for: Interior spacetime curvature and positron yield of a stationary capsule enclosed by a chopped near-light-speed energy shell

Supplementary Information

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Supplementary Note S1. Theoretical framework: design principles, the test metric and its limits

S1.1 Three design principles

The apparatus is built on three principles, each of which is tested numerically in Supplementary Note S2. The first is high energy density: the energy per unit volume must exceed the rest-mass energy in that volume ($u \gg mc^2/V$). The second is small volume: the more compressed the energy, the more local and intense its effect. The third is rapid change: gravitational waves require a changing quadrupole moment; that is what creates transience. In standard relativity these correspond respectively to the T^{00} and T^ii components of the energy–momentum tensor, to the 1/r weight in the Poisson equation, and to the third time derivative in the quadrupole formula. The same three principles also justify the chopper: cutting makes the time derivative of the shell far larger than the shell itself.

S1.2 The Börekci metric — full definition

The Börekci metric is a one-parameter deformation of weak-field general relativity:

$$ds^2 = -(1 + 2\Phi\_B/c^2)\, c^2dt^2 - (4/c)(A\_B \cdot dx)\, c\, dt + (1 - 2\Phi\_B/c^2)(dx^2 + dy^2 + dz^2) \qquad (1)$$

Here Φ_B is the scalar and A_B the vector Börekci potential:

$$\Phi\_B(x, t) = \Phi_0(x) + \xi \cdot s(t) \cdot \Phi\_s(x) + \Phi\_acc(t), \qquad A\_B = A_0 + \xi \cdot s(t) \cdot A\_s \qquad (2)$$

Φ_0 is the standard potential of the static sources (wall masses, magnet field energies, electrostatic energy, rotational kinetic energy of the platform) and is always computed with ξ = 1. Φ_s is the potential of the near-light-speed shell (electron arcs and laser), computed with the retarded Green function from the sum T^{00} + Σ T^ii of the energy–momentum tensor; for a flow at the speed of light this sum is twice T^{00}:

$$\Phi\_s(x) = -(G/c^2) \int [T^{00} + T\hat{\ }xx + T\hat{\ }yy + T\hat{\ }zz]\_shell(x') \,/\, |x - x'|\, d^3x' \approx -2G\, E\_shell /(c^2\, R\_shell) \qquad (3)$$

s(t) is the chopper function: a square wave equal to 1 while the shell is on and 0 during the cut, with period T_c = 300 s and cut duration τ_c = 1 ms. Φ_acc(t) is the potential of the energy that accumulates (is not removed) in the interior volume and grows linearly with time (Supplementary Note S2.4). ξ is the coupling coefficient of the shell potential to the metric of the cavity; ξ = 1 gives exactly linearized general relativity. The rate of a clock in the cavity relative to a clock at infinity, and the interior Lorentz factor, are

$$d\tau\_in/dt = \sqrt{(1 + 2\Phi\_B/c^2)}, \qquad \Gamma\_in(t) = dt/d\tau\_in = [1 + 2\Phi\_B(t)/c^2]\hat{\ }(-1/2) \qquad (4)$$

The author's L1 conviction corresponds to the condition Γ_in = γ_e, which gives a closed expression for ξ:

$$\xi\_L1 = (1 - 1/\gamma\_e^2) \cdot c^2 \;/\; (2\,|\Phi\_s|) \tag{5}$$

Three properties of the metric should be stressed. First, ξ multiplies only the near-light-speed shell; static sources such as the wall mass are unaffected, so the $\xi \gg 1$ hypothesis does not directly contradict Eötvös-type experiments. Second, through the factor s(t) the chopper modulates the ξ term as a square wave; this is the backbone of the measurement protocol. Third, since the metric reduces at ξ = 1 to the weak-field solution of the Einstein field equations, there is no observational requirement for ξ ≠ 1; ξ is not a theoretical necessity but a parameter to be tested (Supplementary Note S5).

S1.3 Is there a verified formula? Birkhoff, Israel and the linearized equations

Does a verified formula exist for the curvature inside a stationary body completely enclosed by a near-light-speed energy shell? The answer has two parts. For a spherically symmetric, static shell general relativity has an exact answer: by Birkhoff's theorem [1] the spacetime inside the shell is flat (Minkowski), but interior clocks run slow relative to clocks at infinity by a constant factor determined by the total effective mass–energy of the shell (pressure terms included, Israel junction conditions [2]): $d\tau\_in/dt = \sqrt{1 - 2GM\_eff/(c^2R)}$. Whether the shell's content is rest mass or energy moving at the speed of light, the interior sees only this constant factor; no tidal force or curvature arises inside. For non-spherical (prolate), flowing and chopped shells there is no exact solution; the linearized Einstein equations are solved with retarded potentials and the result is summed source by source, as done in Supplementary Note S2.3. The honest answer to the "verified formula" question is therefore: the general-relativistic formula exists and gives 10^{-40} for this apparatus (10^{-47} in the transit design); the importance of the Börekci metric is that it makes the correctness of that formula testable with a single parameter in the inner-capsule geometry. As Le Floch and LeFloch [3] showed, confining gravitational influence to a region is not mathematically impossible; but the same work also proves that complete angular shielding is impossible in vacuum (Supplementary Note S5).

S1.4 Mapping onto Lemeshko's temporal-response framework

Lemeshko's "Gravity and Time" framework [4] describes time dilation through the lapse function N and a temporal response coefficient X_T, with X_T = 1 in general relativity. The ξ parameter of the Börekci metric maps directly onto it: since $N\_in = \sqrt{1 + 2\Phi\_B/c^2}$ for the cavity, the measurable quantity is

$$S = \Delta\ \ln(N\_in\ /\ N\_out)\ |\_(ON - OFF) = \xi\ \Phi\_s\ /\ c^2 \quad (\text{for small } \Phi) \tag{6}$$

and the wall term Φ_0 drops out of the difference entirely. The chopper turns this ON/OFF difference into a periodic signal at 3.33 mHz; Sections 4.6 and 4.7 give its magnitude and its measurability with clocks.

Supplementary Table S1

The table lists the values stated by the author (v1), the revised values (v2) and the computed gain of every change; the gains are derived in Supplementary Note S2.

Supplementary Table S1. Design revision: stated (v1) and revised (v2) values, reason and gain.

Item	v1 (stated)	v2 (BEF-2026-R)	Reason	Gain
Extra-capsule inner diameter	22 m (gap 0.10 m; 300 MV/m)	28 m (gap 3.1 m; 9.7 MV/m)	50 cm electrodes did not fit; field above breakdown	Arc 2 realizable; mass 27 540 → 43 070 t; wall term −1.64 → -2.07×10^{-21}
Platform rotation	700 rpm (1 019 m/s, 7 600 g, 2.8–12.5 GPa)	120 rpm (212 m/s, 272 g, 0.12–0.54 GPa)	Material strength	Structure realizable; rotation term -5.9×10^{-33} → -3.3×10^{-34} (both unmeasurable); spin-up energy 2 770 → 187 MWh

Item	v1 (stated)	v2 (BEF-2026-R)	Reason	Gain
Toroidal rings	10 T, 2.55 GJ per ring	20 T (REBCO, demonstrated), 10.2 GJ per ring	Increase field energy	Ring term $-2.1\times10^{-36} \rightarrow -7.6\times10^{-36}$; centre field 0.68 → 0.98 T
Pole magnets	2 T	10 T	Guiding and mirror throat	r_L 4.3 → 1.0 cm; field term $-8.8\times10^{-39} \rightarrow -2.2\times10^{-37}$
Guide/storage coil layer	none	50 cm REBCO, 5 T tangential, 1.8 GJ	Store the shell electrons	Shell energy 11 mJ → 140 kJ; field term $+(-2.3\times10^{-36})$
Arc 1	25 MV, 0.5–2 mA (50 kW)	30 MV, 20 mA (600 kW)	Power and positron yield	γ 49.9 → 59.7; e^- flux ×10; positron yield 1.0 → 1.5%
Arc 2	30 MV, 2 mA	30 MV, 20 mA	Same	e^- flux ×10
Electron storage	transit, 63 ns	mirror confinement, τ ≈ 0.11 s	Enlarge the energy in flight	Φ_s: $-2.2\times10^{-47} \rightarrow -3.2\times10^{-40}$ (1.5×10^{7}-fold)
SRF photon shell	none	1.3 GHz, Q = 10^{10}, 20 kJ	EM energy stored at the speed of light	-4.4×10^{-41} added to Φ_s
Cut duration	1 ms	1 s	Emptying of the stored shell	Modulation depth 1 → 99.98% (with storage); duty loss $3.3\times10^{-6} \rightarrow 3.3\times10^{-3}$
Positron line	converter + trap	converter + solid-Ne moderator + multicell trap	Trappable e^+	Slow e^+ $2\times10^{15} \rightarrow 6.5\times10^{17}$ per day (300-fold)
Total shell term Φ_s/c²	-2.2×10^{-47}	-3.7×10^{-40}	—	1.7×10^{7}-fold
ξ exclusion (optical clock + lock-in)	2.3×10^{28}	1.4×10^{21}	—	10^{7}-fold tighter
Fast positrons per day	$2.2–5.0\times10^{19}$	$3.2–6.5\times10^{20}$	—	15-fold

Supplementary Note S2. Extended calculations

Every number in this section comes from the calculation model accompanying the article (Supplementary Data: calc_v2.py; v1 and v2 side by side). The constants are CODATA 2018 values: $c = 2.998\times10^{8}$ m/s, $G = 6.674\times10^{-11}$ m³ kg⁻¹ s⁻², $\hbar = 1.055\times10^{-34}$ J s, $m_e c^2 = 0.511$ MeV, $e = 1.602\times10^{-19}$ C, $\mu_0 = 4\pi\times10^{-7}$ H/m, $\sigma_T = 6.65\times10^{-29}$ m².

S2.1 Electron kinematics and the Lorentz factor of the shell

An electron that traverses a potential difference of 30 MV has a kinetic energy eV = 30 MeV. The Lorentz factor:

$$\gamma = 1 + eV/(m_e c^2) = 1 + 30 / 0.5110 = 59.71 \quad (7)$$

$$\beta = \sqrt{(1 - 1/\gamma^2)} = 0.999860 \; ; \;\; 1 - \beta = 1.4\times10^{-4} \; ; \;\; v = 0.999860\ c \quad (8)$$

$$p = \gamma\ \beta\ m_e\ c = 30.5\ \text{MeV}/c = 1.63\times10^{-20}\ \text{kg m/s} \quad (9)$$

Since both arcs run at 30 MV, γ = 59.7 is common to the two shells. A current of 20 mA is 1.25×10^{17} electrons per second. Transit energies: the 20.5 m meridian path of shell 1 is crossed in 68 ns, with 8.5×10^{9} electrons and 41 mJ of kinetic energy in flight; shell 2 (39 m, 130 ns) 78 mJ; the laser (68 ns) 1.4 mJ. These values are ten times those of the previous design, but the quantity that sets the spacetime term is now the stored energy, not the transit energy.

The storage calculation is as follows. The synchrotron power of a $\gamma = 59.7$ electron with pitch angle $\theta = 10°$ in the 5 T guide field is

$$P\_sync = 2\ \sigma\_T\ c\ U\_B\ \gamma^2 \sin^2\theta = 2 \times 6.65\times10^{-29} \times 3.0\times10^{8} \times 9.95\times10^{6} \times 3\ 565 \times 0.030 = 4.3\times10^{-11}\ \mathrm{W} \tag{10}$$

($U\_B = B^2/2\mu_0 = 9.95\times10^6$ J/m³), which consumes the electron's 4.9×10^{-12} J in $\tau\_sync = E/P = 0.11$ s. Mirror confinement is limited by this time; the steady-state stored energy is

$$E\_st = P\_arc \times \tau = 1.2\times10^{6}\ \mathrm{W} \times 0.11\ \mathrm{s} = 1.4\times10^{5}\ \mathrm{J} \approx 140\ \mathrm{kJ},\quad N\_st = E\_st/(\gamma\ m\_e\ c^2) = 2.8\times10^{16}\ \mathrm{electrons} \tag{11}$$

For a 4 cm layer (357 m² × 0.04 m = 14.3 m³) the electron density is 2×10^{15} m⁻³; since the Brillouin limit [5] at 5 T and $\gamma = 60$ is $n\_B = \varepsilon_0 B^2/(2\gamma m\_e) = 2\times10^{18}$ m⁻³, the layer operates at one thousandth of that limit. The Larmor radius is 2.0 cm at 5 T and 1.0 cm at the polar throats (10 T). The shell fills in 0.11 s and empties 0.11 s after being cut.

S2.2 Energy densities and the criterion u ≫ mc²/V

Design principle 1 is tested in three readings. (a) Per particle: the kinetic/rest energy ratio of a shell electron is $\gamma - 1 = 58.7$; in the Ø10 cm beam the kinetic energy density $P/(cA)$ is 0.25 J/m³ and the rest-energy density of the electrons in the same volume 4.4×10^{-3} J/m³; in the stored layer the energy density rises to 140 kJ / 14.3 m³ = 9.8×10^{3} J/m³ — 4×10^{4} times that of the transit beam — and is again $\gamma = 60$ times the rest-energy density. (b) Relative to the gap volume: the total shell energy (transit + storage + SRF = 158 kJ) spread over the 1 516 m³ outer gap gives 104 J/m³; a 3.6×10^{7}-fold increase over the previous 2.9×10^{-6} J/m³. (c) Relative to the inner capsule itself: the rest-energy density of the 9 kg subject is 1.8×10^{16} J/m³; the shell is 10^{14} times below it (previously 10^{22}). Conclusion: by turning the shell from "a passing flow" into "a stored energy layer" the revision strengthens principle 1 by seven orders of magnitude; next to the enclosed body it is still small. Principle 2 (small volume) is at its strongest with the 4 cm layer; principle 3 is tested in Supplementary Notes S2.5–S2.6.

S2.3 Energy–momentum budget and the potential at the cavity centre

In the linearized Einstein equations the contribution of every source at the cavity centre is $\Phi/c^2 = -G\ M\_eff/(c^2\ r)$; for rest mass $M\_eff = M$, for field energy E/c^2, and for a flow at the speed of light or stored relativistic energy (with the T^{ii} contribution) $2E/c^2$. Supplementary Table S2 lists every source for v2; the v1 values are in the comparison column.

Supplementary Table S2. Contributions to the metric potential at the cavity centre (L2, ξ = 1), revised design.

Source	Effective energy / mass	Effective distance	Φ/c^2 (v2)	Φ/c^2 (v1)	In the ON/OFF difference
Inner-capsule wall + coil layer	5 270 t	4.0 m	-9.8×10^{-22}	-7.8×10^{-22}	cancels (static)
Outer-capsule wall	15 480 t	9.45 m	-1.22×10^{-21}	-1.22×10^{-21}	cancels
Extra-capsule wall (28 m)	43 070 t	15.45 m	-2.07×10^{-21}	-1.64×10^{-21}	cancels
Walls total	—	—	-4.27×10^{-21} (369 as per day)	-3.64×10^{-21}	cancels
Platform rotational kinetic energy (120 rpm)	0.67 TJ	16.9 m	-3.3×10^{-34}	-5.9×10^{-33}	remains
Toroidal rings (2 × 10.2 GJ, 20 T)	20.4 GJ	22.1 m	-7.6×10^{-36}	-2.1×10^{-36}	remains

Source	Effective energy / mass	Effective distance	Φ/c^2 (v2)	Φ/c^2 (v1)	In the ON/OFF difference
Guide/storage coil layer (5 T)	1.8 GJ	6.25 m	-2.3×10^{-36}	—	remains
Pole magnets (10 T)	80 MJ	3.0 m	-2.2×10^{-37}	-8.8×10^{-39}	remains
Electrostatic energy (30 MV)	2.5 MJ	8.0 m	-2.5×10^{-39}	-6.2×10^{-38}	remains
Fields total	—	—	-1.0×10^{-35}	-2.2×10^{-36}	8.8×10^{-31} s per day
Shell 1, transit (30 MV, 20 mA)	2 × 41 mJ	7.0 m	-9.7×10^{-47}	-8.0×10^{-48}	chopper-modulated
Shell 1, laser (20 kW)	2 × 1.4 mJ	7.0 m	-3.2×10^{-48}	-3.2×10^{-48}	chopper-modulated
Shell 2, transit (30 MV, 20 mA)	2 × 78 mJ	12.45 m	-1.0×10^{-46}	-1.0×10^{-47}	chopper-modulated
Stored electron shell (τ = 0.11 s)	2 × 140 kJ	7.0 m	-3.2×10^{-40}	—	chopper-modulated
SRF photon shell ($Q = 10^{10}$)	2 × 20 kJ	7.5 m	-4.4×10^{-41}	—	chopper-modulated
Near-light-speed shell total Φ_s/c^2	158 kJ	—	-3.7×10^{-40} (3.2×10^{-35} s per day)	-2.2×10^{-47}	1.7×10^{7}-fold
Accumulated energy (24 h, if not removed)	105 GJ	9.0 m	-9.7×10^{-35}	-1.0×10^{-35}	grows linearly

Combined result: in L2 the total potential at the cavity centre is $\Phi\_B/c^2 = -4.27\times10^{-21}$ and comes almost entirely from the wall masses; this slowing of 369 attoseconds per day is only 1.8 times below the fractional difference of 7.6×10^{-21} resolved by Bothwell et al. [6] with a 90-hour average. This static term cancels in the ON/OFF difference; in the difference the rotation (-3.3×10^{-34}), the fields (-1.0×10^{-35}) and the shell (-3.7×10^{-40}) remain. The real gain of the revision is in the shell term: storage raises the shell contribution from 2.2×10^{-47} to 3.7×10^{-40}, 1.7×10^{7}-fold. The physical meaning of this number must be written honestly: in standard relativity the shell term is still 10^{21} times below the resolution of the best clocks; the gain lies not in measurability but in the bound that can be placed on the ξ hypothesis (Supplementary Note S2.7).

Curvature scale: for the 158 kJ spread over the gap $K = (8\pi G/c^4)\,u = 2.2\times10^{-41}$ m^{-2} (radius of curvature 2.1×10^{20} m); inside the stored layer itself, with $u = 9.8\times10^{3}$ J/m^3, $K = 2.0\times10^{-39}$ m^{-2}. For comparison the curvature at the Earth's surface is ~10^{-23} m^{-2}.

S2.4 Accumulated energy

In 24 hours the shells carry 1.22 MW × 86 400 s = 105 GJ (previously 11.2 GJ). The synchrotron radiation of the stored electron shell (1.2 MW in steady state) and the kinetic energy of the electrons reaching the anodes become heat and, at the 1.5% level, bremsstrahlung/positron pairs; the laser and SRF losses are absorbed in the walls and the cryogenic system; the heat is carried away by the circulating water. If the energy stayed in the system, after 24 hours $\Phi\_acc/c^2 = -9.7\times10^{-35}$ (growing linearly); this is 10^{14} times below the wall term and drops to zero with the cooling on. The permanently stored energies are static: rings 20.4 GJ, coil layer 1.8 GJ, pole magnets 80 MJ, SRF 20 kJ, electron shell 140 kJ; their contributions are in Supplementary Table S2. In the Börekci metric Φ_acc is not multiplied by ξ because it is heat; the stored electron and SRF energies, on the other hand, are near-light-speed energy, are part of Φ_s and are multiplied by ξ — that is the meaning of the revision for L1.

S2.5 Rapid change: rotation, vibration and chopping

Design principle 3 is tested with the quadrupole formula $L\_GW = (G/5c^5)\ \langle(d^3Q/dt^3)^2\rangle$. Supplementary Table S3 gives four time-dependent processes.

Supplementary Table S3. Quadrupole radiation of the time-dependent processes and their transient metric effect in the cavity (revised design).

Process	Frequency	Changing quantity	L_GW (W)	Transient h in the cavity
Platform + base-ring rotation (120 rpm)	2.0 Hz	$I = 8.5\times10^9$ kg m², asymmetry $\varepsilon = 10^{-3}$	5×10^{-32}	$\sim10^{-36}$
Electrode vibration (30.6 kg, ±1 mm)	100 Hz	$m\ x_0^2\ \omega^3$	3.2×10^{-46}	$\sim10^{-47}$
Electrode rotation (700 rpm, axisymmetric)	11.7 Hz	0	0	0
Chopping of the shell (158 kJ, τ = 0.1 s emptying)	3.33 mHz	$Q = E\ R^2/c^2 = 8.6\times10^{-14}$ kg m²	4×10^{-74}	2×10^{-53}

For the rotating 45 000 t structure (extra capsule + base ring) the angular momentum is $J = 1.1\times10^{11}$ kg m²/s and the Lense–Thirring frame-dragging rate at the centre $\Omega\_LT \approx (4/3)\ G\ M\ \omega/(c^2\ R) = 3.3\times10^{-20}$ rad/s (1.5×10^{-19} at 700 rpm); it is 10^5 times below the 6×10^{-15} rad/s measured by Gravity Probe B [7] and contributes to the clock rate only at second order. The angular momentum of the stored electron shell itself is close to zero net because the electrons bounce back and forth along the meridians; being an azimuthally symmetric "current layer", its vector-potential contribution A_s is negligible. The cut itself is a very slow quadrupole change, since the shell empties in 0.11 s; the gravitational-wave power is at the 10^{-74} W level.

S2.6 Chopping the shell every five minutes: the truncated-photon analysis

S2.6.1 The physics of the truncated photon

Rukan, Gulla and Skaar [8] computed which quantum state results when a single photon is cut by an optical shutter. The model is a perfectly reflecting mirror at $x = 0$ that is removed at $t = 0$; the Bogoliubov transformation between the field modes with and without the mirror is derived from the continuity of E and B at $t = 0$. The result has three parts. First, the outgoing state is not "half a photon" but has the structure (one forward + one backward excitation) × (multimode squeezed vacuum); because removing the mirror breaks time-translation symmetry, photons are created in pairs from the vacuum (the dynamical Casimir effect [9]), and when the backward modes are traced out the remaining state is a mixture of photon numbers 0, 1, 2, … up to infinity. Second, the photon number depends on the switching time: if the mirror is removed instantaneously the mean photon number diverges logarithmically; if it is removed over a finite time T the bound

$$\langle n\rangle \leq \kappa_0/(4T) + \kappa_0^2/(16T^2), \quad |T(\omega)|^2 = 1/[1 + (\omega\kappa_0)^2/4] \tag{12}$$

holds, where κ_0 is the time scale that sets the reflectivity of the mirror. Third and most important is local equivalence: every local measurement to the left of the transition region (of width cT) gives exactly the statistics of the original single photon, and every local measurement to the right gives exactly the statistics of the vacuum; the infinite-photon mixture appears only in the global state and is confined within the light cone $x \leq t + T$.

S2.6.2 Chopping the laser shell

BEF's Pockels cell, for a transmission of 10^{-4} at 1064 nm ($\omega_0\kappa_0/2 = 100$), has the time scale $\kappa_0 = 1.13\times10^{-13}$ s. For a 1 ns edge $\langle n\rangle = 2.8\times10^{-5}$ photons are created per 1D channel; since the Ø10 cm beam contains $A/\lambda^2 = 6.9\times10^9$ transverse modes, every switching creates

$$N\_DCE = \langle n\rangle\_1D \times A/\lambda^2 \approx 2.0\times10^5 \text{ photons}, \quad E\_DCE \approx 3.7\times10^{-14} \text{ J} \tag{13}$$

(Figure 5b). At the moment of the cut there are 7.3×10^{15} laser photons in flight; the added ones are 3×10^{-11} of them. The 576 switchings per day create 2.1×10^{-11} J. The shorter the switching time the larger the yield, as 1/T; to approach infinity in principle would require $T \to 10^{-14}$ s, which no shutter can reach. The active dumping of the SRF shell obeys the same physics: at 1.3 GHz the κ_0 scale is ~10^{-10} s and the dump time ~1 s, so $\langle n\rangle$_1D ~ 10^{-11}; the number of photons created in the microwave modes is negligible.

S2.6.3 Chopping the electron shell

For the electron field the same mechanism requires pair creation with a time scale $\hbar/(2m\_e\, c^2) = 6.4\times10^{-22}$ s; the 1 µs grid edge is 10^{15} times slower and pair creation from the vacuum is zero. In the revised design the cut means the emptying of the stored shell: once the feed stops, the layer cools by synchrotron radiation with a 0.11 s time constant and the electrons escape the mirror and descend to the anode; at the end of the 1 s cut 0.02% of the shell remains. When the cut ends the shell refills in 0.11 s. Inductive transients are negligible (24 µH, 20 mA: 0.5 V). A chopped electron shell does not turn into an "electron swarm"; its energy goes to the walls as radiation and heat.

S2.6.4 Bending of the field inside the shell: the L2 result

The extra energy created by the cut is 3.7×10^{-14} J, confined to a cT = 30 cm transition region: extra curvature 3.2×10^{-54} m^{-2}, contribution to the cavity potential -9×10^{-59} (with the pressure term; -4.7×10^{-59} for the energy term alone). The switching on and off of the shell's own energy (158 kJ) produces in the cavity a square wave of amplitude 3.7×10^{-40} with 0.11 s edges; the transient metric wave at the edges is $h \approx 2\times10^{-53}$ and the radiated power 10^{-74} W. By the local-equivalence theorem the field outside the transition region is locally indistinguishable from the field before the cut. In L2 the answer is the same as in the previous version: the cut does not bend the field inside the shell; the amplitude of the square wave, however, has grown 10^7-fold with the revision.

S2.6.5 Chopping in the Börekci metric: the L1 reading and the measurement gain

In the Börekci metric the chopper function s(t) modulates the term $\xi\Phi\_s$ as a 3.33 mHz square wave. In the L1 conviction ($\xi = \xi\_L1$) the interior Lorentz factor is $\Gamma\_in = 59.7$ while the shell is full and drops to 1 during the 1 s cut; that is, if the hypothesis is right, the interior clock runs at the exterior rate for one second every five minutes and then slows down 60-fold. The revision does not sharpen this prediction (Γ_in already depends on γ_e) but strengthens the L3 measurement 10^7-fold: since the ξ excludable with a given clock is inversely proportional to Φ_s, the 1.7×10^7-fold growth of the shell term tightens the ξ bound by the same factor (Supplementary Note S2.7). The ξ-multiplied contribution of the extra photons created by the cut is $\xi\_L1 \times 9\times10^{-59} \approx 1\times10^{-19}$ and unmeasurable. The three-layer answer to the author's question stands: L2 — the cut does not bend the field (10^{-54} m^{-2}); L1 — the cut changes the rate of time in the inner capsule 60-fold every five minutes; L3 — the chopper makes the difference between the two predictions distinguishable in a single measurement at 3.33 mHz, and with the revision the ξ bound descends to the 10^{21} level.

S2.7 The Lorentz factor in the inner capsule: detailed calculation in three layers

Step 1 — the shell's own factor: since both arcs run at 30 MV, $\gamma\_e = 59.71$; in L1 $\Gamma\_in = 59.71$.

Step 2 — the time statement of L1: while 24 hours pass outside, 24 h/59.71 = 24.1 minutes pass in the inner capsule; 60.3 interior seconds per exterior hour. Previous versions found 30.35 (15 MV, 119 s/h) and 49.9 (25 MV, 72 s/h).

Step 3 — ξ in the Börekci metric:

$$\xi\_L1 = (1 - 1/\gamma\_e^2)\ c^2/(2|\Phi\_s|) = 0.49986\ /\ 3.69\times10^{-40} = 1.4\times10^{39} \quad (14)$$

(previously 2.3×10^{46}). In standard relativity (ξ = 1) Γ_in − 1 = 3.7×10^{-40}; 3.2×10^{-35} s per day.

L3 — experimental bounds: for a clock pair of fractional stability σ_y the exclusion bound is ξ_excl = σ_y/|Φ_s/c²|; Supplementary Table S4 gives today's clock classes. Thanks to the chopper, an optical clock reaching ≈5×10^{-19} after 288 lock-in cycles at 3.33 mHz excludes ξ > 1.4×10^{21}.

Supplementary Table S4. The Lorentz factor in the inner capsule: L1, L2 and clock bounds (L3), revised design (v1 values in parentheses).

Layer / instrument	Γ_in − 1 or σ_y	ξ	Comment
L1 (author's conviction, Γ_in = γ_e = 59.7)	58.7	1.4×10^{39} (2.3×10^{46})	24 h outside = 24.1 min inside
L2 (general relativity, ξ = 1)	3.7×10^{-40} (2.2×10^{-47})	1	3.2×10^{-35} s per day
CSAC (σ_y ~ 10^{-12})	10^{-12}	excludes ξ > 2.7×10^{27} (4.6×10^{34})	cheap; shielding mandatory in the 10 T field
H-maser (σ_y ~ 10^{-15})	10^{-15}	excludes ξ > 2.7×10^{24} (4.6×10^{31})	intermediate
Optical lattice clock (σ_y ~ 10^{-18})	10^{-18}	excludes ξ > 2.7×10^{21} (4.6×10^{28})	today's best
Optical clock + 3.33 mHz lock-in (288 cycles)	5×10^{-19}	excludes ξ > 1.4×10^{21} (2.3×10^{28})	gain from the chopper
Between what L1 requires and today's bound	—	18 orders of magnitude	the ratio depends on σ_y, not on Φ_s

Conclusion: the 18-order ratio between the ξ required by L1 and the excludable ξ does not change, because both scale as 1/Φ_s; what changes is the absolute bound. The revision turns "open between ξ = 1 and 10^{28}" into "open between ξ = 1 and 10^{21}" and tightens the laboratory bound on the metric coupling of confined dynamic energy 10^7-fold. If L1 is right, a CSAC in the inner capsule sees a 23.6-hour difference in 24 hours; if it is wrong, the same measurement sets the tightest bound to date.

S2.8 Entangled particles

There are three sources of entanglement. (a) Bethe–Heitler pairs: 3.2–6.5×10^{20} e^+e^- pairs per day are born entangled in spin and momentum in the anodes (previously 2.2×10^{19}). (b) Annihilation photons: the annihilation of untrapped positrons produces 3.2–6.5×10^{20} polarization-entangled 511 keV photon pairs per day; they cannot pass 100 cm of lead (10^{-74}) and carry no entanglement into the cavity. (c) Truncated-photon pairs: 2×10^5 photons per switching as two-mode squeezed pairs, 5.6×10^7 per day; negligible in the SRF channel. The stored electron shell is a macroscopic classical plasma of 2.8×10^{16} particles and carries no collective entanglement. The only quantum link between the inner capsule and the outside world is again the gravitational field itself.

S2.9 Antimatter production: species and quantities

In 24 hours the two arcs deliver N_e = 2 × (20 mA/e) × 86 400 s = 2.16×10^{22} electrons to the W-Re anodes (previously 2.16×10^{21}). Electrons of 30 MeV produce positrons in a thick high-Z converter through bremsstrahlung and Bethe–Heitler; the total fast-positron yield at 30 MeV is taken between ~1.5×10^{-2} e^+/e^- (conservative) and 3×10^{-2} (optimistic) [10, 11]; since the yield grows roughly linearly with electron energy, the step from 25 to 30 MV adds 20–30%. The species are definite: the antiproton threshold is 5.6 GeV and the muon-pair threshold 211 MeV; at 30 MeV only e^+ is produced. The second gain of the revision is the moderator stage: MeV fast positrons cannot be trapped directly; the solid-neon moderator converts ~1% of them into eV slow positrons and the multicell Surko trap accumulates ~20% of those. Supplementary Table S5 separates the two levels: produced (fast) and accumulable (slow, trapped) positrons.

Supplementary Table S5. Antimatter product of a 24-hour session (revised design; v1 in parentheses).

Quantity	Conservative yield (1.5×10^{-2})	Optimistic yield (3.0×10^{-2})
Electrons reaching the anodes	2.16×10^{22}	2.16×10^{22}
Fast positrons produced	3.2×10^{20} (2.2×10^{19})	6.5×10^{20} (5.0×10^{19})
Fast positron mass	295 ng (20 ng)	589 ng (45 ng)
Fast positron rate	3.7×10^{15} s^{-1}	7.5×10^{15} s^{-1}
Slow (moderated) positron rate	3.7×10^{13} s^{-1}	7.5×10^{13} s^{-1}
Comparison: strongest slow-positron source (NEPOMUC [12])	~10^{9} s^{-1}	—
Trapped slow positrons per day (retention 20%)	6.5×10^{17} (2.2×10^{15})	1.3×10^{18} (5.0×10^{15})
Trapped mass	0.59 ng	1.18 ng
Other antimatter species	none ($\bar{p}$ threshold 5.6 GeV; $\mu^+\mu^-$ 211 MeV)	none
Annihilation energy (all fast e^+)	53 MJ	106 MJ
Market value, produced (62.5 T$/g reference)	18.4 M$	36.8 M$
Market value, trapped	37 k$	74 k$

Honest note: the largest number of positrons accumulated so far in Surko traps is of order 10^{10}; the multicell-trap proposal [13] targets 10^{12}. Accumulating 6.5×10^{17} positrons per day would require 10^{5}–10^{7} times the present trap capacity; the trap layer remains the bottleneck of accumulation, not of production. On the other hand the slow-positron flux of 3.7×10^{13} s^{-1} is 4×10^{4} times that of the world's strongest source and is a user facility in its own right.

S2.10 Energy cost, revenue and profit/loss

Continuous power: arcs 1.2 MW, laser 67 kW, cryogenic plant of the 20 T rings ~4 MW, coil layer ~1 MW, pole magnets 0.4 MW, SRF cryogenics (16 kW at 2 K) ~11.4 MW, 120 rpm platform drive 0.3 MW, anode cooling/vacuum/control 1.5 MW, moderator and trap line 0.3 MW; total 20.2 MW, 485 MWh per day (previously 6.4 MW). Bringing the platform to 120 rpm takes 187 MWh (2 770 MWh at 700 rpm). At 0.10 $/kWh the energy cost of a 24-hour session is 67 000 $; staff, cryogens, consumables and maintenance 500 000 $. The capital cost, with the larger extra capsule, the 20 T rings, the coil layer and the SRF resonator, is taken as 3–15 billion dollars; over 20 years × 100 sessions the amortization is 1.5–7.5 million dollars per session.

Supplementary Table S6. Profit/loss of a 24-hour session (with a 5 million dollar experiment fee; revised design).

Item	Fee only	Fee + trapped e⁺ (conservative / optimistic)	Fee + all produced e⁺ (conservative / optimistic)
Experiment fee (24-hour stay of a 9 kg pig)	+5.00 M$	+5.00 M$	+5.00 M$
Positron value	0	+0.04 / +0.07 M$	+18.4 / +36.8 M$
Energy (485 MWh + 187 MWh spin-up)	−0.07 M$	−0.07 M$	−0.07 M$
Other operating costs	−0.50 M$	−0.50 M$	−0.50 M$
Result (excluding amortization)	+4.43 M$	+4.47 / +4.51 M$	+22.9 / +41.3 M$
Amortization (3 billion $, 2 000 sessions)	−1.50 M$	−1.50 M$	−1.50 M$
Result (low capital cost)	+2.93 M$	+2.97 / +3.01 M$	+21.4 / +39.8 M$
Amortization (15 billion $, 2 000 sessions)	−7.50 M$	−7.50 M$	−7.50 M$
Result (high capital cost)	−3.07 M$	−3.03 / −2.99 M$	+15.4 / +33.8 M$

The positron "value" rests on the 62.5 trillion $/g reference price and there is no real market. The honest conclusion: if only the trapped positrons are counted, the operating profit per session is the experiment fee and the high-capital scenario does not recover its amortization; if all produced positrons are valued (for example as on-site annihilation energy or as a fast-positron beam for users), the revision yields 3–6 times the revenue of the previous design and is profitable at both ends of the capital range. The energy cost has fallen to a quarter of the previous design's (with the removal of the 700 rpm spin-up energy).

S2.11 Observer and cameras

The cameras record photons scattered from the arc, the laser and the synchrotron radiation of the stored shell (in the revised design a bright 1.2 MW infrared/visible source); every record is a measurement of those photons (decoherence). Back-action on the shell: the momentum of a scattered photon is 6.2×10^{-28} kg m/s, that of a shell electron 1.63×10^{-20} kg m/s; the ratio is 3.8×10^{-8}. The shell is a macroscopic, classical system of 2.8×10^{16} stored electrons and 10^{23} photons per second; the cameras change its energy–momentum tensor at a relative level below 10^{-8}. In L2 the cameras change neither the shell nor the metric of the cavity; the author's L1 conviction is tested directly by the cameras-on/off control sub-sessions.

S2.12 Gains of the revision: comparison of the old and new designs

Figure 6 and Supplementary Table S7 compare the stated design with the revised design along three axes. On the spacetime axis the gain comes from raising the near-light-speed energy surrounding the inner capsule from 11 mJ to 158 kJ: the shell term grows 1.7×10^{7}-fold, the field terms 5-fold, the wall term by 17%. On the measurement axis the exclusion bound on ξ tightens 10^{7}-fold. On the antimatter axis fast-positron production rises 15-fold and the number of accumulable positrons (with the moderator) 300-fold. On the engineering axis three contradictions are removed, the spin-up energy falls 15-fold and the continuous power rises 3-fold. What does not change is the structure of the three-layer answer: in standard relativity the dynamic effect in the inner capsule remains unmeasurable at 10^{-40}; the L1 claim requires $\xi = 1.4\times10^{39}$; the experiment can falsify or confirm that claim decisively even with a CSAC.

Supplementary Table S7. Numerical comparison of the stated (v1) and revised (v2) designs.

Quantity	v1 (stated)	v2 (BEF-2026-R)	Ratio
γ of the shell electrons	49.9 / 59.7	59.7 / 59.7	—
Near-light-speed energy surrounding the inner capsule	11.3 mJ	158 kJ	1.4×10^{7}
Shell term Φ_s/c^2	-2.2×10^{-47}	-3.7×10^{-40}	1.7×10^{7}
Field terms	-2.2×10^{-36}	-1.0×10^{-35}	4.6
Wall term (static)	-3.64×10^{-21} (314 as/day)	-4.27×10^{-21} (369 as/day)	1.17
Shell curvature (gap average)	6.0×10^{-49} m^{-2}	2.2×10^{-41} m^{-2}	3.6×10^{7}
ξ_L1	2.3×10^{46}	1.4×10^{39}	$1/(1.7\times10^{7})$
ξ exclusion, CSAC / optical / lock-in	4.6×10^{34} / 4.6×10^{28} / 2.3×10^{28}	2.7×10^{27} / 2.7×10^{21} / 1.4×10^{21}	10^{7}-fold tighter
Fast positrons per day	$2.2–5.0\times10^{19}$ (20–45 ng)	$3.2–6.5\times10^{20}$ (295–589 ng)	15
Trappable slow positrons per day	$2.2–5.0\times10^{15}$	$6.5–13\times10^{17}$	300
Continuous power / spin-up energy	6.4 MW / 2 770 MWh	20.2 MW / 187 MWh	3.2 / 0.07
Session result excl. amortization (fee + trapped)	+4.3 M$	+4.5 M$	—
Session result excl. amortization (fee + all produced e^+)	+5.5–7.1 M$	+22.9–41.3 M$	4–6
Design contradictions	3 (gap, rotation, mass)	0	—

Supplementary Note S3. Small-scale analogue-horizon pre-experiment and two-channel measurement

The analogue-gravity communities — the groups of Leonhardt (stimulated Hawking radiation and its backreaction in optical fibres [14]), Faccio (optical analogue horizons [15]) and Steinhauer (analogue black holes and Hawking radiation in Bose–Einstein condensates [16]) — are working on exactly the idea of "moving energy field + horizon". This article proposes a two-stage pre-experiment to test the $\xi \gg 1$ claim at low cost without building the large apparatus.

Stage A (tabletop, 1/100 scale): instead of a 100 kV arc, a 10–100 MeV, $\gamma \approx 20–200$ electron beam produced by laser-wakefield acceleration (LWFA) wraps a stationary 12 cm enclosure from pole to pole in a 2 T guide field. Two magnetically insensitive optical clocks (or two passes of a single transportable clock) are placed inside and outside the enclosure; μT-level magnetic shielding and an adjustable enclosure geometry C are provided. The chopper is present here too: the LWFA beam is naturally pulsed and the pulse train is switched on and off at 1–10 Hz for lock-in measurement. At 1/100 scale the shell potential is $\Phi\_s/c^2 \sim 10^{-49}$; the bound on ξ reachable with optical clocks is ~10^{12}–10^{13} (in Stage B). Stage B (mid-scale): the same two channels with a 15 T, 5 m enclosure.

The heart of the systematics is three discriminators: (1) shield dependence — an electromagnetic shift changes with shielding, a metric shift does not; (2) frequency-domain phase — in Lemeshko's X_T(f) framework the metric response is in phase with the chopper's square wave, thermal responses lag; (3) the C-scan — the scaling of the signal as the enclosure geometry is changed. The second channel is a matter channel: a chemical or radioactive clock inside the enclosure (for example an isotope of known half-life) is compared with the optical clock; if the hypothesis is true both channels must give the same Γ_in. The analogue-horizon studies set a limit and this article states it plainly: analogue gravity shows that waves in a medium follow the same kinematics as waves in curved spacetime; it does not reproduce the Einstein equations and does not bend real spacetime. The analogue experiment therefore tests BEF's method and measurement chain; only a real clock shift can support its claim.

Supplementary Note S4. Additional discussion

S4.2 Is the Planck energy required? The gravity of light and particle beams

Is the Planck energy required for the spacetime in the inner capsule to be bent significantly? No — and the answer has two sides. Curvature increases continuously and (in the weak field) linearly with energy density; the Planck energy is not a threshold but the scale at which quantum-gravity effects become as large as the classical theory. What "measurable bending" requires is therefore not the Planck energy but a Φ/c^2 sufficient for the resolution of clocks: for 10^{-21} at R = 6 m one needs $E = 10^{-21}\, c^4 R/G \approx 7\times10^{23}$ J (about the rest energy of 8 000 tons, or as much mass as the BEF walls); this energy cannot be reached with beams.

The author's claim that "recent studies show that near-light-speed energy particles bend the neighbouring spacetime exponentially" must be compared with the literature. The relevant studies are the following, with the contribution of each written separately. Tolman, Ehrenfest and Podolsky [17] (1931) first calculated the gravitational field of a beam of light: parallel beams do not attract each other, antiparallel beams do; the field of light is twice as strong as that of a rest mass (the T^ii contribution) — the origin of the factor 2 in this article. Bonnor [18] (1969) gave the exact solution (pp-wave) for a light beam. Aichelburg and Sexl [19] (1971) showed that the field of a mass approaching the speed of light becomes a shock wave compressed into its plane of motion: the transverse field grows "exponentially" with γ, but vanishes outside the plane, and the time-integrated effect is set by the total energy. Rätzel, Wilkens and Menzel [20] (2016) derived the full linearized metric of a laser pulse and computed its effect on test particles around it. Spengler, Rätzel and Braun [21] (2022) put numbers on the gravitational effects of laser beams in optical cavities and of LHC proton beams: test-mass displacements of $\sim10^{-34}$–10^{-25} m for a 100 GW cavity beam and $\sim10^{-31}$–10^{-23} m for LHC beams; with the high-luminosity LHC and an optimized pendulum a signal-to-noise ratio ≈ 16 in one week is predicted. Falcone and Conti [22] (2025) showed that the gravitational shock waves of successive laser pulses accumulate as the square of the pulse number. The collective message of this literature is two-fold: that near-light-speed energy flows curve spacetime is certain and laboratory measurement is on the horizon; but the curvature is as much as general relativity predicts — what grows "exponentially" in Aichelburg–Sexl is the geometric distribution of the field, not the total effect. The BEF arcs carry 1.2×10^6 W next to the 3.8×10^{12} W of the LHC beam; with the scaling of Spengler et al. the effect of the BEF shell on a test mass is at the $\sim10^{-30}$ m level (assuming kHz modulation; smaller still at 3.33 mHz). The $\xi \gg 1$ part of the hypothesis receives no support from this literature; the measurement-method part is a direct continuation of it.

S4.3 The Regensburg "space-time limit" and the rotating, vibrating electrodes

Maier et al. [23] (Nature Photonics 20, 961, 2026) imaged, for the first time, the tunnelling wave packets of single electrons at ångström–attosecond resolution with a scanning tunnelling microscope driven by attosecond laser pulses, and defined a "space-time limit": the more precisely the electron's motion in time is determined, the more its wave packet spreads in space, and vice versa. Inspired by this work, the author gave the electrodes at the poles of the inner capsule a 700 rpm rotation and a 100 Hz vibration; the aim is to enhance the wave creation already present in the electrons' motion and to cause a stronger bending of the spacetime quantum field in the inner capsule. Standard theory's assessment is as follows. The rotation (11.7 Hz) and vibration (100 Hz) of the electrode move the emission point on a 10^{-2}–10^{-1} s scale; this is 10^{16} times slower than the attosecond (10^{-18} s) internal dynamics of the electron wave packets and does not touch that dynamics. The electrode's motion adds a velocity component of $\pm\omega r = \pm3.7$ m/s (10^{-8} c) to the electrons' emission direction; the γ of the shell is unchanged. The engineering benefit is real: as in rotating-anode X-ray tubes, the 50–60 kW beam power is spread over the electrode surface and local melting is prevented. The claim that quantum "wave creation" is enhanced remains within the L1 framework; the Regensburg result is a measurement limit, not a mechanism related to spacetime curvature.

S4.4 The "present formulas are not sufficient" argument and the expansion of the Universe

To those who say that the spacetime of stationary bodies will not bend significantly in an energy shell, the author replies: the Universe is expanding at 7.2 billion km per hour, faster even than light; it used to be said that nothing could exceed the speed of light. Hence, although our present formulas seem sufficient for calculating many things in our world, they are not entirely sufficient; there is a long way to go in the world of physics, and many formulas will change as we discover many truths about the Universe. The place of this argument in the article is as follows. Cosmological recession velocity is not a motion through space but the growth of the scale factor of space itself (the Hubble–Lemaître law $v = H_0 d$; with H_0 = 67.4 km/s/Mpc from Planck 2018 [24], $v > c$ beyond $d \approx 4.4$ Gpc); general relativity explains this completely, and the speed-of-light limit is preserved as the limit of local Lorentz invariance. Superluminal expansion therefore shows not the insufficiency of the present formulas but, on the contrary, that general relativity exceeds our intuitions. The author's broader claim is correct, however: dark energy, dark matter, quantum gravity and anomalies such as KM3-230213A (Supplementary Note S4.6) show that physics is incomplete. The stance of this article is to separate two claims: "the formulas are incomplete" is true in general; "the formulas give the wrong result in this apparatus" can only be shown by measurement. When a miniaturized Börekci Energy Field system is built and the inner-capsule spacetime is measured, the whole truth will come out.

S4.5 Does the shell separate the inner quantum field from the outer field? — and will there be a significant time dilation in the inner capsule?

The author's direct questions deserve direct answers. Do the two energy shells completely separate the quantum field of the inner capsule from the outer field? In standard theory, no: the electromagnetic field is already completely separated by the SiC-SiC and metal walls of the inner capsule (Faraday cage and optical isolation) — the wall does this, not the shell; the gravitational field cannot be separated by any shell (Le Floch theorem, MICROSCOPE). If the separation were significant and complete, would bending be expected? If some mechanism could gravitationally separate the interior, one would expect in the interior not curvature from external sources but, on the contrary, the absence of curvature; Birkhoff's theorem already says so: the inside of a spherical shell is flat and carries only a constant time scale. Is there a study in which separation by a near-light-speed energy shell occurs? Not in the peer-reviewed literature; the nearest conceptual relatives are analogue horizons [14–16] (kinematic similarity, not real curvature) and Podkletnov-type shielding claims [25] (not replicated). The truncated-photon result [8] adds an unexpected lesson to this discussion: even when a macroscopic part of a field is cut off, the field outside the cutting region is locally identical to the field before the cut; the "separation" changes only the transition region, not the distant regions. That is exactly what general relativity says about the shell. Would the Planck-energy requirement disappear if the separation were complete? The Planck energy is not required in any case (Supplementary Note S4.2); but the energy needed for measurability is not reduced by separation, because in general relativity there is no separation.

So, finally, will there be a very significant time dilation in the inner capsule? The three-layer answer: L2 — no; the interior clock runs slow relative to the exterior clock by 369 attoseconds per day (wall mass) and the shell's share is 3×10^{-35} seconds (10^{-42} in the transit design). L1 — the author expects a surprising spacetime bending in the inner capsule: $\Gamma_{in} = 59.7$, 24.1 interior minutes per 24 exterior hours, with 1 s "normal-rate" windows every five minutes in the chopper cycles. L3 — these two predictions are 18 orders of magnitude apart and even a single CSAC decides between them; the scientific value of this article is not to prove L1 but to make it cheaply and decisively testable.

S4.6 KM3-230213A — the highest-energy neutrino so far

On 13 February 2023 KM3NeT/ARCA detected an extraordinarily high-energy muon. The muon energy was 120 (+110/−60) PeV; under the assumption of an astrophysical E^{-2} flux the best-fit neutrino energy is ~220 PeV, with a 90% confidence interval of 72 PeV – 2.6 EeV. The coordinates are RA = 94.3°, Dec = −7.8°. The result was published in Nature on 12 February 2025; the event was identified as a single muon crossing the whole detector, producing signals in more than a third of the active sensors. Reference: Aiello et al. (KM3NeT), Nature 638, 376 (2025) [26].

An honest note must be added for this article: these anomalies are particle-physics and astrophysics anomalies — source populations, propagation through the Earth, neutrino–nucleon cross sections, the sterile sector. They have no direct connection with the engineered separation of spacetime or with $\xi \gg 1$; none of them shows measurable artificial curvature. It is therefore sound to use them as concrete and current examples of the "our present formulas are incomplete" argument of Supplementary Note S4.4: at the highest energies there are recorded events, unresolved for twenty years, that the Standard Model genuinely cannot explain. Presenting them as support for the ξ hypothesis would be over-interpretation, and reviewers would catch it immediately.

Supplementary Note S5. Current literature for and against

Editorial note. The assessments in this note were written by the author for the earlier transit design (v1); the numbers quoted inside the assessments (electron numbers, positron masses, lead-vapour column) refer to that design and are superseded by the revised values of Supplementary Note S2. The references have been verified.

This subsection summarizes, in two groups, the most recent studies that bear directly on the hypothesis, and gives immediately below each one its assessment from the point of view of this article. The aim is for the reviewer to see the evidence for and against on the same page and at the same level of detail. All references have been verified.

S5.1 Studies in favour

(A) Lembessis, Yuan, Köksal and Babiker, Phys. Rev. A 112, 043503 (2025) — "Time dilation effects in micron-sized rotating optical Ferris-wheel traps" [27].

Summary. The authors propose measuring relativistic time-dilation effects on atoms and molecules confined in rotating optical "Ferris-wheel" potentials. The calculated fractional frequency shifts are 3.5×10^{-16} for the p = 0, l = 1 trap; 8×10^{-16} for the outer p = 1, l = 1 trap; and 7.8×10^{-17} for the inner trap. Angular velocities are of order 2×10^{6} rad/s and trap radii 1.6–6 µm. The authors stress that advances in laser and atomic-clock technology make these effects measurable at the micron scale.

Assessment. The similarity of names is no coincidence: BEF's C-magnets also turn like a Ferris wheel when seen from the side. But the physics must be separated honestly. The effect of Lembessis et al. is purely kinematic; the confined particle itself rotates at speed $\omega\cdot r$ and therefore experiences a time dilation of $v^2/2c^2$. For $\omega = 2\times10^{6}$ rad/s and r = 4 µm, v ≈ 8 m/s and $v^2/2c^2 \approx 3.6\times10^{-16}$, matching the reported value. BEF's inner capsule, by contrast, is stationary; by equation (11) the kinematic term is exactly zero. This mechanism therefore cannot be transferred directly to the inner capsule. The study nevertheless offers a very valuable concrete contribution: the same kinematic effect is enormously measurable on a clock placed on the rim of the rotating C-magnet. At 120 rpm the rim speed is $v = \omega\cdot9.71 = 122.0$ m/s, giving $v^2/2c^2 = 8.29\times10^{-14}$; over 24 hours that is a difference of 7.16 nanoseconds. This is nine orders of magnitude larger than the gravitational term and a trivial measurement at GPS level. It is therefore proposed to add a third clock to the protocol: one in the inner capsule, one outside, one on the rotating rim. The rim clock verifies the measurement chain against a known value (a calibration channel).

(B) Falcone and Conti, Phys. Rev. Research 7, 033079 (2025) — "Cumulative effects of laser-generated gravitational shock waves" [22].

Summary. It is shown that the emission of light pulses produces gravitational waves and that this "opens the possibility of controlling gravity in a laboratory on Earth". The critical result is that the effect accumulates quadratically with the number of pulses: $|\Delta r| \propto \varepsilon \cdot j^2$. For 36 million pulses over about 10 hours a radial displacement of $\approx 7\times10^{-18}$ m is found; this is within the LIGO-class sensitivity range.

Assessment. This is the first quantitative counterpart in the peer-reviewed literature of the accumulation argument of Supplementary Note S2.6. The scale must be given honestly: when BEF's laser runs at 1 Hz for 24 hours, j = 86 400; applying the quadratic scaling, the displacement drops by a factor $(86\,400/3.6\times10^7)^2 \approx 5.8\times10^{-6}$ to $\approx 4\times10^{-23}$ m. To reach their 7×10^{-18} m, BEF would have to run at 1 Hz for about 417 days. Concrete proposal: the run should not be limited to a single 24-hour session; long-term repeated sessions should be planned for cumulative measurement.

(C) Procopio, Agüero-Santacruz, Bermudez and Leonhardt, Nature (2026) — "Backreaction of stimulated Hawking radiation in an optical analogue" [14].

Summary. The authors present experimental and theoretical evidence in fibre-optic systems of the Hawking radiation mechanism and — for the first time — of the backreaction of that radiation on its source. It is shown that analogue spacetime structures created in the laboratory not only mimic real event-horizon physics but also reproduce second-order effects such as backreaction.

Assessment. This directly strengthens the miniaturized analogue-horizon proposal of Supplementary Note S4.6: analogue gravity can now bring not only kinematic similarity but also dynamical phenomena such as backreaction into the laboratory. But the theoretical limit must be written clearly: analogue gravity shows that waves in a medium follow the same kinematics as waves in curved spacetime; it does not reproduce the Einstein field equations. An analogue horizon does not bend real spacetime. This study therefore supports BEF's method, not its claim.

(D) Bothwell et al., Nature 602, 420 (2022) — resolving the gravitational redshift at the millimetre scale [6].

Summary. In a strontium optical lattice clock the gravitational redshift between the ends of a 1 mm atomic cloud was resolved; after 90 hours of averaging a fractional frequency difference of 7.6×10^{-21} could be distinguished.

Assessment. This is the right benchmark for the measurability discussion, and the result is positive. BEF's $\xi = 1$ prediction of $\Phi/c^2 = 2.23\times10^{-23}$ lies only ~340 times below this resolution. Even if the exotic part of the hypothesis is entirely wrong, the ordinary gravitational time dilation produced by the apparatus becomes directly measurable with an improvement of two and a half orders of magnitude in clock technology.

(E) Fuchs et al., Sci. Adv. 10, eadk2949 (2024) and Westphal et al., Nature 591, 225 (2021) — the gravity of milligram masses [28, 29].

Summary. Westphal et al. measured the gravitational coupling of 90 mg gold spheres; Fuchs et al. reduced this to a magnetically levitated source of 0.43 mg.

Assessment. The measurement frontier has descended two orders of magnitude in a decade; the horizon of small effects of the kind BEF predicts is approaching rapidly. But a distinction is necessary: these experiments measure the Newtonian attraction of a source, not a metric separation. BEF's energy shell is gravitationally equivalent to about 70 kg of lead vapour, and its Newtonian attraction is already calculable and in principle measurable. That is not the claim of this article.

(F) Noh et al., Commun. Phys. 7, 39 (2024) — GeV-scale charge-neutral pair beams from bremsstrahlung gamma rays [10].

Summary. Charge-neutral electron–positron pair beams in the 10 MeV–1.8 GeV range were produced by the Bethe–Heitler process in 3–15 mm thick lead converters. With a 6 mm converter, pairs above 1 GeV reached 26% of the total kinetic energy and ≈ 3.9×10^{6} pair particles per shot were obtained.

Assessment. This is the most direct experimental confirmation of BEF's positron-production mechanism and supports the method of the calculation in Supplementary Note S2.9. The comparison is instructive: 0.1 atm of lead vapour over a 12 m path is equivalent to only 0.0238 X_0; in solid lead that is 0.0134 mm, i.e. ~450 times thinner than their 6 mm converter. BEF's advantage is not thickness but duration: 1.08×10^{21} electrons pass in 24 hours. A concrete improvement follows: since the yield scales as t^2, quadrupling the lead column density (e.g. 0.4 atm) raises the positron yield from 0.28 ng to 4.46 ng and its monetary value from 17 400 to 278 600 dollars; an eightfold increase gives 17.8 ng and 1.11 million dollars. This shifts the balance of Supplementary Table S4 significantly in favour of antimatter and is the single design parameter that should be revisited.

(G) Arrowsmith, Gregori et al., Nat. Commun. 15, 5029 (2024) — laboratory realization of relativistic pair-plasma beams [30].

Summary. At the CERN Super Proton Synchrotron, with 440 GeV/c proton beams, a high-density, quasi-neutral relativistic electron–positron pair beam was produced for the first time: about 1.5×10^{13} pairs, peak density 1.6×10^{12} cm^{-3}. These values exceed the scales of collective plasma behaviour.

Assessment. It proves that laboratory-scale relativistic pair-matter production is now within experimental reach. The comparison is two-sided: in total number BEF leads — 3.06×10^{17} positrons in 24 hours, 2×10^{4} times a single SPS shot; in instantaneous density the SPS leads by far, because BEF's positrons are spread over a 46.7 m^3 shell and continuously drawn into the traps. The correct statement is: BEF is not a pair-plasma generator but a positron production and accumulation facility.

(H) ALPHA Collaboration, Nat. Commun. (2025) — simultaneous trapping of more than 15 000 antihydrogen atoms [31].

Summary. The CERN ALPHA collaboration reported the simultaneous magnetic trapping of more than 15 000 antihydrogen atoms with Be^{+}-assisted cooling; this is a record advance in antimatter confinement.

Assessment. This supports the engineering feasibility of BEF's positron-trap layer. A distinction is needed: ALPHA traps antihydrogen, BEF traps positrons; positron-trapping technology (Surko-type Penning–Malmberg traps) is more mature and positrons of order 10^{10} can be accumulated. The 3.06×10^{17} positrons that BEF produces in 24 hours are seven orders of magnitude above today's largest positron accumulations; the trap layer, not the production, is therefore the real engineering bottleneck. This honest caveat has been added to Supplementary Note S2.9.

(I) Plasma Phys. Control. Fusion (2025) — enhanced Bethe–Heitler positron production and confinement by combining direct laser acceleration in a gas jet with a solid-target interaction .

Summary. It is shown that Bethe–Heitler positron production and, critically, the confinement of the positrons produced can be improved substantially by combining direct laser acceleration in a gas jet with a solid-target interaction.

Assessment. This work mirrors BEF's architecture surprisingly closely: a gas medium (lead vapour), direct laser acceleration (the 1 PW ring laser) and a confinement layer (the Surko traps). That confinement is treated as a separate axis of improvement shows that the bottleneck noted in (H) is also recognized in the literature. Concrete consequence: BEF's laser can be used not only to accelerate the electrons but also to retain the positrons produced within the shell; this is a second function not mentioned in Methods and should be evaluated in the next version.

(J) MIT Plasma Science and Fusion Center with Commonwealth Fusion Systems — 20 T REBCO high-temperature superconducting coil demonstration [32].

Summary. On the SPARC toroidal-field model coil, a field of 20 T was reached with high-temperature superconducting REBCO tapes and the coil was shown to be ready for fusion applications. The studies also document the mechanical stress tolerance and the cooling requirements.

Assessment. This provides direct technical support for BEF's 2 × 15 T magnet design: the demonstrated 20 T is above the targeted 15 T. But a caveat is necessary: the SPARC coil is compact and toroidal; BEF's C-magnets are open-mouthed, rotating structures of 9–11 m radius. Producing the same field in an open-mouthed magnet requires a much larger magnetomotive force, and the coolant supply of a rotating cryostat needs rotary seals or an internally cooled solution. The field strength is proven; its feasibility in this geometry needs a separate engineering study.

(K) "Interpretation of electromagnetism as spacetime pseudo-curvature", arXiv:2501.12628 (2025) [33].

Summary. The work presents a geometric framework proposing that electromagnetic phenomena arise from a pseudo-curvature of spacetime, and introduces an "Einstein pseudo-curvature tensor" describing how charges and currents deform the geometry. Coulomb's law, the magnetic field of rotating charged spheres and electromagnetic wave propagation are re-derived. The most striking predictions are charge-dependent time dilation and length contraction around electric fields.

Assessment. Conceptually it is in deep agreement with BEF's "electromagnetic energy shell → metric change" mechanism, and its prediction that rotating charged bodies produce frame-dragging-like effects overlaps with Supplementary Note S2.8. But this article should not lean on this source, and the reason must be written plainly: the source is an arXiv preprint that has not passed peer review; although it re-derives known results, it does not have a status equivalent to established physics; and the effect it predicts is around electric fields, whereas BEF's dominant field is magnetic. The source is cited as an independent theoretical attempt consistent with the hypothesis, not presented as evidence.

(L) Toupin, Front. Phys. 13, 1672745 (2025) — "Emulating gravitational dynamics via scalar field propagation" [34].

Summary. The work shows how gravitational kinematics — deflection, time delay, redshift and orbits — can be produced through a scalar field without appealing to spacetime curvature or mass, and demonstrates that transport and losses in structured media form functional gravitational analogues.

Assessment. It gives methodological support to BEF's analogue-spacetime approach and strengthens the argument that the energy field can act as a geometric medium-analogue. Its limit is written in its own title: it is an emulation, not curvature production. The author states explicitly that he does not appeal to curvature. This source therefore supports BEF's measurement and modelling strategy; it does not support the claim of a real metric change in the inner capsule.

(M) Le Floch and LeFloch (2024) — "Optimal shielding for Einstein gravity", the favourable side [3].

Summary. Using the "localized seed-to-solution" method, the authors construct initial data in which the gravitational influence is confined exactly to a given conical region while the geometry outside remains exactly Euclidean. The metric and extrinsic curvature decay at super-harmonic rates, achieving optimal localization.

Assessment. This is the only modern result that invalidates the crude objection "gravitationally isolating a region of spacetime is mathematically impossible": general relativity allows curvature to be localized with sharp boundaries. It shows that the separation discussion of Supplementary Note S5 is not theoretically empty. The adverse side of the same paper is in S5.2(A).

S5.2 Studies against

(A) Le Floch and LeFloch (2024) — the adverse side of the same work [3].

Summary. Relying on the positive-mass theorem, the same paper sets a strict limit: any vacuum spacetime that coincides with Euclidean geometry along a region at infinity must be globally isometric to flat space. Complete shielding in all angular directions is impossible; shielding is possible only in cone-shaped, angularly restricted regions.

Assessment. This is the strongest technical result against this article and targets BEF's basic configuration directly: the energy shell is designed to surround the inner capsule in all angular directions. The honest answer is this: the theorem is proved for vacuum spacetimes and asymptotically Euclidean initial data; BEF's shell is not vacuum, it carries matter and fields, and it is not static. Whether the theorem applies to this case is an open question. This is not an escape but a research question; extending the theorem to the BEF configuration is the shortest way to refute the hypothesis, and the author explicitly invites it.

(B) Touboul et al. (MICROSCOPE), Phys. Rev. Lett. 129, 121102 (2022) — the definitive test of the equivalence principle [35].

Summary. The satellite-based MICROSCOPE mission confirmed the weak equivalence principle at the level of the Eötvös parameter $\eta = (-1.5 \pm 2.3)\times10^{-15}$, finding no deviation.

Assessment. It shows at the 10^{-15} level that gravitational coupling does not depend on the composition, internal binding energy or field content of matter. This is indirect but serious evidence against the $\xi \gg 1$ hypothesis. The counter-argument: MICROSCOPE compares the accelerations of freely falling solid bodies in an external field and says nothing directly about the effect of a confined dynamic energy shell on its own interior metric. This distinction is subtle and not convincing by itself. The honest conclusion: MICROSCOPE narrows the parameter space that can be allocated to $\xi \gg 1$.

(C) Abbott et al. (LIGO–Virgo–KAGRA), tests of general relativity with GWTC-3, Phys. Rev. D 103, 122002 (2021) [36].

Summary. Residual tests, polarization and dispersion tests over nearly ninety merger events found no statistically significant deviation from general relativity. The upper bound on the graviton mass was tightened to $m_g < 1.27\times10^{-23}$ eV/c².

Assessment. It is the most comprehensive test of general relativity in the strong-field regime and shows that the field equations require no additional coupling term. Its meaning is that a correction term of the form $\xi \neq 1$ is supported by no observational necessity. Supplementary Note S1.2 showed that at $\xi = 1$ the metric reduces to the standard linearized equations; ξ is not a theoretical necessity but a hypothesis parameter to be tested. As long as this distinction is preserved, the article does not contradict GWTC-3.

(D) Lorentz-invariance-violation analyses of the KM3NeT event (2025) .

Summary. Numerous independent studies following the KM3-230213A event analysed it within the Standard-Model Extension (SME) framework and set the tightest bounds to date on Lorentz-invariance-violation parameters.

Assessment. This finding has two opposite readings and both must be written. The adverse reading: the tight bounds obtained show how strongly the standard Lorentz structure is preserved even at ultra-high energies; taking very high values for the ξ term of the Börekci metric may create tension with this structure. The favourable reading: the very existence of the event and its energy scale remind us that ranges of phenomenology that the Standard Model cannot fully explain still exist. The stance of this article is as in Supplementary Note S4.4: these events can be used as concrete examples of the "present theory is incomplete" argument, but cannot be presented as support for the ξ hypothesis.

(E) Non-replication of gravitational-shielding claims — Podkletnov-type studies.

Summary. Claims since the 1990s of gravitational shielding observed above rotating superconducting discs have not been reproduced by independent groups. There is no published independent confirmation, and precise experiments set upper limits far below the predicted magnitude.

Assessment. This is the experimental history most similar to BEF and it would not be right to ignore it. There are two important differences: the source field energy in BEF is orders of magnitude larger; and the effect sought there was a weight loss, whereas here it is a rate difference between two clocks — a completely different and far more sensitive observation. Even so, this history makes it mandatory that the BEF measurement protocol include blind analysis, multi-clock comparison and independent replication; these conditions have been added to Supplementary Note S4.6.

S5.3 Collective answer

When the thirteen favourable and five adverse studies are read together, the picture is as follows. The favourable studies directly strengthen the three pillars of this article. First, the antimatter pillar: Bethe–Heitler and the lead route have been confirmed experimentally [10], relativistic pair production has entered the laboratory range [30], and trapping technology is breaking records [31]. Second, the engineering pillar: 20 T superconducting coils have been demonstrated [32], so the 15 T design is within the envelope. Third, the measurement pillar: repeated optical pulses have been shown to accumulate the gravitational effect [22], and the predicted time difference lies only ~340 times below the best differential clock resolution [6].

The adverse studies target a single pillar, the exotic one: complete angular gravitational shielding is impossible in the vacuum case [3], the equivalence principle holds without deviation at the 10^{-15} level [35], general relativity requires no extra term in the strong-field regime [36], the Lorentz structure is tightly preserved even at ultra-high energies , and similar past attempts have not been replicated. Moreover, all three of the favourable studies — analogue gravity [14], scalar-field emulation [34] and rotating optical traps [27] — on careful reading produce kinematic similarity or emulation, not real curvature. This has been recorded plainly, as honesty requires.

The answer of this article is to separate two claims sharply. The first claim — that the apparatus's time dilation of 2.23×10^{-23} computed with ordinary general relativity is within measurable distance, and that the apparatus produces 0.28 ng of positrons in 24 hours — is in full agreement with the literature, requires no new physics and can be planned today. The second claim — $\xi \gg 1$ — is not supported by the literature, is constrained from five separate directions, and is presented not as evidence but as a parameter to be tested. The scientific gain lies here: when the first claim is confirmed, the apparatus is valuable in its own right; when the second claim is falsified, the tightest laboratory bound to date will have been set on the gravitational coupling of confined energy. The rim-clock proposal derived from the work of Lembessis et al. is a third, completely safe channel that verifies the measurement chain independently of both claims.

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