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FROM CULTURAL PRACTICE TO FORMAL GEOMETRIC REASONING: A PRE-EMPIRICAL HYPOTHETICAL LEARNING TRAJECTORY THROUGH NIAS STONE-JUMPING

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ABSTRACT

Ethnomathematics research has demonstrated the mathematical richness of cultural artefacts; however, cultural contexts are often used primarily to identify or illustrate pre-existing mathematical concepts, leaving unclear how culturally situated knowledge contributes to the development of formal mathematical reasoning. This study examines how the culturally meaningful and material-spatial characteristics of the Nias stone-jumping tradition (Hombo Batu) can be progressively re-represented as formal geometry and theoretically organised through Van Hiele's levels of geometric thinking. A qualitative theoretical-interpretative design was employed, combining structured field observation, semi-structured interviews with three cultural informants, photographic documentation, and theoretical analysis. The analysis proceeded through cultural and structural coding, cultural-geometric interpretation, geometric re-representation, and theoretical mapping onto Van Hiele's framework. The findings indicate that the monument's wider base, narrower upper platform, inclined lateral surfaces, and customary proportions are related to its cultural function, structural stability, and inherited construction knowledge. Through cultural-geometric interpretation, these culturally situated and material-spatial characteristics were subsequently re-represented as an idealised rectangular truncated pyramid. Van Hiele's framework was then used to organise a conjectured progression from holistic visualisation and analysis to relational reasoning, formal deduction, and generalisation. This synthesis resulted in a pre-empirical hypothetical learning trajectory in which Hombo Batu functions as an epistemic resource whose role progressively changes as geometric reasoning becomes more formal. The study contributes a

conceptual account of cultural–geometric transformation that connects cultural understanding, mathematical interpretation, cognitive structuring, and formal generalisation without reducing culture to a contextual illustration. The proposed trajectory requires future empirical validation with learners.

Keywords: ethnomathematics; Hombo Batu; cultural-geometric interpretation; geometric reasoning; Van Hiele theory; hypothetical learning trajectory

INTRODUCTION

In recent decades, the integration of culture into mathematics education has gained increasing attention through the field of ethnomathematics. This perspective challenges the assumption that mathematical knowledge is culturally neutral by recognising that mathematical ideas and ways of reasoning emerge within social practices, artefacts, activities, and systems of knowledge developed by particular communities (Bishop, 1988; D’Ambrosio, 2006). From this perspective, cultural practices and artefacts can provide meaningful resources for mathematics learning because they connect formal mathematical ideas with forms of knowledge and experience situated in learners’ cultural environments (Kabuye Batiibwe, 2024; Rosa & Orey, 2011).

Despite these contributions, an important analytical challenge remains in ethnomathematics research. A substantial body of research has focused on identifying mathematical concepts that can be recognised in cultural artefacts, including geometric shapes, patterns, measurements, and numerical relationships (Deda et al., 2024; Pradana et al., 2022). Such identification is valuable for demonstrating the mathematical richness of cultural practices; however, identifying a formal mathematical concept in a cultural artefact does not by itself explain how the culturally situated characteristics of that artefact are related to the mathematical structure subsequently attributed to it. When an artefact is detached from its cultural function and immediately translated into conventional mathematical terminology, culture risks functioning primarily as a context in which an already established mathematical concept is located rather than as an epistemic resource contributing to mathematical meaning-making.

Addressing this limitation requires attention to the intermediate process through which culturally meaningful practices and material characteristics are interpreted before being re-represented as mathematical relationships. In this study, this intermediate process is conceptualised as cultural–geometric interpretation: the analytical process through which cultural meanings, functional purposes, inherited construction knowledge, and observable material-spatial characteristics are related before conventional geometric terminology is introduced. Cultural–geometric interpretation therefore does not assume that formal geometric categories already exist within community descriptions; rather, it establishes an empirically traceable basis from which researcher-generated mathematical re-representation can subsequently be developed.

This issue is particularly relevant to geometry education because geometric understanding involves progressive changes in how objects and spatial relationships are perceived, described, related, and formally reasoned about. International research has consistently documented students’ difficulties in moving beyond visual recognition toward conceptual and deductive understanding of geometric properties (Clements & Battista, 1992; Hock et al., 2015). These difficulties are closely associated with the hierarchical development of geometric thinking, in which learners progressively reorganise visual and spatial experiences into increasingly relational and formal systems of reasoning (Jones, 2003). Van Hiele’s theory provides a particularly relevant cognitive framework for examining such progression through the levels of visualisation, analysis, informal deduction, formal deduction, and rigour (Hiele, 1986). Because progression across these levels is shaped by instructional experience rather than simply by age (Crowley, 1987), the framework provides a theoretical means of examining how culturally situated spatial experience might be progressively reorganised into formal geometric reasoning.

To translate such a theoretically organised progression into a conjectured pathway for learning, the concept of a hypothetical learning trajectory (HLT) provides a complementary framework. Simon, (1995) conceptualised an HLT as comprising a learning goal, learning activities, and a hypothetical learning process through which learners' thinking is anticipated to develop. Importantly, a hypothetical trajectory represents a teacher's or researcher's conjecture rather than a predetermined or empirically established sequence of learning; it is expected to be examined and revised in relation to learners' actual responses. In the present study, this principle is adapted at a pre-empirical level: Van Hiele's levels organise the anticipated progression of geometric reasoning, while the culturally grounded analysis of Hombo Batu provides the basis for formulating learning goals, hypothetical activities, and anticipated forms of reasoning. The resulting trajectory is therefore proposed as a theoretically grounded conjecture requiring subsequent empirical investigation rather than as evidence of an observed learning progression.

The Nias stone-jumping tradition, locally known as Hombo Batu, provides a culturally situated case through which this relationship can be examined. Hombo Batu is not merely a stone object possessing a recognisable geometric appearance; it forms part of a cultural practice that has been maintained across generations and is associated with cultural meanings, inherited construction knowledge, and the practice of stone jumping. This perspective is consistent with ethnomathematical scholarship that views mathematical knowledge as embedded in cultural practices, ways of knowing, and socially situated activities rather than as knowledge detached from cultural experience (D'Ambrosio, 2006; Rosa & Orey, 2011). Its physical configuration including a broad lower structure, a narrower upper platform, and inclined lateral surfaces is therefore considered in this study in relation to its cultural function and construction rather than solely according to its resemblance to a conventional geometric solid. Such an approach is consistent with research emphasising that culturally situated artefacts, practices, and Indigenous knowledge can serve as meaningful resources through which mathematical ideas are interpreted and connected to learners' cultural worlds. Previous research has identified mathematical concepts associated with Hombo Batu (Sarumaha & Gee, 2021), but limited attention has been given to how culturally and functionally meaningful characteristics of the structure can provide an analytical basis for its progressive geometric re-representation.

Accordingly, the gap addressed in this study lies not simply in identifying additional geometric concepts within Hombo Batu, but in explaining the analytical and cognitive pathway between culturally situated characteristics and formal geometric reasoning. Specifically, it remains necessary to account for (1) how cultural meanings, functions, and construction characteristics are related to observable material-spatial features of the artefact; (2) how these features can be systematically re-represented as geometric relationships without treating the cultural context as analytically irrelevant; and (3) how the resulting relationships can be organised into a theoretically coherent progression of geometric thinking. This position is consistent with culturally responsive and culturally sustaining perspectives in education, which emphasise that learners' cultural knowledge, identities, experiences, and community practices should function as substantive resources for disciplinary learning rather than as peripheral additions to predetermined content (Gay, 2018; Paris & Alim, 2017). In mathematics education specifically, attention to cultural and community knowledge has also been associated with approaches that seek to connect mathematical activity with learners' lived experiences and ways of making sense of the world. Van Hiele's theory is employed here as an organising cognitive framework for this latter process, while ethnomathematical interpretation provides the basis for maintaining the connection between cultural practice and mathematical abstraction.

Therefore, this study aims to examine how the culturally meaningful and material-spatial characteristics of the Nias stone-jumping tradition can be progressively transformed into formal geometric reasoning through Van Hiele's theory of geometric thinking. Rather than empirically measuring students' learning outcomes, the study develops a theoretically grounded, pre-empirical hypothetical learning trajectory that traces a progression from culturally situated perception of Hombo Batu, through geometric analysis and relational reasoning, toward formal deduction and

generalisation. The conceptualisation of a hypothetical learning trajectory is grounded in the view that learning trajectories coordinate mathematical learning goals, learning activities, and conjectures about the development of learners' mathematical thinking (Clements & Sarama, 2004; Simon, 1995; Tzur & Simon, 2004). Accordingly, the pre-empirical designation used in this study distinguishes the theoretically conjectured trajectory from an empirically validated sequence of student learning. The study is guided by the following questions: (1) What cultural, functional, and material-spatial characteristics of Hombo Batu provide the basis for its geometric interpretation? (2) How can these culturally situated characteristics be systematically re-represented as geometric relationships? and (3) How can these relationships be organised through Van Hiele's levels into a hypothetical progression toward formal geometric reasoning? In doing so, the study seeks to contribute a conceptual account of cultural–geometric transformation in which culture is retained as an epistemic foundation of mathematical abstraction rather than functioning solely as a contextual illustration.

METHODS

Research Design

This study employed a qualitative theoretical–interpretative design combining empirical cultural inquiry with theoretical mathematical modelling. This design is consistent with qualitative inquiry in which meaning, practices, and context are examined interpretatively while maintaining close attention to participants' perspectives and situated empirical evidence (Creswell & Clark, 2018; TRacy, 2020). The qualitative component was used to investigate the cultural meanings, functions, inherited construction knowledge, and material-spatial characteristics associated with Hombo Batu, while the theoretical–interpretative component examined how these empirically grounded characteristics could be progressively re-represented in formal geometric terms and organised through Van Hiele's framework. The design was therefore not intended to constitute a full ethnography of the Nias community, nor to evaluate an instructional intervention. Rather, it focused on a bounded cultural artefact and its associated practices as the empirical basis for developing a theoretically conjectured pathway toward formal geometric reasoning.

The analysis maintained a distinction between the empirical and theoretical components of the study. Interview, observational, and photographic data were used to establish culturally situated meanings, functions, construction knowledge, and material-spatial relationships. Cultural–geometric interpretation subsequently connected these empirical characteristics before formal geometric terminology was introduced through researcher-generated geometric re-representation. Van Hiele's theory was then used as an organising cognitive framework for constructing a pre-empirical hypothetical learning trajectory (HLT). Because no learner data or classroom intervention were included, the proposed Van Hiele progression and HLT represent theoretical conjectures rather than observed learning outcomes.

Research Context and Cultural Artefact

The study was situated in Bawomataluo Village, South Nias, Indonesia, where Hombo Batu is maintained as part of the local stone-jumping tradition. The monument was examined not simply as a physical object displaying geometric characteristics but as a material component of a culturally situated practice. The observational framework therefore included both its physical-spatial characteristics and its position and use within the traditional practice.

Hombo Batu was selected as the focal cultural artefact because its material form, cultural function, and inherited construction characteristics provide an opportunity to investigate the transition between culturally situated spatial knowledge and formal geometric representation. The study did not assume a priori that the community conceptualised the monument using formal

geometric terminology. Instead, the formal geometric representation was treated as a researcher-generated abstraction developed from cultural, structural, and spatial evidence.

Cultural Informants and Participant Selection

Three adult cultural informants were purposively selected based on their roles and familiarity with Hombo Batu and the stone-jumping tradition. The selection was intended to obtain complementary perspectives from individuals associated with cultural knowledge, site preservation, and community continuity rather than to achieve statistical representativeness. The participants comprised R1, a 68-year-old traditional elder; R2, a 54-year-old cultural-site caretaker; and R3, a 61-year-old community elder. Participant identifiers R1, R2, and R3 were used throughout the analytical records and manuscript to preserve anonymity.

The three informants were treated as knowledgeable cultural sources whose accounts could provide evidence concerning the cultural meaning, historical and contemporary function, construction practices, and material characteristics of Hombo Batu. Their accounts were not used to infer the views of the Nias community as a whole. Rather, they were triangulated with structured field observation and photographic documentation to establish the culturally situated and material-spatial evidence required for subsequent cultural–geometric interpretation.

Semi-Structured Interview Protocol

The semi-structured interview protocol was organised into three interrelated domains. The first, cultural background, addressed the history, cultural meaning, preservation, and contemporary use of Hombo Batu. The second, physical construction, addressed traditional construction practices, the relationship between the wider base and narrower top, customary dimensions or proportions, and structural modifications. The third, geometric characteristics, elicited informants' descriptions of symmetry, the relationship between the upper and lower surfaces, the inclined lateral surfaces, and the possibility of representing the monument through a geometric form.

These domains were designed to distinguish between culturally situated accounts of the artefact and the subsequent mathematical interpretation undertaken by the researchers. Thus, interview statements concerning stability, customary proportions, cultural function, and construction were not treated as formal mathematical statements. Instead, they provided contextual and structural evidence against which the researchers' geometric abstraction could be examined.

Field Observation and Documentation

Field observation was guided by a structured observation sheet developed to document the physical characteristics of Hombo Batu relevant to subsequent geometric abstraction. The observation focused on the overall three-dimensional form of the monument; the shape of the lower and upper platforms; the number and form of lateral surfaces; visible edges and vertices; symmetry; the relationship between the upper and lower surfaces; inclination of the lateral surfaces; approximate proportions; cultural context; and photographic documentation.

The completed observation record indicated a wider rectangular base and a smaller rectangular upper platform, four inclined lateral surfaces, apparent symmetry between opposite sides, parallel upper and lower surfaces, approximately consistent lateral inclination, and a visibly larger base than the upper platform. The cultural-context record identified the monument as the obstacle used in the Hombo Batu stone-jumping tradition.

Photographs were used to document the monument from multiple viewing directions and its use within the stone-jumping practice. The photographic record therefore served two related purposes: preserving evidence of Hombo Batu within its cultural setting and supporting comparison of its material-spatial characteristics across different viewing perspectives. Photographic evidence

was subsequently triangulated with field notes and interview accounts during cultural–geometric analysis.

Data Analysis

Data analysis was conducted as a staged interpretative process designed to preserve an explicit analytical chain between cultural evidence and formal geometric representation. Rather than beginning with a predetermined geometric classification of Hombo Batu, the analysis proceeded from raw cultural and observational evidence toward progressively more formal interpretation. The qualitative analysis was conducted primarily by the first author, who was directly involved in the field observations, interviews, and documentation of Hombo Batu. Initial coding and interpretation were subsequently discussed and reviewed with the co-authors to examine the consistency of the interpretations with the empirical evidence and the conceptual focus of the study. Rather than seeking statistical intercoder agreement, this collaborative review focused on maintaining traceability between the original cultural and observational data, the emerging analytical interpretations, and the subsequent mathematical re-representations. Where interpretative questions arose, the analysis was reconsidered against the relevant interview transcripts, field notes, and photographic documentation.

Stage 1: Cultural and structural coding. Interview responses and observational records were first examined to identify recurring descriptions related to cultural meaning, cultural function, construction practices, structural characteristics, spatial characteristics, and customary proportional knowledge. Initial codes included, among others, wide base, narrow upper platform, inward-sloping sides, balanced opposite sides, similar orientation of the upper and lower surfaces, structural stability, functional design, and customary proportions. Coding at this stage remained close to the language of the empirical records and deliberately avoided imposing formal geometric classifications. The codes were subsequently grouped into broader analytical categories representing (a) cultural meaning and continuity, (b) cultural function, (c) inherited construction knowledge, (d) structural characteristics, and (e) material-spatial characteristics. These categories provided the empirical basis for the cultural–geometric interpretation undertaken in Stage 2.

Stage 2: Cultural–geometric interpretation. Codes and analytical categories generated in Stage 1 were compared across interview, observational, and photographic sources to identify relationships among cultural meanings, functional purposes, inherited construction knowledge, and observable material-spatial characteristics. Cultural–geometric interpretation refers here to the analytical process through which these culturally situated and material-spatial characteristics were related before conventional geometric terminology was formally introduced. For example, informants' accounts of structural stability were considered alongside observations of the wider supporting base, whereas descriptions of the monument's function as a jumping obstacle were examined in relation to the smaller upper platform and inward-inclining lateral surfaces. At this stage, these characteristics were retained in culturally and materially descriptive terms rather than being classified as formal geometric properties. The purpose was to establish an empirically traceable bridge between cultural evidence and subsequent researcher-generated mathematical abstraction.

Stage 3: Geometric re-representation. The culturally grounded material-spatial relationships established through cultural–geometric interpretation were subsequently re-represented using formal geometric terminology. The wider lower platform was represented as a rectangular base plane; the smaller upper platform as a smaller rectangular plane; the inward-inclining lateral surfaces as trapezoidal faces; and the corresponding orientation of the upper and lower surfaces as parallel planes. These relationships collectively provided the basis for constructing an idealised rectangular truncated-pyramid model of Hombo Batu. This representation was treated as a researcher-generated mathematical abstraction rather than as a formal geometric classification attributed to the cultural informants.

Stage 4: Theoretical mapping through Van Hiele's framework. After the cultural–geometric relationship had been established, the resulting geometric representation was theoretically mapped

onto Van Hiele's hierarchical levels. At Level 0 (visualisation), the artefact was considered as a holistic three-dimensional form. At Level 1 (analysis), attention shifted to faces, edges, vertices, and structural properties. At Level 2 (informal deduction), relationships such as parallelism and relationships among lateral faces were considered. At Level 3 (formal deduction), these relationships provided the basis for deductive reasoning about the surface-area structure. At Level 4 (toward rigour and generalisation), reasoning was conjectured to extend beyond the specific artefact toward increasingly abstract relationships among broader classes of truncated pyramids. The Van Hiele mapping was theoretical rather than empirical: the levels represent conjectured forms of geometric reasoning that could be supported through engagement with the artefact, not levels demonstrated by actual learners in this study.

Stage 5: Construction of the pre-empirical hypothetical learning trajectory. The cultural–geometric interpretation, geometric re-representation, and Van Hiele mapping were synthesised into a pre-empirical hypothetical learning trajectory (HLT). Drawing on the HLT framework (Simon, 1995), the trajectory was constructed around three interrelated components: learning goals, hypothetical learning activities, and anticipated forms of geometric reasoning. Because the present study did not involve learners, Simon's hypothetical learning process was operationalised here as anticipated forms of geometric reasoning rather than as observed cognitive development. For each stage, the analysis also specified the cultural or material reference through which the proposed activity could be situated and the changing epistemic role of Hombo Batu as reasoning became increasingly formal. Learning goals were formulated from the type of geometric reasoning theoretically associated with each Van Hiele stage; hypothetical activities were designed to connect these goals with the culturally grounded geometric representation; and anticipated reasoning described the forms of thinking that learners might exhibit when engaging with those activities.

Because no learner data were collected, neither the proposed activities nor the anticipated reasoning represent observed classroom processes or demonstrated learning outcomes. The trajectory is therefore designated pre-empirical: it constitutes a theoretically grounded conjecture about possible relationships among learning goals, culturally situated activities, and anticipated geometric reasoning that requires subsequent empirical investigation and revision through classroom-based research.

Across these analytical stages, cultural–geometric interpretation and geometric re-representation were treated as distinct but connected processes. The former established relationships among cultural meaning, function, construction knowledge, and material-spatial evidence, whereas the latter introduced formal mathematical terminology to model those relationships. This distinction was maintained throughout the analysis to avoid treating researcher-generated geometric categories as indigenous cultural classifications.

Trustworthiness and Theoretical Validity

Trustworthiness was addressed through data-source triangulation, collaborative interpretative review, analytical traceability, and repeated checking against the original empirical evidence. Interview accounts, structured observational records, and photographic documentation were compared to examine convergence and divergence in descriptions of the cultural meaning, function, construction characteristics, and material-spatial configuration of Hombo Batu. Initial coding and interpretation were conducted by the first author and subsequently discussed and reviewed with the co-authors. When questions or alternative interpretations emerged during this review, the relevant interview transcripts, field notes, and photographic records were revisited before the interpretation was retained or refined. This procedure was intended to strengthen interpretative consistency while preserving traceability to the original data rather than to establish statistical intercoder reliability.

Interpretative validity was further supported by maintaining an explicit distinction between emic cultural descriptions and etic mathematical re-representations. Informants' accounts were used to establish cultural meaning, functional purposes, inherited construction knowledge, customary

proportional knowledge, and relationships between function and physical configuration. Formal concepts such as rectangular planes, trapezoidal lateral faces, parallel planes, and the rectangular truncated pyramid were introduced only during the geometric re-representation stage and were treated as researcher-generated analytical constructs. Accordingly, these formal mathematical terms and classifications are not attributed to the cultural informants or presented as traditional mathematical terminology used by the community.

Analytical transparency was maintained through a cultural–geometric analysis matrix linking data source, empirical evidence, initial code, analytical category, cultural–geometric interpretation, and subsequent geometric re-representation. This matrix provided an audit trail showing how cultural, observational, and material evidence contributed to the development of the idealised geometric model. The same distinction between empirical evidence and theoretical interpretation was maintained during Van Hiele mapping: the Van Hiele levels were used to organise anticipated forms of geometric reasoning rather than to classify observed learners. Consequently, the resulting hypothetical learning trajectory is presented as pre-empirical and theoretically conjectured rather than as an empirically validated sequence of student learning.

RESULTS

Cultural Meaning, Function, and Construction of Hombo Batu

Analysis of the interview and observational data indicates that Hombo Batu is understood within the local cultural context not simply as a physical monument but as a material representation of cultural practice, identity, and continuity. Across the three cultural informants, the monument was consistently associated with inherited cultural traditions and with values attached to the historical practice of stone jumping. R1 described Hombo Batu as having been used across generations as part of the preparation of young men before adulthood, whereas R2 emphasised its status as one of the recognisable symbols of Bawomataluo Village. R3 similarly described the monument as an important representation of Nias cultural heritage whose original function has changed but whose cultural significance remains.

The cultural meanings attributed to the monument included courage, responsibility, discipline, respect for ancestral traditions, cultural identity, and intergenerational preservation. R1, for example, characterised Hombo Batu as symbolising “courage, responsibility, discipline, and readiness to contribute to the community,” while the other informants emphasised bravery, cultural identity, and the responsibility of younger generations to preserve Nias cultural heritage. These accounts position the monument within a continuing cultural practice rather than as an isolated material object.

Although the historical function of Hombo Batu has changed, its relationship with the practice of stone jumping remains visible. Informants reported that the monument continues to be used in demonstrations, cultural festivals, ceremonies, tourism activities, and educational contexts. Photographic documentation likewise records the monument both as a material structure and as an obstacle over which stone jumping is performed (Figure 1).



Figure 1. Hombo Batu within the cultural practice of stone jumping

The interview data further revealed that the physical configuration of Hombo Batu is connected with culturally inherited construction knowledge. Informants described the monument as being constructed from natural stones carefully selected and arranged by experienced builders using knowledge transmitted across generations. Particularly important for the subsequent geometric analysis was the relationship between the monument's function and its physical configuration. R1 explained:

"A wider base provides stability, while the narrower upper part creates the obstacle that participants must jump over."

R2 similarly associated the larger base with supporting the weight of the monument and preventing structural movement, while R3 characterised the configuration as both structurally stable and functionally suitable for stone jumping. These accounts indicate that the difference between the lower and upper parts of the monument is not treated analytically as an arbitrary geometric feature. Rather, the configuration is associated with structural stability and the cultural function of the monument as a jumping obstacle.

A further culturally relevant characteristic concerns proportion. Informants did not report formal written numerical standards governing the monument's dimensions. Instead, they described proportions as inherited from earlier generations and maintained through customary construction practices. R1 stated that builders followed proportions inherited from previous generations; R2 reported that these proportions were maintained when repairs were undertaken; and R3 explicitly distinguished customary proportions from formal measurement standards. Thus, proportional consistency constitutes part of the culturally transmitted construction knowledge surrounding the artefact, while its later expression through formal geometric measurement represents a subsequent mathematical re-representation.

Material-Spatial Characteristics of the Cultural Artefact

Structured field observation was used to examine how the culturally and functionally meaningful configuration described by informants was materially realised in the monument. Observation from multiple viewing directions showed a three-dimensional structure characterised by a broad lower section, a smaller upper platform, and four lateral surfaces inclined inward from the base toward the top. The lower and upper surfaces appeared rectangular and approximately parallel, while opposite lateral sides displayed a balanced or symmetrical configuration. These material-spatial characteristics did not themselves constitute formal geometric categories. Their mathematical significance was established through cultural-geometric interpretation, in which observational characteristics were considered together with informants' accounts of cultural function, structural stability, and inherited construction knowledge before formal geometric terminology was introduced.

Photographic documentation from the front, back, right, and left sides was used to compare these characteristics across viewing directions (Figure 2).



Figure 2. Material-spatial characteristics of Hombo Batu from four viewing directions: (a) front, (b) back, (c) right, and (d) left

The observational evidence was consistent with the interview accounts. Informants described the front and back as similar and the left and right sides as balanced; they also reported that the upper platform was positioned approximately parallel to the lower surface and that the lateral surfaces were intentionally inclined toward the smaller upper platform.

Importantly, these observations were initially retained as descriptions of the material-spatial configuration rather than immediately classified using formal geometric terminology. The broad lower platform, narrower upper platform, inward inclination, balanced opposite sides, and relative orientation of the upper and lower surfaces therefore constituted the empirical basis for the subsequent geometric re-representation.

From Cultural–Spatial Evidence to Geometric Re-representation

The cultural analysis matrix was used to trace how raw observational and interview evidence was progressively interpreted as formal geometric relationships. This analytical step distinguished the culturally situated and material descriptions of Hombo Batu from the mathematical terminology subsequently introduced by the researchers.

Table 1. Analytical pathway from cultural and observational evidence to geometric re-representation

Data source	Empirical evidence	Initial code	Analytical category	Cultural–geometric interpretation	Subsequent geometric re-representation
Interview R1 + observation	“A wider base provides stability...” + visibly wider lower section	Wide supporting base	Functional/structural relationship	Width difference is associated with culturally functional structural stability	Larger rectangular base plane
Interview R1–R3 + observation	Narrower upper section associated with jumping function	Narrow upper platform	Cultural function/material form	Upper narrowing relates physical configuration to its function as an obstacle	Smaller rectangular upper plane
Observation + photographs	Four sides incline inward toward the top	Inward-sloping sides	Material-spatial characteristic	Inclination connects lower and upper sections while preserving the monument's functional form	Four trapezoidal lateral faces
Interview	Proportions inherited	Customary	Inherited	Proportional relationship	Proportional

R1–R3	and maintained through customary practice	proportion	construction knowledge	represents transmitted construction knowledge rather than formal measurement rules	relationship between corresponding dimensions
Observation + photographs	Upper and lower surfaces maintain similar orientation	Similar surface orientation	Material-spatial relationship	Spatial organisation provides the basis for formal relational abstraction	Parallel rectangular planes

Note. The table summarises the cultural analysis matrix used to document the movement from raw evidence to researcher-generated geometric interpretation.

This analytical chain is supported by the original cultural analysis matrix, which records, for example, the movement from a “broad lower platform” through the code wide base to a rectangular base interpretation, and from four inward-inclining side surfaces through sloping sides to trapezoidal lateral faces. Interview evidence was used alongside observation rather than as independent evidence of formal geometry. For example, the statement that the wider base contributes to stability was coded as a construction principle and used to contextualise the larger supporting base rather than to attribute the mathematical concept of a rectangular plane to the informant.

Taken together, the cultural, observational, and photographic evidence provided the basis for representing the monument as an idealised rectangular truncated pyramid. This representation constitutes an etic, researcher-generated mathematical model rather than an emic cultural classification. Indeed, one informant explicitly noted that the monument was not traditionally described using mathematical terminology, although its form could be represented as a three-dimensional object with a broad base, smaller upper surface, and sloping sides. Figure 3 therefore represents a deliberate transition between two modes of representation: the culturally situated material artefact and its idealised geometric model.

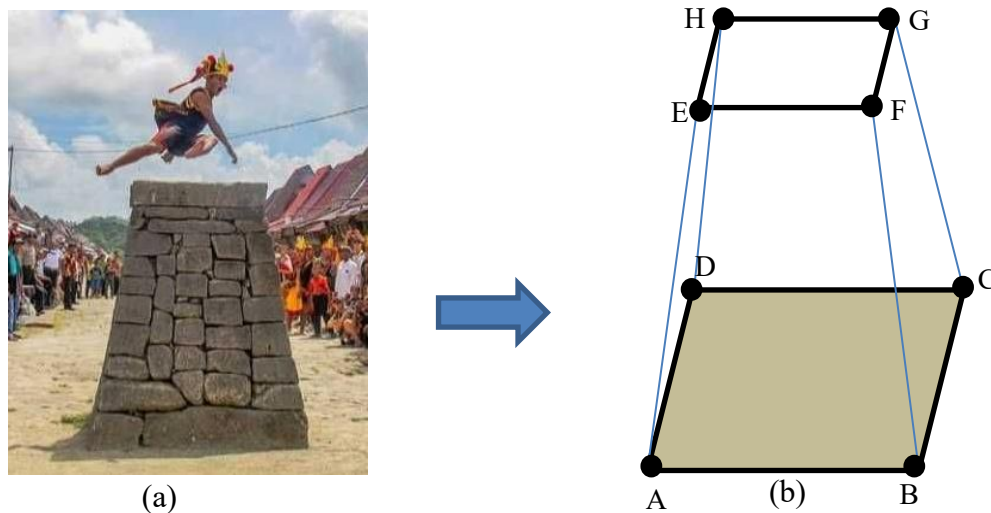


Figure 3. Mathematical re-representation of Hombo Batu: (a) cultural artefact and (b) idealised rectangular truncated-pyramid model

Within this model, the monument can be represented by eight vertices (A–H), twelve edges, and six faces. The upper and lower surfaces are represented as two parallel rectangular planes of different dimensions, while the four inclined lateral surfaces are represented as trapezoidal faces. This abstraction provides the formal geometric structure required for the subsequent Van Hiele analysis while retaining an explicit analytical connection to the cultural and material evidence from which it was generated.

Cultural–Geometric Progression through Van Hiele’s Levels

Following geometric re-representation, the culturally grounded spatial characteristics of Hombo Batu were theoretically organised through Van Hiele’s levels of geometric thinking. The mapping does not represent observed learner performance. Rather, it identifies forms of geometric reasoning that could theoretically be elicited when the artefact and its geometric representation are used as successive epistemic references.

Level 0: Visualisation

At the visualisation level, Hombo Batu is encountered as a culturally situated three-dimensional whole. Attention is initially directed to its overall appearance and its recognisable role within the stone-jumping practice rather than to formal geometric properties. The learner can distinguish the broad lower section, smaller upper section, and inclined appearance of the monument without yet identifying these characteristics through formal terminology.

The cultural artefact remains particularly important at this stage because visual recognition begins with the physical monument rather than with a pre-given textbook representation of a truncated pyramid. In the theoretical trajectory, the first geometric meaning therefore emerges from engagement with the culturally familiar or culturally contextualised whole.

Level 1: Analysis

At the analysis level, attention shifts from holistic recognition to examination of individual material-spatial characteristics. The broad base, smaller upper platform, inclined sides, corners, and boundaries can be re-represented as geometric components. Within the idealised model, these correspond to eight vertices (A, B, C, D, E, F, G, and H), twelve edges (AB, AD, BC, CD, AE, BF, CG, DH, EF, EH, FG, and GH), and six faces: front (ABEF), back (CDGH), left (ADEH), right (BCFG), top (EFGH), and bottom (ABCD) (Figure 3b).

At this stage, the culturally situated structure functions as a reference against which the abstract components can be interpreted. The aim is therefore not simply to label geometric elements but to recognise how formal geometric components are produced through re-representation of observable characteristics of Hombo Batu.

Level 2: Informal Deduction

At the informal deduction level, reasoning moves from individual components toward relationships among them. The upper and lower surfaces can be recognised as parallel rectangular planes of different dimensions, while opposite lateral surfaces can be compared in terms of their corresponding forms and spatial arrangement. These relationships allow the artefact to be understood as a coherent geometric structure rather than as an assemblage of independent faces and edges. The net of the idealised structure makes these relationships particularly visible (Figure 4).

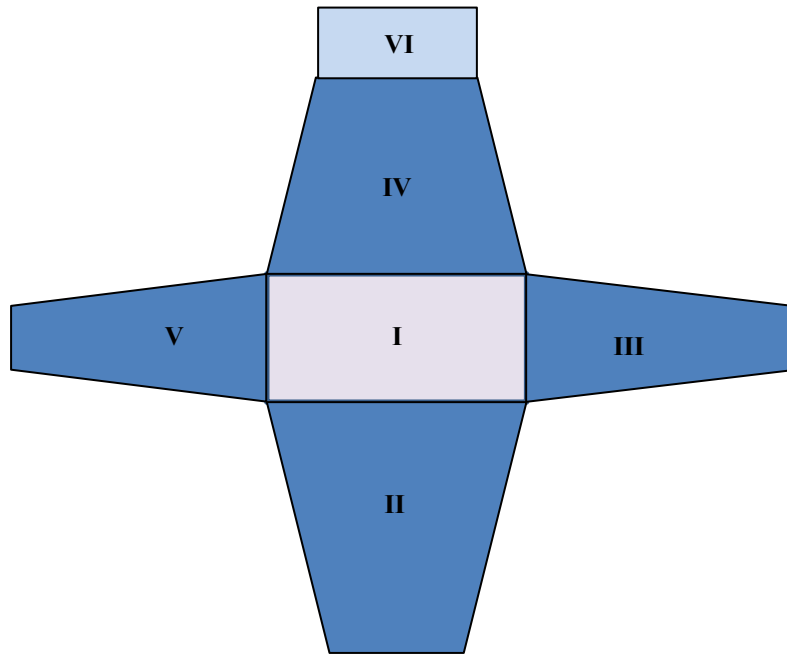


Figure 4. Net of the idealised rectangular truncated-pyramid representation of Hombo Batu

In the net, faces II and IV and faces III and V form corresponding pairs of lateral surfaces, while faces I and VI represent the lower and upper rectangular planes. The transition from the physical monument to its net therefore constitutes a further representational transformation: material-spatial relationships observed in Hombo Batu are reorganised into a two-dimensional configuration through which relationships among the constituent faces can be examined.

Level 3: Formal Deduction

At the formal deduction level, relationships established at the preceding levels provide a basis for deductive reasoning about the surface structure of the idealised model. Because the rectangular truncated pyramid can be decomposed into its constituent faces, its total surface area can be expressed as the sum of the areas of the lower base, upper plane, and four lateral faces. For the rectangular-base model represented in Figure 4:

$$SA = A_{Base} + A_{Top} + 2A_{Lateral\ Pair\ 1} + 2A_{Lateral\ Pair\ 2}$$

The important theoretical transition at this level is not simply the production of a surface-area formula. Rather, the formula follows from relationships progressively established through the earlier stages: the artefact is first recognised as a whole, decomposed into components, and then reorganised according to relationships among its constituent surfaces. Accordingly, the mathematical derivation remains analytically traceable to the cultural artefact even though the reasoning itself has become increasingly formal.

Level 4: Toward Rigour and Generalisation Beyond the Artefact

At the highest conjectured stage of the trajectory, reasoning moves toward the abstraction associated with Van Hiele's rigour level by deliberately extending beyond the particular configuration of Hombo Batu. The rectangular truncated-pyramid representation is treated as one instance within a broader class of truncated pyramids formed by two parallel polygonal planes connected by lateral faces. Learners could compare this representation with triangular-, square-, pentagonal-, and more generally n -gonal truncated pyramids to examine which structural relationships remain invariant and which depend on the form of the bases.

For an idealised case in which equivalent lateral faces are assumed, the structural relationship may be expressed as:

$$SA = A_{Base} + A_{Top} + nA_{Lateral\ Face}$$

This activity is not claimed to constitute full empirical attainment of Van Hiele's rigour level. Rather, it represents a conjectured movement toward greater abstraction and generalisation beyond the culturally derived case. In the proposed trajectory, generalisation therefore occurs after the cultural artefact has supported visual recognition, analysis, relational reasoning, and deduction, rather than through its immediate removal from context.

Construction of a Pre-Empirical Hypothetical Learning Trajectory

The preceding analysis was synthesised into a pre-empirical HLT describing a theoretically conjectured progression from culturally situated experience toward formal geometric abstraction. The trajectory does not claim that learners in the present study actually progressed through these stages; rather, it identifies anticipated forms of activity and reasoning that can subsequently be examined through classroom-based research. The synthesis of the cultural–geometric analysis and Van Hiele mapping was used to formulate a pre-empirical HLT comprising learning goals, hypothetical learning activities, and anticipated forms of geometric reasoning. The trajectory also specifies the cultural/material reference available at each stage and the changing epistemic role of Hombo Batu. These components should be interpreted as theoretically coordinated conjectures rather than as an empirically observed sequence of learning. Table 2 summarises this proposed trajectory.

Table 2. Pre-empirical hypothetical learning trajectory from culturally situated experience to formal geometric reasoning

Van Hiele stage	Learning goal	Cultural/material reference	Hypothetical learning activity	Anticipated geometric reasoning	Epistemic role of Hombo Batu
Level 0: Visualisation	Recognise Hombo Batu as a three-dimensional whole and describe its overall spatial configuration without relying on formal properties	Hombo Batu within the stone-jumping practice; photographs of the monument	Observe the monument or photographs and describe its cultural use and overall appearance, including the broad lower section, narrower upper section, and inclined sides	Holistic recognition of a three-dimensional form based primarily on overall appearance rather than explicit properties	Primary experiential reference: cultural meaning and physical form provide the initial basis for spatial recognition
Level 1: Analysis	Identify and describe constituent spatial and geometric characteristics of the idealised representation	Physical artefact, photographs, and idealised geometric model	Compare the artefact with its idealised model; identify surfaces, boundaries, corners, upper and lower sections, and lateral components	Recognition of individual properties and components, subsequently expressed as faces, edges, vertices, rectangular surfaces, and trapezoidal lateral faces	Source for geometric decomposition: material characteristics remain traceable as formal components are introduced
Level 2: Informal Deduction	Establish relationships among the geometric components	Idealised model, photographs, and geometric net	Compare upper and lower surfaces and opposite lateral faces; construct or	Recognition and coordination of relationships such as parallelism, correspondence	Reference for relational reasoning: observed structural relationships provide a basis for

Van Hiele stage	Learning goal	Cultural/material reference	Hypothetical learning activity	Anticipated geometric reasoning	Epistemic role of Hombo Batu
	derived from the artefact		examine the net; discuss relationships among constituent surfaces	among lateral faces, and structural relationships within the solid	coordinating formal properties
Level 3: Formal Deduction	Use established geometric relationships to construct deductive reasoning about the surface structure of the idealised model	Idealised rectangular truncated-pyramid model and its net	Decompose the model into constituent faces and reason from their relationships to formulate the total surface-area structure	Deductive coordination of base, top, and lateral face areas into a justified surface-area relationship	Cultural origin retained, formal model foregrounded: reasoning increasingly operates through the mathematical representation derived from the artefact
Level 4: Toward Rigour and Generalisation	Extend reasoning beyond the particular Hombo Batu model toward more abstract classes of truncated pyramids	Hombo Batu-derived model compared with other truncated-pyramid representations	Compare truncated pyramids with triangular, rectangular, pentagonal, and more general polygonal bases; examine which structural relationships remain invariant	Movement toward abstraction and generalisation of structural and surface-area relationships beyond the original case	Deliberately transcended through abstraction: Hombo Batu becomes the originating case from which reasoning extends to broader mathematical structures

This trajectory indicates a progressive change in the epistemic role of the cultural artefact. At the initial levels, Hombo Batu functions as the principal experiential and spatial reference. At the analytical and relational levels, it becomes a source for identifying and coordinating geometric relationships. At formal deduction, the idealised representation increasingly mediates reasoning, while at rigour the particular artefact is deliberately transcended through mathematical generalisation. The Van Hiele mapping matrix supports this theoretical progression from holistic recognition, through identification of properties and relationships, to deductive reasoning and generalisation.

Conceptual Model of Cultural–Geometric Transformation

Synthesising the empirical and theoretical analyses, the study proposes a conceptual model in which formal geometric understanding is not produced through an immediate extraction of geometry from a cultural object. Instead, the transformation proceeds through a sequence of analytically distinguishable stages:

- Cultural meaning and practice
 - culturally functional construction
 - material-spatial characteristics
 - cultural–geometric interpretation
 - geometric re-representation
 - Van Hiele cognitive structuring
 - pre-empirical HLT
 - formal geometric reasoning and generalisation.

In this model, the transition from culture to mathematics is mediated by interpretation rather than by direct equivalence. Cultural accounts establish the meanings, functions, and construction knowledge associated with Hombo Batu; observation and photographic documentation establish its material-spatial characteristics; researcher-generated mathematical re-representation formalises these characteristics; and Van Hiele's theory organises the anticipated cognitive progression toward increasingly formal reasoning.

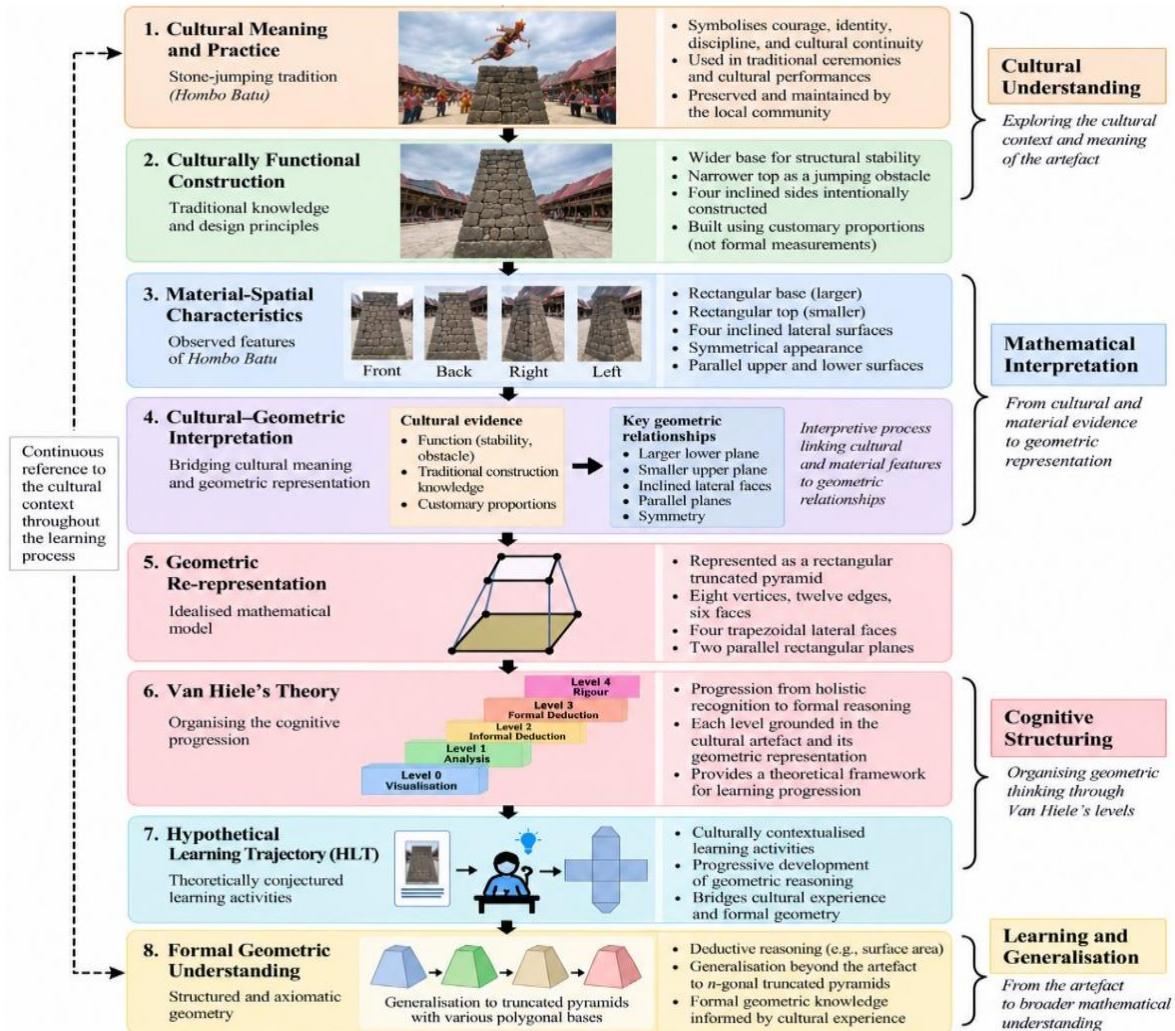


Figure 5. Conceptual model of culturally grounded transformation toward formal geometric reasoning

This model distinguishes between the cultural origin of the mathematical representation and the formal mathematical reasoning subsequently developed from that representation. Consequently, the artefact is neither treated as merely decorative context nor assumed to contain formal school geometry in an unmediated form. Instead, culturally situated knowledge and material-spatial characteristics provide the epistemic basis for a sequence of re-representations through which formal geometry can theoretically be developed.

DISCUSSION

The findings of this study suggest that the mathematical educational potential of Hombo Batu lies not merely in its visual resemblance to a conventional geometric solid, but in the analytically traceable relationship between cultural practice, inherited construction knowledge, material-spatial characteristics, and their subsequent geometric re-representation. The wider base, narrower upper section, inclined lateral surfaces, and customary proportions were associated with

the monument's structural stability, functional role in stone jumping, and construction knowledge transmitted across generations. Thus, the spatial characteristics subsequently interpreted geometrically were not approached as isolated physical features but were first understood in relation to the cultural practices and functions through which the artefact exists.

This finding shifts the emphasis from merely identifying school mathematics in a cultural object toward explaining how culturally situated knowledge can provide a basis for mathematical interpretation. This position is consistent with recent ethnomathematical research showing that mathematical ideas can be explored through culturally embedded artefacts and practices, such as the Batak Toba tandok (Marbun et al., 2026) and Minangkabau Marosok trading (Nurjanah et al., 2021). More broadly, Rawat & Nagaraju, (2026) identify cultural artefacts, Indigenous knowledge, pedagogical applications, and culturally responsive approaches as prominent themes in contemporary ethnomathematics research. The present study extends this literature by making explicit the interpretative process between cultural evidence and formal geometry rather than moving directly from artefact recognition to mathematical classification.

Central to this process is cultural–geometric interpretation, which functions as an analytical bridge between culturally situated knowledge and formal geometric representation. The proposed transformation proceeds through cultural meaning and practice → culturally functional construction → material-spatial characteristics → cultural–geometric interpretation → geometric re-representation. Hombo Batu was therefore not immediately classified as a rectangular truncated pyramid. Cultural accounts, structured observation, and photographic evidence first established relationships among cultural function, construction knowledge, and observable spatial characteristics; only subsequently were these characteristics re-represented through concepts such as rectangular planes, trapezoidal lateral faces, parallelism, and ultimately an idealised rectangular truncated pyramid. Importantly, this representation is an etic mathematical interpretation grounded in emic cultural and material evidence, rather than a claim that the community traditionally conceptualises Hombo Batu through formal geometric terminology. Its analytical function is therefore not to translate cultural knowledge directly into school mathematics, but to make explicit why particular material-spatial characteristics become relevant to subsequent mathematical abstraction.

This distinction is consistent with recent culturally responsive and situated mathematics research. Turner et al., (2026) show that culturally responsive mathematical modelling can incorporate learners' identities and out-of-school resources into mathematical activity, while Jung et al., (2026) emphasise the need for substantive connections between mathematical tasks and learners' cultural and social contexts. In geometry specifically, Apsari and Abrahamson (2026) demonstrate how mathematical meaning can emerge through situated interactions among activity, signs, and tools, while da-Silva-Branco et al., (2026) illustrates how mathematical cognition can be embedded within a living cultural practice. Although these studies employ theoretical perspectives different from the present study, they support the argument that culture can participate in mathematical meaning-making rather than functioning merely as an external or motivational context.

Accordingly, Hombo Batu is conceptualised in this study as an epistemic resource whose role changes as geometric reasoning becomes increasingly formal. At the earlier Van Hiele levels, the cultural artefact provides the experiential and spatial reference for recognising and analysing its material structure. At the relational level, its structural characteristics support reasoning about relationships among surfaces and other geometric components. At the deductive level, the idealised geometric representation increasingly mediates reasoning, and at the rigour level the particular artefact is deliberately transcended through generalisation to broader classes of truncated pyramids. Culture therefore does not disappear immediately after abstraction; rather, its epistemic role gradually shifts as formal mathematical reasoning develops.

This progression is compatible with research on geometric learning progressions and Van Hiele-based instruction. Seah & Horne, (2021, 2026) emphasise the progressive development of geometric reasoning from less formal ways of recognising spatial characteristics toward

increasingly coordinated and sophisticated reasoning. Similarly, Baranović & Batarelo Kokić, (2026) and Ndungo et al., (2026) demonstrate the continuing relevance of Van Hiele-informed instruction for organising the development of geometric thinking. The present study, however, differs from these empirical investigations because no learners were assessed or classified according to Van Hiele levels. Here, Van Hiele's theory functions as an organising cognitive framework applied after the culturally grounded geometric representation has been established. It neither explains the cultural meaning of Hombo Batu nor directly converts cultural knowledge into mathematics; rather, it structures the forms of geometric reasoning that could theoretically develop from the resulting representation.

The synthesis of these processes produces a pre-empirical hypothetical learning trajectory in which learning goals, proposed activities, and anticipated forms of geometric reasoning are theoretically coordinated across the Van Hiele stages. Unlike a descriptive learning progression, the trajectory specifies not only increasingly formal forms of reasoning but also the culturally situated activities through which such reasoning is conjectured to develop. It begins with engagement with Hombo Batu rather than a pre-given textbook solid and progresses through material-spatial noticing, cultural-geometric interpretation, geometric re-representation, relational and deductive reasoning, and formal generalisation. The designation pre-empirical is essential because the trajectory represents a theoretically grounded conjecture rather than an observed sequence of student learning. This distinction is consistent with research demonstrating that geometric learning progressions depend on learners' actual engagement with structured experiences and therefore require empirical investigation (Baranović & Batarelo Kokić, 2026; Seah & Horne, 2026).

Taken together, the principal theoretical contribution of this study is therefore not the identification of a truncated pyramid within Hombo Batu, but an account of how culturally meaningful characteristics can be systematically interpreted, mathematically re-represented, and cognitively organised toward formal geometric reasoning. The resulting model integrates four interconnected layers: Cultural Understanding, Mathematical Interpretation, Cognitive Structuring, and Learning and Generalisation. Within these layers, cultural-geometric interpretation occupies a critical mediating position, preventing the transition from culture to formal mathematics from being treated as immediate or self-evident. In this sense, the model positions culture as an epistemic foundation for mathematical abstraction rather than merely as a contextual illustration.

These conclusions should nevertheless be interpreted within the scope of the study. The cultural analysis concerns one artefact in a specific cultural setting and draws on three cultural informants; therefore, it does not represent the entirety of Nias cultural or mathematical knowledge. Moreover, the rectangular truncated-pyramid representation is an idealised researcher-generated model of a materially irregular stone structure. Most importantly, the proposed learning trajectory has not yet been empirically tested with learners. Future classroom research should therefore examine whether students actually establish the proposed relationships among cultural meaning, material-spatial characteristics, geometric representation, and formal reasoning, and whether their progression corresponds to or diverges from the hypothesised Van Hiele trajectory. Comparative studies involving other cultural artefacts would further help determine which components of the proposed cultural-geometric transformation are specific to Hombo Batu and which may be transferable to culturally grounded geometry education.

CONCLUSION

This study provides a theoretically grounded account of how the culturally situated characteristics of the Nias stone-jumping tradition (Hombo Batu) can be progressively re-represented and organised toward formal geometric reasoning. Rather than treating Hombo Batu merely as a cultural illustration of a pre-existing geometric concept, the analysis establishes an explicit pathway from cultural meaning and practice, culturally functional construction, and material-spatial characteristics to cultural-geometric interpretation and formal geometric re-representation. Through this process, the idealised representation of Hombo Batu as a rectangular

truncated pyramid is understood as a researcher-generated mathematical model grounded in cultural, observational, and material evidence, rather than as a formal mathematical classification attributed to the cultural community itself.

Building on this culturally grounded representation, Van Hiele's theory provides a cognitive framework for organising a conjectured progression from holistic recognition and analysis of geometric properties to relational reasoning, formal deduction, and generalisation. The resulting pre-empirical hypothetical learning trajectory coordinates learning goals, hypothetical activities, and anticipated forms of reasoning while positioning Hombo Batu as an epistemic resource whose role progressively changes as mathematical reasoning becomes more formal. The principal contribution of the study therefore lies not in identifying geometric concepts within a cultural artefact, but in proposing a conceptual mechanism through which cultural understanding, mathematical interpretation, cognitive structuring, and formal generalisation can be analytically connected. In particular, cultural-geometric interpretation serves as the mediating analytical process through which culturally situated meanings, functions, and material-spatial characteristics remain traceable within the subsequent process of mathematical abstraction.

Because the proposed learning trajectory has not yet been empirically examined with learners, it should be regarded as a theoretically grounded conjecture rather than evidence of actual cognitive progression. Future research should therefore implement and refine the trajectory in classroom settings, examining how students negotiate the transition from cultural meaning and material-spatial experience to geometric representation, deduction, and generalisation. Such investigation would provide empirical evidence for evaluating the proposed model and determining its potential applicability to other cultural artefacts and culturally grounded contexts of geometry learning.

AUTHOR CONTRIBUTIONS

Efrata Gee: Conceptualised the study, developed the theoretical framework integrating ethnomathematics and Van Hiele's theory, conducted data collection and analysis, and drafted the manuscript; **Turmudi:** Provided academic supervision, contributed to the refinement of the research design, and reviewed the manuscript for theoretical consistency; **Nanang Priatna:** Contributed to methodological guidance, particularly in qualitative research and cultural interpretation, and provided critical feedback on data interpretation; **Bambang Avip Priatna:** Supported the development of the mathematical and didactic analysis and reviewed the manuscript to ensure conceptual clarity and academic rigor.

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ETHICAL STATEMENT

This study involved non-invasive qualitative research with adult cultural informants. Prior to data collection, participants were informed about the purpose of the study and the use of interview and documentary data for research purposes. Informed consent was obtained from all participants before the interviews were conducted. Participant identities were anonymised in the research records and manuscript using the identifiers R1, R2, and R3. The research was conducted with respect for local cultural knowledge, practices, and heritage associated with Hombo Batu.

DATA AVAILABILITY DECLARATION

The data supporting the findings of this study, including anonymised interview transcripts, field observation records, and supporting documentation, are publicly available in the Zenodo repository at <https://doi.org/10.5281/zenodo.21505312>.

DECLARATION OF INTEREST

The authors declare that there is no conflict of interest regarding the content of this publication.

DATA SHARING STATEMENT

Data supporting the findings and conclusions are available upon request from the corresponding author.

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