

Scaling Laws for Rainfall Infiltration in Unsaturated Soils: Derivation and Validation Using Centrifuge Model Tests

Xiaochuan Liu

Yan'an University

Gaofei Kong

Yan'an University

Junxi He

Yan'an University

Haijie He

11312013@zju.edu.cn

Zhejiang University

Yunfang Wu

Bo'ao Construction Co., Ltd

Chao Zhang

Fuzhou University

Article

Keywords: unsaturated soil, rainfall infiltration, scaling law, centrifugal model test, wetting front

Posted Date: September 21st, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-10956957/v1>

License:   This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Additional Declarations: No competing interests reported.

Scaling Laws for Rainfall Infiltration in Unsaturated Soils: Derivation and Validation Using Centrifuge Model Tests

Xiaochuan Liu ¹, Gaofei Kong ², Junxi He ¹, Haijie He ^{3,*}, Yunfang Wu ⁴, and Chao Zhang ^{5,*}

¹ College of Civil Engineering and Architecture, Yan'an University University, Yan'an 716000, China
² College of yanan medical, Yan'an University University, Yan'an 716000, China
³ College of Civil and Architectural Engineering, Zhejiang University, Hangzhou 310058, China
⁴ Bo'ao Construction Co., Ltd, Taizhou 318020, China
⁵ College of Civil Engineering, Fuzhou University, Fuzhou 350000, China
* Correspondence: 11312013@zju.edu.cn (H.H.); zhangchao1985@fzu.edu.cn(C.Z.)

Abstract

Centrifuge model testing can reproduce the in-situ stress state of soils. It has been widely used in studies of foundation settlement. However, the scaling laws for rainfall infiltration have not been fully validated by systematic experimental evidence. In this study, the scaling laws for rainfall infiltration are derived from the classical Green–Ampt model and the first similarity theorem. These scaling laws are then validated by centrifuge model tests on unsaturated soil foundations under different g-levels. The results show clear prototype-to-model scaling relationships. The scaling laws for wetting-front migration velocity, saturated hydraulic conductivity in the wetted zone, and infiltration time are 1, 1:1, and N^2 :1, respectively. These results agree well with the theoretical derivations. The Gardner equation is also verified as an effective criterion for evaluating suction similarity. Suction similarity can be maintained when the centrifuge-induced suction calculated from the Gardner equation is smaller than the initial soil suction. However, suction similarity is no longer valid when the calculated suction exceeds the initial soil suction. In this case, the centrifugal acceleration increases the initial soil suction.

Keywords: unsaturated soil; rainfall infiltration; scaling law; centrifugal model test; wetting front

Academic Editor: Firstname Last-name

Received: date
Revised: date
Accepted: date
Published: date

Citation: To be added by editorial staff during production.

Copyright: © 2025 by the authors. Submitted for possible open access publication under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

The reliability of centrifuge model tests depends on appropriate scaling laws. These laws allow the measured model response to be extrapolated to the prototype scale. For rainfall infiltration in unsaturated soils, the main physical quantities include seepage velocity, infiltration time, wetting-front depth, hydraulic conductivity, water content, and suction.

Previous studies have examined scaling laws for several unsaturated seepage problems. Cargill et al. [1] studied transient seepage in an earth dam during water-level rise and drawdown under different centrifugal accelerations. Their results showed that the scaling ratios of seepage velocity and seepage time were 1 and N^2 :1, respectively. These results were generally consistent with theoretical predictions. Culligan et al. [2] investigated finger flow at 1g and 10g. They found that the finger width in the model was about 1/N of that in the prototype, and the finger-flow velocity in the model was about N times

that in the prototype. Rezzoug et al. [3] conducted centrifuge model tests on capillary rise in soils with different densities and g-levels. Their results showed that the scaling ratios of rise height, rise time, and rise velocity were 1, $N^2:1$, and 1, respectively. These scaling laws were further confirmed by numerical simulations and centrifuge model tests reported by Crancon et al. [4] and Thorel et al. [5].

Theoretical studies have also provided important support for unsaturated seepage scaling. Dell'Avanzi et al. [6] derived the governing equation for unsaturated seepage based on Darcy's law and Richards' equation. Their derivation assumed constant soil volume and permeability independent of the acceleration field. They showed that the suction profile in the model can be identical to that in the prototype under a nearly uniform acceleration field. They also obtained scaling ratios of 1, $N:1$, and $N^2:1$ for seepage velocity, flow rate, and seepage time, respectively. Gragnano [7] validated these scaling laws by comparing numerical results with centrifuge model test data. Later studies have widely adopted these relationships in centrifuge model tests involving unsaturated seepage [8 – 10].

These studies have improved the understanding of transient unsaturated flow in centrifuge modeling. They mainly focused on water-level variation in earth dams, finger flow, and capillary rise. However, rainfall infiltration has a different hydraulic boundary condition and a distinct wetting-front evolution process. Many centrifuge model tests have been conducted on rainfall-induced slope instability. In these tests, the existing scaling laws for unsaturated seepage were usually adopted directly. Direct and systematic experimental validation of the scaling laws for rainfall infiltration remains limited.

Suction similarity is another key issue in centrifuge modeling of unsaturated soils. Centrifugal acceleration may change the water content and suction of the model soil. This change can alter the initial hydraulic state of the model. It may also weaken the similarity between the model and the prototype. Therefore, suction similarity is a prerequisite for reliable simulation of rainfall infiltration in unsaturated soils. Gardner [11] first proposed a relationship between centrifuge angular velocity and suction. Gens [12] noted that consolidation under hypergravity may reduce soil suction. With the development of miniature tensiometers [13,14] and micro-TDR probes [15], real-time monitoring of suction and water content in centrifuge tests has become possible. However, a clear criterion for suction similarity is still lacking. Experimental validation of such a criterion is also insufficient.

This study derives the scaling laws for rainfall infiltration in unsaturated soils based on the classical Green – Ampt model and the first similarity theorem. The derived scaling laws are then validated using centrifuge model tests on unsaturated horizontal soil foundations under different g-levels. The scaling relationships for wetting-front migration velocity, infiltration time, and saturated hydraulic conductivity in the wetted zone are examined. In addition, the Gardner equation is used to calculate the maximum suction induced by centrifugal acceleration. The calculated suction is compared with tensiometer measurements. This comparison is used to evaluate whether the Gardner equation can serve as a criterion for suction similarity in centrifuge modeling of rainfall infiltration.

2. Derivation of Scaling Laws

2.1. Scaling Laws for Rainfall Infiltration

The Green-Ampt model assumes a homogeneous soil and the existence of a distinct horizontal wetting front during infiltration. Ahead of the wetting front (i.e., in the unsaturated zone), the soil water content remains at the initial water content θ_i (hereinafter, water content refers to volumetric water content; volumetric water content equals gravimetric water content multiplied by the soil dry density). Behind the wetting front (i.e., in the wetted zone), the soil is saturated with a water content θ_s . Water flow in the wetted zone follows Darcy's law, and the saturated permeability coefficient k_s in the wetted zone is assumed constant. The suction head at the wetting front s_f is also assumed constant. Moisture diffusion in the unsaturated zone is neglected, and only one-dimensional vertical flow is considered [16-19].

In this model, the constant water head at the ground surface is denoted as H , and the depth of wetting front penetration is z_f , which is a function of infiltration time t . The expression for the suction head at the wetting front s_f is as follows:

$$s_f = \frac{\psi}{\rho_w \cdot g} \quad (1)$$

In Equation (1), ψ denotes the suction at the wetting front. The parameter g is the gravitational acceleration, and ρ_w is the density of water.

The origin of the z -axis is set at the ground surface. The positive z -direction is upward. The hydraulic head at the ground surface is H . The hydraulic head at the wetting front is $-(s_f + z_f)$. Therefore, the hydraulic gradient across the wetted zone is $(s_f + z_f + H)/z_f$. According to Darcy's law, the infiltration rate at the ground surface i can be expressed as:

$$i = k_s \frac{z_f + s_f + H}{z_f} \quad (2)$$

The soil layer thickness and the constant water head H are scaled by a prototype-to-model ratio of $N:1$. Based on this known scaling relationship, the scaling laws for the physical quantities in Equation (2) can be derived using the first similarity theorem. For the prototype, the infiltration rate i_p is expressed as:

$$i_p = k_{s_p} \frac{z_{f_p} + s_{f_p} + H_p}{z_{f_p}} \quad (3)$$

For the model, the infiltration rate i_m can be expressed as:

$$i_m = k_{s_m} \frac{z_{f_m} + s_{f_m} + H_m}{z_{f_m}} \quad (4)$$

The following scaling factors are defined:

$$B_i = \frac{i_p}{i_m} ; \quad B_{k_s} = \frac{k_{s_p}}{k_{s_m}} ; \quad B_{z_f} = \frac{z_{f_p}}{z_{f_m}} ; \quad B_{s_f} = \frac{s_{f_p}}{s_{f_m}} ; \quad B_H = \frac{H_p}{H_m} \quad (5)$$

where each scaling factor represents the ratio of the corresponding prototype quantity to the model quantity. Substituting the scaling factors in Equation (5) into the prototype equation (Equation (3)) and rearranging the terms gives:

$$i_m = \frac{B_{k_s}}{B_i} k_{s_m} \frac{z_{f_m} + \frac{B_{s_f}}{B_{z_f}} s_{f_m} + \frac{B_H}{B_{z_f}} H_m}{z_{f_m}} \quad (6)$$

For Equation (6) to have the same form as the model equation (Equation (4)), the following similarity condition must be satisfied:

$$\frac{B_{k_s}}{B_i} = \frac{B_{s_f}}{B_{z_f}} = \frac{B_H}{B_{z_f}} = 1 \quad (7)$$

According to the scaling law for saturated seepage, $B_{k_s}=1/N$. Based on Equation (7), geometric similarity between the prototype and the model requires $B_i=1/N$. Since $B_H=N$, both B_{z_f} and B_{s_f} are also equal to N .

The scaling factor for gravitational acceleration is:

$$B_g = \frac{g_p}{g_m} = \frac{1}{N} \quad (8)$$

From Equation (1), the scaling factor for suction is obtained as:

$$B_\psi = \frac{\psi_p}{\psi_m} = 1 \quad (9)$$

The relationship between infiltration time t and wetting-front penetration depth z_f is expressed as:

$$t = \frac{\theta_s - \theta_i}{k_s} \left[z_f - (s_f + H) \ln \frac{z_f + s_f + H}{s_f + H} \right] \quad (10)$$

The first similarity theorem is then applied to Equation (10). The same soil is assumed for the prototype and the model. Centrifugal acceleration is also assumed not to change θ_i . Therefore, B_{θ_i} and B_{θ_s} . It follows that the scaling factor for infiltration time is $B_t=N^2/1$.

The wetting-front migration velocity v_f is related to the wetting-front penetration depth and infiltration time as:

$$v_f = \frac{dz_f}{dt} \quad (11)$$

Applying the first similarity theorem to Equation (11) gives $B_{v_f}=1/N$. Table 1 summarizes the scaling laws obtained from the above derivation.

Table 1. Scaling laws for physical quantities involved in rainfall infiltration in unsaturated soils.

-	Physical quantity	Dimension	Prototype	Model
Known scaling laws	Soil layer thickness	L	N	1
	Constant water head H	L	N	1
	Initial water content in unsaturated zone θ_i	Dimensionless	1	1
	Saturated water content in wetted zone θ_s	Dimensionless	1	1
Derived scaling laws	Wetting front penetration depth z_f	L	N	1
	Suction head at wetting front s_f	L	N	1
	Suction at wetting front ψ	ML-1T-2	1	1
	Wetting front migration velocity v_f	L/T	1	N
	Infiltration time t	T	N ²	1
	Saturated permeability coefficient in wetted zone k_s	L/T	1	N
	Infiltration rate at ground surface i	L/T	1	N

2.2. Criterion for the Suction Scaling Law

As discussed above, the derivation of the scaling laws assumes that centrifugal force does not change the initial water content θ_i . Under this assumption, the scaling ratios for water content, suction, and suction head are all 1:1. However, pore water may drain from the soil during centrifuge rotation. If the initial water content changes under hypergravity, the suction in the model will no longer be similar to that in the prototype. This breaks the similarity of the initial hydraulic conditions. It may also affect the similarity of the test results.

Figure 1 illustrates the basic concept of the Gardner equation. Consider a small, initially saturated soil specimen under free-drainage boundary conditions. If the specimen is placed at the rotation axis of the centrifuge, its rotation radius r is zero. The centrifugal force is then zero. In this case, the suction remains equal to its initial value of 0 kPa, namely $\psi_0=0$ kPa.

If the initially saturated specimen is placed at a finite rotation radius r , drainage occurs under centrifugal force. Drainage continues until equilibrium is reached. The suction at equilibrium is denoted as ψ . Gardner [11] proposed the following relationship between the centrifuge angular velocity ω and the induced suction ψ :

$$\frac{d\psi}{dr} = \frac{\rho r \omega^2}{g} \quad (12)$$

where ρ is the density of the pore fluid, and g is the gravitational acceleration. Integrating Equation (12) from 0 to r gives:

$$\psi = \int_0^r \frac{\rho r \omega^2}{g} = \frac{\rho r^2 \omega^2}{2g} \quad (13)$$

In actual centrifuge model tests, the soil at a rotation radius r is usually unsaturated. Its initial suction ψ_0 is therefore not equal to 0 kPa. Assume that the applied centrifugal acceleration is N_1g at the same rotation radius. The maximum suction ψ_1 that can be generated in an initially saturated specimen at this radius can be calculated using Equation (13).

If $\psi_1 > \psi_0$, the centrifugal force at N_1g can increase the initial suction of the soil. As noted above, the scaling ratio for suction should be 1:1. In this case, the suction in the model is not similar to that in the prototype. If $\psi_1 < \psi_0$, the centrifugal force at N_1g does not change the initial suction of the soil. The suction in the model is then similar to that in the prototype.

The Gardner equation [11] can therefore be used as a criterion for suction similarity. In centrifuge model tests, the angular velocity ω is usually not given directly. The applied g-level N is usually provided instead. The corresponding reference radius is generally the centrifuge arm length R . Thus, Equation (13) can be rewritten as follows. Given:

$$Ng = R\omega^2 \quad (14)$$

Since ω does not vary with changes in the rotation radius, it follows that:

$$\frac{Ng}{R} = \omega^2 \quad (15)$$

The distance r from the soil to the centrifuge axis is generally smaller than the centrifuge arm length R . Substituting Equation (15) into Equation (13) gives:

$$\psi_0 = \frac{\rho N r^2}{2R} \quad (16)$$

Equation (16) is equivalent to Equation (13). It can be used to evaluate whether suction similarity is satisfied for a soil with a given initial suction under a specified centrifugal g-level. The validity of this criterion is examined in the following sections.

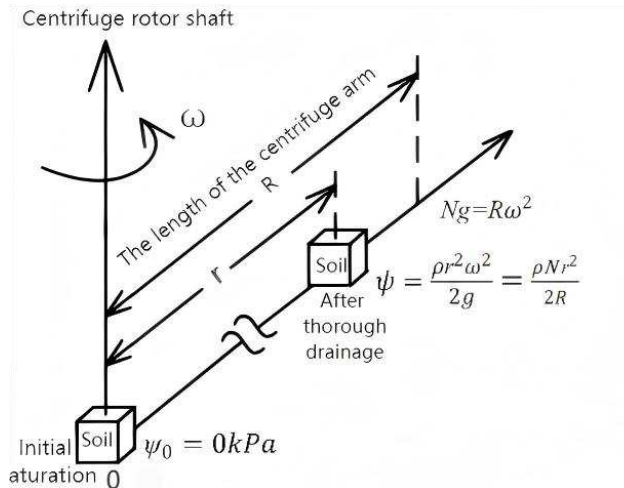


Figure 1. Schematic diagram of the Gardner equation

3. Experimental Validation

The derived scaling laws and the proposed criterion for suction similarity were validated by centrifuge model tests. The tests simulated rainfall infiltration in horizontal un-

saturated soil foundations. This section briefly describes the test soil, test equipment, monitoring instruments, test program, model preparation, and test procedure. Detailed information can be found in a previous study by the authors [20].

3.1 Test soil

Table 2 presents the physical properties of the loess used in the tests. The soil was collected from Xi'an, Shaanxi Province, China. It is classified as clayey silt. Figure 2 shows the drying and wetting soil-water characteristic curves (SWCCs) of the compacted loess with a dry density of 1.45 g/cm³.

Table 2. Physical properties of the test loess

Property	Value
Percentage of sand (>0.075 mm)	2.20%
Percentage of silt(0.075–0.005 m)	71.80%
Percentage of clay(<0.005 mm)	26%
Specific gravity	2.71
Maximum dry density (g/cm ³)	1.705
Optimum water content	19.30%
Plastic limit	22%
Liquid limit	36.70%
Plasticity index	14.7
Saturated permeability coefficient (dry density 1.45 g/cm ³)(m/s)	1.29×10 ⁻⁷

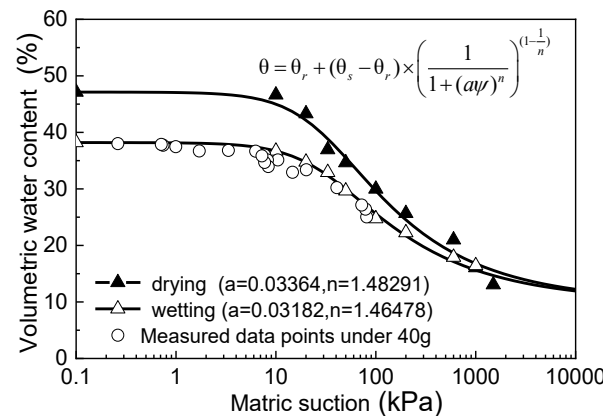


Figure 2. SWCCs of compacted loess with a dry density of 1.45 g/cm³

3.2 Test equipment

The tests were conducted on the ZJU-400 centrifuge at Zhejiang University [21] (Figure 3), which has an arm length R of 4.5m. A self-developed onboard rainfall simulator [22] (Figure 4) was used.



Figure 3. ZJU-400 centrifuge



Figure 4. Photograph of the rainfall simulator

Water content monitoring was carried out using miniature TDR probes [15] developed by the authors' research group, as shown in Figure 5(a). Suction monitoring was performed using miniature tensiometers developed by Chen Rui et al. [14], as shown in Figure 5(b).



a. Miniature TDR probe b. Miniature tensiometer

Figure 5. Instruments for monitoring soil response used in the tests

3.3 Test scheme

Figure 6 shows the overall test layout. The test simulated rainfall infiltration in a horizontal loess foundation. The model foundation was 50 cm long, 40 cm wide, and 28 cm thick, as shown in Figure 7(a). The constant water head was 3 cm. The dry density of the compacted foundation was 1.45 g/cm³. The target water content of the prepared loess was 27.55%.

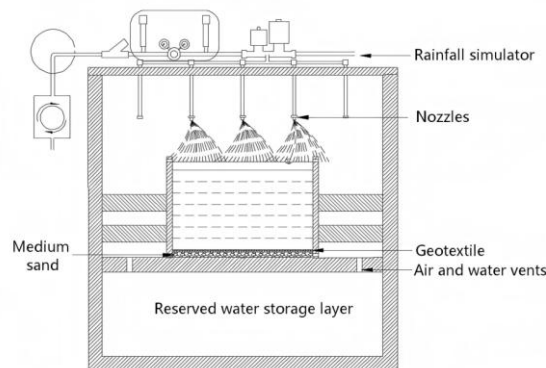
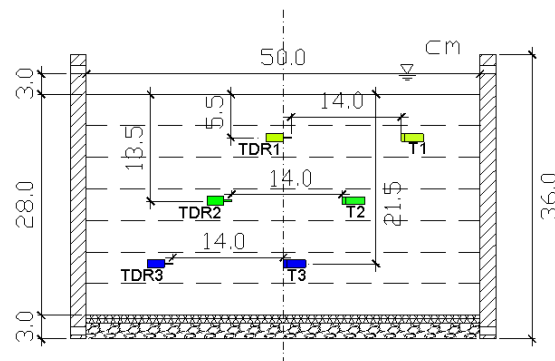
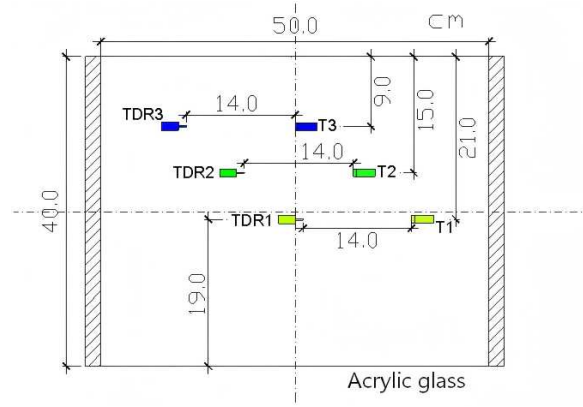


Figure 6. Schematic diagram of the overall test layout

Figure 7(a) shows the cross-sectional layout of the monitoring instruments. The instruments were arranged in three layers: upper, middle, and lower layers. One TDR probe and one tensiometer were placed in each layer. Figure 7(b) shows the plan view of the instrument layout. The TDR probes in the three layers were arranged in a staggered pattern.



a. Cross-sectional view



b. Top view

Figure 7 Layout of monitoring instruments in the model foundation (T: tensiometer)

The tests included three parallel groups. The only variable was the centrifugal acceleration level. The selected g-levels were 20g, 40g, and 60g. The 40g test is used here as an example. At the model scale, the rainfall simulator applied a rainfall intensity of 387 mm/h. This value corresponds to a prototype rainfall intensity of 9.7 mm/h. It is equivalent to a hydraulic conductivity of 1.1×10^{-4} m/s. At 1g, the measured saturated hydraulic conductivity of the loess with the same dry density was 1.8×10^{-7} m/s. According to the theoretical scaling law for saturated seepage, the saturated hydraulic conductivity at 40g should be 40 times the value at 1g. The corresponding value is 7.2×10^{-6} m/s. The rainfall intensity at the model scale was much higher than this value. Therefore, surface ponding formed rapidly on the foundation.

When the ponded water depth exceeded 3 cm, water began to drain through the overflow holes on the baffles at both sides of the foundation, as shown in Figure 7(a). Continuous rainfall was then maintained to keep a constant water head of 3 cm. During constant-head infiltration, soil water content and suction were recorded using the TDR probes and tensiometers. The centrifuge was decelerated and stopped after all monitoring layers showed clear responses and the measured values became stable.

4. Test results

Figure 8 shows the measured responses during rainfall infiltration at 40g. The solid line represents the suction measured by the tensiometer with a sampling interval of 1 s. This interval allowed accurate determination of the wetting-front arrival time. The wetting-front arrival time is defined as the time when the wetting front reached the monitoring point. The solid symbols represent the water content measured by the TDR probes.

Figures 9 and 10 show the time histories of suction and water content at 20g and 60g, respectively. The wetting-front arrival times and wetting-front migration velocities are summarized in Table 3. The wetting-front migration velocities showed only small differences among different depth intervals. This result indicates that the wetting-front migration velocity in the horizontal foundation remained nearly stable during rainfall infiltration. As shown in Figure 2, the measured suction and water content data at 40g agree well with the soil-water characteristic curve obtained from the pressure plate test at 1g. This agreement confirms the reliability of the miniature tensiometer and TDR measurements under hypergravity.

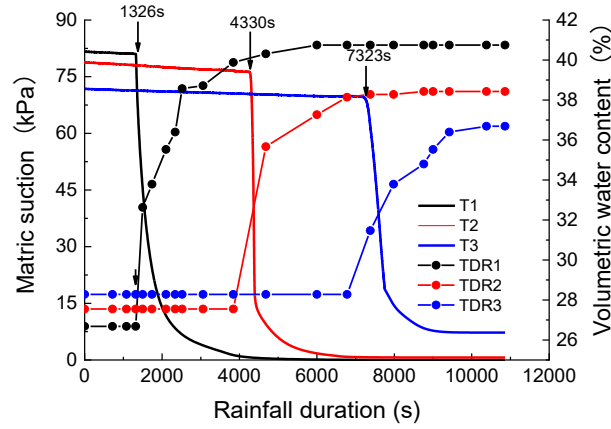


Figure 8. Measured suction and water content at 40g

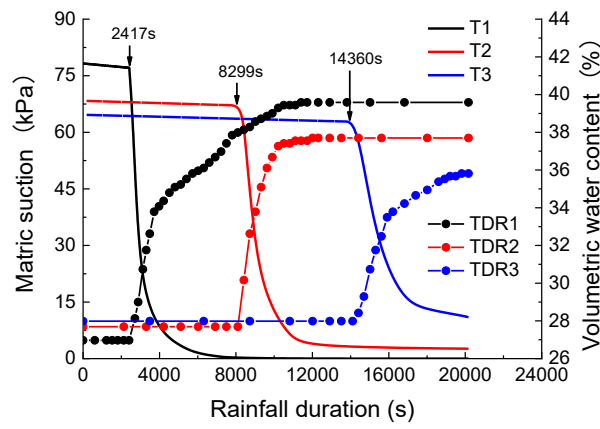


Figure 9. Measured suction and water content at 20g

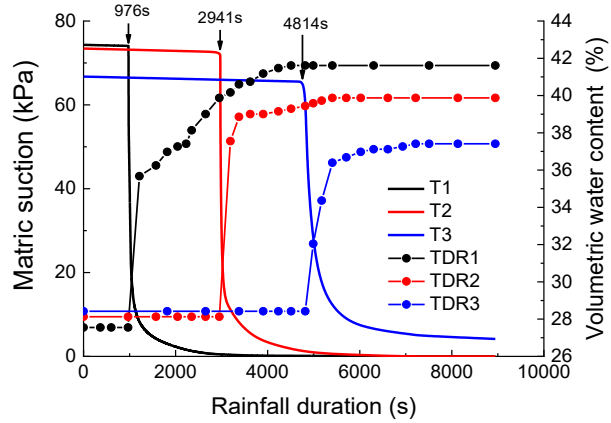


Figure 10. Measured suction and water content at 60g

Table 3. Wetting front arrival time and migration velocity measured by tensiometer

g-level	Wetting front arrival time (s)			Wetting front migration velocity(m/s)		
	at 5.5 cm	at 13.5 cm	at 21.5 cm	5.5-13.5cm	13.5-21.5cm	5.5-21.5cm
20	2417	8299	14360	1.36×10^{-5}	1.32×10^{-5}	1.34×10^{-5}
40	1326	4330	7323s	2.66×10^{-5}	2.67×10^{-5}	2.67×10^{-5}
60	976	2941	4814	4.07×10^{-5}	4.27×10^{-5}	4.17×10^{-5}

5. Validation of scaling laws

5.1 Validation of scaling laws for rainfall infiltration

Table 4 presents the wetting-front migration velocity v_f at different g-levels. It also presents the water content and the corresponding degree of saturation before and after the tests. The results show that the ratios of wetting-front migration velocities are close to the ratios of the applied g-levels. This agreement validates the scaling law for v_f listed in Table 1:

$$B_{v_f} = \frac{v_{fp}}{v_{fm}} = \frac{1}{N} \quad (17)$$

Slight deviations are observed for the 20g/60g and 40g/60g cases. These deviations may be related to differences in the initial water content of the model soil at different g-levels. The wetting-front migration velocity at 60g is slightly higher than the values expected from the 20g/60g and 40g/60g ratios. This result corresponds to the slightly higher initial water content of the model soil at 60g, as shown in Table 4. A higher initial water content can increase the unsaturated hydraulic conductivity of the soil. It can therefore lead to a faster wetting-front migration velocity.

Table 4. Wetting front migration velocity, water content, and corresponding degree of saturation

g-level l	v_f (m/s)	Average water con- tent be- fore test(%)	Average water content after test(%)	Average degree of saturation after test (%)	k_s (m/s)	θ_s (%)	s_f (cm)
20	1.34×10^{-5}	27.6	37.7	81.1	1.12×10^{-6}	39.5	0.774
40	2.67×10^{-5}	27.5	38.6	83	2.26×10^{-6}	39.1	-0.087
60	4.17×10^{-5}	28.0	39.6	85.2	3.61×10^{-6}	38.7	-1.124

Equation (10) was used to derive the scaling laws for the other physical quantities. The ponding water depth, (H), was 3 cm in all tests. The suction time-history curves at the three monitoring points were used to determine the suction breakpoints. These breakpoints represent the wetting-front arrival times t . They correspond to three wetting-front penetration depths, (z_f), which were equal to the burial depths of the monitoring points: 0.055 m, 0.135 m, and 0.215 m. Thus, each monitoring point provided one equation.

The 40g test is used as an example. As shown in Figure 8, the water contents at the three monitoring points before the test were 26.68%, 27.55%, and 28.30%, respectively. After the test, the corresponding values were 40.75%, 38.43%, and 36.69%, respectively. The water contents were not identical before and after the test. This condition does not fully satisfy the assumption of the Green-Ampt model. The difference in water content among the three monitoring points became larger after the test. This was mainly caused by the limited infiltration duration. The three monitoring points did not reach the theoretical saturated state. Therefore, the monitoring point closer to the constant water head had a higher water content after infiltration.

To reduce calculation errors, the measured values of θ_i were substituted into Equation (10). The saturated water content θ_s was treated as the first unknown. According to Equation (1), the suction head at the wetting front s_f varies with the g-level. Therefore s_f was treated as the second unknown. The saturated hydraulic conductivity of the wetted zone k_s also varies with the g-level. Thus k_s was treated as the third unknown. The known and unknown quantities were then substituted into Equation (10). This procedure produced a system of three equations. The three unknowns θ_s , s_f , and k_s were obtained by solving this system.

$$\begin{cases} 1326 = \frac{\theta_s - 0.2668}{k_s} \left[0.055 - (s_f + 0.03) \ln \frac{0.055 + s_f + 0.03}{s_f + 0.03} \right] \\ 4330 = \frac{\theta_s - 0.2755}{k_s} \left[0.135 - (s_f + 0.03) \ln \frac{0.135 + s_f + 0.03}{s_f + 0.03} \right] \\ 7323 = \frac{\theta_s - 0.2830}{k_s} \left[0.215 - (s_f + 0.03) \ln \frac{0.215 + s_f + 0.03}{s_f + 0.03} \right] \end{cases} \quad (18) \quad 356$$

The same procedure was applied to the 20g and 60g tests. To reduce errors caused by differences in initial water content among different g-levels, the initial water contents for the 20g and 60g tests were set equal to those used in the 40g test. The corresponding systems of equations are given in Equations (12) and (13), respectively.

$$\begin{cases} 2417 = \frac{\theta_s - 0.2668}{k_s} \left[0.055 - (s_f + 0.03) \ln \frac{0.055 + s_f + 0.03}{s_f + 0.03} \right] \\ 8299 = \frac{\theta_s - 0.2755}{k_s} \left[0.135 - (s_f + 0.03) \ln \frac{0.135 + s_f + 0.03}{s_f + 0.03} \right] \\ 14360 = \frac{\theta_s - 0.2830}{k_s} \left[0.215 - (s_f + 0.03) \ln \frac{0.215 + s_f + 0.03}{s_f + 0.03} \right] \end{cases} \quad (19) \quad 363$$

$$\begin{cases} 973 = \frac{\theta_s - 0.2668}{k_s} \left[0.055 - (s_f + 0.03) \ln \frac{0.055 + s_f + 0.03}{s_f + 0.03} \right] \\ 2941 = \frac{\theta_s - 0.2755}{k_s} \left[0.135 - (s_f + 0.03) \ln \frac{0.135 + s_f + 0.03}{s_f + 0.03} \right] \\ 4814 = \frac{\theta_s - 0.2830}{k_s} \left[0.215 - (s_f + 0.03) \ln \frac{0.215 + s_f + 0.03}{s_f + 0.03} \right] \end{cases} \quad (20) \quad 365$$

The calculation results from Equations (18)-(20) are presented in Tables 4 and 5. The ratio of saturated hydraulic conductivity in the wetted zone at different g-levels is equal to, or slightly smaller than, the ratio of the g-levels. This trend is similar to that observed for the wetting-front migration velocity. It indicates that the saturated hydraulic conductivity in the wetted zone k_s follows the theoretical scaling law:

$$B_{k_s} = \frac{k_{sp}}{k_{sm}} = \frac{1}{N} \quad (21)$$

The calculated saturated water contents in the wetted zone are very close at the three g-levels. They also differ only slightly from the measured water contents after rainfall infiltration, as shown in Table 4. This agreement supports the reliability of the calculated values. It also indicates that the saturated water content in the wetted zone follows the theoretical scaling law:

$$B_{\theta_s} = \frac{\theta_{sp}}{\theta_{sm}} = \frac{1}{1} \quad (22)$$

The calculated suction heads at the wetting front are negative for the 40g and 60g tests, as shown in Table 4. However, the suction head at the wetting front should be positive in the Green-Ampt model. This discrepancy may be caused by errors in the measured suction breakpoint times at 40g and 60g. The calculation results are sensitive to the wetting-front arrival time t . A small change in t can cause large variations in the saturated hydraulic conductivity k_s , the saturated water content in the wetted zone θ_s , and the suction head at the wetting front s_f .

From Table 5, the ratio of wetting front migration velocities at 20g and 40g is:

$$\frac{v_{f20}}{v_{f40}} = \frac{1}{2} \quad (23)$$

The distances between the three monitoring points were the same in the 20g and 40g tests. Therefore:

$$\frac{z_{f20}}{z_{f40}} = \frac{1}{1} \quad (24)$$

Based on the expression for the wetting front migration velocity v_f (Equation (11)), it follows that:

$$\frac{t_{20}}{t_{40}} = \frac{2}{1} \quad (25)$$

Table 5 compares the wetting-front arrival times at the three monitoring points under different g-levels. As the depth of the monitoring point increased, the ratio of wetting-front arrival time between different g-levels became closer to the inverse ratio of the g-levels. This result indicates that the error in the arrival-time ratio decreased with increasing infiltration depth and infiltration time. Therefore, the infiltration time t , follows the theoretical scaling law.

Table 5. Comparison of physical quantities between different g-levels

Index	20g/40g	20g/60g	40g/60g
v_f	1.0/2.0	1.0/3.1	1.0/1.6
k_s	1.0/2.0	1.0/3.2	1.0/1.6
θ_s	1.0/1.0	1.0/1.0	1.0/1.0
Wetting front arrival time t at 5.5 cm depth	1.8/1.0	2.5/1.0	1.4/1.0
Wetting front arrival time t at 13.5 cm depth	1.9/1.0	2.8/1.0	1.5/1.0
Wetting front arrival time t at 21.5 cm depth	2.0/1.0	3.0/1.0	1.5/1.0

5.2 Validation of the suction scaling law

The suction scaling law was evaluated using the Gardner equation, as given in Equation (16). Figures 11-13 show the time histories of water content, suction, and centrifugal acceleration during centrifuge acceleration. The target g-levels were 20g, 40g, and 60g, respectively. As stated above, the centrifuge arm length R was 4.5 m. The rotation radius r at each monitoring point was determined from the positions of the three tensiometers in the model foundation. These positions corresponded to the upper, middle, and lower layers shown in Figure 7(a).

Table 6 presents the measured suction values at different g-levels. It also presents the suction values calculated using the Gardner equation. At 20g and 40g, the calculated suction values were smaller than the measured suction values at all monitoring points. This result indicates that the centrifugal force at 20g and 40g did not change the initial suction of the soil. Therefore, the suction in the model soil remained similar to that in the prototype. The measured results support this judgment. The maximum suction decreases at the same monitoring point were only 0.6 kPa at 20g and 1.1 kPa at 40g. These changes were very small compared with the initial suction values at 1g. Thus, the Gardner equation provides a reasonable criterion for evaluating suction similarity in these two tests.

At 60g, the suction values calculated using the Gardner equation were larger than the measured suction values at all monitoring points. This result indicates that the centrifugal force at 60g could change the initial suction of the soil. Therefore, the suction in the model soil was not similar to that in the prototype. The measured results show that the suction at each monitoring point increased by 7.8–8.5 kPa compared with the initial value. This again supports the judgment based on the Gardner equation.

Although suction similarity was not satisfied at 60g, Table 5 shows that the wetting-front migration velocity ratios at different g-levels still followed the theoretical scaling laws. This result indicates that the suction increase of 7.8–8.5 kPa at 60g did not cause a significant error in the wetting-front migration velocity. Further research is needed to determine the allowable suction difference between Ng and 1g without causing significant errors in the scaling laws. According to the Gardner equation, the suction at 60g should theoretically increase by 26.8–43.1 kPa. However, the measured increase was only 7.8–8.5 kPa. This difference may be related to the drying behavior of the soil. The drying process at 60g is similar to that at 1g. A higher soil suction requires a longer time for further drying.

Figure 13 shows that the measured suction decreased during acceleration from 0g to 40g. This phenomenon may be explained by two factors. First, the sudden increase in soil pressure during centrifuge acceleration may have caused slight deformation of the tensiometer casing. This deformation could compress the water in the tensiometer chamber and generate positive pressure. In this case, the measured suction decreased, although the actual soil suction may not have changed.

Second, before the g-level reached 40g, the theoretical suction calculated using the Gardner equation was smaller than the initial soil suction. Therefore, centrifugal force did not induce soil drying during this stage. After the g-level exceeded 40g, the theoretical suction became larger than the initial soil suction. Centrifugal force then caused soil drying. As a result, the suction increased more rapidly. After the model was held at 60g for about 5 min, the rate of suction increase gradually decreased. The soil then entered a slow drying stage.

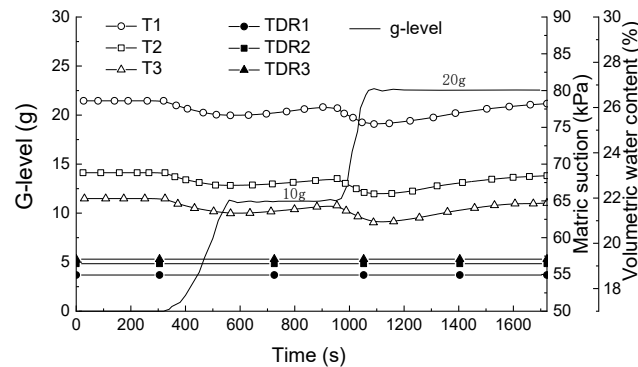


Figure 11. Measured water content and suction during centrifuge acceleration at 20g

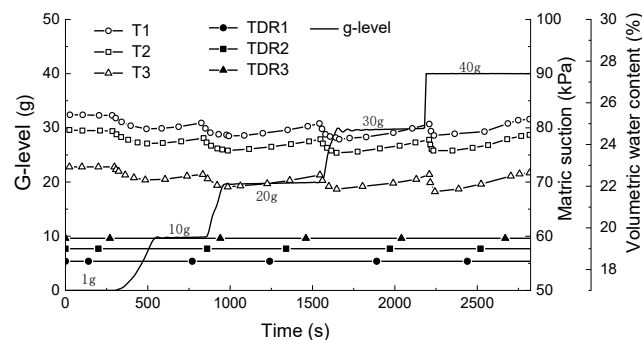


Figure 12. Measured water content and suction during centrifuge acceleration at 40g

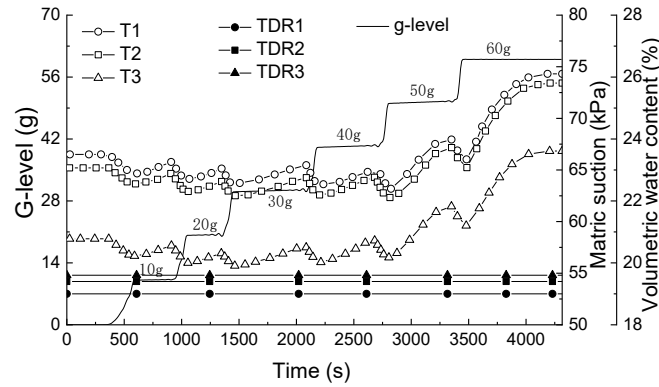


Figure 13. Measured water content and suction during centrifuge acceleration at 60g

Table 6. Measured suction and suction calculated by the Gardner equation under different g-levels

g-level	Measuring point	r (m)	Measured suction (kPa)			Suction from Gardner equation	Similarity judgment
			at 1g	at Ng	Difference (Ng-1g)		
20	at 5.5 cm	3.74	78.6	78.2	-0.4	31.1	Similar
	at 13.5 cm	3.82	68.8	68.4	-0.4	32.4	
	at 21.5 cm	3.9	65.3	64.7	-0.6	33.8	
40	at 5.5 cm	3.74	82.4	81.6	-0.8	62.2	
	at 13.5 cm	3.82	79.6	78.7	-0.9	64.9	
	at 21.5 cm	3.9	72.8	71.7	-1.1	67.6	
60	at 5.5 cm	3.74	66.5	74.3	7.8	93.3	Dissimilar
	at 13.5 cm	3.82	65.2	73.4	8.2	97.3	
	at 21.5 cm	3.9	58.3	66.8	8.5	101.4	

6. Conclusions

This study derived the theoretical scaling laws for rainfall infiltration based on the Green-Ampt model and the first similarity theorem. The derived scaling laws were validated using centrifuge model tests on rainfall infiltration in unsaturated soil foundations under different g-levels. The suction scaling law was evaluated using the Gardner equation. The main conclusions are as follows:

- (1) For the 20g/40g tests, the ratios of wetting-front migration velocity at different monitoring points were all 1/2.0. These ratios were consistent with the ratio of the g-levels. For the 20g/60g and 40g/60g tests, the ratios were also close to the corresponding g-level ratios. These results show that the scaling law for wetting-front migration velocity under different g-levels is 1:N. This agrees with the derived theoretical scaling law.
- (2) The ratio of saturated hydraulic conductivity in the wetted zone at different g-levels was equal to, or slightly smaller than, the corresponding g-level ratio. This trend was similar to that observed for the wetting-front migration velocity. The results indicate that the scaling law for the saturated hydraulic conductivity in the wetted zone is 1:N. This also agrees with the derived theoretical scaling law.
- (3) As the depth of the monitoring point increased, the ratio of wetting-front arrival time between different g-levels approached the inverse ratio of the g-levels. This result shows that the error in the wetting-front arrival-time ratio decreased with increasing infiltration depth and infiltration time. The scaling law for wetting-front arrival time t is therefore $N^2:1$. This result agrees with the derived theoretical scaling law.

- (4) At 20g and 40g, the suction values calculated using the Gardner equation were smaller than the measured suction values at all monitoring points. The maximum suction decreases were 0.6 kPa and 1.1 kPa, respectively. These changes were very small compared with the initial suction values at 1g. At 60g, the suction values calculated using the Gardner equation were larger than the measured suction values at all monitoring points. The measured suction increased by 7.8–8.5 kPa compared with the initial value. These results verify the effectiveness of the Gardner equation for evaluating suction similarity.
- (5) Although suction similarity was not fully satisfied at 60g, the ratios of wetting-front migration velocity at different g-levels still followed the theoretical scaling laws. This indicates that the suction increase of 7.8–8.5 kPa at 60g did not cause a significant error in the wetting-front migration velocity. Further research is needed to determine the allowable suction difference between Ng and 1g that does not cause significant errors in the scaling laws.

Author Contributions: Conceptualization, C.Z. and Y.W.; methodology, C.Z. and Y.W. and H.H.; validation, X.L. and J.H.; Formal analysis, X.L. and G.K.; Resources, C.Z. and Y.W.; Data curation, J.H. and G.K.; Writing-original draft, X.L. and J.H.; Writing-review & editing, C.Z. and Y.W. and H.H.; Supervision, C.Z. and Y.W.; Project administration, C.Z. and Y.W.; Funding acquisition, C.Z. and Y.W. All authors have read and agreed to the published version of the manuscript.

Funding: The work that is described in this paper has been supported by the grants awarded by the Yan'an Science and Technology Project (No. 2025SLJBZ-001) and the Yulin Science and Technology Project (No. 2025-CXY-077).

Data Availability Statement: The data that support the findings of this study are available from the corresponding author upon reasonable request.

Acknowledgments: I would like to declare on behalf of my co-authors that the work described was original research that has not been published previously, nor has it been published in whole or in part elsewhere. All the authors listed have approved the manuscript that is enclosed.

Conflicts of Interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

- [1] Centrifugal modeling of transient water flow: Cargill, K W; Ko, H Y J Geotech Engng Div ASCE, V109, NGT4, April 1983, P536–555[J]. International Journal of Rock Mechanics & Mining Sciences & Geomechanics Abstracts, 1983, 20(4):119.
- [2] Culligan P J, Barry D A, Parlange J Y. Scaling unstable infiltration in the vadose zone[J]. Canadian Geotechnical Journal, 1997, 34(3):466-470.
- [3] Rezzoug A, Konig D, Triantafyllidis T, et al. Scaling laws in centrifuge modelling for capillary rising in soils[J]. 2000.
- [4] Crancon P, Guy C, Pili E, et al. Modeling of capillary rise and water retention in centrifuge tests using time domain reflectometry[C]// Proceeding of the International Symposium on Physical Modelling and Testing in Environmental Geotechnics. 2000.
- [5] Thorel L, Noblet S, Garnier J, et al. Capillary rise and drainage flow through a centrifuged porous medium[C]//Proceeding of the International Symposium on Physical Modelling and Testing in Environmental Geotechnics. 2000: 251-258.
- [6] Dell'Avanzi E, Zornberg J G, Cabral A R. Suction profiles and scale factors for unsaturated flow under increased gravitational field[J]. Soils and foundations, 2004, 44(3): 79-89.
- [7] Gragnano C G, Pirone M, Giretti D, et al. Numerical investigations of a transient seepage process in the design and validation of a river embankment centrifuge test[M]//Physical Modelling in Geotechnics. Moonkyung Chung Sung-Ryul Kim Nam-Ryong Kim Tae-Hyuk Kwon Heon-Joon Park Seong-Bae Jo Jae-Hyun Kim, 2022: 924-927.
- [8] Li S, Xu Q, Tang M, et al. Centrifuge modeling and the analysis of ancient landslides subjected to reservoir water level fluctuation[J]. Sustainability, 2020, 12(5): 2092.
- [9] Ouzzine B, de Sauvage J, Madabhushi G, et al. Centrifuge modelling of an energy pile group with groundwater flow[J]. International Journal of Physical Modelling in Geotechnics, 2024, 24(3): 150-160.

- [10] Jayakody S H S, Uzuoka R, Ueda K, et al. Unsaturated slopes behavior under antecedent intermittent rainfall patterns: centrifuge and numerical study[J]. *Acta Geotechnica*, 2023, 18(11): 5773-5790. 533
534
- [11] Gardner, R.A. 1937. The method of measuring the capillary tension of soil moisture over a wide moisture range. *Soil Science*, 43: 277-283. 535
536
- [12] Gens A. Soil–environment interactions in geotechnical engineering[J]. *Géotechnique*, 2010, 60(1): 3-74. 537
- [13] Take W A, Bolton M D. A new device for the measurement of negative pore water pressures in centrifuge models[C]//International Conference on Physical Modelling in Geotechnics–ICPMG. 2002, 2: 89-94. 538
539
- [14] CHEN R, LIU J, Ng, et al. Development of a new assembled tensiometer[J]. *Rock and Soil Mechanics*, 2013, (10): 3028-3032. 540
- [15] Chen Y M, Wang H L, Chen R P, Chen Yun. A Newly Designed TDR Probe for Soils with High Electrical Conductivities[J]. *Geotechnical Testing Journal*, 2014, 37(1):1-10. 541
542
- [16] LIU S S, BAI M J, XU D, et al. Parameter simplification of Green-Ampt model and its relationship with soil physical parameters[J]. *Transactions of the Chinese Society of Agricultural Engineering*, 2012, 28(1): 106-110. 543
544
- [17] Lv T, Zhang J, Xue J F, et al. Study on determination method of permeability coefficient in Green-Ampt model[J]. *Rock and Soil Mechanics*, 2015(S1): 341-345. 545
546
- [18] Wang W Y, Wang Z R, Wang Q J, et al. Improvement and verification of Green-Ampt infiltration model in loess[J]. *Journal of Hydraulic Engineering*, 2003(5): 30-34. 547
548
- [19] Zhang G H, Shao M A, et al. Estimating suction parameter S_f in Green-Ampt infiltration model using soil physical properties[J]. *Acta Pedologica Sinica*, 2000, 37(4): 553-557. 549
550
- [20] Zhan L T, Hu Y T, Liu X C, et al. Centrifugal model test and multi-physics joint monitoring of rainfall infiltration in unsaturated loess foundation[J]. *Rock and Soil Mechanics*, 2019, 40(7): 2478-2486. 551
552
- [21] Chen Y M, Han C, Ling D S, et al. Development of ZJU400 centrifuge and performance evaluation of its shaking table[J]. *Chinese Journal of Geotechnical Engineering*, 2011, 33(12): 1887-1894. 553
554
- [22] Zhan L T, Liu X C, Tai P, et al. Centrifugal model tests on rainfall-induced silt slope instability and verification of rainfall intensity-duration warning curves[J]. *Chinese Journal of Geotechnical Engineering*, 2014, 36(10): 1784-1790. 555
556

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content. 557
558
559
560