

Boundary layer wind turning and cross-isobaric mass flux from an idealised single column model framework perspective

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Abstract

The atmospheric boundary layer (ABL) wind turning, the angle between the direction of the surface friction and the wind just above the ABL, plays an important role in the formation and lifespan of synoptic-scale cyclones. Analysis of this wind turning and the accompanying mass flux are used as diagnostics to evaluate how parametrization of sub-grid processes affect climate and numerical weather prediction models. A suite of single column model experiments, where a large parameter space is explored, are performed and analysed. The first GEWEX Atmospheric Boundary-Layer Study (GABLS1) is used as inspiration as the initial and forcing conditions, turbulence parametrization parameters and different vertical resolutions are simulated until quasi-steady state. Scaling of vertical profiles from the simulation suite identifies controlling parameters of the wind turning and cross-isobaric mass flux to be wind speed, roughness length and the ABL thickness. Systematic differences in the vertically integrated cross-isobaric mass flux between associated with vertical resolutions and choice of turbulence closure are found. The differences are greatest for simulations representative of locations in mid-latitude climate that often experience near-neutral conditions. The results can partly explain why state-of-the-art numerical weather prediction and climate models show very

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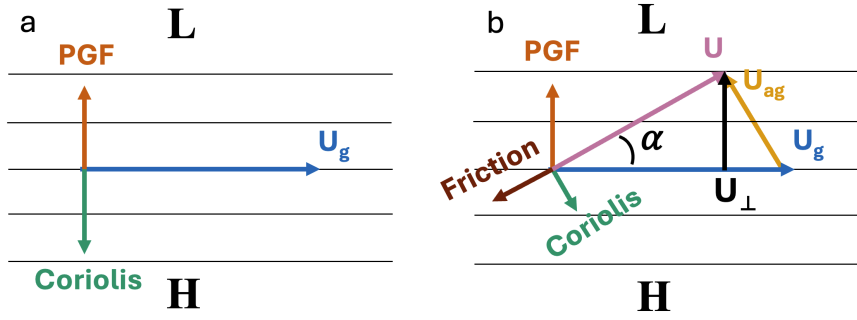


Fig. 1 Balanced flow above (a) and within (b) the ABL showing the geostrophic (U_g) and ageostrophic (U_{ag}) wind from the force balance between pressure gradient (PGF), Coriolis and friction. The isobar-perpendicular component of the near surface wind ($U_{\perp}$) and the wind turning angle (α) are also included.

different ABL wind turnings and mass fluxes that differs from observations and large eddy simulations.

Keywords NWP and Climate models · Cross-isobaric flow · Large scale circulation · Turbulence parametrisation · Boundary layer wind turning

1 Introduction

Over a flat and homogeneous surface, a balance between the pressure-gradient, Coriolis effect and frictional force governs the quasi-stationary flow in the atmospheric boundary layer (ABL). Above the ABL, in the free atmosphere, the effects of friction diminishes and due to this change in force balance the wind rotates clockwise (in the NH and counter clockwise in the SH) with height over the layer. This is commonly known as Ekman turning (Ekman 1905) and is schematically shown in Fig. 1. This turning of the wind can be used to analyze the effects of friction on the flow in the ABL as it is directly connected to cross-isobaric mass flow, $U_{\perp}$ (Fig. 1), throughout the ABL. This mass transport has an impact on development of e.g. cyclones and large-scale flow which can therefore affect medium-range weather forecasts and climate integrations (Beare 2007; Holton and Hakim 2013). Note that the ageostrophic wind ($U_{ag} = U - U_g$) is not the same as the cross-isobaric component of the near surface wind as it is the truly perpendicular component of the wind compared to the free stream wind that moves mass across isobars.

The analytical solution of the Ekman equations, assuming a constant eddy viscosity, gives a turning angle between the surface and the free stream wind of 45° , while a more realistic approach results in much lower angles (Grisogono 2011). This wind turning is generally underestimated in reanalysis and models in comparison to observations (e.g. Blackadar 1962; Hess and Garratt 2002a; Lindvall and Svensson 2019). Brown et al. (2005) takes on the comparison of the wind turning across the marine ABL and compares the ERA-40 reanalysis

to radio sonde data from oceanic station in the North Atlantic. They identify significant underestimation of the turning in the reanalysis compared to the observations. Lindvall and Svensson (2019) create a global climatology of the wind turning and cross-isobaric mass fluxes over land from radio sondes and compare to the ERA-interim reanalysis. The same pattern is visible as both the absolute value of the turning and the mass flux are underestimated in the reanalysis, as well as its range of values. Furthermore, they identified that the analyzed data does not follow the surface Rossby number similarity theory of Lettau (1962). The theory is valid for neutral and barotropic ABLs but Lindvall and Svensson (2019) finds a dependence on stratification even in the strong wind cases that are usually assumed to be near-neutral in the observed ABL. The Rossby number similarity theory connects the turning of the wind to the surface layer stability. This similarity has been shown to fit observational data well (Hess and Garratt 2002a), but less correlated with data from models with sophisticated turbulence closure (Hess and Garratt 2002b).

The stably stratified ABL has for long time been a challenge for numerical models (Sandu et al. 2013). The first Global Energy and Water Cycle Experiment (GEWEX) Atmospheric Boundary Layer Study (GABLS1) is a model inter-comparison that describes a weakly stable ABL developing in time with a decreasing ground temperature under moderately strong shear forcing. The case is assumed to be statistically homogeneous in the horizontal direction enabling the so-called single-column model (SCM) approximation and suitable for large eddy simulations (LES), see Cuxart et al. (2006) and Beare et al. (2006), respectively. The SCM case is run to a quasi-stationary state with many different research and operational numerical weather prediction (NWP) models using different boundary layer schemes. The results from this model inter-comparison show a large spread for e.g. the ABL thickness, which is related to the sensitivity of the difference turbulence schemes and they are also different from the LES results (Cuxart et al. 2006). Further analysis of the GABLS1 case reveals that the models generally underestimate the turning of the wind compared to the LES, and present very different magnitude for the ABL vertically integrated cross-isobaric mass flux (Svensson and Holtslag 2009). These discrepancies are linked to differences in turbulence parametrisation as well as the vertical resolution in the numerical model. Moreover, the numerical results are contaminated by simplified boundary conditions, corresponding assumptions and inadequate near-surface resolution (Grisogono 2011). Svensson and Holtslag (2009) also pointed out that the governing equations directly give the turning of the wind with height and the related cross-isobaric mass flux is solely governed by surface turbulent stress in the direction of the geostrophic wind for quasi-steady state conditions.

While the turning of the wind with height in models and observations has been studied, including the cross-isobaric mass flux, the conditions controlling these properties are not yet fully understood. Extension or corrections of existing similarity theories or drag laws have been attempted (e.g. Liu et al. 2021) by realising that truly neutral ABLs in the nature are rare to non-existent. Existing similarity and laws are most likely build upon observations of conven-

tionally neutral boundary layers (CNBLs), i.e. neutral ABLs with a capping inversion. A connection to the CNBLs can be provided via the layer thickness as a parameter, which is not taken into account in the theory by Lettau (1962).

As existing similarity theories for the turning angle over the ABL and the corresponding mass flux have limited applicability we suggest an alternative approach to examine the role of these in state-of-the-art NWP and climate models. A SCM, while based on parametrized turbulence, provides a controlled framework that includes the essential processes active in a non-cloudy ABL. In this study, we build a database by performing a large ensemble of SCM simulations across a wide range of idealized ABL conditions to systematically relate key environmental parameters to wind turning and cross-isobaric mass flux.

The database consists of simulations of a variety of cases (amount $\mathcal{O}10^4$) inspired by previous studies, observations and reanalysis data to examine the vertical structure of the near neutral and stable ABL. The parameter space for the simulations include a variation of thermally stratified conditions, selection of surface roughness and free stream wind speeds, informed by observations and reanalysis. The column is placed at different latitudes while certain components of the turbulence closure are varied. The influence of vertical resolution is also examined along with the turbulence representation. We investigate the vertical distribution of the ABL wind turning and mass flux and which parameters or components in the model that are important for it, as well as compare the diagnostics to similarity theories.

Lastly, ERA5 reanalysis from three climatically distinct locations is used to construct probability distributions within the SCM parameter space. Sampling the SCM ensemble according to these distributions allows us to quantify the climatological difference arising in the cross-isobaric mass flux due to the turbulence scheme and vertical resolution.

The results from the simulation sets are explored in the light of how they would perform in a global model setting. Therefore, we focus the analysis on two main diagnostics that provide information on their potential influence on the large-scale dynamics; the wind turning and the cross-isobaric mass flux. We analyze the structural behaviour of the reference set of simulation in relation with existing theories and then extend the discussion to the role of closure constants and vertical resolution. Figures are presented with data from one or more simulation sets, where parts of each figure are discussed separately in the different sections of the results. Later sections then revisit earlier figures for comparison and completion of discussion of the data they present.

2 The single-column model equations and simulation setup

The governing equations for a dry ABL (see Appendix) are solved numerically assuming statistical homogeneity in the horizontal directions. The vertical solution of the velocity and potential temperature is developing in time from a set of initial conditions and driven by a constant geostrophic forcing. The system

of equations include a first order closure for modelling the turbulence similar to the formulation used operationally in the European Centre for Medium Range Weather Forecast (ECMWF) Integrated Forecast System (IFS) for neutral and stably stratified boundary layers. The eddy viscosities for momentum and heat are derived based on the local mean variable gradient, a turbulent length scale and stability functions (ECMWF 2023). It is a widely used method of the turbulence closure in operational state-of-the-art NWP and climate models.

The stability functions for momentum (F_m) and heat (F_h) are given by:

$$\begin{aligned} F_m(Ri) &= \frac{1}{1 + 2bRi(1 + dRi)^{-1/2}} \\ F_h(Ri) &= \frac{1}{1 + 2bRi(1 + dRi)^{1/2}}. \end{aligned} \quad (1)$$

These are formulated using a bulk Richardson number (Ri, see Appendix) as stability parameter and contain two constants, b and d . The level of turbulence is efficiently controlled by the constants and the mixing length. In the surface layer, it is $l = \kappa z$ and is bounded by the asymptotic length scale λ_0 which here is taken to be 30 m in both simulation sets, a value motivated by the measurements of Tjernström (1993). In the current version of IFS λ_0 is restricted based on the boundary-layer height ECMWF (2023), not adopted here. Both the stability function constants and the asymptotic length scale influence the level of turbulence and has been used to tune the performance of the IFS Sandu et al. (2013) and their sensitivity clearly connects the local friction to the global scale performance.

The SCM computations are performed using a general one-dimensional solver written in Python, based on the same principles as an earlier Maple-based solver (Mårtensson and Wallin 2004). The problem is formulated as a system of time-dependent, coupled one-dimensional partial differential equations, defined symbolically together with boundary conditions. The equations are automatically discretised and a problem-specific Python solver is generated in a preprocessing step. Spatial derivatives are approximated using second-order central differences on a collocated grid, while time integration is based on an implicit first-order scheme. The resulting non-linear system is solved iteratively using a tri-diagonal matrix structure. This approach enables rapid implementation of complex models while reducing the risk of human-induced errors. The resulting solver is computationally efficient, as symbolic processing is performed during preprocessing and the generated code utilises the NumPy and SciPy libraries.

The boundary conditions for wind (U,V) implies the logarithmic law for fully rough surfaces, which means that $U = V = 0$ at the surface roughness length z_0 . The vertical coordinate is shifted so that $z = 0$ at the boundary condition. The ground temperature is here set at the same location, z_0 , but can in principal be shifted to a different temperature roughness height z_{0t} . The eddy viscosities K_m and K_h are set at the boundary to be consistent with the ground momentum and heat fluxes (Equation 2)

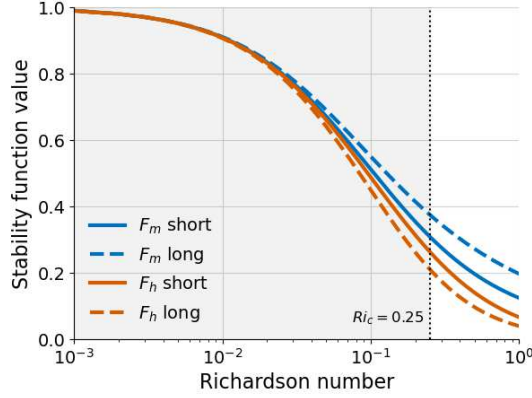


Fig. 2 The momentum (blue) and heat (red) stability functions as function of Richardson number used in the turbulence closure for neutral and stable regimes (Eq. 1). The short and long tail versions are shown with solid and dashed lines, respectively. Note the logarithmic x-scale and values below the critical Richardson number ($Ri_c = 0.25$) is shaded in grey.

$$K_m \frac{\partial U}{\partial z} = u_\tau^2, \quad K_h \frac{\partial T}{\partial z} = u_\tau \theta^*, \quad (2)$$

where u_τ and θ^* are evaluated from the log law. Here the first cell centre, at $z = z_1$, is used:

$$U_1 = \frac{u_\tau}{\kappa} \ln \left(\frac{z_1 + z_0}{z_0} \right), \quad T_1 - T_{sfc} = \frac{Pr_t \theta^*}{\kappa} \ln \left(\frac{z_1 + z_0}{z_{h0}} \right), \quad (3)$$

where $Pr_t = 0.7$ is the Prandtl number, $\kappa = 0.41$ is the von Kármán constant and the roughness length for momentum (z_0) and heat (z_{h0}) are set to the same value.

The vertical grid of the model is manually generated with a stretching function, that uses the surface roughness length, the selected domain height and a stretching factor (see Appendix) to define the first cell centre height and the remaining grid.

3 Simulations

Two sets of simulations are performed, using two specifications of the closure constants b and d in Eq. 1. The first simulation set, from now on called short, uses $b = 5$ and $d = 1$, which are currently used in IFS ECMWF (2023). The second set uses $b = d = 5$, called long from here on. Figure 2 shows the stability functions for momentum and heat. It is clear that the short formulation for the momentum approaches zero faster than the long, i.e. it has a shorter tail.

Each of the two simulations sets is furthermore split up in three sets with different vertical resolution (see Table 1). From here on, the simulation set

Table 1 The vertical grid of the simulation sets.

Simulation set	Number of levels	Height to first level (m)	Levels below 100 m	Levels below 500 m
Reference	300	1 (2) ¹	37	100
Medium	90	10	5	17
Coarse	30	30	2	5

¹When z_0 is large**Table 2** The simulation parameter space with the GABLS1 values (Cuxart et al. 2006) in bold.

z_0 (m)	U_g (m s ⁻¹)	Latitude (deg)	Initial neutral layer depth (m)	Lapse rate above ABL (K m ⁻¹)	Surface cooling (K h ⁻¹)
0.03	4	30	100	0.001	0.0
0.1	8	45	200	0.003	0.05
1.0	16	60	300	0.005	0.1
2.0	24	73	400	0.007	0.25
			500	0.009	0.375
					0.5
					1.0

using the highest vertical resolution and short tail functions are called the reference simulations, assuming that this is the set where the physical processes are well resolved. It is used as the reference in comparison to the other simulation sets. The medium vertical resolution is chosen as it is close to operational global NWP. A very coarse resolution is used in the final set of simulations mimicking coarse resolution climate models. For the high-resolution simulations that have a roughness length of $z_0 = 2$ m, the height of the first cell centre is moved from just above 1 m to 2 m. Common for all simulation is the domain top at 14 km and a timestep of 60 s. The simulations reach quasi-stationary state after 9-11 hours, varying with latitude. All simulations are run for 12 hours and the mean of the last hour is used in the analysis.

Following the GABLS1 setup (Cuxart et al. 2006), the direction of free stream geostrophic wind is along the x-axis, consequently, the V component is the isobar-perpendicular part (see Fig. 1). The initial surface temperature is set to 265 K with a cooling of 0.25 K h⁻¹ throughout the simulation, and weak inversion of 1 K km⁻¹ placed at 100 m height. This simulation setup is extended to explore a wide range of conditions through varying parameters (Tab. 2). The choice of values for the span of each parameter is inspired by, but not limited to, studies of SCM simulations which were sampled from reanalysis and analysis of observations (e.g. Kelly and van der Laan 2023; Lanzilao and Meyers 2024; Liu et al. 2021; Sullivan et al. 2016).

Simulations spanning the parameters in Table2 are performed for both the short and long tail sets, and the three vertical resolutions. This allows exploration of the relative importance of both the numerical resolution as well

as the turbulence representation for the simulated wind turning and mass flux. Each simulation set contains 21,000 simulations, with a combined total of 126,000 simulations.

4 Method and definitions

The wind turning over the ABL is defined as:

$$\alpha = \alpha_{BLH} - \alpha_{sfc} \quad (4)$$

where α_{sfc} is the surface angle (i.e. first model level assuming no turning in the surface layer) and α_{BLH} is the wind direction at the top of the ABL. The turning is adjusted to fall within $\alpha \in [-180; 180]$ degrees, i.e. a positive value when the wind veers (turns clockwise with height) in the ABL and a negative one when the wind backs (turns anti-clockwise with height). When the wind veers in the ABL, the cross-isobaric wind component points in the direction of the pressure gradient but opposite direction when the wind backs.

Two methods are used to diagnose the BLH, based on turbulent fluxes and mean profiles, respectively. The method based on the stress profile described in Cuxart et al. (2006) is used to diagnose the SCM simulated BLH. A bulk Ri number, calculated from the surface to the height where $Ri > Ri_c$, is used to gauge the BLH from the ERA5 reanalysis data (Hersbach et al. 2020) as described in (ECMWF 2023). The BLH and the corresponding wind vector is linearly interpolated between model levels.

The vertically integrated cross-isobaric mass flux $\dot{M}$ is the vertical integration of the cross-isobaric wind vector times the air density (units of $[\dot{M}] = \text{kg} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$), from the surface up to the BLH (Equation 5). It acts as a measure of total cross-isobaric momentum transport done in the ABL.

$$\dot{M} = \int_{sfc}^{BLH} \rho V \, dz = \int_{sfc}^{BLH} M \, dz \quad (5)$$

where ρ is air density, V is the cross-isobaric wind component and height z .

Data bins for presenting statistics of the SCM simulations for representing varying stability is best represented by equal size bins in the stability function f_m since this quantity directly controls the turbulence intensity in the closure

$$f_m = [0, 0.2, 0.4, 0.6, 0.8, 1].$$

Corresponding bins for the bulk Richardson number, Ri , are made from the stability function curves in Fig. 2 for both long and short and result in the following bin boundaries

$$Ri = [\infty, 4.88 \cdot 10^{-1}, 1.62 \cdot 10^{-1}, 6.89 \cdot 10^{-2}, 2.53 \cdot 10^{-2}, 0]$$

for the short tail f_m , and

$$Ri = [\infty, 9.66 \cdot 10^{-1}, 2.16 \cdot 10^{-1}, 7.87 \cdot 10^{-2}, 2.66 \cdot 10^{-2}, 0]$$

for the long tail f_m . These Ri number bins are later used in the analysis of the SCM data.

ERA5 data from three different locations are used to sample the SCM parameter space according to its probability distribution, thereby quantifying the differences between the simulation sets. The three locations are: 1) Keflavik in Iceland, a coastal international airport in the North Atlantic located at roughly 64 °N which has an estimated surface roughness length of $z_0 \approx 0.1$ m; 2) Lindenberg in Germany, which is a continental observation site at ca 52 °N with $z_0 \approx 0.75$ m; and 3) Sodankylä in Finland, an Arctic inland observation site at 67°N with $z_0 \approx 1.8$ m. These locations are chosen as they represent different conditions and mid-latitude climates. Two years, 2019 and 2020 are included. Only the near neutral and stably stratified conditions enters the analysis which is 92, 67 and 50% of the hourly ERA5 data at the three locations, respectively. Figure 3 shows the probability distribution of the large-scale forcing conditions for the boundary layer at the three locations along with the parameter space covered by the SCM experiments. The contours are cumulative probability contours. The low value percentage contours enclose the most common conditions in ERA5 which appear in the centre of the distribution with rarer cases appearing towards the edges. Higher percentage value contours therefore cover a bigger range of the data, with the 100% contour enclosing the entire ERA5 dataset considered.

5 Exploring the behaviour of the wind turning and cross-isobaric mass flux

5.1 Scaling of vertical profiles and diagnostics

As introduced in Section 1, a similarity theory for the wind turning angle and Rossby (Ro) number has been derived from observations (Lettau 1962) applicable for the neutral ABL:

$$Ro = \frac{U_g}{|f| \cdot z_0}, \quad (6)$$

$$\alpha = 173.58 \log_{10}(Ro) - 3.03. \quad (7)$$

The similarity relationships (Equations 6 and 7) present a dependency of the wind turning angle, α , on the Coriolis effect, f , (latitude dependence), the free stream wind speed, U_g , and the surface roughness length, z_0 (see grey lines in Fig. 4).

Here, we examine a subset of CNBLs (i.e. no surface cooling forcing) from our reference set (Fig. 4), that all have the same stratification as GABLS1 above the ABL (see Tab. 2). The simulations generally have a lower value of wind turning than the Lettau (1962) theory. Similar dependencies are seen in the SCM simulations with the roughness length and the latitude while almost no dependency of the Ro number is present for the free stream wind speed. The

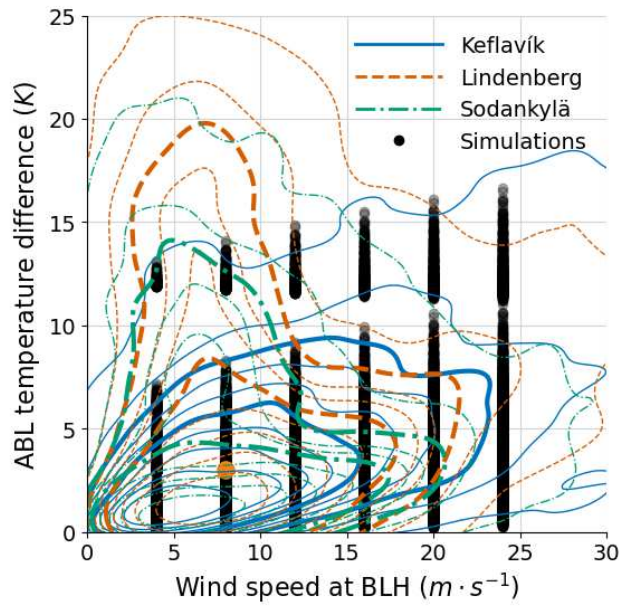


Fig. 3 Probability density space based on ERA5 wind speeds at the BLH and difference in air temperature over the ABL from a (blue) coastal location in the North Atlantic (Keflavík, Iceland), (red) a continental site in Europe (Lindenberg, Germany) and (green) an Arctic inland site (Sodankylä, Finland). Contours show every 10% from 0% to 100%, increasing from the centre towards the edges. Bold lines represent 50% and 80% contours. The black dots cover the reference set of simulations with the GABLS1 case marked with an orange dot.

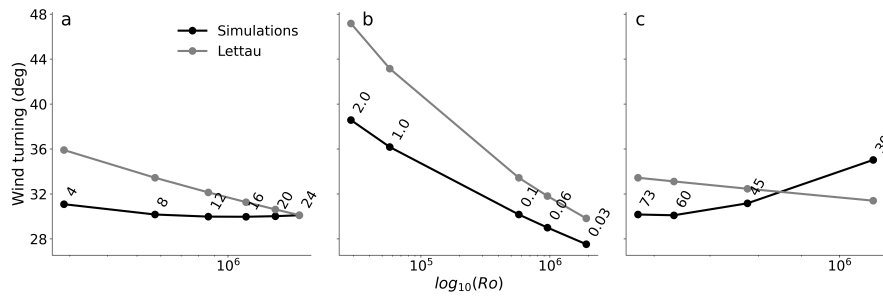


Fig. 4 Wind turning angle (deg) over the ABL as function of Ro number (Eq. 6) from SCM simulations for CNBLs with a stratification of 0.001K m^{-1} above the ABL (black) along with the theoretical angle from Eq. 7 (gray). Simulation parameter variations in a) free stream wind speed (U_g , m s^{-1}), b) surface roughness (m), and c) latitude ($^{\circ}\text{N}$). Annotations of values in each panel.

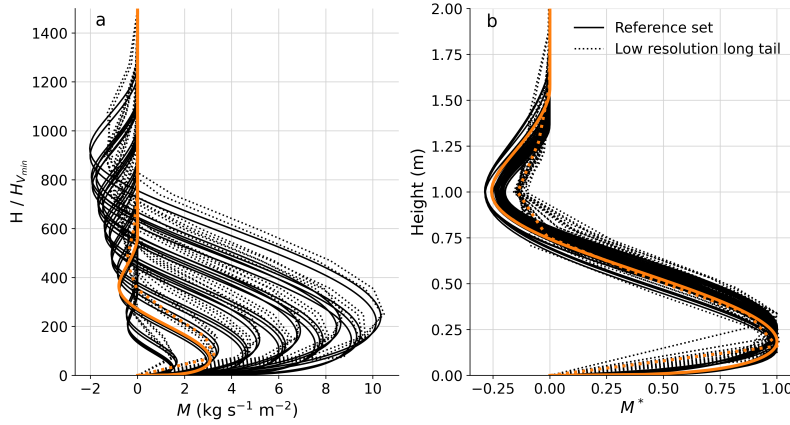


Fig. 5 Unscaled (a) and scaled (b) mass flux for simulations of varying wind speed forcing and surface roughness. Other parameters set to the values of GABLS1 (Tab. 1). The GABLS1 case marked in orange.

largest range in predicted turning angle is for varying roughness length. The latitude dependence of the simulations is, however, opposite of the similarity but the dependency is weak. The same overall dependencies are seen for truly neutral ABL simulations (not shown). However, fully neutral conditions in the atmosphere are rare or even none existent as the free atmosphere above the ABL is stratified, often even a capping inversion is present. Thus, the similarity is for CNBL and likely some of the latitude dependence is the result of climate variable stratification above the ABL, a signal embedded in the observations used in Lettau (1962).

Additional dependencies on buoyancy, stratification and wind speed have so far been identified as reasons for deviation from the similarity theory (e.g., Liu et al. 2021; Lindvall and Svensson 2019; Kelly and van der Laan 2023). To investigate important parameters for the wind turning and the cross-isobaric mass flux, a non-dimensional scaling of our CNBL simulations is derived. The integrand in Equation 5 is scaled with its value at the height of the maximum wind speed, i.e. $V_{max}\rho V_{max}$:

$$M^* = (V \cdot \rho) \cdot V_{max}^{-1} \cdot \rho_{V_{max}}^{-1}. \quad (8)$$

Profiles of this scaled mass flux, $M^*(z)$, from the reference set of SCM are shown in Fig. 5 with the vertical coordinate scaled with the height of the minimum value (most negative) of V . With this scaling, all profiles of M (Fig. 5a) collapse onto a reasonably self-similar curve as can be seen in Fig. 5b, independent of their initial conditions or forcing.

The self-similarity of the scaled profiles indicates that the vertical distribution of cross-isobaric mass flux and wind turning has a universal character despite large variations in forcing and environmental conditions. The fact that the appropriate height scale coincides with the minimum in V further suggests

that the mass flux is dependent on the stratification of the ABL as well as the other components of the Lettau (1962) theory.

The integral measure of the cross-isobaric mass flux (Eq. 5) can be scaled similarly as the profiles of mass flux, using either inner or outer ABL parameters:

$$\begin{aligned}\dot{M}^*_{inner} &= \dot{M} \cdot V_{max}^{-1} \cdot \rho_{V_{max}}^{-1} \cdot z_0^{-1} \text{ (inner scaling),} \\ \dot{M}^*_{outer} &= \dot{M} \cdot U_g^{-1} \cdot \rho_{H_{\dot{M}_{max}}}^{-1} \cdot H_{\dot{M}_{max}}^{-1} \text{ (outer scaling).}\end{aligned}\tag{9}$$

The outer scaling uses the the free stream wind speed forcing (U_g) and $H_{\dot{M}_{max}}$ is the height above which the integral cross-isobaric mass flux, $\dot{M}$, becomes constant with height, i.e. the height where the wind velocity returns to the magnitude and direction of U_g . The density taken from the height $H_{\dot{M}_{max}}$.

These scalings, which can be seen from the profiles in Fig. 5 and Eq. 9, show that in the ABL the cross-isobaric mass flux is greatest where the cross-isobaric wind component is strongest. Thus, the lower half of the ABL provides the majority of the contribution to the ABL integral. The maximum values are found close to the position of $H/H_{V_{min}} = 0.25$ (Fig. 5a); closer to the ground the magnitude of the cross-isobaric component becomes smaller due to the no-slip condition at z_0 . The scaling with inner parameters in Eq. 9 points towards the governing parameters behind the mass flux, the cross-isobaric wind component and the roughness length, while the outer scaling points towards the free stream wind speed and the total thickness of the layer. Thus, the free stream wind speed and surface roughness, which induces the cross-isobaric wind, and the thickness of the layer are important for the development of the cross-isobaric mass flux and the integral measure.

5.2 Relation of wind turning and mass flux to forcing parameters

Profiles of the wind turning show a pattern where the turning is non-monotonic with height, with back-turning in the region of the low level jet (Fig. 6). The turning tends to be greater in more stably stratified cases than in near neutral and conventionally neutral cases. The turning gradient is also increasing with stability (not shown). The wind turning for simulations with different setup can result in similar values of wind turning (Fig. 6a) while the integrated cross-isobaric mass flux for these cases are very different (Fig. 6b). That is mostly due to the different geostrophic wind speed forcing and a deeper boundary-layer. Likewise, very similar simulation setups can result in very different values of wind turning but similar values of cross-isobaric mass flux (not shown). The magnitude of the total ABL turning and its gradient compare well to the results of Kelly and van der Laan (2023) that explored the wind turning from SCM modelling and observations from locations in northern Europe.

The integrated cross-isobaric mass flux increases with increasing U_g (Fig. 6b), as this increases the cross-isobaric component V as well, (see Fig. 5). The increasing stratification, which leads to shallower ABL, reduces the integrated

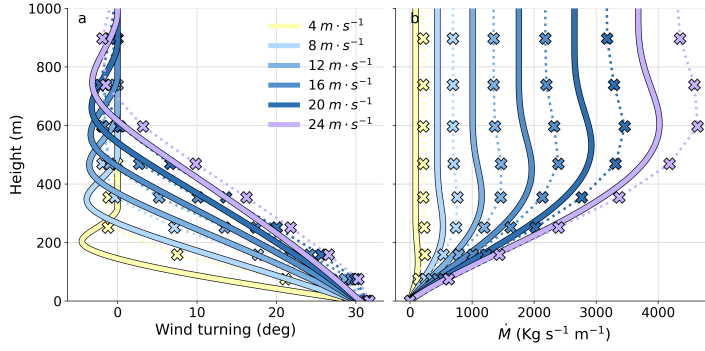


Fig. 6 Wind turning (a) and vertically integrated mass flux (b) from SCM simulations with varying free stream wind speed. Reference data with solid lines and long tail low resolution data with dotted and crossed lines. Other parameters are set to the same as GABLS1 case (Tab. 1).

mass flux. One exception are simulations where both stratification and free stream wind are large; wind speed is the main driving factor of the mass flux. However, those cases are rare in reality according to Fig. 3.

The level of turbulence which is causing the wind turning and thereby the governs the mass flux, is dependent on both shear and buoyancy. The relative importance of shear and buoyancy can be examined by comparing simulations that differ only in free stream wind or the surface cooling condition. Removing or doubling the surface cooling forcing condition for the GABLS1 case setup has much less impact on where on the stability curve the case lands than halving or doubling the wind speed (not shown). Cases with higher mass flux tend to be on the near neutral end while the cases with lower mass flux stack towards the more stably stratified end.

As discussed above, the cross-isobaric mass flux is primarily related to the wind turning, the geostrophic wind forcing and the boundary-layer depth. But, their relations are not linear, simple nor straightforward. Absolute values of the wind turning and cross-isobaric mass flux have little to no correlation at all (not shown). Using the outer scaling of vertically integrated cross-isobaric mass flux (Eq. 9), a an organized relationship emerges for simulations of CNBLs, see Fig. 7a. The relationship becomes near-linear between the scaled mass flux and the wind turning and the data clusters with values of roughness length. Such a relationship, or correlation, is absent in the Lettau (1962) formulation (Eq. 7 and Fig. 7a). When including the more stably stratified cases, a gradual decrease of this relationship is seen (Fig. 7b). For the largest cooling rates the correlation is lost, similar to Lettau (1962).

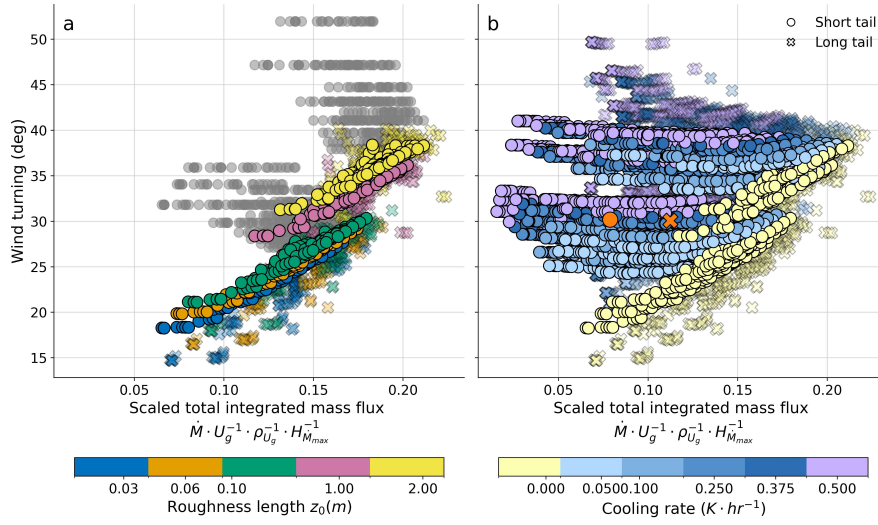


Fig. 7 a) Outer scaled mass flux and wind turning from CNBL SCM simulations colored by roughness length and b) same as (a) but of varying stability colored with surface cooling forcing. Short tail data displayed with a dot and long tail with crosses. Only simulations from 73 deg N showcased for clarity. Theoretical values (Eq. 7) from Lettau (1962) are shown in grey in panel (a) and crosses refer to simulations using long tails in both panels.

6 Effects of vertical resolution and turbulence closure constants on the diagnostics

Not only the variable physical conditions, represented by the simulation setup and forcing conditions, influence the magnitude of the integrated cross-isobaric mass flux. The choice of vertical resolution and the details of the turbulence parametrisation of physical processes also has an influence. Here we investigate systematic changes of these numerical representations on the cross-isobaric mass flux. All simulations have the same time-step length (Tab. 1) although a coarser resolution normally would lead to a longer one. Thus, the investigated sensitivity is likely underestimated.

The short and long tail stability functions from the turbulence parametrisation have a different shape and tail towards the stable regime (Fig. 2). They start to deviate in the near neutral regime and it is interesting to note that the order of the stability functions for momentum and heat switch for the two tails. While many state of the art NWP and climate models have a more sophisticated turbulence parametrisation, it is of interest to find out how the stability functions, that have been extensively used, affect the variables examined here. The results are viewed in perspective from inter-comparison studies where similar closures are used with lower vertical resolution and simpler first-order closures for the turbulence parametrisations (e.g. Bretherton and Park 2009; Seland et al. 2020; Lin and Yu 2022). Long tail stability functions has

been known to be a way of artificially increasing mixing in stable regimes as they tend to improve performance in global models (Sandu et al. 2013). Thus, short and long tail simulation sets are used to showcase the sensitivity and role of the turbulence parametrisation and to explore its effects on the mean profiles and our diagnostics.

When comparing mass flux from the different simulation sets, there is an increase of, on average, 11 % between the lowest and highest resolution. This increase is seen for both sets of turbulence stability functions. However, if the resolution is kept constant, the mass flux amount increases by 3-4 % from short to long tail functions. There is an overall increase in the mass flux of 15 % when the resolution is degraded from highest to lowest and the tails are long rather than short (e.g. Figures 5 and 6b). This is a systematic change across the parameter space where no SCM simulations have a decrease in mass flux with change in either resolution or stability function. The relative difference of mass flux between the different resolutions and/or the stability functions tends to be highest for simulations that has stronger stratification, have low wind and are at a high latitude (not shown). Note that it is relative difference but not absolute number of amount of mass transported across isobars.

The modified resolution and stability function affects the mass flux through changes in the BLH. Lower resolution and long tails degrade the organized behaviour seen in the reference set in Fig. 7. There are wider spread of the data, with inconsistency in the clustering of the CNBLs simulations with different roughness length (Fig. 7a) and for the more stably stratified cases we see both over and underestimation of the wind turning for a given value of the scaled mass flux, compared to the reference set (Fig. 7b). The same degradations in organized behaviour is noted when analysing other parameters in similar manner (not shown), such as relationships to the wind turning and the mass flux to the BLH.

Moreover, a shift in stability at low levels is seen when altering the resolution and tails (Fig. 8). Simulations with coarser resolution tend to be more neutral than high resolution cases and simulations from the long set are more neutral than ones from the short. The latter is explainable by the design of the stability functions (Fig. 2). However, with coarser resolution the vertical gradients of mean variables become more ill resolved and result in a more neutral stratification, which also may be influenced by the constant time step chosen for all simulation sets. The same is true for the cross-isobaric mass flux ($\dot{M}$) where the amount of cases with higher mass flux in near neutral conditions increases when resolution is decreased (Fig. 8). The difference between short and long tail simulation statistics within each resolution can be explained by the different bins behind the data selection. The long tail functions give larger bins compared to the short tail ones as the same range of value of the momentum stability function f_m results in a wider span of Richardson number for long tail compared to short tail ones (Fig. 2 and section 4). When the wind turning is examined in the same manner, it exhibits a pattern where, with coarser resolution the turning is decreased for near neutral cases but increased for more stably stratified ones (not shown).

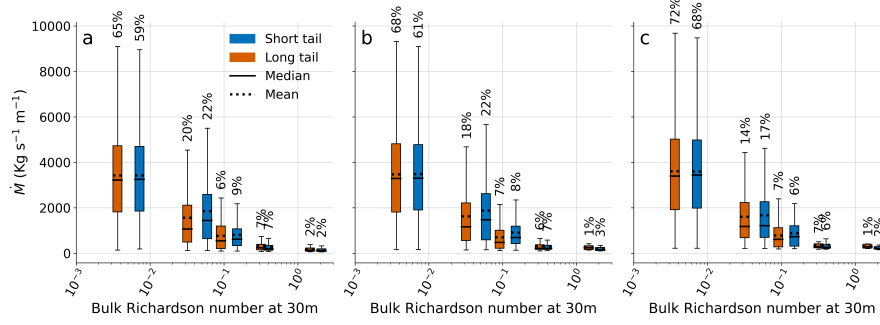


Fig. 8 Histograms of vertically integrated cross-isobaric mass flux binned based on the bulk Richardson number evaluated over 30 m closest to the surface for a) high, b) medium, c) low resolution for both short (blue) and long (red) tail simulations. The data is presented in five bins over the bulk Richardson number from short and long tail stability functions (see section 4). Annotated numbers indicate amount of simulations within each bin. Coloured area of the box plot span the 25th to 75th percentile and the median and mean with a solid and dotted black lines, respectively.

The systematic effect of the vertical resolution on the mean solution can be traceable through analyzing the eddy coefficients for heat and momentum, K_m and K_h , respectively, along with the local vertical gradient of the mean components (see Appendix). With coarser resolution, the local mean vertical gradient is less resolved and the K_m and K_h are larger compared to the reference simulations (Fig. 9). Due to the design of the stability functions, simulations with long tail provide more mixing of momentum than short tails, but vice versa for heat. Although there is a difference between the medium and high resolution, these two are much more similar to each other than when compared with the low resolution. That must be due to the medium resolution simulations' capability of resolving the local gradients better as it has more than double or triple amount of vertical levels below 100 or 500 m, respectively (Tab. 1).

7 Quantifying effects of variations in resolution and turbulence closure on the diagnostics projected at three sites

Weather and climate variations result in differences in stability regimes as exemplified by ERA5 reanalys at three locations (Fig. 3). At Keflavík higher free stream wind speed is more frequent than Lindenberg and Sodankylä where larger temperature difference over the ABL is more common. The free stream wind speed, which is an initial and forcing condition in the SCM simulations and taken as the wind speed at the BLH in ERA5, span similar range in both datasets. The ABL temperature difference is calculated similarly in ERA5 and the SCM but the comparison might be influenced by other processes in ERA5 than the surface cooling used in the SCM. The construction of the idealized SCM simulations makes it hard to represent the very stable ABLs that occur

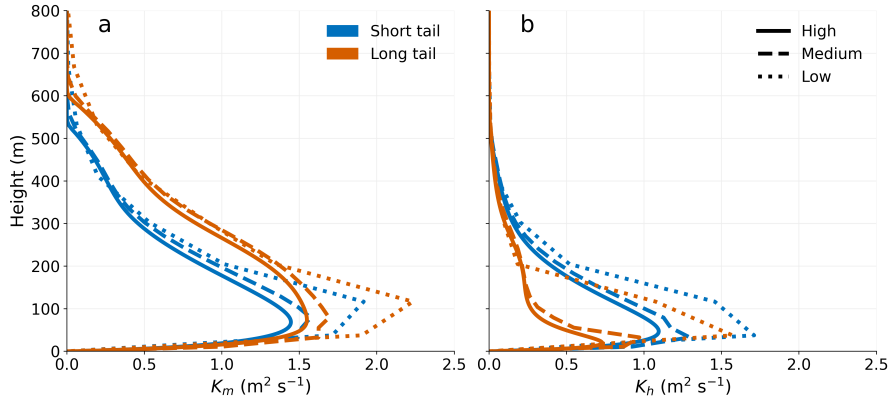


Fig. 9 The eddy diffusivity coefficients of a) momentum (K_m) and b) heat (K_h) from the GABLS1 case. High resolution simulations (solid lines), medium (dashed) and coarse (dotted), short tail data set in blue and long tail data in red.

in Lindenberg and Sodankylä. Furthermore, the BLH in the reanalysis data is diagnosed using a bulk Richardson number method (ECMWF 2023), while in the simulations we use a method based on the stress in the ABL as described in Cuxart et al. (2006). Overall the SCM simulations fall well within the parameter space of the reanalysis data, especially so for Keflavík. The GABLS1 case represents one of the most common conditions found at all three sites.

The ERA5 derived probabilities (Fig. 3) are used to sampled data from the SCM simulations at the three locations. We select only the SCM simulations with similar latitudes and roughness lengths, for the latter we use the reanalysis forecast roughness length. In summary, the sampling is based on four variables: wind speed, temperature difference, roughness length and latitude. These include all the Lettau (1962) similarity theory components together with the additional governing parameter, stratification. The simulations are divided into bins of equal likelihood of occurrence (10%) using the ERA5 probability contours in Fig. 3 for each location. Then, the cumulative mass flux, F , is calculated according to Equation 10 and the results are presented in Fig. 10.

$$F_n = \sum_1^n \left(F_{n-1} + \frac{f_n}{0.1} \right) \quad (10)$$

Where $F_0 = 0$ and f_n is the mean of all the cross-isobaric mass flux from each bin, then divided by its likelihood (here 10%).

The concave shape of the curves in Fig. 10 indicates that the simulations within the last bins contribute more to the total mass flux than the simulations from the lower percentage bins. The cases outside the 70% contour (Fig. 3) contribute about half of the total transport at all three locations. Most of these cases have strong free stream wind forcing but some have weaker winds and strong cooling. The strongest wind cases contribute most to the total transport

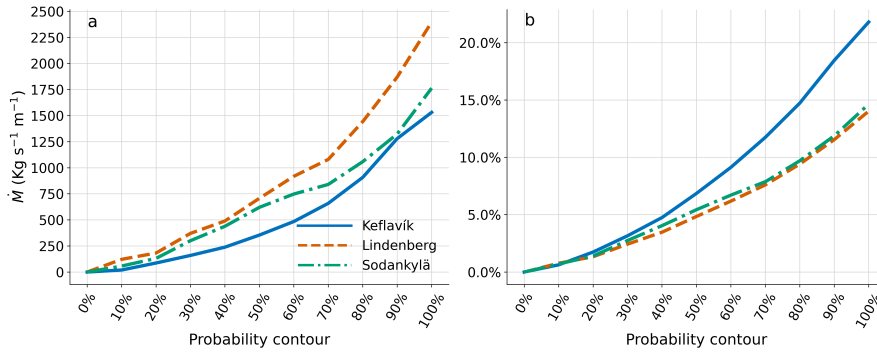


Fig. 10 a) Summed averages of mass flux from high resolution simulations with short tails falling within each probability contour at each ERA5 location from Fig. 3 scaled with probability (Eq. 10). b) Same as (a) but difference in mass flux between the set data and long tail simulations with low resolution, shown as a percentage increase and scaled with the total mean from (a).

despite that the wind turning over the layer is less, see Section 5. The wind turning is generally larger with increasing stability, however, the strong winds and deeper layers dominate the accumulated transport.

Fig. 10b shows a systematic increase for all locations; largest for Keflavík. The increase is similar for Lindenberg and Sodankylä, as for the entire SCM simulation set on average as discussed in section 6, but about 5% larger for Keflavík. It is again the contributions from the strongest free wind cases that matter the most. According to Fig. 3, Sodankylä experiences relatively more stably stratified ABLs and less cases of high winds than the other two locations. Even though Lindenberg has many cases that have low winds and stably stratified ABLs, it also has cases that are more neutrally stratified and higher winds, more so than in Sodankylä.

This leads to the conclusion that changing the resolution and tails matters less in high latitude and continental climates than a mid-latitude location. These results indicate, that for the cross-isobaric mass flux, the choice of modelling of the turbulence has a stronger effect in a location that more often experiences neutral conditions, deeper ABLs and higher winds than in stably stratified, shallow ABLs with low winds. This does not mean that there is no difference in the more stable regimes, on the contrary the relative change can be very large in such conditions with consequences for the surface energy budget (Sandu et al. 2013).

Figure 11 shows similar behaviour for all six simulation sets at Keflavík with increasing differences towards the outer probability contours. Furthermore, differences between vertical resolutions are larger than between tails which agrees with results from section 6.

Previous studies have found that choice of modelling of the physical processes and general numerical setup influence the cross isobaric mass flux. Sensitivity of near surface winds to horizontal model resolution were found in global

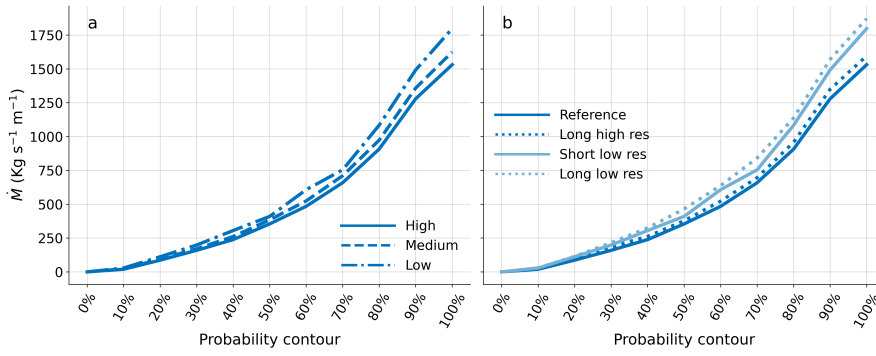


Fig. 11 Same as Fig. 10 but only for the Keflavik location showing a) the three different resolutions using short tail simulations only and b) the two different stability function tails from high and low resolution simulations.

high resolution climate models by Huusko et al. (2025). There, most significant change in parameters connected to changes in global circulation were found in the Southern Ocean storm tracks during austral winter. Furthermore, Pyykkö and Svensson (2023) compared the ABL vertically integrated cross-isobaric mass flux from nine different climate models. They found big differences between models in trade wind and mid-latitude storm regions. Those climate models have differences in the numerical setup, horizontal and vertical resolution as well as how to represent physical processes. Given these results, in combination with those from Fig. 10 and 11 we can conclude that representation of processes in the boundary-layer are important for the ABL vertically integrated cross-isobaric mass flux.

8 Conclusions

We have performed a large ensemble of idealized single-column model simulations for near neutral to stable conditions of dry atmospheric boundary layer with different vertical resolution and turbulence parametrisation constants. The simulations span a wide parameter space motivated by previous studies, observations and reanalysis data. Moreover, six different set of data are produced through varying the vertical resolution and the constants within the turbulence closure. Diagnostics of wind turning and vertically integrated cross-isobaric mass flux were calculated for all simulations and used for evaluating these variations.

Governing parameters for these diagnostics were identified through scaling. Previous theory and studies identified that stratification plays a role for both these diagnostics. Here, further analysis showed that the free stream wind forcing is the most important parameter for the cross-isobaric mass flux, but effects of stratification are more visible for the wind turning. The dependencies

and variations of these diagnostics are not straightforward nor simple as shown by scaling analysis.

Decreasing vertical resolution results in degenerated correlations due to the decreased numerical accuracy. More importantly, a systematic difference in the diagnosed cross-isobaric mass flux is found. Coarsening the resolution and modifying the stability function (from short to long tail versions) of the parametrisation results to up to 15% increase in cross-isobaric mass flux, where the influence of the resolution is dominating over the stability function. The largest absolute change in mass flux occur for near neutral cases with deep boundary layers and high winds. Locations that experience such conditions are therefore more sensitive to these changes, even though the relative difference might be greater in low wind and stably stratified conditions. This was shown mapping the parameter space of the simulated data onto a probability density space derived using two years of reanalysis data.

These results show how changes in the vertical resolution and equation closure can play a big role for the cross-isobaric mass flux in the boundary layer of climate and numerical weather prediction models, which connects to the atmospheric general circulation. The cross-isobaric mass flux is therefore a strong evaluation tool for the boundary layer modelling schemes in numerical weather prediction and climate models.

The simulations are idealized for a dry boundary layers, with constant forcing, run to quasi-stationary state. In reality such conditions rarely exist and other properties and conditions also influence the boundary layer turbulence, the wind turning and cross-isobaric mass flux. Nevertheless, the present study provides insight into the behaviour of these diagnostics and the influence of vertical resolution and turbulence closure. It can be used to further understand these parameters in a fully coupled model and more complex situations.

For future work, integrating such diagnostics as a part of on-line calculations within numerical weather prediction and climate models may be useful for assessing the influence of different parametrizations. This would enable a more complete analysis of the truly cross-isobaric wind vector within the fully coupled system.

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Appendix: System equations, closure and vertical grid

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1 System equations and closure

The single column approximation system equations read:

$$\begin{aligned}\frac{DU}{Dt} &= -\frac{\partial \overline{u'w'}}{\partial z} - f(V_g - V), \\ \frac{DV}{Dt} &= -\frac{\partial \overline{v'w'}}{\partial z} + f(U_g - U), \\ \frac{D\theta}{Dt} &= -\frac{\partial \overline{\theta'w'}}{\partial z}.\end{aligned}$$

With the turbulent flux closure:

$$\begin{aligned}\overline{u'w'} &= -K_m \frac{\partial U}{\partial z} \\ \overline{v'w'} &= -K_m \frac{\partial V}{\partial z} \\ \overline{w'\theta'} &= -K_h \frac{\partial \theta}{\partial z}\end{aligned}$$

where U and V are the mean horizontal wind components, θ is the potential temperature and the prime stands for the variables fluctuation about its mean.

The eddy diffusivity coefficients (K_m and K_h) are modelled with a mixing length and stability functions with a first order closure on the turbulence for momentum and heat respectively

$$\begin{aligned}K_m &= l_m^2 \frac{\partial \mathbb{V}}{\partial z} f_m \\ K_h &= l_m l_h \frac{\partial \mathbb{V}}{\partial z} f_h\end{aligned}$$

where l_m, l_h are momentum and heat mixing lengths and f_m, f_h stability functions and $\mathbb{V} = \sqrt{U^2 + V^2}$ the wind speed. The mixing lengths are represented as:

$$\frac{1}{l_{m,h}} = \frac{1}{\kappa z} + \frac{1}{\lambda_0}$$

with $\lambda_0 = 30m$ as observed in Tjernström (1993). The stability functions read

$$f_m = \frac{1}{1 + 2bRi(1 + dRi)^{-1/2}}$$

$$f_m = \frac{1}{1 + 2bRi(1 + dRi)^{1/2}}$$

606 where $b = 5$ and $d = 1$ for short tails and $b = d = 5$ for long tails. The
 607 Richardson number is calculated using the Brunt-Väisälä frequency and the
 608 stress

$$Ri = N^2/S^2$$

609 with

$$N^2 = \frac{2g}{\theta} \frac{\partial \theta}{\partial z}$$

and

$$S^2 = \left(\frac{\partial U}{\partial z} \right)^2 + \left(\frac{\partial V}{\partial z} \right)^2.$$

610 2 The vertical grid

611 The vertical grid is built from a function that takes in the number of vertical
 612 levels, the domain top and the roughness length and returns a stretching factor
 613 k needed to satisfy the condition derived here. That condition is, the biggest
 614 stretching factor k so that the first cell centre will be located higher than
 615 the roughness length z_0 . If the lowest model level is lower than the roughness
 616 length the no-slip condition at z_0 is contradicted. The geometric sum

$$\sum_0^n k^n = \frac{1 - k^{n+1}}{1 - k}$$

represents the exponential form k^n . The lowest level in the model made with
 the resolution function will be

$$z = \frac{k^0 h}{\sum_0^n k^n}$$

and the distance to the first cell centre has to be greater than the roughness
 length. The height of the first cell centre is then

$$z = \frac{\frac{k^0 h}{\sum_0^n k^n} - 0}{2}$$

617 so then

$$\frac{\frac{k^0 h}{\sum_0^n k^n} - 0}{2} > z_0$$

gives that

$$\frac{k^0 h}{2z_0} > \sum_0^n k^n = \frac{1 - k^{n+1}}{1 - k}.$$

References

- Beare RJ (2007) Boundary layer mechanisms in extratropical cyclones. *Quarterly Journal of the Royal Meteorological Society* 133(623):503–515, DOI 10.1002/qj.30
- Beare RJ, Macvean MK, Holtslag AAM, Cuxart J, Esau I, Golaz JC, Jimenez MA, Khairoutdinov M, Kosovic B, Lewellen D, Lund TS, Lundquist JK, McCabe A, Moene AF, Noh Y, Raasch S, Sullivan P (2006) An intercomparison of large-eddy simulations of the stable boundary layer. *Boundary-Layer Meteorology* 118(2):247–272, DOI 10.1007/s10546-004-2820-6
- Blackadar AK (1962) The vertical distribution of wind and turbulent exchange in a neutral atmosphere. *Journal of Geophysical Research* 67(8):3095–3102, DOI 10.1029/JZ067i008p03095
- Bretherton CS, Park S (2009) A new moist turbulence parameterization in the community atmosphere model. *Journal of Climate* 22(12):3422 – 3448, DOI 10.1175/2008JCLI2556.1
- Brown AR, Beljaars ACM, Hersbach H, Hollingsworth A, Miller M, Vasiljevic D (2005) Wind turning across the marine atmospheric boundary layer. *Quarterly Journal of the Royal Meteorological Society* 131(607):1233–1250, DOI 10.1256/qj.04.163
- Cuxart J, Holtslag AAM, Beare RJ, Bazile E, Beljaars A, Cheng A, Conangla L, Ek M, Freedman F, Hamdi R, Kerstein A, Kitagawa H, Lenderink G, Lewellen D, Mailhot J, Mauritsen T, Perov V, Schayes G, Steeneveld GJ, Svensson G, Taylor P, Weng W, Wunsch S, Xu KM (2006) Single-Column Model Intercomparison for a Stably Stratified Atmospheric Boundary Layer. *Boundary-Layer Meteorology* 118(2):273–303, DOI 10.1007/s10546-005-3780-1
- ECMWF (2023) IFS Documentation CY48R1 - Part IV: Physical Processes. ECMWF, DOI 10.21957/02054f0fbf
- Ekman VW (1905) On the influence of the earth’s rotation on ocean currents. *Arkiv för Matematik, Astronomy Och Fysik* 2(11):1–53
- Grisogono B (2011) The angle of the near-surface wind-turning in weakly stable boundary layers. *Quarterly Journal of the Royal Meteorological Society* 137(656):700–708, DOI 10.1002/qj.789
- Hersbach H, Bell B, Berrisford P, Hirahara S, Horányi A, Muñoz-Sabater J, Nicolas J, Peubey C, Radu R, Schepers D, Simmons A, Soci C, Abdalla S, Abellan X, Balsamo G, Bechtold P, Biavati G, Bidlot J, Bonavita M, De Chiara G, Dahlgren P, Dee D, Diamantakis M, Dragani R, Flemming J, Forbes R, Fuentes M, Geer A, Haimberger L, Healy S, Hogan RJ, Hólm E, Janisková M, Keeley S, Laloyaux P, Lopez P, Lupu C, Radnoti G, de Rosnay P, Rozum I, Vamborg F, Villaume S, Thépaut JN (2020) The ERA5 global reanalysis. *Quarterly Journal of the Royal Meteorological Society* 146(730):1999–2049, DOI 10.1002/qj.3803
- Hess GD, Garratt JR (2002a) Evaluating Models of The Neutral, Barotropic Planetary Boundary Layer using Integral Measures: Part I. Overview. *Boundary-Layer Meteorology* 104(3):333–358

- Hess GD, Garratt JR (2002b) Evaluating Models Of The Neutral, Barotropic Planetary Boundary Layer Using Integral Measures: Part II. Modelling Observed Conditions. *Boundary-Layer Meteorology* 104(3):359–369, DOI 10.1023/A:1016525332683
- Holton JR, Hakim GJ (2013) *An Introduction to Dynamic Meteorology*, 5th edn. Academic Press, DOI 10.1016/C2009-0-63394-8
- Huusko L, Pórarinnsson PA, Pyykkö J, Svensson G (2025) Resolution dependence of the turbulent atmospheric boundary layer in global storm-resolving climate simulations. *Quarterly Journal of the Royal Meteorological Society* 151(768):e4940, DOI <https://doi.org/10.1002/qj.4940>
- Kelly M, van der Laan MP (2023) From shear to veer: theory, statistics, and practical application. *Wind Energy Science* 8(6):975–998, DOI 10.5194/wes-8-975-2023
- Lanzilao L, Meyers J (2024) A parametric large-eddy simulation study of wind-farm blockage and gravity waves in conventionally neutral boundary layers. *Journal of Fluid Mechanics* 979, DOI 10.1017/jfm.2023.1088
- Lettau HH (1962) Theoretical wind spirals in the boundary layer of a barotropic atmosphere. *Beiträge zur Physik der Atmosphäre* 35:195–212
- Lin QJ, Yu JY (2022) The potential impact of model horizontal resolution on the simulation of atmospheric cloud radiative effect in cmip6 models. *Terrestrial, Atmospheric and Oceanic Sciences* 33(1):21, DOI 10.1007/s44195-022-00021-3
- Lindvall J, Svensson G (2019) Wind turning in the atmospheric boundary layer over land. *Quarterly Journal of the Royal Meteorological Society* 145(724):3074–3088, DOI 10.1002/qj.3605
- Liu L, Gadde SN, Stevens RJ (2021) Geostrophic drag law for conventionally neutral atmospheric boundary layers revisited. *Quarterly Journal of the Royal Meteorological Society* 147(735):847–857, DOI 10.1002/qj.3949
- Mårtensson GE, Wallin S (2004) A general 1d-solver for partial differential equations. in: *Licentiate thesis of g. e. mårtensson*. KTH Royal Institute of Technology, Tech rep
- Pyykkö J, Svensson G (2023) Wind turning in the planetary boundary layer in cmip6 models. *Journal of Climate* 36(17):5729 – 5742, DOI 10.1175/JCLI-D-22-0705.1
- Sandu I, Beljaars A, Bechtold P, Mauritsen T, Balsamo G (2013) Why is it so difficult to represent stably stratified conditions in numerical weather prediction (NWP) models? *Journal of Advances in Modeling Earth Systems* (2):117–133, DOI 10.1002/jame.20013
- Seland Ø, Bentsen M, Olivié D, Toniazzo T, Gjermundsen A, Graff LS, Debernard JB, Gupta AK, He YC, Kirkevåg A, Schwinger J, Tjiputra J, Aas KS, Bethke I, Fan Y, Griesfeller J, Grini A, Guo C, Ilicak M, Karset IHH, Landgren O, Liakka J, Moseid KO, Nummelin A, Spensberger C, Tang H, Zhang Z, Heinze C, Iversen T, Schulz M (2020) Overview of the norwegian earth system model (noresm2) and key climate response of cmip6 deck, historical, and scenario simulations. *Geoscientific Model Development* 13(12):6165–6200, DOI 10.5194/gmd-13-6165-2020

- Sullivan PP, Weil JC, Patton EG, Jonker HJJ, Mironov DV (2016) Turbulent winds and temperature fronts in large-eddy simulations of the stable atmospheric boundary layer. *Journal of the Atmospheric Sciences* 73(4):1815 – 1840, DOI 10.1175/JAS-D-15-0339.1
- Svensson G, Holtslag AAM (2009) Analysis of Model Results for the Turning of the Wind and Related Momentum Fluxes in the Stable Boundary Layer. *Boundary-Layer Meteorology* 132(2):261–277, DOI 10.1007/s10546-009-9395-1
- Tjernström M (1993) Turbulence length scales in stably stratified free shear flow analyzed from slant aircraft profiles. *Journal of Applied Meteorology and Climatology* 32(5):948 – 963, DOI 10.1175/1520-0450(1993)032<0948:TLSISS>2.0.CO;2