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SEISMIC STABILITY ANALYSIS OF MULTISTAGE LOESS SLOPES BASED ON THE RIGOROUS LIMIT EQUILIBRIUM METHOD

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Abstract

This paper presents a rigorous limit equilibrium method for analyzing the seismic stability of multi-level loess slopes. Based on the vertical slice method, the study introduces inter-slice forces under seismic conditions using a pseudo-static approach. The slope surface and slip surface are described using piecewise functions, and the horizontal and vertical seismic forces are incorporated into the force equilibrium equations. By adopting assumptions from classical earth pressure theory and Spencer's method, expressions for inter-slice normal and shear forces are derived. The safety factor is explicitly formulated by solving the equilibrium equations, incorporating the Mohr–Coulomb failure criterion. A case study of a three-level homogeneous loess slope is analyzed using the proposed method, with results compared to those from GEO SLOPE and PLAXIS 3D. The computed safety factors are in close agreement, validating the proposed approach. The method offers a simplified yet accurate analytical solution for evaluating the seismic stability of multi-level slopes with general slip surface shapes.

Keywords: Slope stability; Seismic analysis; Limit equilibrium method; Multistage loess slope; Safety factor;

1. Introduction

Seismic action is a critical trigger for slope instability. Intense ground motions can drastically alter the stress field within a slope, leading to the degradation of geotechnical strength and potentially resulting in catastrophic sliding or collapse. For multi-level loess slopes with their complex structural morphology, the interplay between their distinctive stepped topography and the collapsible, fissured nature of loess makes them particularly susceptible to progressive damage and discontinuous deformation under cyclic seismic loading, thereby raising serious stability concerns.

In both theoretical analysis and engineering practice, the integration of the pseudo-static method with limit equilibrium methods has become an established approach for evaluating slope stability under seismic conditions. Over the years, researchers have endeavored to refine these methodologies, developing more precise techniques to enhance the theoretical framework and practical applicability of seismic slope stability assessment. Ye et al. [1,2] investigated the dynamic response and stability of loess slopes, revealing key influencing factors and response patterns. Luo et al. [3,4] developed stability models for multi-level slopes, incorporating pseudo-static upper-bound analysis and random ground motion effects. To enhance analytical rigor, Zheng et al. [5,6] focused on improving slip

surface stress characterization and establishing theoretical bounds for the safety factor. The application scope of limit equilibrium methods has been significantly expanded by Fan et al. [7-13], who addressed complex scenarios involving 3D slopes, anti-slide pile reinforcement, unsaturated soils, and rock slopes. Furthermore, Wang et al. [14-16] introduced uncertainty analysis into slope stability evaluation using stochastic fields and fuzzy set theory. Concurrently, refinements to the pseudo-dynamic method for more realistic seismic analysis were contributed by Jiang et al. [17-19]. Additionally, Yang et al. [20] used FLAC3D to compare dynamic responses under uniform and traveling wave excitations. They found that the slip surface formed through the connection of tensile-shear failure zones under both excitation types, but the traveling wave excitation better reflected actual failure scenarios. Wang [21] took the high fill slope of an airport in Yunnan as the object, compared the numerical method with the limit equilibrium method, revealed the law of safety factor of different algorithms, and clarified that the construction stress field had a significant impact on the results, which provided a reference for similar projects. Han et al. [22] investigated the stability of three-dimensional layered soil slopes under rainfall infiltration, and calculated the three-dimensional slope safety factor considering rainfall infiltration using the Hovland three-dimensional limit equilibrium method.

However, significant challenges persist in the stability analysis of multi-level loess slopes. Key unresolved issues include quantifying the influence of complex slope morphology on the safety factor and determining reasonable inter-slice forces under seismic conditions when using slice-based methods. To address these challenges, this study proposes a rigorous limit equilibrium slice method based on the pseudo-static approach. The method incorporates the normal stress on the slip surface and defines the morphological functions of the multi-level slope profile and the potential slip surface. By introducing reasonable assumptions for the seismic-induced shear and normal forces between soil slices, an explicit expression for the safety factor of multi-level loess slopes under earthquake action is ultimately derived.

2. Multi-level slope ground vibration stability analysis based on rigorous vertical slice method

2.1 Positive Stress Analysis of Sliding Surface

Fig. 1 presents the slope analysis model based on the vertical slice method, assuming the slope slip surface is $y = s(x)$, the slope function is $y = g(x)$, and the gravity of the micro soil slice is $w(x) = \gamma[g(x) - s(x)]$. In the traditional vertical slice method, the soil is divided into n slices, and the force at the bottom of each slice is considered as a concentrated force. The safety factor is calculated as the ratio of the sum of sliding moments to the sum of resisting moments under static equilibrium conditions. In this paper, the seismic effect is treated as equivalent to static loading, and inspired by the traditional slice method, normal and shear stresses are applied on the potential sliding surface. The soil on the sliding surface is divided into countless tiny vertical soil slice units, each with a width of dx in the vertical direction, and their forces are analyzed.

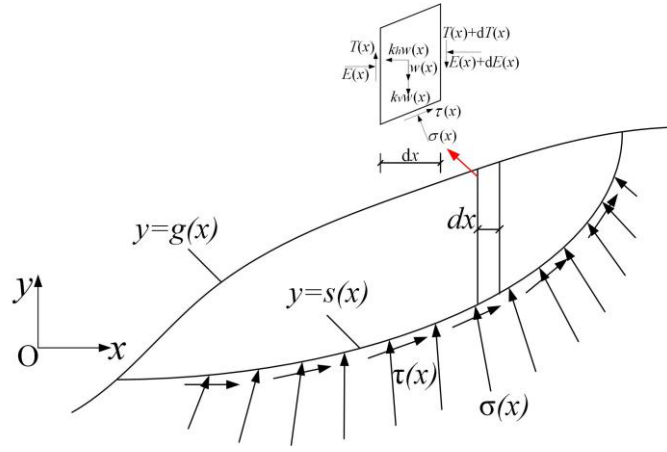


Fig. 1. Model of the vertical slice method for a multistage slope

Due to the extremely small width of the soil slice sampled, the normal and shear stresses at the base of the soil slice can be considered as small concentrated forces, with all forces satisfying static equilibrium:

$$dE(x) + k_h w(x) + \sigma(x) \sin \theta = \tau(x) \cos \theta \quad (1)$$

$$dT(x) + (1 + k_v) w(x) = \sigma(x) \cos \theta + \tau(x) \sin \theta \quad (2)$$

$$\sigma(x) = \frac{dT(x) + (1 + k_v) w(x) - [dE(x) + k_h w(x)] \tan \theta}{\sin \theta \tan \theta + \cos \theta} \quad (3)$$

$$\sigma(x) = \frac{dT(x) + (1 + k_v) w(x) - [dE(x) + k_h w(x)] s'}{\sin \theta s' + \cos \theta} \quad (4)$$

$$\sigma(x) = \frac{(1 + k_v - k_h s') w(x)}{s' \sin \theta + \cos \theta} + \frac{dT(x) - dE(x) s'}{s' \sin \theta + \cos \theta} \quad (5)$$

$$\sigma(x) = \sigma_w(x) + \sigma_s(x) \quad (6)$$

$$\sigma_w(x) = \frac{(1 + k_v - k_h s') w(x)}{\sin \theta s' + \cos \theta} \quad (7)$$

$$\sigma_s(x) = \frac{dT(x) - dE(x) s'}{s' \sin \theta + \cos \theta} \quad (8)$$

where dE is the normal force increment between vertical micro soil slice units; dT is the tangential force increment between vertical micro soil slice units; k_h is the coefficient of seismic acceleration in the horizontal direction; k_v is the coefficient of seismic acceleration in the vertical direction; σ is the positive stress at the bottom of the micro soil slice units; τ is the tangential stress at the bottom of the micro soil slice units; and θ is the angle between the bottom of the micro soil slice units and the horizontal direction.

2.2 Determination of interslice forces in vertical slices under seismic action

According to the existing research, the inter-slice force can be converted into equivalent horizontal force and equivalent vertical force. The horizontal force between the soil slices is the normal force between the slices, which is distributed on the side of the soil or retaining wall in the form of lateral force, as shown in Fig. 2. The state of force at a point M at any depth z . The vertical force is equal to the gravitational force of the soil column, $\sigma_c = \gamma z$, γ is the gravity of the soil body, $p_0 = K_0 \sigma_c = K_0 \gamma z$, p_0 is the static earth pressure strength, $E_0 = 0.5 \gamma H^2 K_0$, E_0 is the static earth pressure, K_0 is the static earth pressure coefficient, and H is the height of the soil body. The vertical seismic force on the micro soil slice under seismic action is $k_v w(x)$, the vertical force is the combined force of the vertical

seismic force $k_v w(x)$ and the gravitational force $w(x)$ of the slice, and the horizontal interslice force on the vertical soil slice is the component force of the vertical combined force, $\sigma_{c1} = \gamma(1+k_v)y$, $p_{01} = \zeta\gamma(1+k_v)y$, $E = 0.5\zeta\gamma(1+k_v)y^2$, in this paper, $y = g - s$, therefore, $dE = \zeta(1+k_v)(g' - s')w$.

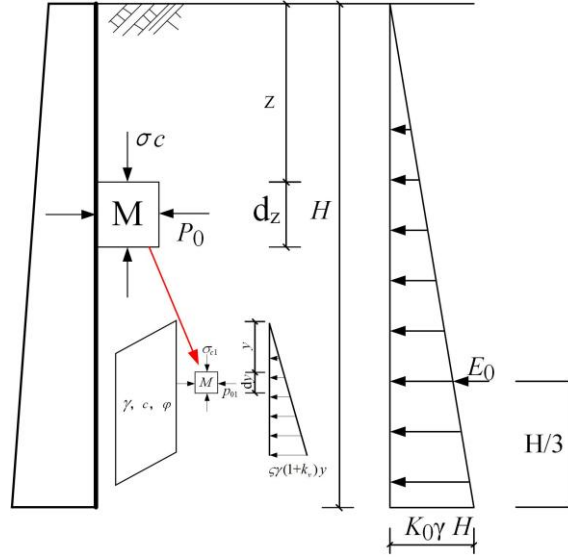


Fig. 2. Static earth pressure calculation sketch

In the static earth pressure calculation formula, K_0 is the static earth pressure coefficient, and according to the principle of earth pressure calculation, there also exists a horizontal interslice force coefficient ζ equivalent to the static earth pressure coefficient, and the gravity of the micro-soil slice is known to be $w(x) = \gamma[g(x) - s(x)]$, so the horizontal interslice force on the soil slice is $E = 0.5\zeta\gamma(g - s)^2$. Since the vertical seismic force and the interslice force are seldom considered in the study of the slice division method, and the seismic action is generally not expressed in the interslice force, the study of the interslice force under seismic action is very little. In fact, the vertical seismic force is used as a force equivalent to gravity in the proposed static method, and according to the principle of earth pressure calculation, the vertical seismic force has some influence on the interslice force.

In slope stability analysis, most of the studies have not fully considered the role of interslice force, which can be called simplified slice division method, in comparison, there are fewer studies on the interslice force of slice division method, and all of them start from the assumptions of Spencer's method, M-P method and so on. In this paper, the tangential force between slices is obtained according to the assumptions of Spencer's method, see the basic assumptions.

2.3 Basic assumptions

1. The slope surface morphology function, $g(x)$, is a segmented function, and the slip surface morphology, $s(x)$, is a known function, which allows the angle of points on the slip surface to be known, as well as the vertical micro soil slice unit weight for a width of dx ;

2. Assuming the existence of interslice force, the interslice force can be considered as follows:

(1) According to the classical earth pressure theory, the lateral horizontal force of the earth slice belongs to the lateral force of the earth slice with the earth pressure, so using the principle of calculation of the classical earth

pressure, taking into account the seismic force, the horizontal force assumption on the side of the soil slice under seismic action is obtained as $E = 0.5\zeta\gamma(1+k_v)(g-s)^2$, ζ is the coefficient to be determined, The width of the micro soil slice unit is dx .

$$\text{Then, } dE = \zeta(g'-s')(1+k_v)w dx, s' = \frac{ds}{dx} = \tan \theta, g' = \frac{dg}{dx}.$$

(2) Drawing on Spencer's assumptions about interslice forces, it is argued that there is a fixed constant relationship between the incremental tangential force and the incremental normal force on the slice, and that the incremental normal force on the soil slice under seismic effects is dE , Thus the increment of tangential force between micro soil slice units is $dT = \eta dE$, Then, we can get the formulation: $dT = \eta\zeta(g'-s')(1+k_v)w dx = \lambda(g'-s')(1+k_v)w dx$.

Substituting the above assumptions about the force between the slices into the expression for the stress distribution function yields:

$$\begin{aligned} \sigma(x) &= \frac{(1+k_v-k_h s')w}{s' \sin \theta + \cos \theta} + \frac{dT(x) - dE(x)s'}{s' \sin \theta + \cos \theta} \\ &= \frac{(1+k_v-k_h s')w + (\lambda - \zeta s')(g'-s')(1+k_v)w}{s' \sin \theta - \cos \theta} dx \end{aligned} \quad (9)$$

2.4 Strength criterion and solution of the safety factor

On the sliding surface $y = s(x)$ by the linear Moore Cullen criterion:

$$\tau = \frac{c + \sigma \tan \varphi}{F_s} \quad (10)$$

where: F_s is the factor of safety; c is the cohesive force of the soil on the potential slip surface; and φ is the angle of internal friction of the soil on the potential slip surface.

The potential sliding body needs to satisfy the static equilibrium condition along with moment equilibrium in the horizontal and vertical directions and around the origin $(0, 0)$:

$$\int_a^b (\tau \cos \theta - \sigma \sin \theta - k_h w) dx = 0 \quad (11)$$

$$\int_a^b (\sigma \cos \theta + \tau \sin \theta) dx - \int_a^b (1+k_v)w dx = 0 \quad (12)$$

$$\int_a^b (1+k_v)wx dx - \int_a^b [(\sigma \sin \theta - \tau \cos \theta + k_h w)s + (\sigma \cos \theta + \tau \sin \theta)x] dx = 0 \quad (13)$$

Substituting Eq. (9) into Eqs. (11), (12), and (13) and associating Eqs. (11) to (13) gives:

$$\lambda \cdot (A_1 + \frac{1}{F_s} A'_1) + \zeta \cdot (A_2 + \frac{1}{F_s} A'_2) = A_3 + \frac{1}{F_s} A'_3 \quad (14)$$

$$\lambda \cdot (B_1 + \frac{1}{F_s} B'_1) + \zeta \cdot (B_2 + \frac{1}{F_s} B'_2) = B_3 + \frac{1}{F_s} B'_3 \quad (15)$$

$$F_s = \frac{D_1 \lambda + D_2 \zeta + D_3}{D'_1 \lambda + D'_2 \zeta + D'_3} \quad (16)$$

The relevant parameters are in integral form, as shown in Eq. (17).

$$\left\{ \begin{array}{l}
A_1 = -\int_a^b \sin \theta \frac{(g' - s')(1 + k_v)w}{s' \sin \theta + \cos \theta} dx \quad A'_1 = \int_a^b \tan \varphi \cos \theta \frac{(g' - s')(1 + k_v)w}{s' \sin \theta + \cos \theta} dx \\
A_2 = \int_a^b \sin \theta \frac{s'(g' - s')(1 + k_v)w}{s' \sin \theta + \cos \theta} dx \quad A'_2 = -\int_a^b \tan \varphi \cos \theta \frac{s'(g' - s')(1 + k_v)w}{s' \sin \theta + \cos \theta} dx \\
A_3 = \int_a^b (k_h w + \sigma_w \sin \theta) dx \quad A'_3 = -\int_a^b \cos \theta (\sigma_w \tan \varphi + c) dx \\
B_1 = \int_a^b \cos \theta \frac{(g' - s')(1 + k_v)w}{s' \sin \theta + \cos \theta} dx \quad B'_1 = \int_a^b \tan \varphi \sin \theta \frac{(g' - s')(1 + k_v)w}{s' \sin \theta + \cos \theta} dx \\
B_2 = -\int_a^b \cos \theta \frac{s'(g' - s')(1 + k_v)w}{s' \sin \theta + \cos \theta} dx \quad B'_2 = -\int_a^b \tan \varphi \sin \theta \frac{s'(g' - s')(1 + k_v)w}{s' \sin \theta + \cos \theta} dx \\
B_3 = \int_a^b [(1 + k_h w) - \sigma_w \cos \theta] dx \quad B'_3 = -\int_a^b \sin \theta (\sigma_w \tan \varphi + c) dx \\
D_1 = \int_a^b \frac{(g' - s')(1 + k_v)w}{s' \sin \theta + \cos \theta} \tan \varphi (x \sin \theta - s \cos \theta) dx \quad D'_1 = -\int_a^b \frac{(g' - s')(1 + k_v)w}{s' \sin \theta + \cos \theta} (x \cos \theta + s \sin \theta) dx \\
D_2 = -\int_a^b \sin \theta \frac{s'(g' - s')(1 + k_v)w}{s' \sin \theta + \cos \theta} \tan \varphi (x \sin \theta - s \cos \theta) dx \quad D'_2 = \int_a^b \frac{s'(g' - s')(1 + k_v)w}{s' \sin \theta + \cos \theta} (x \cos \theta + s \sin \theta) dx \\
D_3 = \int_a^b (c + \sigma_w \tan \varphi) (x \sin \theta - s \cos \theta) dx \quad D'_3 = \int_a^b [(1 + k_v)wx - k_h ws - \sigma_w (x \cos \theta + s \sin \theta)] dx
\end{array} \right. \quad (17)$$

The joint Eqs. (14) and (15), solved with respect to the parameters to be determined and obtained:

$$\left\{ \begin{array}{l}
\lambda = \frac{M_1 + \frac{1}{F_s} M_2 + \frac{1}{F_s^2} M_3}{N_1 + \frac{1}{F_s} N_2 + \frac{1}{F_s^2} N_3} \\
\varsigma = \frac{M'_1 + \frac{1}{F_s} M'_2 + \frac{1}{F_s^2} M'_3}{N_1 + \frac{1}{F_s} N_2 + \frac{1}{F_s^2} N_3}
\end{array} \right. \quad (18)$$

The parameters are as follows:

$$\left\{ \begin{array}{l}
M_1 = A_3 B_2 - A_2 B_3 \quad M'_1 = A_1 B_3 - A_3 B_1 \\
M_2 = A_3 B'_2 + A'_3 B_2 - A_2 B'_3 - A'_2 B_3 \quad M'_2 = A_1 B'_3 + A'_3 B_1 - A_1 B'_3 - A'_3 B_1 \\
M_3 = A'_3 B'_2 - A'_2 B'_3 \quad M'_3 = A'_1 B'_3 - A'_3 B'_1 \\
N_1 = A_1 B_2 - A_2 B_1 \\
N_2 = A_1 B'_2 + A'_2 B_1 - A_1 B'_2 - A'_2 B_1 \\
N_3 = A'_1 B'_2 - A'_2 B'_1
\end{array} \right. \quad (19)$$

Substituting the parameters and into the parameter expression (17), A cubic equation for one can be obtained:

$$F_s^3 + t_3 F_s^2 + t_2 F_s + t_1 = 0 \quad (20)$$

where

$$\left\{ \begin{array}{l}
t_1 = -\frac{D_1 M_3 + D_2 M'_3 + D_3 N_3}{D'_1 M_1 + D'_2 M'_1 + D'_3 N_1} \\
t_2 = \frac{D'_1 M_3 + D'_2 M'_3 + D'_3 N_3 - D_1 M_2 - D_2 M'_2 - D_3 N_2}{D'_1 M_1 + D'_2 M'_1 + D'_3 N_1} \\
t_3 = \frac{D'_1 M_2 + D'_2 M'_2 + D'_3 N_2 - D_1 M_1 - D_2 M'_1 - D_3 N_1}{D'_1 M_1 + D'_2 M'_1 + D'_3 N_1}
\end{array} \right. \quad (21)$$

The cubic equation expression for then can again be written as:

$$(F_s - \frac{t_3}{3})^3 + p_1 (F_s - \frac{t_3}{3}) + q_1 = 0 \quad (22)$$

Among the parameters:

$$\begin{cases} p_1 = -\frac{t_3^2}{3} + t_2 \\ q_1 = -\frac{t_3^2}{27} - \frac{t_2 t_3}{3} + t_1 \end{cases} \quad (23)$$

The final expression for the factor of safety of the slope can be obtained as:

$$F_s = \frac{t_3}{3} + \sqrt[3]{-\frac{q_1}{2} + \sqrt{\left(\frac{q_1}{2}\right)^2 + \left(\frac{p_1}{3}\right)^3}} + \sqrt[3]{-\frac{q_1}{2} - \sqrt{\left(\frac{q_1}{2}\right)^2 + \left(\frac{p_1}{3}\right)^3}} \quad (24)$$

The above Equation is of analytical significance and is an explicit answer to the strict limit equilibrium method for slopes under seismic action using the proposed static method, which is applicable to slip-fractured surfaces of general shape.

Fig. 3 shows a sketch of a four-stage slope, and the segmentation function that represents its slope morphology is shown in Eq. (25).

$$g(x) = \begin{cases} x \tan \theta_1; & 0 < x \leq h_1 \cot \theta_1 \\ h_1; & h_1 \cot \theta_1 < x \leq h_1 \cot \theta_1 + b_1 \\ (x - b_1 - h_1 \cot \theta_1) \tan \theta_2 + h_1; & h_1 \cot \theta_1 + b_1 < x \leq h_1 \cot \theta_1 + b_1 + h_2 \cot \theta_2 \\ h_1 + h_2; & h_1 \cot \theta_1 + b_1 + h_2 \cot \theta_2 < x \leq h_1 \cot \theta_1 + b_1 + h_2 \cot \theta_2 + b_2 \\ (x - b_1 - h_1 \cot \theta_1 - b_2 - h_2 \cot \theta_2) \tan \theta_3 + h_1 + h_2; & h_1 \cot \theta_1 + b_1 + h_2 \cot \theta_2 + b_2 < x \leq h_1 \cot \theta_1 + b_1 + h_2 \cot \theta_2 + b_2 + h_3 \cot \theta_3 \\ h_1 + h_2 + h_3; & h_1 \cot \theta_1 + b_1 + h_2 \cot \theta_2 + b_2 + h_3 \cot \theta_3 < x \leq h_1 \cot \theta_1 + b_1 + h_2 \cot \theta_2 + b_2 + h_3 \cot \theta_3 + b_3 \\ (x - b_1 - h_1 \cot \theta_1 - b_2 - h_2 \cot \theta_2 - b_3 - h_3 \cot \theta_3) \tan \theta_4 + h_1 + h_2 + h_3; & h_1 \cot \theta_1 + b_1 + h_2 \cot \theta_2 + b_2 + h_3 \cot \theta_3 + b_3 < x \leq h_1 \cot \theta_1 + b_1 + h_2 \cot \theta_2 + b_2 + h_3 \cot \theta_3 + b_3 + h_4 \cot \theta_4 \\ h_1 + h_2 + h_3 + h_4; & h_1 \cot \theta_1 + b_1 + h_2 \cot \theta_2 + b_2 + h_3 \cot \theta_3 + b_3 + h_4 \cot \theta_4 < x \leq x_c \end{cases} \quad (25)$$

where: θ_1 、 θ_2 、 θ_3 、 θ_4 are the angles between the slope surface and the horizontal direction of the first, second, third and fourth level slopes, respectively, h_1 、 h_2 、 h_3 、 h_4 are the slope heights of the first, second, third and fourth level slopes, respectively, b_1 、 b_2 、 b_3 is the width of the unloading platform above the first, second and third level slopes; and x_c is the distance from the intersection point of the slip surface and the top of the slope to O.

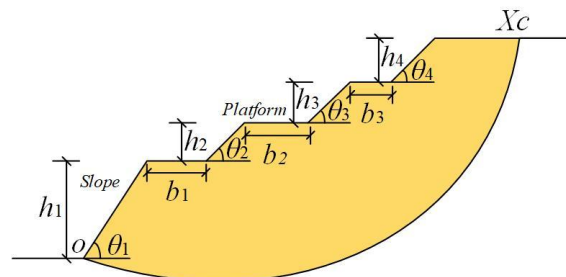


Fig. 3. Sketch of a Four-stage slope

Assuming that the multilevel slope is divided into a total of n levels, according to the relationship between the

slope height, platform width and slope inclination, the multiband function representing the slope morphology of multilevel slopes will be extended to the slope expression function of n-level slopes, and the slopes can be uniformly expressed as:

$$\begin{cases} g(x) = (x - b_1 - h_1 \cot \theta_1 - b_2 - h_2 \cot \theta_2 - \dots - b_{i-1} - h_{i-1} \cot \theta_{i-1}) \tan \theta_i \\ + (h_1 + h_2 + \dots + h_{i-1}); \\ h_1 \cot \theta_1 + b_1 + h_{i-1} \cot \theta_{i-1} + b_{i-1} < x \\ \leq h_1 \cot \theta_1 + b_1 + h_2 \cot \theta_2 + b_2 + \dots + h_{i-1} \cot \theta_{i-1} + b_{i-1} + h_i \cot \theta_i \end{cases} \quad (26)$$

The platform can be expressed as:

$$\begin{cases} g(x) = h_1 + h_2 + h_3 + \dots + h_i ; \\ h_1 \cot \theta_1 + b_1 + h_{i-1} \cot \theta_{i-1} + b_{i-1} + h_i \cot \theta_i < x \\ \leq h_1 \cot \theta_1 + b_1 + h_2 \cot \theta_2 + b_2 + \dots + h_i \cot \theta_i + b_i \end{cases} \quad (27)$$

Consolidated as:

$$g(x) = \begin{cases} [x - \sum_i^n (b_{i-1} - h_{i-1} \cot \theta_{i-1})] \tan \theta_i + \sum_i^n h_{i-1}; & \text{The } i\text{-th stage slope} \\ \sum_i^n (h_{i-1} \cot \theta_{i-1} + b_{i-1}) < x \leq \sum_i^n (h_{i-1} \cot \theta_{i-1} + b_{i-1}) + h_i \cot \theta_i \\ \sum_i^n h_i; & \text{The } i\text{-th unloading platform} \\ \sum_i^n (h_{i-1} \cot \theta_{i-1} + b_{i-1}) + h_i \cot \theta_i < x \leq \sum_i^n (h_i \cot \theta_i + b_i) \\ i = n, \quad \sum_n^n (h_{n-1} \cot \theta_{n-1} + b_{n-1}) + h_n \cot \theta_n < x \leq x_C \end{cases} \quad (28)$$

where: h_i is the height of the slope at level i ; b_i is the width of the platform above the slope at level i ; θ_i is the angle between the slope at level i and the horizontal direction; h_n is the height of the slope at level n ; θ_n is the angle between the slope at level n and the horizontal direction.

The above formula multi-level slope segmentation function, in the need to note that when $i=n$ that segmentation function positive value of the last level of slope to reach the top of the slope, in fact, there is no unloading platform of the n th level of slope, here for the limitations of x for the former in the slip surface and the top of the slope at the intersection of the point X_C (as shown in Fig. 4), x can not be more than the value of the slip surface range.

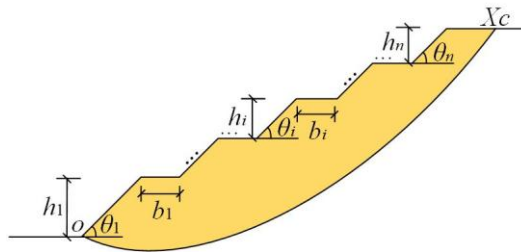


Fig. 4. Sketch of a multi-stage slope

3. Example analysis of homogeneous soil slope project

3.1 Project profiles

A homogeneous tertiary homogeneous loess slope in Lanzhou with a slope height of 24 m, a slope ratio of 1:1, a soil gravity of 16 kN/m³, a soil cohesion of 16.0 kPa and an internal friction angle of 23° (as shown in Table 1). As

this is all the information needed for computational validation, the rest of the irrelevant details are not repeated in this paper.

Table 1 Physical and mechanical parameters of the soil

Soil layer	T/(m)	γ /(kN/m ³)	Soil layer	T/(m)	γ /(kN/m ³)
Loess	>48.0	16.0	16.0	23.0	10.0

3.2 Numerical simulation modelling

The PLAXIS 3D model was built according to this condition to analyze it by numerical simulation. For the models built with GEO and PLAXIS 3D, the Mohr-Coulomb strength criterion was chosen for the soil failure strength criterion, while for the calculation of the factor of safety, PLAXIS 3D used the strength reduction method, and the gridding was chosen to be of fine grade. Since the calculation method is based on the derivation of the anthropomorphic hydrostatic method, in order to reduce the interference of other factors, the equivalent seismic action of the anthropomorphic hydrostatic method is also used in the software, and the model boundary ranges: the length from the top of the slope to the trailing edge of the slope is 2.5 times the height of the slope, the distance from the foot of the slope to the leading edge of the slope is 1.5 times the height of the slope, and the overall height of the model is 2 times the height of the slope. Therefore, a model of size 138 m $\times$ 15 m $\times$ 48 m was built as shown in Fig. 5.

3.3 Calculations

Fig. 5 shows the calculation results of the slope model, where Fig. 5 (a) shows the deformation grid map, which can reflect the deformation of the soil cell. In PLAXIS 3D shear strain represents the shear strain increment generated in the last step of the calculation phase, which reflects the shear zone that may have occurred on the slope. In order to observe the damage mechanism of multistage loess slopes under seismic action, the shear strain increment cloud plots before and after the slope damage are compared. Fig. 5(b) shows the shear strain increment cloud plots before the calculation of the safety factor, before the damage, and the damage is not yet done, from which it is seen that the damage of slopes has a tendency to extend from the foot of the slope upwards, which indicates that the foot of the slope may be the first to experience damage. Fig. 5(c) shows the cloud plot of shear strain increment after calculating the factor of safety, i.e. at the time of damage. Since PLAXIS 3D calculates the factor of safety by using the strength reduction method, i.e. step by step, the shear strength of the soil body is reduced until the slope reaches a stable state of damage, the slope is already approaching the state of instability at this time, and the slip surface is from penetrating to the top of the slope. Fig. 5(d) and Fig. 5(e) show the horizontal displacement cloud before and after calculating the safety factor, and it can be seen that the maximum displacement in Fig. 5(d) is concentrated in the first level of slope, and in Fig. 5(e), when the slope reaches the state of destruction, the maximum displacement moves upward from the foot of the slope, and the range is extended to all levels of slope. Fig. 5(f) shows the overall displacement vector cloud map before and after the slope damage, through the displacement vector cloud map can be seen that the slope damage is developing, and can see the direction and contour of the slope soil damage sliding, mainly sliding forward in a circular arc.

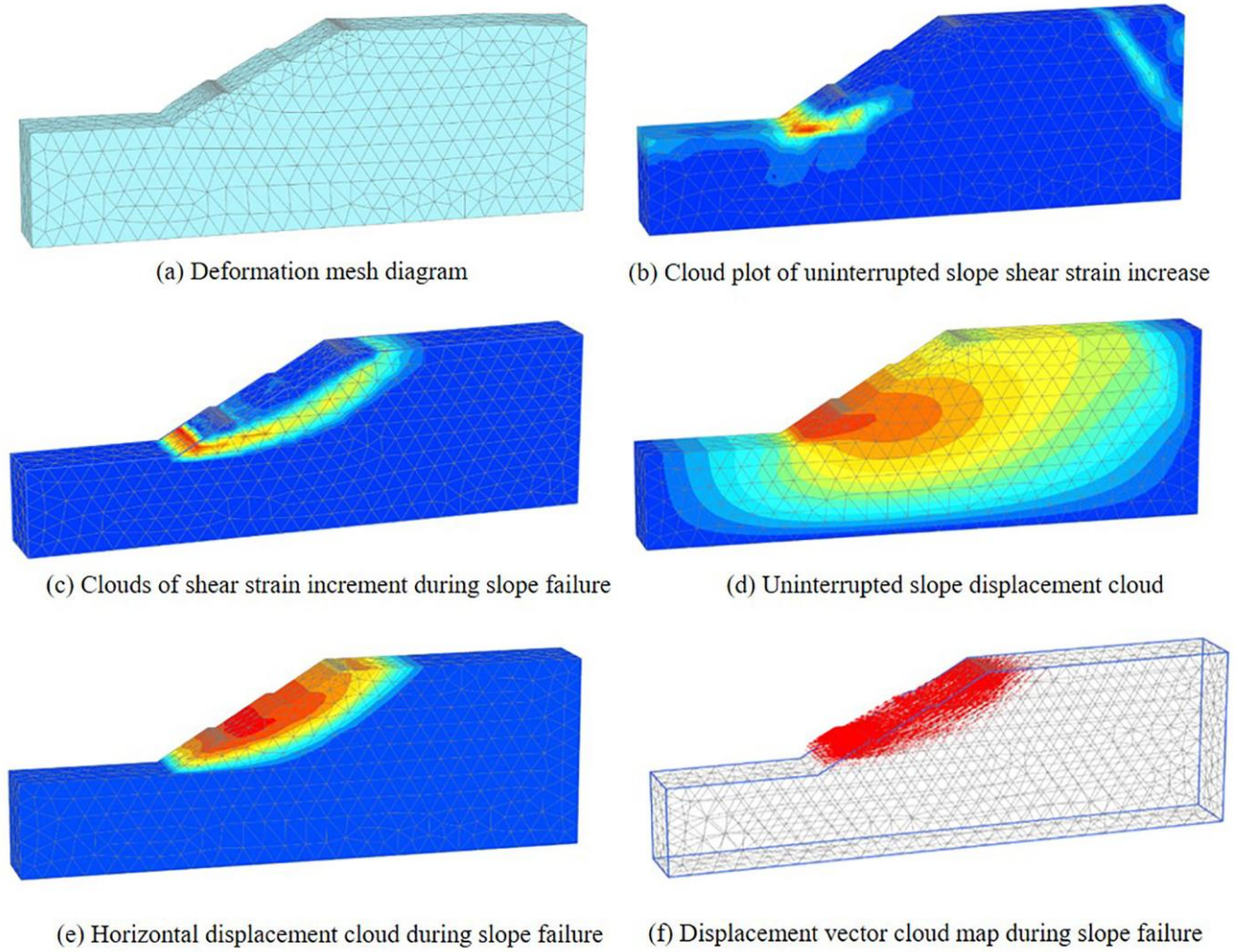


Fig. 5. Simulation results of tertiary homogeneous loess slopes

3.4 Comparative analyses

After the calculation of slope stability, the factor of safety obtained by using the method in this paper is 1.130, which meets the requirement of reasonableness test. As shown in Table 2, the results calculated using the Newmark method in GEO SLOPE [23] and the strength discount method in PLAXIS 3D are 1.127 and 1.137 respectively, which are basically consistent with the results in this paper.

Table 2 Comparison of calculation results of slope safety factor

Different calculation methods	Safety factor
Newmark method	1.127
PLAXIS3D(Discounted strength method)	1.137
Rigorous vertical slice method	1.130

From the comparison of the calculation results, it can be seen that the safety coefficients calculated by the rigorous vertical slice method and the strength reduction method are very close to each other, but the calculation process of the rigorous slice method is simpler, and it can more intuitively show the relationship between the morphological characteristics of the appearance of multilevel slopes (e.g., slope rate at each level, width of the platform, and height of the slopes at each level) and the safety coefficients. The coefficients of safety of the rigorous slice method are slightly smaller, although the former method does not need to assume the potential slip surface pattern, and the latter method is also a slice method, but neither of them considers the interslice force under the

seismic action, which makes the results of the three calculations have errors. The two methods that do not consider the interslice forces are slightly more conservative than the rigorous slice method.

4. Conclusions

Based on the method of slices in limit equilibrium analysis, this paper proposes a rigorous limit equilibrium slice method. By analyzing the forces acting on soil slices under seismic conditions and introducing specific assumptions regarding inter-slice forces, equilibrium equations incorporating both shear and normal forces between slices are established, leading to the derivation of an explicit safety factor expression. The feasibility and rationality of the proposed rigorous vertical slice method are validated through an engineering case study, demonstrating its broad applicability for slope stability analysis in practical engineering scenarios.

(1) This study considers seismic action and inter-slice forces, adopts the pseudo-static method to correlate seismic effects with inter-slice forces, conducts independent analysis of vertical soil slice units, and establishes limit equilibrium equations to derive the rigorous vertical slice method.

(2) Based on classical earth pressure theory, the normal inter-slice forces for vertical soil slices under seismic action are hypothesized. Following Spencer's assumption, the tangential inter-slice forces are determined, and relevant constant coefficients are introduced, thereby enhancing the accuracy of the vertical slice method.

(3) The geometric characteristics of multi-level loess slopes—including platform width, slope ratios at various levels, slope heights, and the number of tiers—are effectively represented using piecewise functions, distinguishing this approach from methods designed for single-level slopes.

(4) Traditional limit equilibrium methods require complex analysis of inter-slice force distribution, while the present approach simplifies safety factor determination through integration of these forces with slip surface normal stress, significantly improving computational efficiency.

Data availability statement: The data supporting this study's findings are available from the corresponding author upon reasonable request.

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Conflicts of Interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Figure

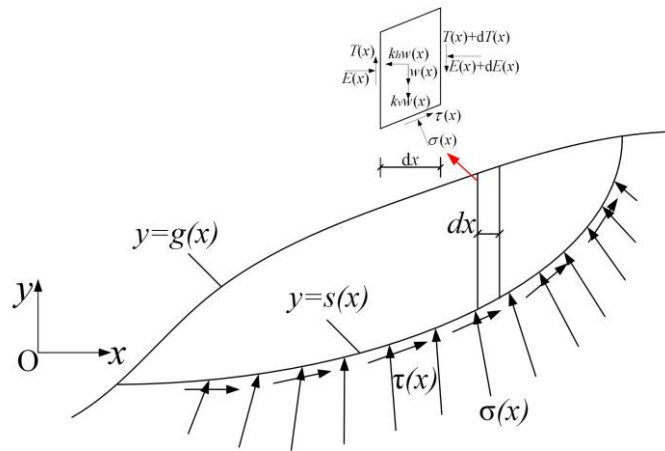


Fig. 1. Model of the vertical slice method for a multistage slope

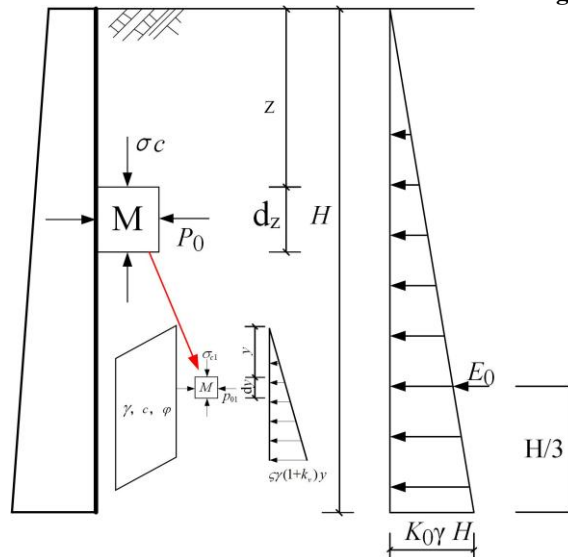


Fig. 2. Static earth pressure calculation sketch

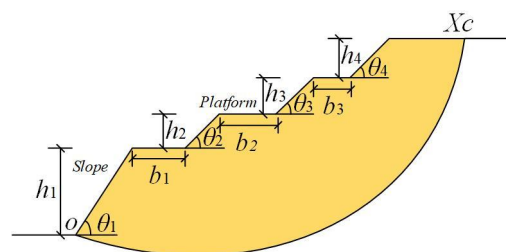


Fig. 3. Sketch of a Four-stage slope

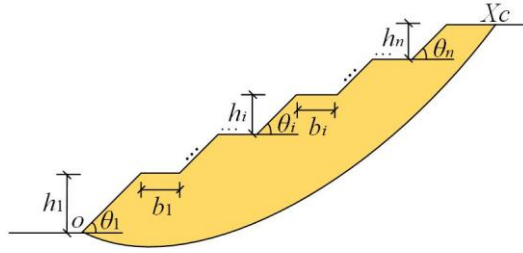
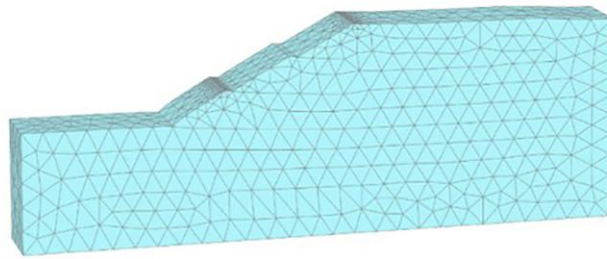
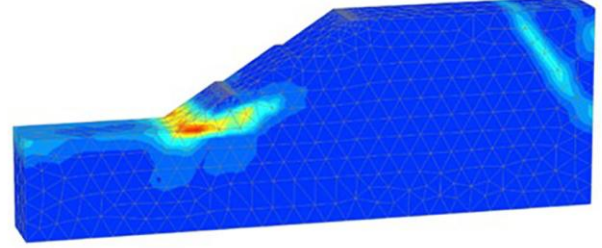


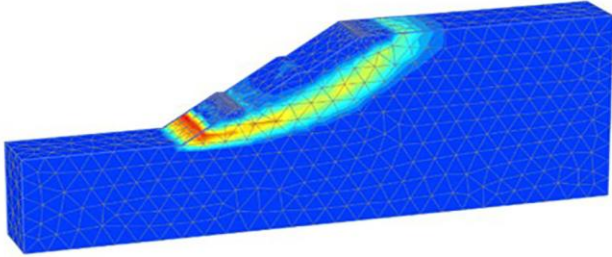
Fig. 4. Sketch of a multi-stage slope



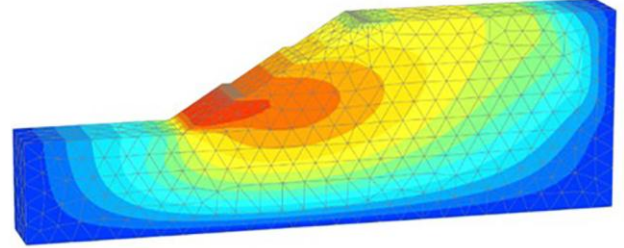
(a) Deformation mesh diagram



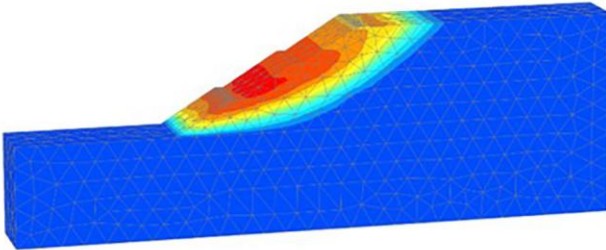
(b) Cloud plot of uninterrupted slope shear strain increase



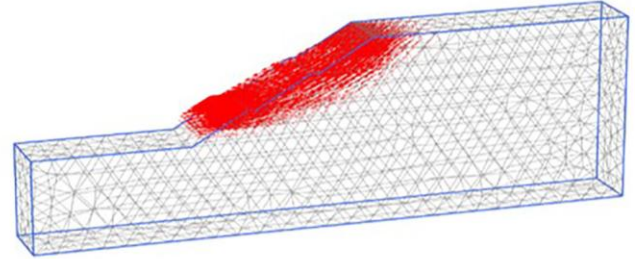
(c) Clouds of shear strain increment during slope failure



(d) Uninterrupted slope displacement cloud



(e) Horizontal displacement cloud during slope failure



(f) Displacement vector cloud map during slope failure

Fig. 5. Simulation results of tertiary homogeneous loess slopes