

Global Benford Deviation Clustering (GBDC): A Spatial Framework for Unsupervised Radiometric Anomaly Detection in Multispectral Imagery

Shaho Piroti

pirotushaho@gmail.com

Kharazmi University

Parviz Zeaieanfiroozabadi

Kharazmi University

Saman Ghafari

Tehran Polytechnic

Research Article

Keywords: Remote Sensing, Benford's Law, Unsupervised Anomaly Detection, Statistical Filtering

Posted Date: September 8th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-10945153/v1>

License: © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License. [Read Full License](#)

Additional Declarations: No competing interests reported.

Abstract

Unsupervised anomaly detection in Earth observation data remains constrained by the reliance on labeled training samples and site-specific threshold calibration. This study investigates the empirical feasibility of Benford's Law, a statistical principle describing the logarithmic distribution of leading digits in natural datasets, as a generalized, threshold-free radiometric filter. We introduce the Global Benford Deviation Clustering (GBDC) framework, which projects a global first-digit deviation signal back onto each pixel via its leading digit, converting an image-wide Benford statistic into a per-pixel anomaly classification, whereas conventional Benford-based approaches terminate at a single global statistic with no per-pixel decision rule. By mapping the mathematical collapse of the theoretical first-digit distribution directly onto the spatial grid, GBDC enables interpretable anomaly localization without prior training or manual parameterization. Using the CloudSEN12 dataset as a standardized case study for dense meteorological anomalies, the framework was evaluated across 500 heterogeneous spatial contexts and five Sentinel-2 spectral bands, yielding a baseline mean F1-score of 0.347. These empirical results demonstrate that projecting Benford deviations into the spatial domain provides a fully automated, training-free heuristic for spatial anomaly screening in multispectral archives.

1. Introduction

Optical Earth observation systems continuously capture the radiometric heterogeneity of the planet's surface. A fundamental requirement in processing these extensive data archives is the accurate identification of radiometric anomalies, transient physical phenomena such as active wildfire fronts, heavy aerosol plumes, snow cover, or dense meteorological formations that locally saturate and mask the underlying terrestrial signal. Detecting these anomalies is critical for downstream land cover analysis, atmospheric correction, and dynamic environmental monitoring. However, automating this extraction across diverse global biomes and uncalibrated top-of-atmosphere (TOA) data remains mathematically and computationally demanding. Conventional anomaly detection architectures rely predominantly on supervised machine learning or physical rule-based thresholding algorithms. While supervised models achieve high precision for specific targets, their operational scalability is heavily constrained by the necessity for exhaustive, pixel-level annotated training datasets, which are labor-intensive to generate. Conversely, physical thresholding requires rigorous atmospheric correction and site-specific calibration, making such approaches brittle when applied to unfamiliar landscapes. Consequently, a structural need exists for threshold-free, unsupervised statistical filters capable of isolating anomalous radiometric behavior without prior target-specific knowledge or labeled samples.

Benford's Law, or the First-Digit Law, establishes an empirical statistical baseline for unconstrained, naturally occurring datasets. It posits that the leading digits in such data are not uniformly distributed but follow a specific logarithmic probability curve. This statistical phenomenon has been established as a robust, unsupervised indicator of structural anomalies across diverse scientific domains; Sambridge et al. (2010) demonstrated its validity across 15 datasets from physics, astronomy, and geophysics, establishing it as a widespread baseline in the natural sciences. In geophysics, it has been successfully applied for earthquake detection (Das et al. 2026) and debris flow identification from seismic signals (Zhou et al. 2024), proving its utility as a computationally inexpensive, training-free anomaly detection tool. Its applications extend to forensic accounting for fraud detection (Nigrini 2012; Durtschi et al. 2004) and signal processing (Cumbrera-Conde et al. 2024). Furthermore, in the domain of digital image processing, early investigations by Jolion (2001) linked Benford's Law to natural image statistics, while subsequent studies such as Fu et al. (2007) leveraged these principles to detect tampered or manipulated images, providing a foundational bridge to visual data analysis. Within the remote sensing literature, the deployment of Benford's Law has been highly restricted. Recent applications have primarily focused on no-reference image quality assessment. For instance, Hao et al. (2024) utilized Benford's Law to detect fusion distortions in hyperspectral imagery without reference data. However, these conventional Benford-based approaches including the methodology proposed by Hao et al. (2024) merely report a singular global statistical metric or an image-wide quality index. The fundamental gap lies in the absence of a framework that structurally translates these global statistical deviations into spatially explicit, pixel-level anomaly maps. The use of Benford's Law as a generalized spatial filter for detecting and localizing physical, on-ground radiometric anomalies in multispectral satellite imagery has remained largely unexplored.

Bridging this gap requires transitioning Benford's Law from a passive global metric into an active, spatially explicit tool for radiometric anomaly extraction.

To address this methodological gap, this study presents an empirical feasibility analysis of Benford's Law as a fully automated, threshold-free spatial anomaly detector. We introduce the GBDC framework, an architecture designed to flag anomalous pixels by quantifying their deviation from the theoretical first-digit distribution and mapping this mathematical collapse directly onto the spatial grid. To ensure this spatial translation is entirely data-driven and threshold-free, the framework integrates a weighted statistical formulation to autonomously extract the anomaly footprint. To rigorously evaluate the algorithmic mechanics, we utilize the Sentinel-2 CloudSEN12 dataset strictly as a standardized case study for dense radiometric anomalies (clouds). Through a comprehensive spatial-spectral ablation study across 500 heterogeneous contexts, this paper evaluates the theoretical limits, spectral dependencies, and sensitivity of the algorithm to radiometric manipulation, assessing its viability as a general-purpose, training-free spatial screening tool. The remainder of this paper is structured as follows. Section 2 reviews the related literature on statistical anomaly detection and established applications of Benford's Law. Section 3 details the theoretical foundation of the proposed GBDC architecture and the experimental configuration of the case study. Section 4 presents the quantitative results of the spatial-spectral ablation study. Section 5 provides a physical discussion of the findings, including the impact of radiometric manipulations on the weighted Benford distribution and inherent methodological limitations. Finally, Section 6 concludes the study and outlines necessary directions for future research.

2. Related Work

2.1. Benford's Law in the Natural Sciences and Signal Processing

The application of Benford's Law has expanded significantly beyond its origins in forensic accounting and socioeconomic auditing. In the natural sciences, it serves as a robust statistical baseline for unconstrained numerical datasets. Sambridge et al. (2010) established this foundation by demonstrating the compliance of 15 diverse scientific datasets ranging from geomagnetic reversals to seismic velocities with the theoretical first-digit distribution, thereby confirming Benford's Law as a widespread phenomenon in physical systems. Recent advancements in geophysics have leveraged this baseline to identify structural disruptions caused by physical anomalies. For instance, Das et al. (2026) utilized Benford's Law to extract earthquake signatures from background noise, validating it as a training-free and computationally inexpensive heuristic for seismic signal analysis. Similarly, Zhou et al. (2024) applied the first-digit distribution to detect debris flow events from continuous seismic records, achieving detection capacities comparable to supervised models while relying on a fraction of the computational overhead. In the broader context of numerical series, Cumbreira-Conde et al. (2024) formulated anomaly detection frameworks based on signal processing theories integrated with Benford deviations. Collectively, these studies validate that severe physical perturbations induce a measurable mathematical collapse in natural variance, positioning Benford's Law as a highly effective, unsupervised screening mechanism.

2.2. Benford's Law in Digital Image Analysis

In the domain of digital image processing, the statistical mechanics of Benford's Law have been progressively adapted to analyze visual data structures. Jolion (2001) provided early empirical evidence linking the gradient magnitudes of natural images to Benford distributions, establishing a theoretical bridge between radiometric variance and logarithmic probabilities. This principle was subsequently operationalized in image forensics by Fu et al. (2007), who identified that manipulated JPEG coefficients systematically deviate from the Benford curve, thereby enabling the detection of synthetic tampering. Within the remote sensing literature, applications remain largely confined to no-reference image quality assessment. Hao et al. (2024) demonstrated this by employing Benford's Law to quantify fusion distortion in hyperspectral sharpened images, successfully evaluating spectral degradation without relying on reference data. However, a critical methodological limitation remains across these applications: existing Benford-based frameworks including the hyperspectral assessment by Hao et al. (2024) are engineered strictly to extract a singular global statistic or an image-wide quality index. While these approaches successfully quantify the macroscopic presence of an anomaly, they fundamentally fail to localize it spatially. The structural

translation of these empirical deviations into explicit spatial mapping has not been achieved, leaving a significant gap in the operational utility of the law for pixel-level anomaly extraction.

2.3. Unsupervised Anomaly Detection in Earth Observation

The accurate extraction of physical anomalies in optical remote sensing heavily relies on algorithms that model background terrestrial variance. While supervised classification networks deliver high precision, they demand exhaustive annotated datasets, limiting their scalability across uncalibrated global archives. Consequently, unsupervised architectures have become standard for exploratory data analysis. The benchmark Reed-Xiaoli (RX) detector (Reed and Yu 1990) isolates anomalies by computing the Mahalanobis distance between a test pixel and the local background covariance matrix. Subsequent optimizations, such as Collaborative Representation for hyperspectral anomaly detection (CRD) (Li and Du 2015) and Low-Rank and Sparse Matrix Decomposition (LRaSMD) (Xu et al. 2016), have significantly improved background suppression and detection sensitivity. Despite these mathematical advancements, conventional unsupervised detectors exhibit systemic vulnerabilities. They often require rigorous atmospheric corrections, assume Gaussian background distributions (which frequently fail in highly heterogeneous scenes), and rely on iterative thresholding or high computational complexity to isolate targets. This highlights an ongoing structural requirement for a threshold-free, parameter-light statistical filter. By leveraging the scale-invariant properties of the first-digit distribution, the proposed GBDC framework circumvents the need for background covariance modeling. Consequently, it provides a fundamentally new, threshold-free alternative that explicitly maps the spatial collapse of radiometric variance, seamlessly bridging the gap between global statistical theory and pixel-level anomaly extraction.

3. Methodology

3.1. Theoretical Foundation: Benford's Law

Benford's Law, also known as the First-Digit Law (Newcomb 1881; Benford 1938), establishes that in naturally occurring, scale-invariant datasets, the leading significant digits are not uniformly distributed but instead follow a logarithmic probability distribution. For a leading digit $d \in \{1, 2, \dots, 9\}$, the expected probability $P(d)$ is defined as:

$$P(d) = \log_{10} \left(1 + \frac{1}{d} \right) \quad (1)$$

This formulation predicts that the digit 1 appears as the leading digit in approximately 30.1% of observations, while the digit 9 appears in only 4.6%. The distribution is monotonically decreasing and independent of measurement units or scale, making it particularly suitable for analyzing uncalibrated radiometric data, such as TOA digital numbers (DN) in satellite imagery.

For a given two-dimensional image band $I \in \mathbb{R}^{H \times W}$ (where H and W denote the spatial height and width of the scene, respectively), each pixel value $x_{i,j} > 0$ at spatial coordinates (i, j) within the spectral band can be reduced to its first significant digit using the following relation:

$$d_{i,j} = \left\lceil \frac{x_{i,j}}{10^{\log_{10}(x_{i,j})}} \right\rceil \quad (2)$$

GBDC assumes that a radiometrically heterogeneous scene characterized by diverse land cover types, variable surface reflectance, and natural atmospheric scattering yields an empirical first-digit frequency O_d approximating the theoretical distribution P_d . We stress that heterogeneity alone does not guarantee this: a scene is a superposition of spectrally distinct land-cover modes, each often confined to a narrow DN range, and the sum of such narrow-range modes need not satisfy the log-scale-invariance that Benford's Law presupposes. Approximate compliance is therefore expected only where the pooled reflectance spans a sufficient dynamic range for individual modal biases to average out; the baseline is treated here as a working assumption rather than an empirically verified property of the specific patches evaluated (see Section 5). However, when a physical anomaly such as a dense cloud, active fire, or heavy aerosol plume is present, the local pixel reflectance

becomes saturated or homogenized, forcing a large number of pixels into a narrow numerical range. This localized saturation artificially concentrates the leading digit distribution, thereby inducing a measurable structural deviation between the observed frequencies and the Benford baseline (Fig. 1).

This structural deviation serves as the foundational statistical signal exploited by the GBDC framework. Unlike conventional approaches that merely compute a global deviation magnitude, the proposed methodology leverages per-digit deviations to rank the digits and subsequently reproject these ranks back into the spatial domain, enabling pixel-level anomaly localization.

3.2. GBDC Architecture

To transition Benford's Law from a global statistical metric into an active spatial filter, we introduce the GBDC architecture, as illustrated in Fig. 2. The core objective of GBDC is to quantify the structural collapse of natural radiometric variance and map this collapse back to the spatial domain without relying on user defined thresholds or prior training. The algorithm operates on uncalibrated single band imagery through a sequence of fully automated analytical stages, logically structured across three main functional processing layers:

We note that Stage 5 introduces no new spatial information: the arrangement of the output map is inherited directly from the raw DN image. The methodological contribution lies in automatically deriving a per-pixel classification decision from a global deviation signal, not in generating novel spatial topological relationships.

- Stage 1: Global Frequency Extraction

For a given single-band image, valid TOA reflectance values (excluding zero and no data masks) are extracted. The leading significant digit for each valid pixel is computed, and the empirical observed frequency, $O(d)$, for each digit $d \in \{1, \dots, 9\}$ is calculated across the entire spatial extent of the scene.

- Stage 2: Weighted Statistical Deviation

A fundamental flaw in naive anomaly detection frameworks is the utilization of absolute error metrics (e.g., Mean Absolute Error), which treat all digit deviations equally. However, under Benford's distribution, a perturbation in a low-probability digit (e.g., $d = 9$) indicates a far more severe structural anomaly than an equivalent perturbation in a high-probability digit (e.g., $d = 1$). To mathematically account for this inherent probability weighting and to isolate the spatial footprint of the anomaly accurately, GBDC employs a probabilityweighted proportional deviation score to quantify the deviation. The score for each digit, E_d , is defined as:

$$E(d) = \frac{(O(d) - P(d))^2}{P(d) + \epsilon}$$

where O_d is the empirical relative frequency (the proportion of valid pixels whose leading digit is d), P_d is the theoretical Benford probability, and ϵ is a negligible constant (10^{-10}) introduced to ensure numerical stability. This formulation heavily penalizes artificial clustering in naturally sparse numerical intervals.

- Stage 3: Spatial Reprojection and Ranking

The digits are sorted in descending order based on their computed deviation scores, $E(d)$. A ranking function assigns an integer rank from 1 (maximum deviation) to 9 (minimum deviation) to each digit. These global statistical ranks are subsequently reprojected onto the original image grid. Each spatial pixel is assigned the rank corresponding to its extracted leading digit, generating a continuous spatial anomaly map where lower values indicate higher statistical perturbation.

- Stage 4: Automated Thresholding via Inflection Point

To guarantee a strictly threshold-free architecture, GBDC avoids manual Top K selection. Instead, it determines the optimal cutoff dynamically by identifying the maximum structural drop (inflection point) in the sorted deviation curve. Let $E_{(r)}$

represent the deviation score at rank r . The absolute drop between consecutive ranks is computed as:

$$\Delta(r) = |E_{(r)} - E_{(r+1)}|$$

The optimal, data-driven threshold K_{auto} is defined as the rank preceding the most precipitous drop in deviation magnitude:

$$K_{auto} = \operatorname{argmax}_r \Delta(r)$$

- Stage 5: Binary Mask Generation

Finally, the continuous spatial map is binarized. All pixels possessing a structural rank less than or equal to the dynamically computed K_{auto} are flagged as anomalous (e.g., dense meteorological formations), producing a spatially explicit anomaly mask without any manual parameterization.

3.3 Experimental Configuration: Case Study on CloudSEN12

To rigorously evaluate the GBDC framework, we utilize the Level1C (L1C) TOA reflectance data from the CloudSEN12 dataset. While CloudSEN12 is fundamentally designed for cloud masking algorithms, this study employs it strictly as a standardized testing ground for dense radiometric anomalies. L1C data preserves the uncalibrated raw radiometric values required to analyze empirical digit distributions prior to any nonlinear atmospheric corrections. To ensure a sufficient statistical presence of the target anomaly within the background variance, the dataset was filtered to exclude clear-sky and near-clear scenes, isolating patches (512×512 pixels) with greater than 5% anomaly coverage. We emphasize that this 5% boundary is not proposed as an operational detection threshold or a mathematically derived sensitivity floor. Rather, it functions strictly as a conservative experimental inclusion criterion. Its sole purpose is to ensure that the target class is minimally represented within the spatial subset, thereby preventing the quantitative evaluation from being mathematically skewed by entirely empty scenes. A subset of 500 diverse spatial contexts was systematically selected and processed to form the evaluation baseline. The spatial mapping capacity of the framework was evaluated across five distinct Sentinel-2 spectral bands, selected to represent varying degrees of surface reflectance and atmospheric absorption: Coastal Aerosol (B1: 443 nm), Blue (B2: 490 nm), Near-Infrared (B8: 842 nm), Cirrus (B10: 1375 nm), and Short-Wave Infrared (B11: 1610 nm). Because Benford's Law is theoretically sensitive to data manipulation, a spatial-spectral ablation study was designed comprising six distinct methodological configurations. This experimental setup evaluates the resilience of the autonomous mathematical formulation against standard spatial filtering and radiometric distortion:

- M1_Baseline: Raw uncalibrated TOA data input with no spatial preprocessing or postprocessing applied.
- M2_PreMedian: Application of a 5×5 median spatial filter prior to the global frequency extraction to suppress localized, high-frequency noise while preserving structural edges.
- M3_PreContrast: Application of a 2%–98% linear contrast stretch before distribution extraction. This configuration deliberately manipulates and expands the dynamic range to assess the response and penalization mechanics of the Chi-Square formulation.
- M4_PostMorph: Baseline extraction followed by a binary morphological opening operation using a 5×5 kernel to eliminate isolated false-positive noise in the final spatial mask.
- M5_PostMajority: Baseline extraction followed by a 3×3 uniform majority filter to smooth the boundaries of the generated anomaly mask.
- M6_MedMaj: The combined integration of the pre-median (5×5) and post-majority (3×3) spatial filters.

In all configurations, the threshold for anomaly delineation was dynamically determined by the autonomous K_{auto} inflection mechanism introduced in Section 3.2, completely isolating the evaluation from any manual parameterization or forced rank selection.

3.4. Evaluation Metrics

To ensure a rigorous and transparent quantitative evaluation of the spatial mapping outputs, the multiclass expert annotations provided in the CloudSEN12 dataset were binarized to align with the binary spatial footprint generated by the GBDC framework. Specifically, pixels annotated as clear-sky background regions were designated as the negative class (0), while pixels representing cloud formations (e.g., thick and thin clouds) were aggregated into the positive anomaly class (1). Cloud shadows were retained in the negative class in the present evaluation protocol. We explicitly acknowledge that this assignment introduces a systematic source of evaluation bias. Because shadows exhibit lower-reflectance DN values where high leading digits are physically rare, the mathematical formulation penalizes this natural absence (as detailed in Section 5.3), thereby inflating spatial false alarms and systematically depressing the aggregate Precision metric. Similarly, incorporating semi-transparent thin clouds into the positive anomaly class imposes a structural ceiling on the Recall metric. Because mixed-radiance pixels lack the requisite radiometric saturation to induce a complete distribution collapse, the framework naturally omits them, systematically elevating the False Negative rate. Consequently, the aggregate evaluation metrics must be interpreted with an awareness of these opposing binarization biases.

The threshold-free spatial maps produced by the autonomous K_{auto} mechanism were evaluated pixelbypixel against this binarized ground truth. The quantitative assessment was conducted using four standard classification metrics: Precision, Recall, F1Score, and the Jaccard Index (Intersection over Union, IoU). These metrics are mathematically derived from the extraction of True Positives (TP), False Positives (FP), and False Negatives (FN):

$$Precision = \frac{TP}{TP + FP} \quad (3)$$

$$Recall = \frac{TP}{TP + FN} \quad (4)$$

$$F1Score = 2 \times \frac{Precision \times Recall}{Precision + Recall} \quad (5)$$

$$IoU = \frac{TP}{TP + FP + FN} \quad (6)$$

Given the completely unsupervised architecture of the GBDC framework and the inherent spatial imbalance between anomalous targets and terrestrial backgrounds in remote sensing imagery, the F1Score and IoU were prioritized as the primary performance indicators. These harmonic metrics provide a balanced evaluation of the algorithm's capability to delineate the spatial boundaries of the anomaly. They strictly penalize both the overestimation of the anomaly footprint (spatial false alarms, reflected in low Precision) and the underestimation of the anomaly extent (missed spatial detections, reflected in low Recall).

4. Results

4.1. spatial-spectral Ablation Analysis

To empirically validate the GBDC framework's capacity to translate one-dimensional statistical deviations into two-dimensional spatial maps, an extensive ablation study comprising 15,000 independent computational evaluations was conducted. The framework was systematically tested across 500 radiometrically diverse spatial contexts extracted from the evaluation dataset. It is imperative to note that this dataset functions exclusively as a standardized validation environment to quantify the structural collapse of Benford's distribution under radiometric saturation, rather than a training corpus.

Table 1

Mean F1-Score matrix evaluating 15,000 computational configurations across five spectral bands and six spatial processing strategies.

Spectral Band	M1_Baseline	M2_PreMedian	M3_PreContrast	M4_PostMorph	M5_PostMajority	M6_MedMaj
B1_Coastal	0.3585	0.3590	0.3290	0.3582	0.3585	0.3590
B2_Blue	0.3582	0.3588	0.3219	0.3446	0.3581	0.3588
B8_NIR	0.3947	0.3961	0.3751	0.3725	0.3945	0.3959
B10_Cirrus	0.3298	0.3304	0.3083	0.3294	0.3294	0.3302
B11_SWIR	0.2960	0.2964	0.2902	0.2955	0.2954	0.2962

Table 1 delineates the mean F1-Score matrix across five Sentinel-2 spectral bands and six morphological configurations. Spectrally, the Near-Infrared band (B8) consistently demonstrated the highest anomaly delineation capacity, yielding a mean F1-Score of approximately 0.394 across baseline and filtered configurations. This behavior is consistent with reflectance contrast in the NIR spectrum: the difference between dense atmospheric anomalies (e.g., thick clouds) and terrestrial backgrounds may induce a more severe structural perturbation in the leading-digit distribution, amplifying the χ^2 deviation signal. We caution, however, that interband F1 differences cannot be cleanly attributed to differential anomaly-induced collapse alone. Each band's characteristic clearsky DN range imposes its own persistent leading-digit bias. Without a perband clearsky Benford baseline, the anomaly signal and this intrinsic background bias remain partially confounded. We further note that this structural limitation is most consequential for B10 (Cirrus, 1375 nm), where near-total atmospheric absorption drives clear-land digital numbers toward zero and severely restricts dynamic range. Under such specific conditions, the χ^2 deviation may be uninterpretable independent of any physical anomaly. From a spatial processing perspective, the baseline configuration operating on raw TOA reflectance (M1_Baseline) achieved an overall mean F1-Score of 0.3474. The integration of localized spatial smoothing, specifically the 5×5 median filter (M2_PreMedian), yielded mathematically negligible improvements, peaking at a mean F1-Score of 0.3482. This marginal variance empirically proves the inherent structural robustness of the GBDC mathematical core; the algorithm effectively localizes radiometric saturation without the necessity of external spatial filtering.

Conversely, the application of a linear contrast stretch (M3_PreContrast) precipitated a distinct degradation in overall performance, reducing the mean F1-Score to 0.3249 and significantly suppressing the mean Recall metric to 0.3653. This empirical degradation rigorously validates the core theoretical premise of the study: Benford's Law relies strictly on the natural, scale-invariant variance of uncalibrated optical data. Artificial manipulation of the dynamic range forces a uniform numerical redistribution, directly destroying the inherent logarithmic probability. Consequently, the statistical gap between the background and the anomaly narrows, leading to elevated omission errors.

4.2. Quantitative Performance and Spatial Error Anatomy

To critically evaluate the threshold-free delineation capability of the GBDC framework, the aggregated classification metrics Precision, Recall, F1-Score, and Intersection over Union (IoU) were analyzed across the filtered validation subset restricted to spatial contexts with more than 5% anomaly coverage. Table 2 presents the summary statistics for the evaluated configurations averaged across all spatial contexts.

Table 2

Quantitative summary of the GBDC configurations averaged across 500 spatial contexts exhibiting > 5% anomaly coverage. The table details the mean and standard deviation of the F1-Score, alongside mean Precision, Recall, and Intersection over Union (IoU) metrics

Method Configuration	Mean F1Score	Std. F1-Score	Mean Precision	Mean Recall	Mean IoU
M1_Baseline	0.3474	0.2488	0.3721	0.4695	0.2416
M2_PreMedian	0.3482	0.2494	0.3717	0.4714	0.2423
M3_PreContrast	0.3249	0.2331	0.3919	0.3653	0.2191
M4_PostMorph	0.3400	0.2497	0.3736	0.4533	0.2362
M5_PostMajority	0.3472	0.2493	0.3722	0.4696	0.2415
M6_MedMaj	0.3480	0.2495	0.3716	0.4712	0.2422

To contextualize the reported global mean F1-score of 0.3474, we note that GBDC is an unsupervised, single-band, calibration-free screen and is not intended to compete with supervised or multi-band cloud-detection architectures. Absolute performance should be read strictly as a proof-of-concept lower bound. We do not benchmark against a random-chance, trivial-classifier, or unsupervised comparator on the identical filtered subset, so the reported F1-score cannot presently be interpreted as demonstrating detection mathematically above chance. Rather, it establishes only the computational feasibility of utilizing mathematical variance collapse as a spatial filter, not a state-of-the-art accuracy result. The mathematically optimal configuration, defined as the integration of a 5×5 median pre-filter (M2_PreMedian), yielded a global mean F1-score of 0.3482, with a mean Precision of 0.3717 and a mean Recall of 0.4714. While the aggregated harmonic mean reflects a moderate global performance, it is imperative to interpret this metric within the context of a strictly unsupervised, global statistical framework operating without localized tuning or adaptive thresholding. In highly heterogeneous scenes containing fragmented or translucent anomalies (e.g., thin cirrus clouds), the natural radiometric variance of the terrestrial background dominates the statistical distribution, thereby diluting the localized χ^2 deviation signal and inducing spatial underestimation (reflected in the elevated False Negative rate).

A structural tradeoff between Precision and Recall is empirically evident when examining the linear contrast stretch configuration (M3_PreContrast). Although this configuration degraded the overall F1-score (0.3249), it achieved the highest mean Precision (0.3919) at the cost of a severe drop in Recall (0.3653). Physically, the artificial expansion of the dynamic range suppresses the statistical deviations induced by low-intensity anomalies, effectively forcing the framework into a highly conservative delineation state. This behavior systematically reduces spatial false alarms (False Positives) but significantly underestimates the actual footprint of the anomaly (False Negatives). To transition the quantitative metrics into a physical context, the spatial error distribution was mapped at the pixel level. Figure 3 illustrates the spatial error anatomy comprising True Positives (TP), True Negatives (TN), False Positives (FP), and False Negatives (FN) across three distinct radiometric scenarios: successful anomaly extraction, severe underestimation, and critical overestimation.

The spatial error maps confirm that the core structural limitation of the global Benford approach manifests primarily at the diffuse boundaries of the anomalies, where radiometric saturation gradually transitions back to natural variance. In scenarios dominated by sparse or semitransparent clouds (e.g., Patch 194), the lack of dense saturation prevents a complete collapse of the digit distribution, leading to systematic underestimation (False Negatives). Conversely, highly reflective but natural terrestrial features, such as urban surfaces or bright barren lands (e.g., Patch 247), can falsely trigger the distribution collapse, causing overestimation (False Positives). However, in spatial contexts characterized by dense, cohesive anomalies (e.g., Patch 5), the mathematical collapse of the first-digit distribution successfully isolates the target footprint, providing robust, threshold-free spatial mapping driven purely by the χ^2 statistical deviation.

5. Discussion

5.1. Methodological Shift: From Global Statistics to Spatial Mapping

Historically, the deployment of Benford's Law in digital image analysis and remote sensing has been structurally confined to macroscopic evaluations. Existing frameworks inherently compress the radiometric variance of an entire scene into a singular diagnostic scalar, typically utilized for no-reference quality assessment or fraud detection. While statistically valid for determining the dataset-wide integrity of an image, this absolute aggregation fundamentally eradicates spatial context. It renders the statistical signal incapable of localizing the physical coordinates of the radiometric perturbations. The GBDC architecture addresses this theoretical gap by fundamentally altering how the first-digit distribution is operationalized. Rather than deriving a singular image-wide metric, the framework quantifies the deviation of each individual digit and maps this mathematical collapse directly back onto the spatial grid. By assigning the weighted statistical rank of the leading digit to its corresponding spatial pixel, GBDC translates a global probabilistic baseline into a spatially explicit anomaly map. This methodological shift establishes that the structural deviation from Benford's distribution, induced by severe radiometric saturation, is not merely a descriptive statistical artifact. Instead, it constitutes a mappable physical footprint. Consequently, the approach successfully transitions Benford's Law from a passive, global indicator of dataset conformity into an active, pixel-level spatial filter.

A key confound remains unresolved by the present evaluation. Because dense anomalies saturate pixels into high DN ranges that inherently share a narrow set of leading digits, the observed detections are consistent with two mechanisms: a genuine breakdown of scale-invariant first-digit structure, or a convoluted global brightness threshold that flags high-DN pixels via the leading-digit space. The current experimental design cannot definitively distinguish these two pathways, and we therefore refrain from claiming the χ^2 collapse as the exclusive causal detection mechanism.

5.2. Theoretical Resilience and the Threshold-Free Paradigm

A persistent vulnerability in conventional statistical anomaly detection frameworks is the reliance on empirical thresholding. Isolating a target from the background typically requires site-specific calibration or prior assumptions regarding the background covariance structure, which frequently fail in highly heterogeneous uncalibrated data. The GBDC architecture eliminates this dependency through the calculation of an automated inflection point within the sorted deviation curve. By dynamically identifying the maximum structural drop in the χ^2 deviation magnitudes, the algorithm forces a mathematically driven separation between the saturated anomaly footprint and the natural variance of the terrestrial background, strictly avoiding manual parameterization. The spatial-spectral ablation analysis provided definitive physical evidence regarding the algorithm's operational boundaries and its reliance on natural radiometric states. The negligible variance in classification accuracy upon applying localized spatial filters (such as the median filter) indicates that the anomaly extraction is driven primarily by global statistical divergence rather than local spatial continuity. The mathematical core of GBDC localizes anomalies based on their disruption of the global logarithmic probability, rendering the framework inherently resilient to high-frequency spatial noise that typically affects traditional edge-based or texture-based detectors.

Conversely, the systematic degradation of detection accuracy following linear contrast stretching underscores a fundamental theoretical constraint of Benford-based spatial filtering. Benford's Law is strictly predicated on the scale-invariant distribution of unconstrained datasets. Artificially expanding the radiometric dynamic range through contrast stretching forces a numerical redistribution of the pixel values. This artificial manipulation flattens the empirical first-digit distribution, thereby suppressing the magnitude of the χ^2 statistical divergence induced by actual physical anomalies. As a result, the statistical margin between the background and the anomaly narrows, leading to severe underestimation and high omission errors. This failure mechanism rigorously confirms that the framework mandates the use of raw, linearly unmanipulated topofatmosphere (TOA) data to preserve the natural structural variance required for the distribution collapse. By construction, the automated $K_{\{auto\}}$ mechanism is unconditional. Because r ranges over the eight transitions of the nineelement digit space, $K_{\{auto\}}$ is implicitly bounded on $[1, 8]$ and always returns a rank, thereby flagging a nonempty spatial subset. It possesses no nulldetection branch to abstain when a genuine anomaly is absent. GBDC thus operates strictly as a relative partitioner of the digitdeviation curve rather than a binary detector with an intrinsic present/absent decision. Furthermore, because the

evaluation corpus was restricted to patches with greater than 5% anomaly coverage, the framework's false-alarm behavior on completely anomaly-free scenes is not characterized here and remains an open constraint for operational deployment.

5.3. Physical Boundaries of the First-Digit Distribution in Optical Data

The spatial error topography generated by the GBDC framework reveals that the algorithm's operational limits are fundamentally governed by the radiometric density of the target anomaly relative to the underlying background. The mathematical collapse of the first-digit distribution requires absolute localized saturation to override the natural variance of the scene. When this physical condition is not met, the framework exhibits systematic, predictable failure modes. Underestimation, or the generation of extensive False Negatives, predominantly occurs in the presence of diffuse, sparse, or semitransparent anomalies, such as thin cirrus formations or dissipating aerosol plumes. In these environments, the optical signal of the terrestrial background bleeds through the translucent anomaly. This radiometric mixing prevents the localized reflectance values from fully homogenizing. Consequently, the natural, scale-invariant variance remains partially intact, and the resulting statistical deviation is insufficient to trigger a definitive structural drop. The anomaly fails to surpass the autonomously derived inflection point ($K_{\{auto\}}$), leading the framework to classify these regions as stable background. This dictates that Benfordbased spatial filtering is intrinsically insensitive to lowdensity radiometric perturbations.

Conversely, the framework is susceptible to critical overestimation, yielding clustered False Positives, when applied to unmasked, highly reflective terrestrial features. Natural land covers such as bright barren rock formations, localized snow, or dense urban concrete exhibit high-albedo profiles that physically mimic the radiometric saturation of meteorological anomalies. These stable terrestrial features naturally concentrate the leading-digit distribution into a narrow numerical range, inducing a severe χ^2 statistical deviation. Because the GBDC architecture operates purely as a variance-based filter without explicit spectral signature matching, it inherently lacks the capacity to distinguish between an anomalous atmospheric saturation and a stable terrestrial saturation. This structural limitation confines the uncalibrated application of the framework to scenarios where the anomaly's reflectance fundamentally exceeds the maximum natural albedo of the underlying biome. Beyond radiometric saturation, two structural biases inherent to the mathematical formulation further constrain the framework. First, a source of spatial overestimation arises from the global-to-local reprojection (Stage 3). Because the leading-digit extraction discards the order-of-magnitude exponent, background pixels in a lower decade (e.g., DN 200) are swept up alongside genuine anomaly pixels in a higher decade (e.g., DN 2000) if they share the same leading digit. Second, a statistical confound emerges from the $\frac{1}{P_d}$ weighting (Stage 2), which heavily penalizes deviations in the rarest digits (7–9). Over lowreflectance surfaces (e.g., water bodies), digital numbers occupy a restricted low decade where high leading digits are physically absent under normal conditions. The weighting scheme registers this natural physical absence as a large deviation, structurally mimicking an anomaly. These cross-magnitude digit sharing and lower-reflectance penalty mechanisms represent intrinsic false-positive sources distinct from high-albedo mimicry.

5.4. Generalizability and Broad Earth Observation Applications

The operational utility of the GBDC framework should not be evaluated as a direct substitute for high-fidelity supervised segmentation architectures. Supervised models fundamentally outperform statistical filters in extracting precise morphological boundaries for known target classes. However, the reliance on exhaustive annotated datasets heavily constrains their scalability. The primary architectural advantage of the GBDC framework lies in its absolute independence from training data, site-specific calibration, and prior target-specific knowledge. Because the algorithm functions purely by quantifying the structural collapse of natural variance, it is inherently agnostic to the physical origin of the anomaly. This characteristic positions GBDC as a density-conditional heuristic for detecting "unknown unknowns" unpredictable radiometric events that supervised models are not trained to identify. Beyond dense meteorological formations, any localized phenomenon that induces severe radiometric saturation, such as active wildfire fronts, sudden industrial accidents, or even localized sensor corruption, will trigger a measurable deviation in the first-digit distribution. Consequently, the framework is intended as a first-pass spatial screening tool operating on fixed-size image tiles within uncalibrated satellite archives (e.g., Level1C data). We emphasize that all reported performance is conditional on the 512×512 evaluation window. Because the global χ^2 deviation is driven by the anomalytobackground pixel ratio, the detection signal is inherently sensitive to the

spatial extent over which digits are pooled. A patchsize sensitivity analysis was not conducted; therefore, deployment on larger windows or full Sentinel2 tiles, where a given anomaly occupies a far smaller pixel fraction, may substantially attenuate the deviation. The reported metrics should not be assumed to transfer to alternative tiling strategies without further validation.

6. Conclusion

This study evaluated the empirical feasibility of employing Benford's Law as a threshold-free spatial filter for radiometric anomaly detection in optical Earth observation data. By introducing the GBDC framework, the analysis structurally projected the mathematical collapse of the global logarithmic distribution back onto the spatial grid, deriving a per-pixel classification decision directly from an image-wide statistical deviation. Evaluated across 15,000 automated configurations, the architecture demonstrated that Pearson's χ^2 deviation, integrated with a dynamic inflectionpoint thresholding function, can isolate dense radiometric anomalies from natural terrestrial variance without requiring labeled training samples, localized spatial filtering, or site-specific calibration. The spatial-spectral ablation analysis verified that the framework's mathematical integrity is strictly dependent on the preservation of raw, uncalibrated TOA reflectance. Artificial dynamic range manipulation, specifically linear contrast stretching, systematically redistributes the inherent first-digit distribution, thereby suppressing the statistical deviation signal and precipitating a collapse in target delineation. While GBDC achieves high spatial precision in environments dominated by cohesive saturation particularly within the Near-Infrared spectrum where target-background contrast is maximized its operational envelope is fundamentally bound by physical radiometric density. The architecture systematically underestimates diffuse, semitransparent anomalies due to background optical mixing, and generates elevated false-positive rates over stable, high-albedo terrestrial features (e.g., barren rock and urban surfaces) because it operates purely on statistical variance rather than spectral signature matching.

Consequently, future research must address these structural constraints to enhance the framework's operational utility. Crucially, to validate the core detection mechanism, future work should prioritize controlled synthetic injection experiments: generating patches with a known Benford-compliant background and injecting anomaly pixels drawn from a high-DN range, varying the anomaly fraction, to test whether detection tracks the χ^2 deviation independently of mean scene brightness. Beyond this foundational validation, a logical methodological progression involves the integration of secondary, lightweight spectral indices (e.g., normalized difference vegetation or snow indices) to construct a hybrid filtering architecture. By applying rudimentary spectral masking prior to the extraction of the empirical firstdigit distribution, the spatial false alarms induced by stable highalbedo land covers could be effectively neutralized. Furthermore, investigating multiband statistical aggregation, rather than isolated single-band extraction, is necessary to determine if a multidimensional Benford analysis can increase the framework's sensitivity to low-density radiometric perturbations.

Declarations

Funding: The authors declare that no funds, grants, or other support were received during the preparation of this manuscript.

Competing Interests: The authors have no relevant financial or non-financial interests to disclose.

Author Contributions

Conceptualization: Shaho Piroti ; Methodology: Shaho Piroti, Parviz Zeaieanfiroouzabadi ; Formal analysis: Shaho Piroti, Parviz Zeaieanfiroouzabadi; Software: Shaho Piroti, Saman Ghafari; Data curation: Shaho Piroti; Writing - original draft preparation: Shaho Piroti, Parviz Zeaieanfiroouzabadi ,Saman Ghafari; Writing - review and editing: Shaho Piroti, Parviz Zeaieanfiroouzabadi ,Saman Ghafari.

Data Availability

The Sentinel-2 Level-1C TOA reflectance data utilized in this study are derived from the publicly available CloudSEN12 benchmark dataset. The GBDC algorithms and data processing scripts developed for this research are available from the

References

1. Benford F (1938) The law of anomalous numbers. *Proc Am Philos Soc* 78(4):551–572. <https://www.jstor.org/stable/984802>
2. Cumbreira-Conde I, Silva-Ramírez EL, Cumbreira FL (2024) Signal processing analysis for detection of anomalies in numerical series. *Expert Syst Appl* 238(Part C):121857. <https://doi.org/10.1016/j.eswa.2023.121857>
3. Das B, Datta A, Ghosh A, Chalakkatta V (2026) Earthquake detection using Benford's law. *arXiv:2607.27821*. <https://doi.org/10.48550/arXiv.2607.27821>
4. Durtschi C, Hillison W, Pacini C (2004) The effective use of Benford's law to assist in detecting fraud in accounting data. *J Forensic Account* 5(1):17–34.
5. Fu D, Shi YQ, Su W (2007) A generalized Benford's law for JPEG coefficients and its applications in image forensics. In: *Proc SPIE* 6505:65051L. <https://doi.org/10.1117/12.704723>
6. Hao X, Li X, Wu J, Wei B, Song Y, Li B (2024) A no-reference quality assessment method for hyperspectral sharpened images via Benford's law. *Remote Sens* 16(7):1167. <https://doi.org/10.3390/rs16071167>
7. Jolion JM (2001) Images and Benford's law. *J Math Imaging Vis* 14(1):73–81. <https://doi.org/10.1023/A:1008363415314>
8. Li W, Du Q (2015) Collaborative representation for hyperspectral anomaly detection. *IEEE Trans Geosci Remote Sens* 53(3):1463–1474. <https://doi.org/10.1109/TGRS.2014.2343955>
9. Newcomb S (1881) Note on the frequency of use of the different digits in natural numbers. *Am J Math* 4(1):39–40. <https://doi.org/10.2307/2369148>
10. Nigrini MJ (2012) *Benford's Law: Applications for Forensic Accounting, Auditing, and Fraud Detection*. Wiley, Hoboken. <https://doi.org/10.1002/9781119203094>
11. Reed IS, Yu X (1990) Adaptive multiple-band CFAR detection of an optical pattern with unknown spectral distribution. *IEEE Trans Acoust Speech Signal Process* 38(10):1760–1770. <https://doi.org/10.1109/29.60107>
12. Sambridge M, Tkalčić H, Jackson A (2010) Benford's law in the natural sciences. *Geophys Res Lett* 37(22):L22301. <https://doi.org/10.1029/2010GL044830>
13. Xu Y, Wu Z, Li J, Plaza A, Wei Z (2016) Anomaly detection in hyperspectral images based on low-rank and sparse representation. *IEEE Trans Geosci Remote Sens* 54(4):1990–2000. <https://doi.org/10.1109/TGRS.2015.2493201>
14. Zhou Q, Tang H, Turowski JM, Braun J, Dietze M, Walter F, Yang CJ, Lagarde S (2024) Benford's law as debris flow detector in seismic signals. *J Geophys Res Earth Surf* 129(9):e2024JF007691. <https://doi.org/10.1029/2024JF007691>

Figures

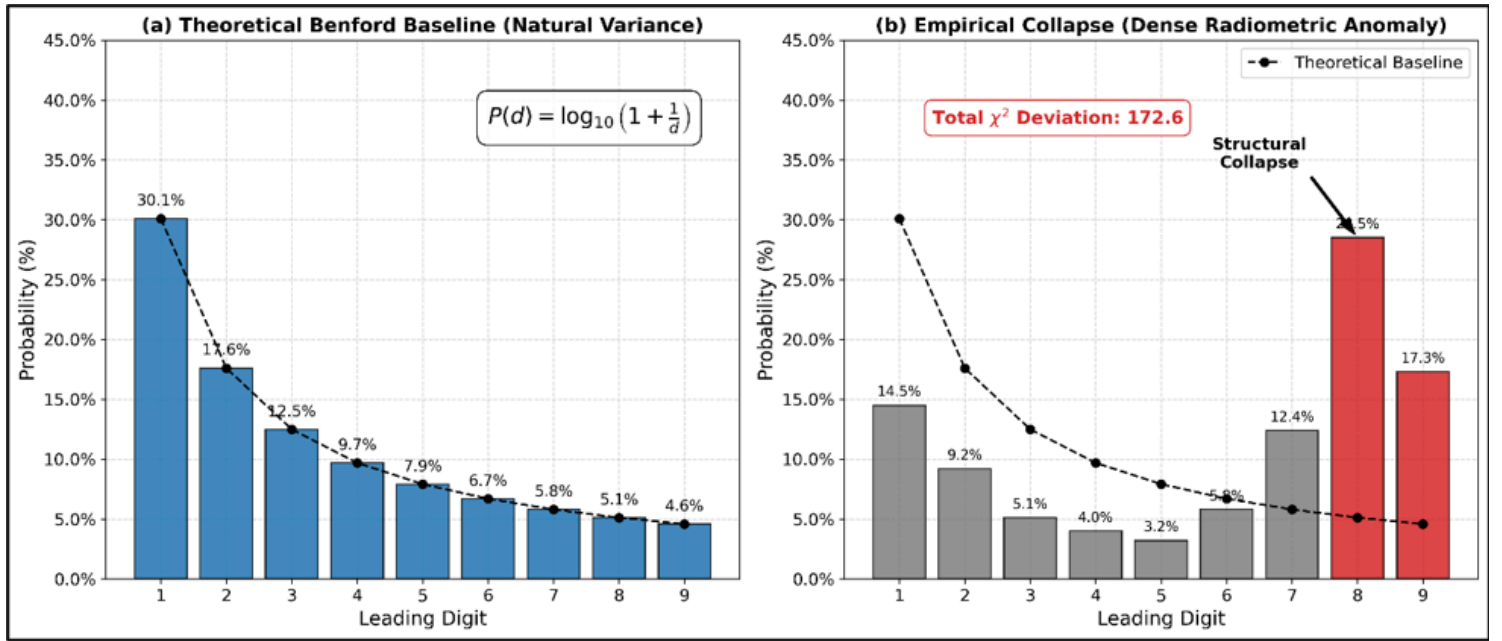


Figure 1

Conceptual illustration of Benford's Law and its structural collapse under radiometric anomaly conditions. (a) Theoretical Benford distribution. (b) Empirical distribution collapse induced by a dense radiometric anomaly (e.g., thick cloud cover). The Chi Square statistic quantifies the deviation from the theoretical baseline

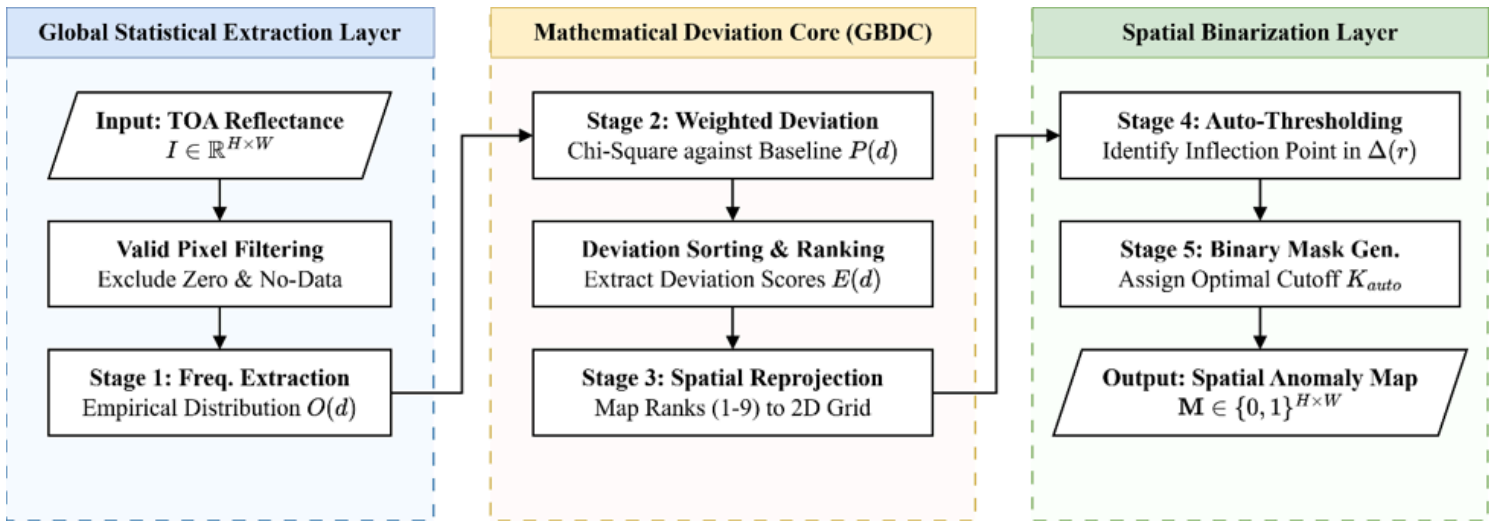


Figure 2

Schematic representation of the proposed GBDC framework. The architecture is structurally divided into three functional layers: (1) global statistical extraction from uncalibrated TOA reflectance, (2) the mathematical deviation core utilizing Pearson's Chi Square to quantify structural collapse, and (3) spatial binarization driven by an automated inflection point threshold

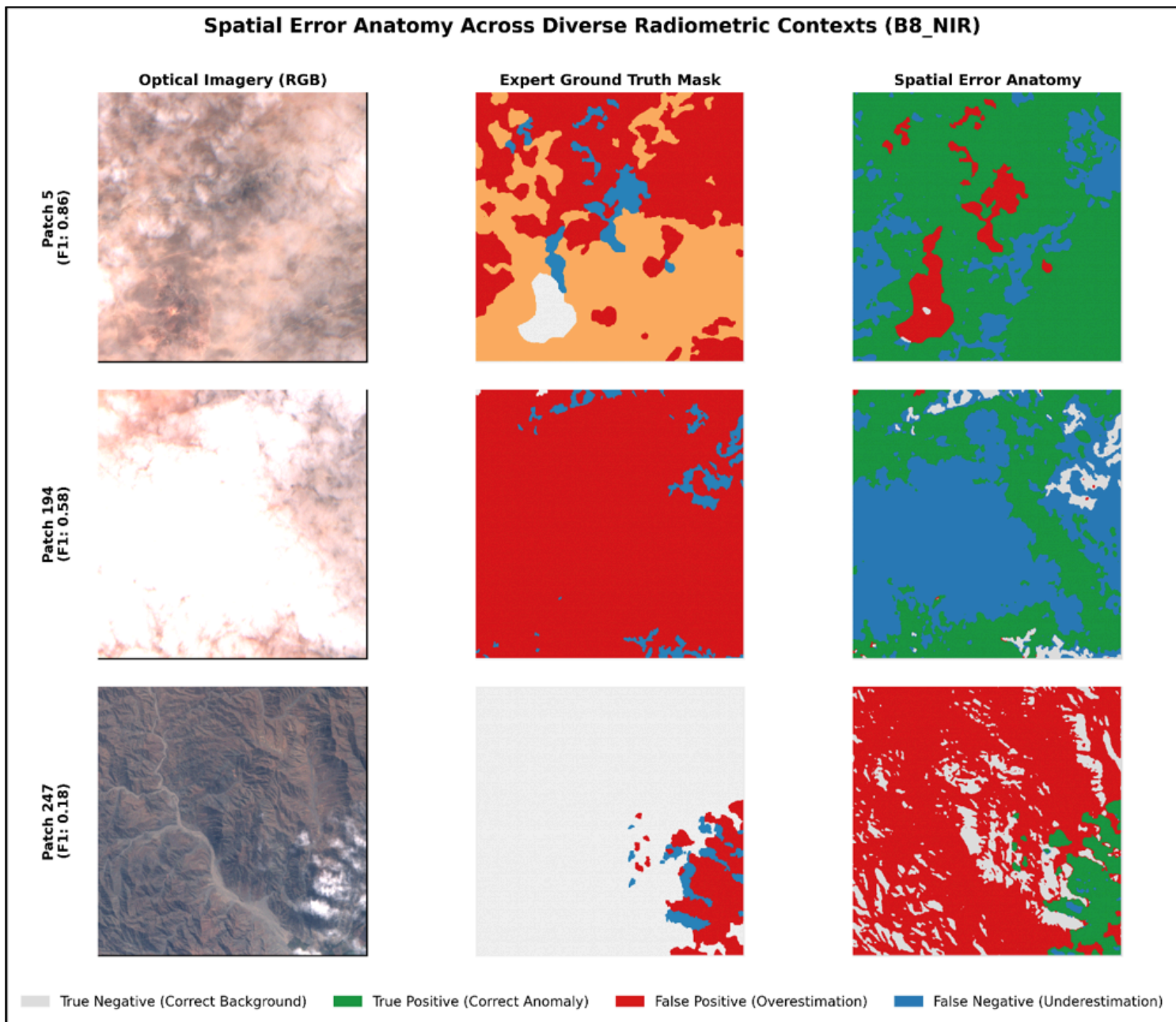


Figure 3

Spatial error anatomy of the GBDC algorithm applied to the B8 (NIR) band across three representative radiometric contexts. The rows illustrate varying degrees of anomaly delineation accuracy: successful extraction (Patch 5, F1: 0.86), systematic underestimation resulting in high False Negatives (Patch 194, F1: 0.58), and critical overestimation triggered by highly reflective barren terrain resulting in extensive False Positives (Patch 247, F1: 0.18)