented; the model therefore describes the seasonal envelope of the flux rather than its weather-scale response, and the depth of the minimum should not be interpreted as a response to the abruptness of monsoon onset.

5.2 Comparison with Literature Values

The modelled annual-mean flux of $\sim 245 \text{ Bq m}^{-2} \text{ h}^{-1}$ ($\approx 0.068 \text{ Bq m}^{-2} \text{ s}^{-1}$) lies within the broad range of reported soil exhalation rates globally, which typically span one to three orders of magnitude depending on radium content, soil texture and climate. For the Murree and broader Himalayan foothills region, Ali et al. [10] [10, 11] characterised indoor radon and water radon activities consistent with moderately elevated radium source terms. The silt-loam Ra-226 activity of 48 Bq kg^{-1} used here, combined with the modelled flux, is broadly consistent with the global average soil exhalation estimate of $\sim 0.020\text{--}0.060 \text{ Bq m}^{-2} \text{ s}^{-1}$ cited by Nazaroff [1], with the somewhat higher value in our model reflecting the higher-than-average Ra-226 activity representative of the outer Himalayan geology. The near-linear scaling of flux with A_{Ra} , ρ_b

and f_{em} (Fig. 9) means the absolute level can be directly adjusted to match site-measured radionuclide data without retuning the dynamic model structure.

The $\sim 14\%$ temperature-only summer enhancement of soil-gas radon embedded in the emanation model (Eq. 10) is consistent with the estimate of Arvela et al. [4] derived from Finnish house measurements. In the present idealized monsoon experiment, the factorial attribution (Section 4.5) shows that this thermal signal is not overwhelmed but is comparable to though somewhat smaller than the moisture-driven contribution ($\approx 40\%$ versus $\approx 60\%$ of the seasonal variance), with the two combining nearly additively; moisture is nonetheless what imposes the bimodal shape and the monsoon minimum. We refrain from generalizing this partition to continental monsoon climates as a whole, because the balance between the two controls is likely to shift in arid or semi-arid settings, where large temperature swings occur with little moisture variation, and in high-latitude environments where precipitation falls predominantly as snow. The model is well-suited to explore such climate contrasts, and direct comparison with the Peruvian agroecosystem results of Palacios et al. [8], where an ANN–Monte Carlo approach identified meteorological variables as key predictors of exhalation, would be a productive extension. Both modelling frameworks identify moisture and temperature as primary drivers but differ in their mechanistic transparency; forward physics-based models such as the one presented here can be used to benchmark and interpret the relative feature importances derived from statistical approaches.

5.3 Soil Texture Effects

The soil-type comparison (Fig. 10) reveals that the bimodal seasonal structure is robust across all three texture classes, but the magnitude and seasonal amplitude both increase from sand to silt loam to clay. This ordering reflects the combined effects of higher Ra-226 activity, higher reference emanation coefficient and higher total porosity in fine-textured soils (Table 1), which collectively raise the equilibrium concentration via Eq. (5). Clay soils, despite having lower thermal diffusivity (which reduces the temperature signal penetration depth), show the largest absolute flux and the largest seasonal range. The separation between texture classes widens during the pre-monsoon season when the diffusive pathway is most open, and narrows during the monsoon minimum when all soil types are similarly constrained by high saturation. These findings support the recommendation that soil texture should be characterised as carefully as radium content when designing field monitoring programmes, since texture governs both the seasonal amplitude and the soil's response to rainfall events.

5.4 Implications for Climate–Radon Interactions

The framework may also be applied to climate-change questions, framed here as hypotheses for future testing rather than as results of the present study. Rathebe et al. [7] reviewed evidence that shifting precipitation patterns, extreme rainfall events, permafrost degradation and altered soil freeze–thaw cycles may modify radon emissions, and called for integrated climate–radon modelling. Because the flux responds non-linearly to saturation, the present framework could in principle be used to test how changes in monsoon timing or intensity propagate to the exhalation signal. We emphasise, however, that no climate perturbation is imposed in the present study: the temperature and precipitation forcings are

fixed seasonal climatologies. Quantitative statements about the effect of, for example, an earlier or more intense monsoon would require dedicated perturbation experiments with event-resolved meteorological forcing, and are left as explicit hypotheses for future work.

The parallel to the coupled heat–moisture–air wall model of Chen et al. [9] is instructive: both approaches demonstrate that radon exhalation depends critically on the coupled evolution of moisture and temperature, not on either variable in isolation. The key difference is that for building walls the diurnal cycle and vapour-pressure gradients are the primary forcing, while for field soils the seasonal cycle and precipitation are dominant. Extending the present 1-D column model to include diurnal forcing would bridge this gap and allow direct comparison of wall and soil exhalation contributions to indoor radon in a single framework.

5.5 Model Limitations and Future Work

Several simplifications should be borne in mind when interpreting these results. The coupling sub-models, while literature-based in form, use illustrative parameter values and a calibrated emanation–temperature relation rather than site-fitted constants; the results are therefore a physically consistent demonstration, not a measured prediction. The Rogers–Nielson diffusion relation, although widely used, is a semi-empirical fit to limited experimental data and may not adequately represent all soil structures, particularly those with strong preferential flow paths or macropore networks in which water content and gas diffusivity are decoupled.

Soil moisture is assumed uniform with depth and is generated by a simple leaky-bucket model rather than ingested from observed or reanalysis time series. In reality, moisture profiles are highly non-uniform immediately after rainfall events, and the near-surface layer which most directly controls the exhalation flux can cycle through wet and dry states on timescales of hours to days that are not resolved by monthly forcing data. Future work should explore coupling to a multi-layer Richards equation solver or assimilation of soil-moisture reanalysis data (e.g., ERA5-Land), which would substantially improve both the realism of the forcing and the model's ability to reproduce episodic flux events observed with high-frequency detectors.

The thermal field uses a constant thermal diffusivity, which neglects the moisture dependence of thermal conductivity and heat capacity. Studies of soil heat transport have shown that this can introduce non-trivial errors in the depth and phase of the thermal wave, particularly near field capacity [20]. Similarly, the advection term assumes a steady, uniform pore velocity, omitting barometric pumping the pressure-gradient-driven advection caused by atmospheric pressure fluctuations which can be a significant radon transport mechanism in shallow, permeable soils [12, 13]. Including a time-varying barometric forcing would be straightforward within the existing Crank–Nicolson framework.

Conclusions

We have presented and verified an open-source, transient, climate-forced finite-difference model of soil radon transport that couples the diffusion–advection–decay equation to soil moisture and temperature through established sub-models. The Crank–Nicolson solver reproduces the exact analytical steady state to within 0.0003% of the equilibrium concentration ($\text{RMSE} = 0.136 \text{ Bq m}^{-3}$). Applied to a monsoon climate representative of the Murree region, the outer Himalayan Foothills, the model produces a bimodal annual exhalation signal in which a warm, dry pre-monsoon peak (June maximum $\sim 300 \text{ Bq m}^{-2} \text{ h}^{-1}$) gives way to a sharp monsoon minimum (August $\sim 208 \text{ Bq m}^{-2} \text{ h}^{-1}$) as wet soil traps radon at depth. This bimodality reconciles the apparently contradictory observations that wetting raises pore-air radon concentration while simultaneously reducing surface exhalation the two processes act on the source and transport pathway respectively. Sensitivity analysis identifies radium content, emanation and bulk density as the dominant controls on the absolute flux magnitude (each with a near-linear $\pm 20\%$ response). The seasonal shape of the flux is governed jointly by soil moisture and temperature; the factorial attribution indicates that in this idealized experiment moisture contributes somewhat more than temperature ($\sim 60\%$ versus $\sim 40\%$ of the seasonal variance) and sets the bimodal structure, with the two acting nearly additively. The soil-type comparison confirms that fine-textured soils produce higher fluxes with larger seasonal amplitudes, and that the bimodal structure is robust across soil classes. The tool provides a reproducible, physically interpretable framework for: (i) interpreting time-series exhalation measurements from LR-115 or accumulation-chamber detectors; (ii) generating and testing hypotheses about how projected changes in monsoon timing, intensity, and warming might reshape soil radon fluxes; and (iii) benchmarking statistical and machine-learning approaches to exhalation prediction. Substituting site-measured soil parameters and field exhalation time series is the immediate path to a fully validated, site-specific application, and remains the priority next step.

Declarations

Code and Data Availability

The complete Python code and the data associated with this paper will be made available by the corresponding author (manjum@uidaho.edu/mav.mavia14@gmail.com) upon a reasonable request.

Author Contributions

Mavia Anjum: Conceptualisation, methodology, software, validation, Simulation, writing original draft and review and editing. **Ansar Abbas**: methodology, software, validation, writing original draft. **M.A.M.Azmin**: Simulation, methodology, software. **Arbaz Khan**: Conceptualisation, methodology. **Inza Tariq**: Software, Review and editing. **Hannan Younis**: Supervision, Review and editing.

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Ethics approval and consent to participate

Not applicable. This study investigates environmental physical parameters and does not involve human participants, animal subjects, or clinical trials.

Consent for publication

Not applicable.

Competing interests

The authors declare no competing interests

Language Statement

Software's (Quill Bot and Grammarly) were used to improve the language quality of the manuscript.

References

1. Nazaroff WW (1992) Radon transport from soil to air. *Rev Geophys* 30(2):137–160.
<https://doi.org/10.1029/92RG00055>
2. Savović S, Djordjević A (2008) Numerical solution of the diffusion equation describing the flow of radon through concrete. *Appl Radiat Isot* 66(4):552–555.
<https://doi.org/10.1016/j.apradiso.2007.08.018>
3. Chitra N, Bala Sundar S, Jose MT, Sivasubramanian K, Venkatraman B (2019) A simple model to simulate the diffusion pattern of radon in different soil media. *J Radioanal Nucl Chem* 322(2):1151–1158. <https://doi.org/10.1007/s10967-019-06820-2>
4. Arvela H, Holmgren O, Hänninen P (2016) Effect of soil moisture on seasonal variation in indoor radon concentration: modelling and measurements in 326 Finnish houses. *Radiat Prot Dosimetry* 168(2):277–290. <https://doi.org/10.1093/rpd/ncv182>
5. Hosoda M, Shimo M, Sugino M, Furukawa M, Fukushima M (2007) Effect of soil moisture content on radon and thoron exhalation. *J Nucl Sci Technol* 44(4):664–672.
<https://doi.org/10.1080/18811248.2007.9711855>
6. Faheem M (2008) Matiullah. Radon exhalation and its dependence on moisture content from samples of soil and building materials. *Radiat Meas* 43(8):1458–1462.
<https://doi.org/10.1016/j.radmeas.2008.02.023>
7. Rathebe PC, Mphaga KV, Masekameni DM (2025) Climate change and environmental radioactivity: a review of studies on climate conditions in variation on indoor radon concentrations. *Environ Monit Assess* 197(4):446. <https://doi.org/10.1007/s10661-025-13889-8>
8. Palacios D (2025) Edaphic and meteorological parameters as determinants of radon exhalation and its environmental implication in Peruvian agroecosystems. *Sci Rep* 15(1):22140.
<https://doi.org/10.1038/s41598-025-08108-w>

9. Chen YC, Xie D, Jia QR, Zeng J, Chen GJ, Yang Y (2025) Numerical simulation study on the radon exhalation mechanism of building walls influenced by coupled heat–moisture–air transfer. *J Environ Radioact* 285:107675. <https://doi.org/10.1016/j.jenvrad.2025.107675>
10. Ali N, Khan EU, Akhter P, Khan F, Waheed A (2010) Estimation of mean annual effective dose through radon concentration in the water and indoor air of Islamabad and Murree. *Radiat Prot Dosimetry* 141(2):183–191
11. Anjum M, Siddique N, Younis H, Faiz Y, Anjum M (2024) Assessment of natural radioactivity in spring water and radiation shielding properties of soil from a tourist hub in outer Himalayas. *J Radioanal Nucl Chem* 333(10):4715–4732. <https://doi.org/10.1007/s10967-024-09695-0>
12. Holford DJ, Schery SD, Wilson J, Phillips FM (1993) Modeling radon transport in dry cracked soil. *J Phys Res* 98(B1):567–580. <https://doi.org/10.1029/92JB01845>
13. Schery SD, Gaeddert DH, Wilkening MH (1984) Factors affecting exhalation of radon from a gravelly sandy loam. *J Phys Res* 89(D5):7299–7309. <https://doi.org/10.1029/JD089iD05p07299>
14. Rogers VC, Nielson KK (1991) Multiphase radon generation and transport in porous materials. *Health Phys* 60(6):807–815
15. Weigel F, Radon (1978) *Chem Ztg* 102(9):287–299
16. Markkanen M, Arvela H (1992) Radon emanation from soils. *Radiat Prot Dosimetry* 45(1–4):269–272. <https://doi.org/10.1093/rpd/45.1-4.269>
17. LeVeque RJ (2007) *Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-State and Time-Dependent Problems*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA. <https://doi.org/10.1137/1.9780898717839>
18. Crank J, Nicolson P (1947) A practical method for numerical evaluation of solutions of partial differential equations of the heat-conduction type. *Mathematical Proceedings of the Cambridge Philosophical Society*. ;43(1):50–67. <https://doi.org/10.1017/S0305004100023197>
19. Fuller EN, Schettler PD, Giddings JC (1966) A new method for prediction of binary gas-phase diffusion coefficients. *Industrial Eng Chem* 58(5):18–27. <https://doi.org/10.1021/ie50677a007>
20. Carslaw HS, Jaeger JC (1959) *Conduction of Heat in Solids*, 2nd edn. Clarendon, Oxford

Figures

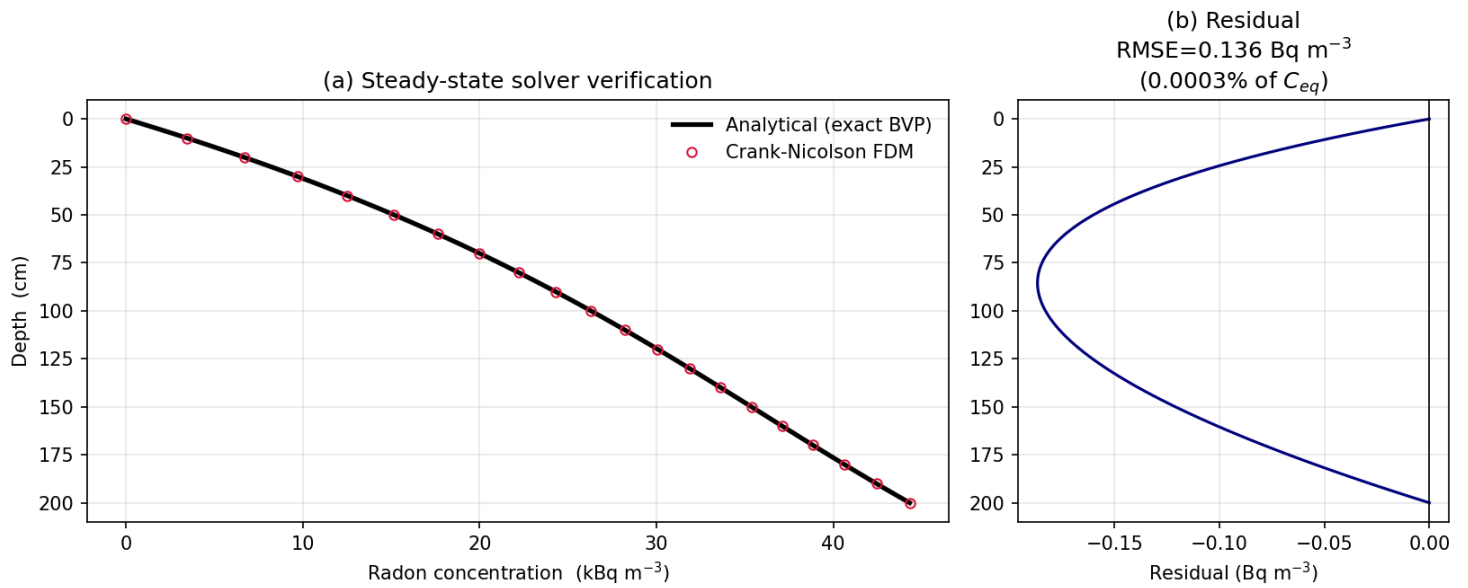


Figure 1

Verification of the Crank–Nicolson solver against the exact analytical steady-state solution. (a) Concentration–depth profile: analytical solution (line) versus finite-difference nodes (circles). (b) Node-by-node residual; the root-mean-square error is 0.0003% of the equilibrium concentration.

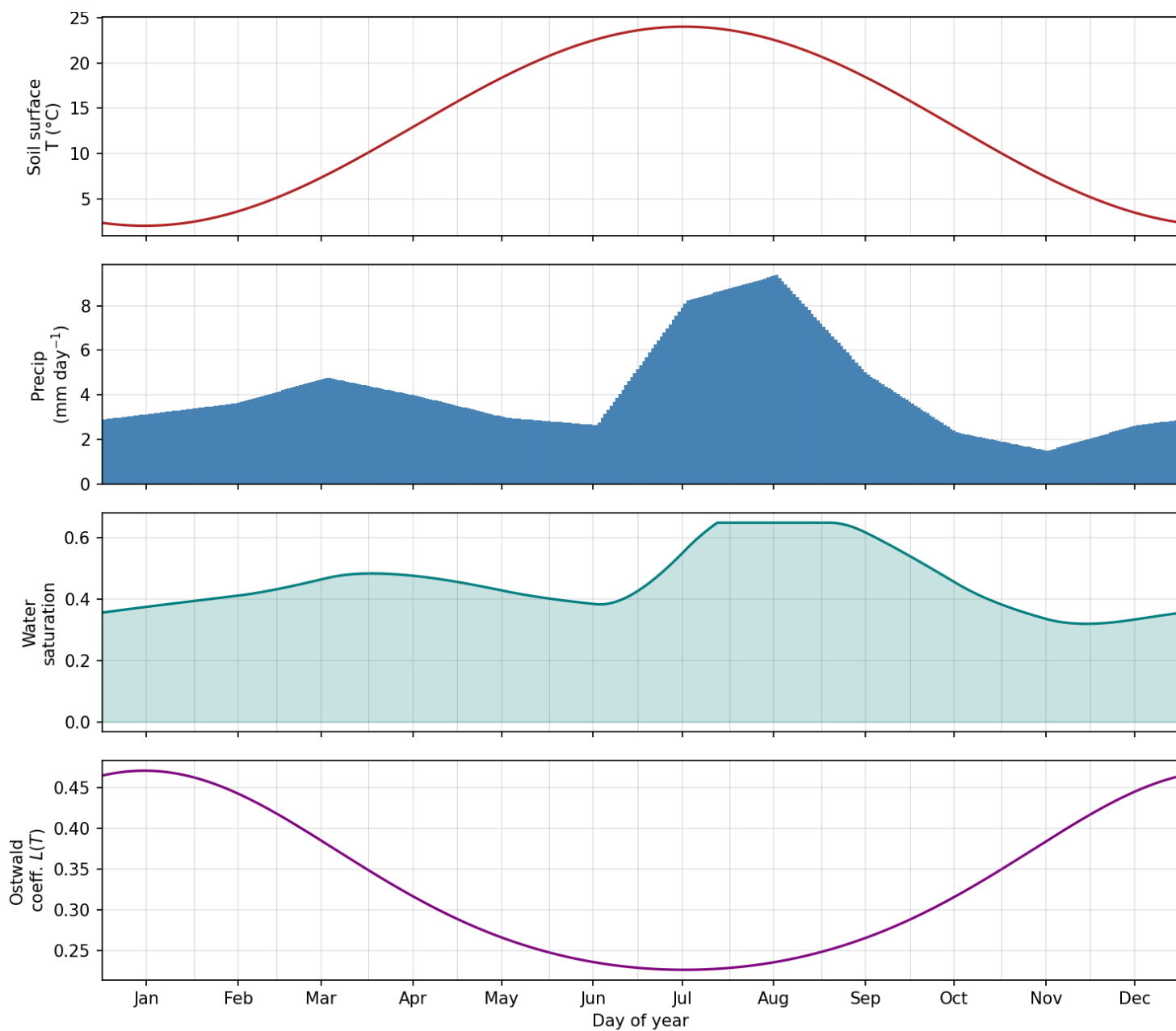


Figure 2

Seasonal climate forcing for the analysed year. From top: soil-surface temperature, daily precipitation rate, soil-water saturation, and the temperature-dependent Ostwald solubility coefficient.

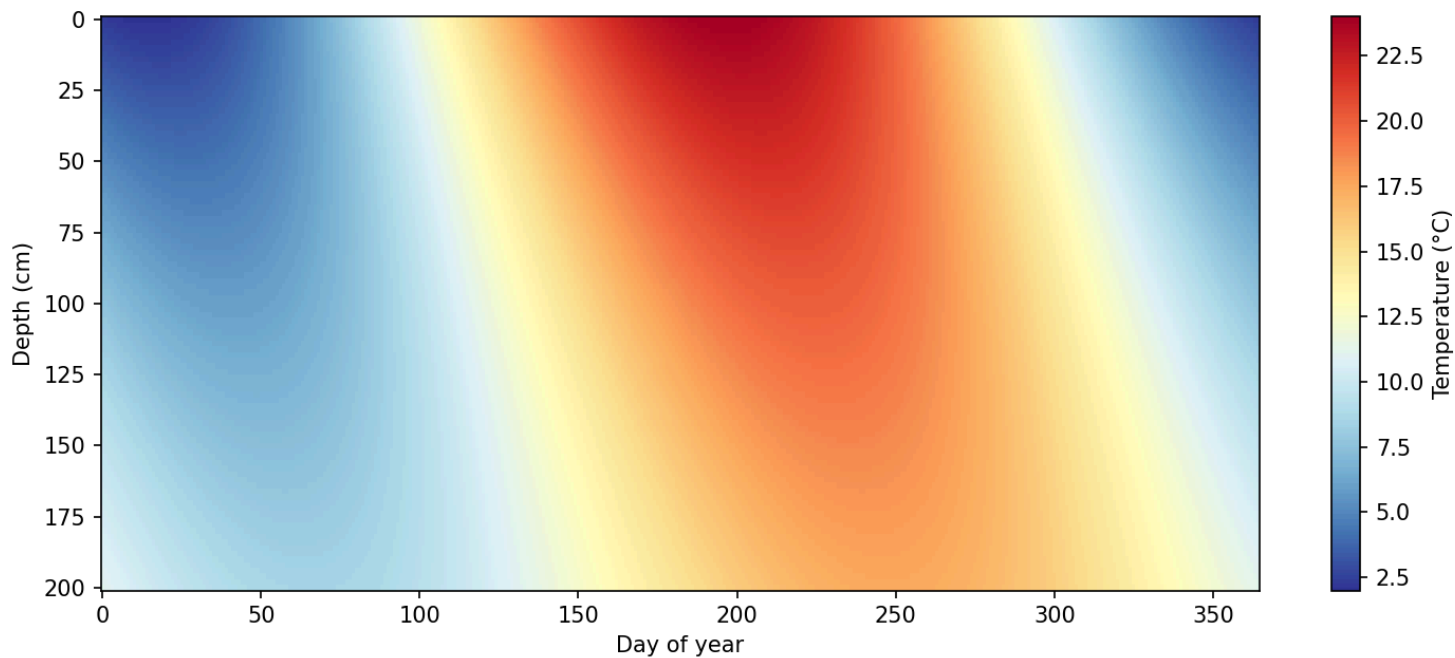


Figure 3

Soil temperature field $T(z,t)$ over the analysed year, showing the damped, phase-lagged annual thermal wave penetrating the 0–200 cm column.

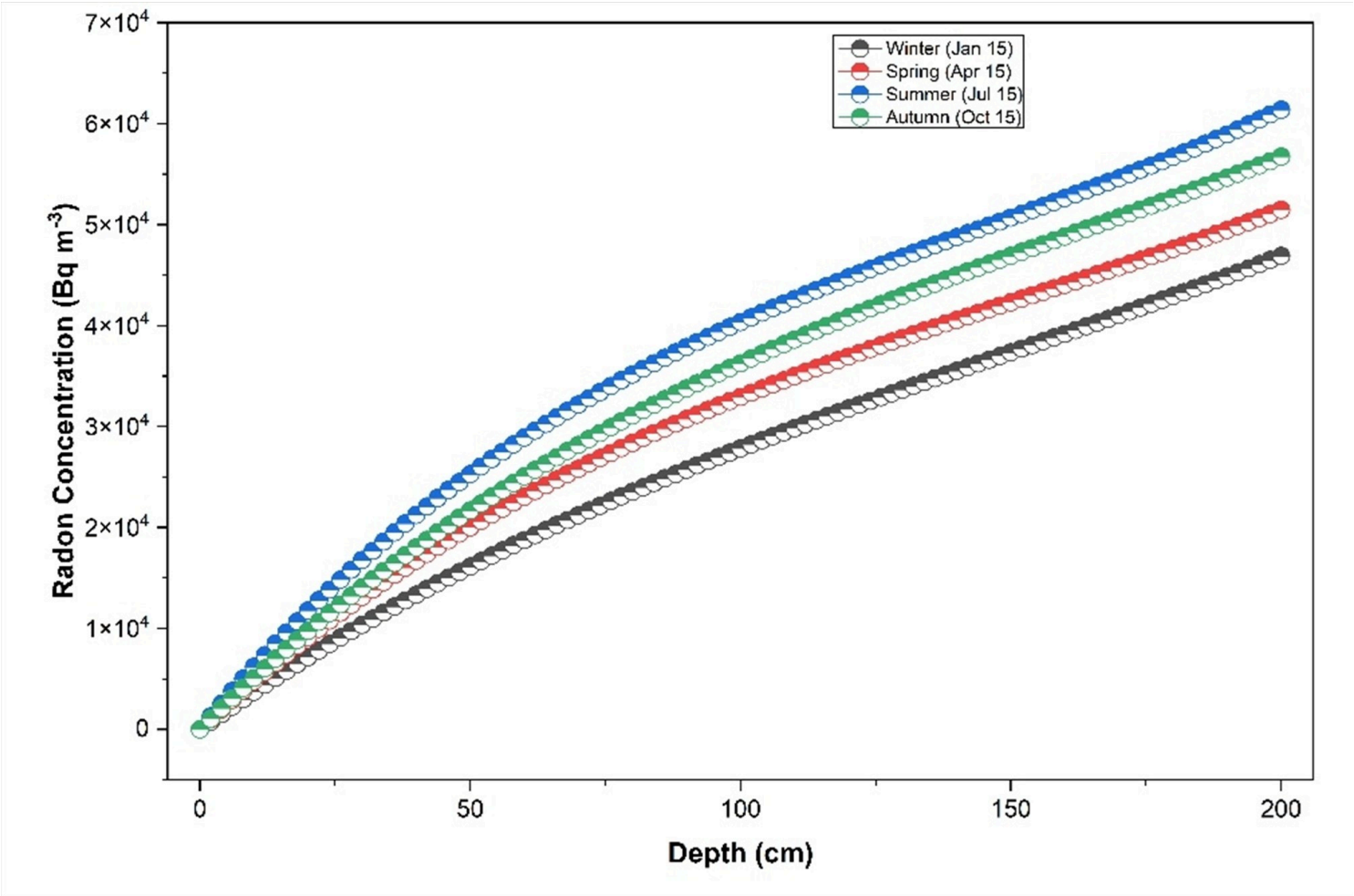


Figure 4

Soil-gas radon concentration–depth profiles for four seasons (winter: Jan 15; spring: Apr 15; summer: Jul 15; autumn: Oct 15).

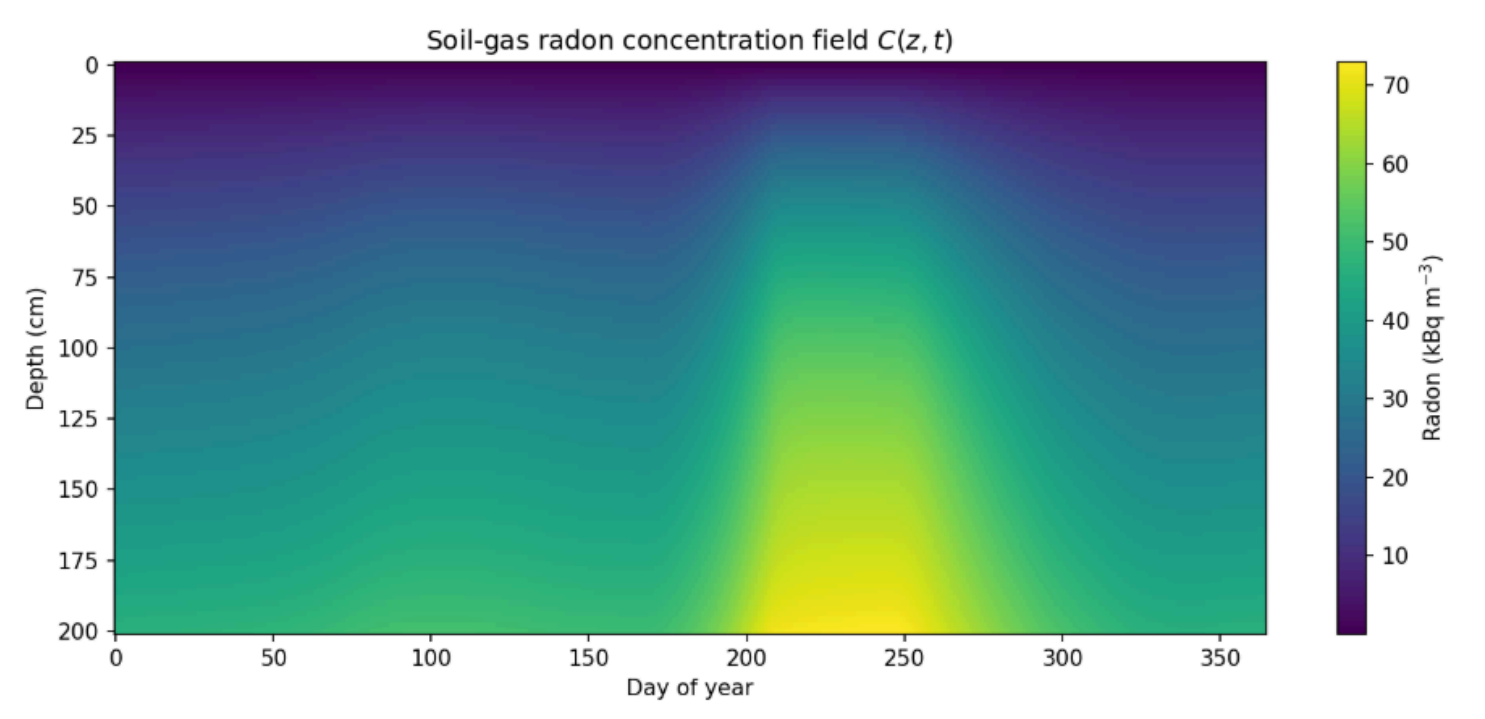


Figure 5

Soil-gas radon concentration field $C(z,t)$. Radon accumulates at depth during the July–September monsoon, when high water saturation suppresses diffusive escape.

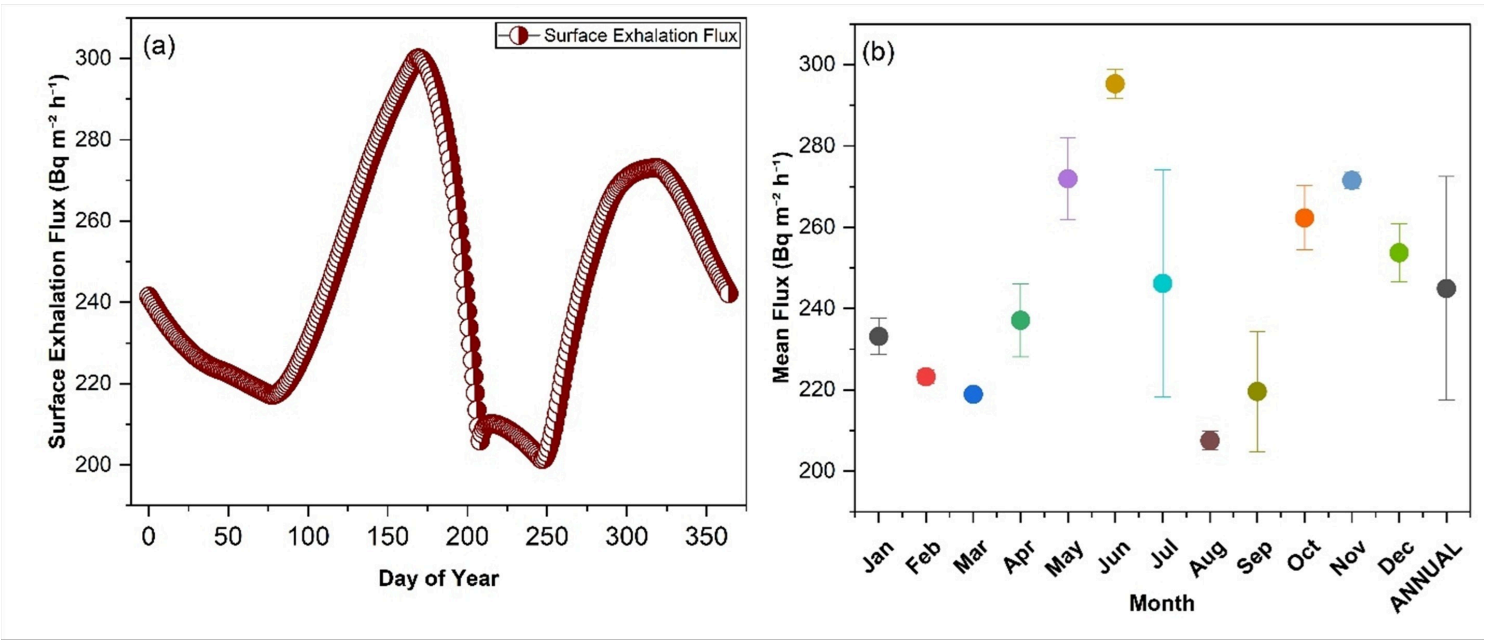


Figure 6

Time-resolved surface radon exhalation flux for the analysed year and months. The bimodal signal reflects variation between thermal enhancement (June peak) and monsoon-moisture suppression (August minimum).

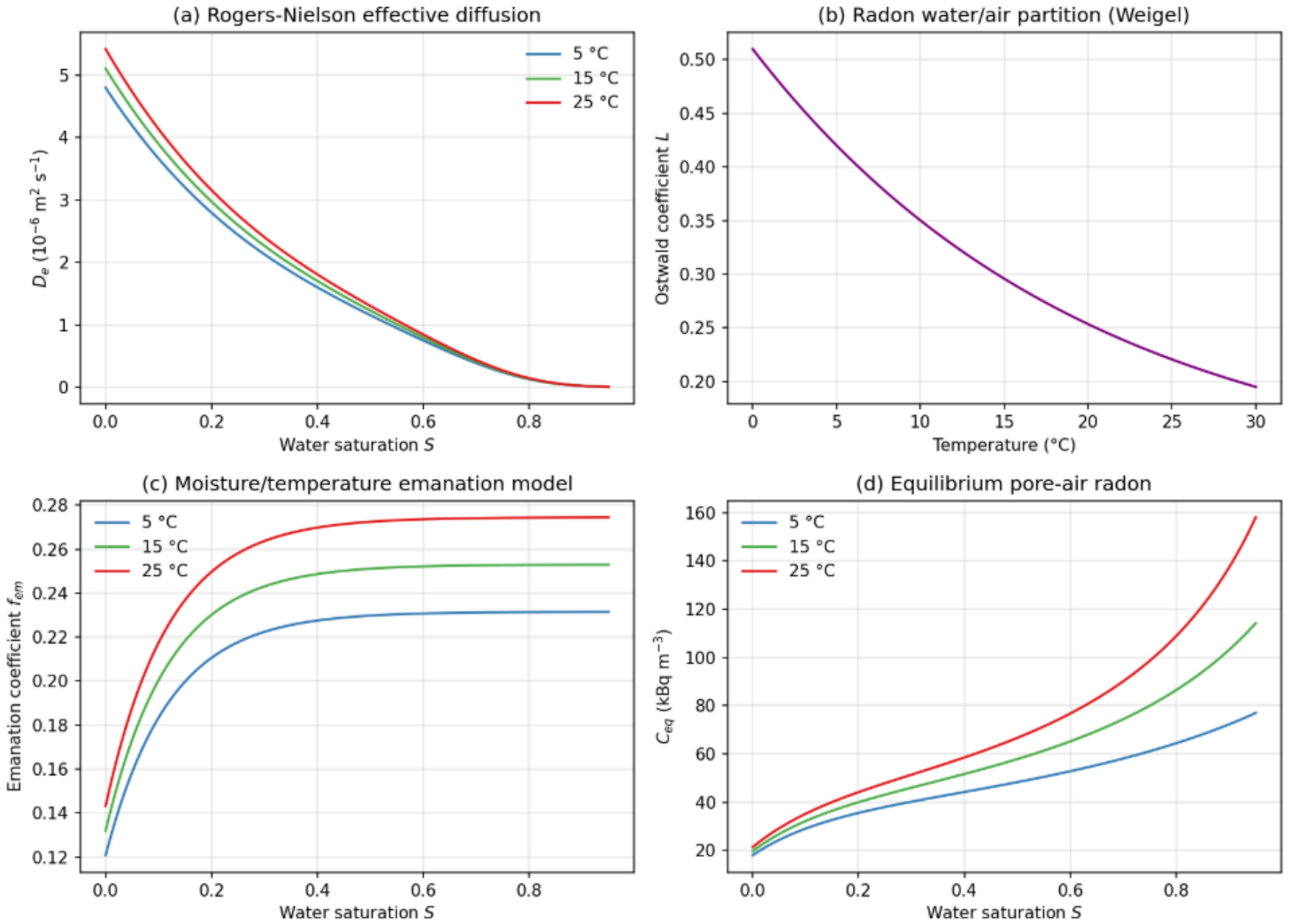


Figure 7

Coupling sub-models. (a) Rogers–Nielson effective diffusion coefficient versus saturation at three temperatures; (b) Weigel Ostwald solubility coefficient versus temperature; (c) emanation coefficient versus saturation; (d) equilibrium pore-air radon versus saturation.

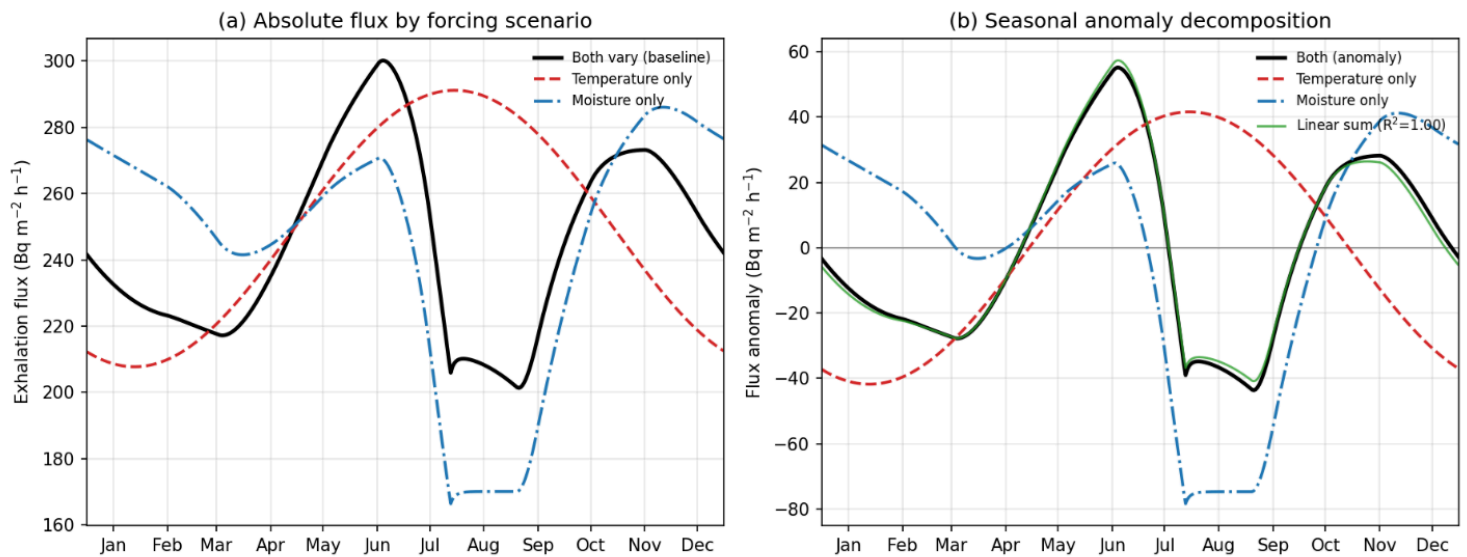


Figure 8

Factorial attribution of the seasonal exhalation cycle (silt loam). (a) Absolute surface flux for three forcing scenarios: both temperature and moisture varying (baseline), temperature only (moisture held at its annual mean), and moisture only (temperature held at its annual mean). (b) Seasonal anomalies about each scenario's own annual mean; the temperature-only and moisture-only anomalies sum almost exactly to the combined anomaly (green; $R^2 = 0.997$), confirming that the two controls act nearly additively. Moisture sets the bimodal shape and the monsoon minimum, whereas temperature alone produces a single broad summer maximum.

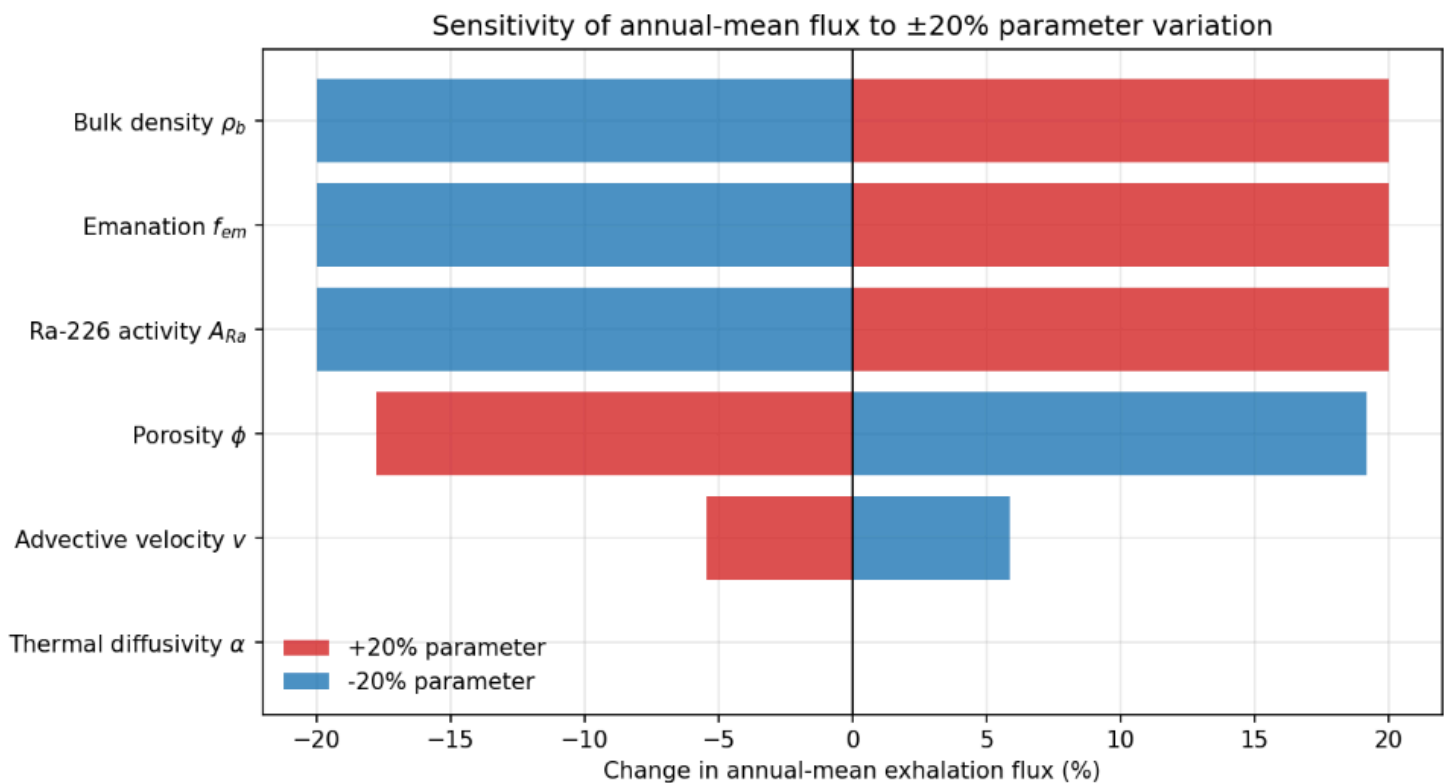


Figure 9

Tornado plot of the sensitivity of the annual-mean exhalation flux to a $\pm 20\%$ variation in each model parameter.

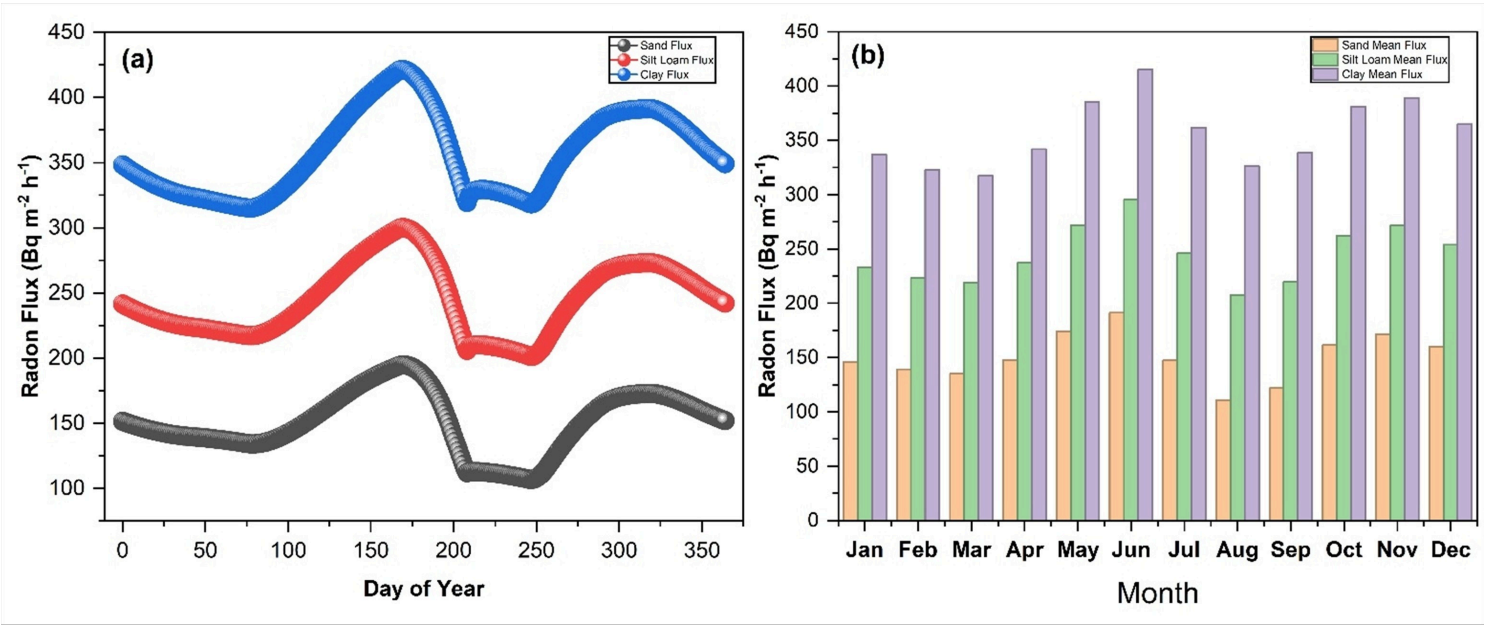


Figure 10

Exhalation flux by soil texture class (sand, silt loam, clay) over the annual cycle. Clay soils produce the highest absolute flux and greatest seasonal amplitude due to their higher Ra-226 activity and emanation coefficient.