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Supporting Information for
Viscoelastic memory selects hierarchical folding in growing tissues

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Supplementary Note 1. Surface wrinkling of the mucosal epithelium and submucosa.

1.1 Surface Wrinkling of Purely Elastic Epithelial Tissues without Growing.

Assuming biological tissues exhibit purely elastic behavior, the governing equation, describing growth-induced surface instability of epithelial tissues, is derived from the Föppl–von Kármán plate theory¹ and the classical fourth-order thin plate equation^{2,3}:

$$D_c \nabla^4 w + N_g h \frac{d^2 w}{dx^2} + p = -\nabla^2 M_g, \quad (\text{S1.1})$$

where $D_c = E_c h^3 / 12(1 - \nu_c^2)$ is the bending stiffness of the cortical layer with the elastic modulus $E_{c,s}$ and Poisson's $\nu_{c,s}$ and the mucosal epithelium thickness h . In the above equation, $w(x, y)$ is the transverse deflection function. The in-plane internal force induced by cortical growth is expressed as $N_g = E_c \int_{h/2}^{-h/2} \varepsilon_g(x, y) dy$ and $p(x)$ represents the transverse reaction force of the submucosa due to deflection. $M_g = E_c \int_{h/2}^{-h/2} y \varepsilon_g(x, y) dy$ represents the bending moment generated by non-uniform growth across the layer thickness ($M_g = 0$ for uniform growth, as assumed herein).

For this planar instability problem, the deflection function is assumed to follow a harmonic form (dominant mode for small perturbations),

$$w(x) = A \cos(kx). \quad (\text{S1.2})$$

In which, the instability wave number is $k = 2\pi/\lambda$ with the instability wavelength λ . Considering the submucosa thickness H , the lateral force of the base is expressed as

$$p(x) = p_0 \cos(kx), \quad p_0 = kf(kH, \nu_s) E_s A / [2(1 - \nu_s^2)], \quad (\text{S1.3})$$

where,

$$f(kH, \nu_s) = \frac{(3 - 4\nu_s) \cosh(2kH) + 5 - 12\nu_s + 8\nu_s^2 + 2k^2 H^2}{(6 - 8\nu_s) \sinh(2kH) - 4kH}. \quad (\text{S1.4})$$

34 Notably, $f(kH, \nu_s)$ decreases with kH and approaches a constant $f(\infty, \nu_s) = 1/2$ ^{4, 5},
 35 when the subcortical substrate thickness H is much larger than the cortical thickness h
 36 ($H \gg h$).

37 Substituting Eqs. (S1.3) and (S1.4) into Eq. (S1.2), the in-plane internal force of the
 38 cortex is derived as

$$39 \quad N_g = \frac{1}{12} \frac{E_c}{1-\nu_c^2} (hk)^2 + \frac{1}{2} \frac{E_s}{1-\nu_s^2} \frac{1}{hk}. \quad (\text{S1.5})$$

40 To determine the critical instability condition, the critical wave number k_{cr} is first
 41 solved by minimizing N_g with respect to k ,

$$42 \quad \frac{dN_g}{dk} = \frac{1}{6} \frac{E_c}{1-\nu_c^2} (h^2 k) - \frac{1}{2} \frac{E_s}{1-\nu_s^2} \frac{1}{hk^2} \doteq 0. \quad (\text{S1.6})$$

43 Solving Eq. (S1.7) yields the critical wave number,

$$44 \quad k_{cr} = \frac{1}{h} \sqrt[3]{3 \frac{1-\nu_c^2}{1-\nu_s^2} \frac{E_s}{E_c}}. \quad (\text{S1.7})$$

45 Substituting k_{cr} from Eq. (S1.9) into Eq. (S1.5), the critical growth-induced in-plane
 46 force N_g^{cr} and critical wavelength λ_{cr} can be obtained,

$$47 \quad N_g^{cr} = \frac{3}{4} \frac{E_s}{1-\nu_s^2} \sqrt[3]{\frac{1-\nu_s^2}{1-\nu_c^2} \frac{E_c}{3E_s}}, \quad \lambda_{cr} = 2\pi h \sqrt[3]{\frac{1-\nu_s^2}{3} \frac{E_c}{1-\nu_c^2} \frac{E_s}{E_c}}, \quad (\text{S1.8})$$

48 which define the critical conditions for surface instability of purely elastic epithelial
 49 tissues.

50 **1.2 Surface wrinkling of purely elastic epithelial tissues with growing.**

51 The above elastic solution can be extended to describe surface instability of elastic
 52 epithelial tissues undergoing growth. Based on Eq. (S1.1), the total transverse
 53 deflection A_{total} of the growing tissue is decomposed into elastic A_e and growth-
 54 induced deflection A_g , $w(x) = A_{total} \cos(kx) = (A_e + A_g) \cos(kx)$. Correspondingly, the
 55 total curvature $\dot{A}_{total} = \dot{A}_e + \dot{A}_g$, where $\dot{A}_e = 2\dot{p}_0(1-\nu_s^2)/E_s k$ and
 56 $\dot{A}_g = G_s A_e = 2p_0(1-\nu_s^2)G_s/E_s k$, with G_s denoting the submucosa growth rate. The
 57 relationship between the submucosa's transverse reaction force and deflection is

analogous to the Maxwell viscoelastic stress-strain relation. For the growing elastic submucosa, this relationship becomes ⁶,

$$\dot{p}_0 + G_s p_0 = \frac{E_s}{2(1-\nu_s^2)} k \dot{A}. \quad (\text{S1.9})$$

Assuming $A(t) = W_0 \exp(\tau_{cr} t)$ with $dA/dt = \tau_{cr} A(t)$ and τ_{cr} being the characteristic wrinkling time ⁷, the transverse force $p(x, t)$ can be expressed as

$$\begin{aligned} p(x, t) &= \frac{E_s}{1-\nu_s^2} k \int_{-\infty}^t \exp[-G_s(t-\tau)] \frac{dA(\tau)}{d\tau} d\tau \cos(kx) \\ &= \frac{E_s k}{2(1-\nu_s^2)} \frac{\tau_{cr}}{\tau_{cr} + G_s} A(t) \cos(kx) \end{aligned} \quad (\text{S1.10})$$

Following the same solution procedure as the above purely elastic case, the in-plane internal force of the cortex during growth is,

$$N_g = \frac{1}{12} \frac{E_c}{1-\nu_c^2} (hk)^2 + \frac{1}{2} \frac{E_s}{1-\nu_s^2} \frac{\tau_{cr}}{\tau_{cr} + G_s} \frac{1}{hk}. \quad (\text{S1.11})$$

Solving $dN_g/dk = 0$, the critical wave number k_{cr} , critical in-plane force N_g^{cr} and critical wavelength λ_{cr} for growing tissues become

$$k_{cr} = \frac{1}{h} \sqrt[3]{3\gamma_s \frac{\bar{E}_s}{\bar{E}_c}}, \quad (\text{S1.12})$$

$$N_g^{cr} = \frac{3}{4} \frac{E_s}{1-\nu_s^2} \gamma_s \sqrt[3]{\frac{3}{\gamma_s} \frac{\bar{E}_c}{\bar{E}_s}}, \quad (\text{S1.13})$$

$$\lambda_{cr} = 2\pi h \sqrt[3]{\frac{1}{3\gamma_s} \frac{\bar{E}_c}{\bar{E}_s}}, \quad (\text{S1.14})$$

where $\bar{E}_c = E_c/(1-\nu_c^2)$, $\bar{E}_s = E_s/(1-\nu_s^2)$, and $\gamma_s = \tau_{cr}/(\tau_{cr} + G_s)$.

To calculate the critical instability time t_{surf} and characteristic time τ_{cr} , the epithelial tissues uniform growth with growth rate G_c , and the growth-induced in-plane force of the epithelial tissue is

$$N_g = \bar{E}_c \varepsilon_g = \bar{E}_c G_c t_{surf}. \quad (\text{S1.15})$$

77 Equating Eqs. (S1.15) and (S1.13) and defining $\tau_{cr} = 1/t_{surf}$, a fifth-order polynomial
 78 equation for t_{surf} is derived,

$$79 \quad (t_{surf})^5 + \frac{2}{G_s}(t_{surf})^4 + \frac{1}{G_s^2}(t_{surf})^3 - \frac{9}{64} \frac{\bar{E}_s^2}{\bar{E}_c^2} \frac{1}{G_c^3 G_s^2} \doteq 0. \quad (S1.16)$$

80 Solving Eq. (S1.18) and combining with Eqs. (S1.14 to S1.16) yield to the complete
 81 solution for surface instability of elastic epithelial tissues during growth.

82 **1.3 Surface wrinkling of viscoelastic epithelial tissues with growing**

83 The analysis is further extended to the case of viscoelastic epithelial tissues during
 84 growth, where the time-dependent mechanical response is captured by power-law
 85 relaxation. For viscoelastic tissues, the governing equation for surface instability (Eq.
 86 S1.3) should be modified to account for time-dependent bending stiffness (Eq.1 in the
 87 main text):

$$88 \quad \int_0^t D_c(t-\tau) \frac{d(\nabla^4 w)}{d\tau} d\tau + N_x^{\text{total}} h \frac{\partial^2 w}{\partial x^2} = p(x, t), \quad (S1.17)$$

89 where $D_c(t)$ denotes the time-dependent cortical bending stiffness. For the
 90 viscoelastic submucosa, the transverse reaction force (analogous to Eq. (S1.12))
 91 becomes,

$$92 \quad p(x, t) = f(kH, v_s) k \gamma_s \int_0^t \bar{E}_s(t-\tau) \frac{d(\partial w / \partial x)}{d\tau} d\tau. \quad (S1.18)$$

93 Laplace transforms are applied to Eqs. (S1.17) and (S1.18) to simplify the convolution
 94 integral inherent to viscoelasticity. In the frequency domain, the transformed governing
 95 equation and transverse force are,

$$96 \quad s \tilde{D}_c(s) \nabla^4 \tilde{w}(x, s) + N_g h \frac{d^2 \tilde{w}(x, s)}{dx^2} + \tilde{p}(x, s) = 0, \quad (S1.19)$$

$$97 \quad \tilde{p}(x, s) = \frac{1}{2} k \gamma_s s \tilde{E}_s(s) \tilde{w}(x, s). \quad (S1.20)$$

98 Following the previous method, the expression for the internal force within the cortical
 99 layer in the frequency domain is given as

$$100 \quad \tilde{N}_g = \frac{1}{12} s \tilde{E}_c(s) (hk)^2 + \frac{1}{2} s \tilde{E}_s(s) \gamma_s \frac{1}{hk}. \quad (S1.21)$$

101 Similarly, letting $d\tilde{N}_g/dn=0$, the transformed in-plane internal force $\tilde{N}_g^{cr}$ and
 102 wavelength $\tilde{\lambda}_{cr}$ can be obtained

$$103 \quad \tilde{N}_g^{cr} = \frac{3}{4} \gamma_s s \tilde{E}_s(s) \sqrt[3]{\frac{\tilde{E}_c(s)}{3\gamma_s \tilde{E}_s(s)}}, \quad \tilde{\lambda}_{cr} = 2\pi h \sqrt[3]{\frac{\tilde{E}_c(s)}{3\gamma_s \tilde{E}_s(s)}}. \quad (S1.22)$$

104 For epithelial tissues with power-law viscoelastic behaviors^{8, 9}, the transforms of the
 105 relaxation moduli in frequency domain are $\tilde{E}_c(s) = \bar{E}_c \Gamma(1-\alpha) s^{\alpha-1}$ and
 106 $\tilde{E}_s(s) = \bar{E}_s \Gamma(1-\beta) s^{\beta-1}$ with $\Gamma(\cdot)$ being the Gamma function, where α and β denote
 107 the power-law exponents of the mucosa epithelium and submucosa, respectively. Then,
 108 the critical conditions in the time domain can be obtained as (i.e., Eq.2 in the main text),

$$109 \quad N_g^{cr} = \frac{3}{4} \rho_\alpha \bar{E}_s \left(\frac{c_\alpha}{3} \gamma_s^2 E_r \right)^{\frac{1}{3}} t_{surf}^{\frac{\alpha+2\beta}{3}}, \quad \lambda_{cr} = 2\pi h \kappa_\alpha \left(\frac{c_\alpha}{3\gamma_s} E_r \right)^{\frac{1}{3}} t_{surf}^{\frac{\alpha-\beta}{3}}, \quad (S1.23)$$

$$\left(t_{surf} + \frac{1}{G_s} \right)^2 t_{surf}^{3-2(\alpha-\beta)} = \frac{9}{64} \frac{1}{E_r^2} \frac{c_\alpha \rho_\alpha^3 (1-\alpha)^3}{G_c^3 G_s^2}$$

110 where,

$$111 \quad c_\alpha = \frac{\Gamma(1-\alpha)}{\Gamma(1-\beta)}, \quad \rho_\alpha = \frac{\Gamma(1-\beta)}{\Gamma\left(1-\frac{\alpha+2\beta}{3}\right)}, \quad \kappa_\alpha = \frac{1}{\Gamma\left(1-\frac{\alpha-\beta}{3}\right)}, \quad (S1.24)$$

112 and

$$113 \quad N_g(t) = \int_0^{t_{cr}} G_c \bar{E}_c (t-\tau)^{-\alpha} d\tau = \frac{\bar{E}_c G_c}{1-\alpha} t_{surf}^{1-\alpha} \doteq N_g^{cr}(t). \quad (S1.25)$$

114 1.4 Post-wrinkling of viscoelastic epithelial tissues during growth.

115 Following wrinkling ($t \geq t_{surf}$), the post-buckling morphological evolution
 116 exhibiting a pronounced time-dependent behavior and geometric nonlinearity with
 117 cumulative growth. The nonlinear membrane force $N_m(x, t)$ becomes non-negligible
 118 for rapid growth wrinkle amplitude, the spatial average expressed as
 119 $N_m^{avg}(t) = \int_0^t \bar{E}_c(t-\tau) d\left\langle (\partial w / \partial x)^2 / 2 \right\rangle / d\tau d\tau$. Then, the equilibrium equation can be
 120 expressed as,

$$121 \quad \int_0^t D_f(t-\tau) \frac{\partial}{\partial \tau} \left(\frac{\partial^4 w}{\partial x^4}(x, \tau) \right) d\tau - N_x^{\text{total}}(t) \frac{\partial^2 w}{\partial x^2} + p(x, t) = 0, \quad (\text{S1.26})$$

122 where the deflection function is $w(x, t) = A(t) \cos(kx)$, the submucosa force is

$$123 \quad p(x, t) = \frac{k}{2} \gamma_s \int_0^t E_s(t-\tau) \frac{\partial w}{\partial \tau}(x, \tau) d\tau \quad \text{with} \quad \gamma_s = (1/t_{\text{surf}}) / (G_s + 1/t_{\text{surf}}) \quad \text{and the total in-}$$

124 plane force is $N_x^{\text{total}}(t) = N_x^g(t) + N_x^m(t)$ with $N_x^g(t) = h \int_0^t \bar{E}_c(t-\tau) G_c d\tau$ and

$$125 \quad N_x^m(t) = \frac{hk^2}{4} \int_0^t \bar{E}_c(t-\tau) \frac{d}{d\tau} [A^2(\tau)] d\tau. \quad \text{Then, the governing equation for amplitudes}$$

126 can be expressed as

$$127 \quad \begin{aligned} & k^4 \int_0^t D_f(t-\tau) \dot{A}(\tau) d\tau - k^2 A(t) \left[N_x^g(t) + \frac{hk^2}{4} \int_0^t E_f(t-\tau) \frac{d}{d\tau} [A^2(\tau)] d\tau \right] \\ & + \frac{k}{2} \gamma_s \int_0^t E_s(t-\tau) \dot{A}(\tau) d\tau = 0 \end{aligned} \quad (\text{S1.27})$$

128 The above equation represents a complete integral-differential equation concerning the

129 amplitude. Within the dynamic assumption, the system evolves slowly, and the

130 convolution integral is approximated as $\int_0^t K(t-\tau) \dot{A}(\tau) d\tau \approx K_{\text{eff}}(t) \dot{A}(t)$ with

$$131 \quad K_{\text{eff}}(t) = \int_0^t K(\tau) d\tau. \quad \text{For viscoelastic tissues, one has} \quad D_f^{\text{eff}}(t) = \int_0^t D_f(\tau) d\tau \quad \text{and}$$

$$132 \quad E_s^{\text{eff}}(t) = \int_0^t E_s(\tau) d\tau. \quad \text{Then, the Landau-type amplitude evolution equation is expressed}$$

133 as,

$$134 \quad \dot{A}(t) = \frac{k^2 N_x^g(t) A(t)}{k^4 D_f^{\text{eff}}(t) + \frac{k}{2} \gamma_s E_s^{\text{eff}}(t) - \frac{hk^4}{2} E_f^{\text{eff}}(t) A^2(t)}. \quad (\text{S1.28})$$

135 Under small deformation conditions, the nonlinear terms can be neglected and then

136 there is $\dot{A}(t) = \psi(t) A(t)$ with $\psi(t) = k^2 N_x^g(t) / \left(k^4 D_f^{\text{eff}}(t) + \frac{k}{2} \gamma_s E_s^{\text{eff}}(t) \right)$. Taking into

137 account the nonlinear large deformation term, the complete nonlinear Landau equation

138 is expressed as (i.e., Eq.3 in the main text),

$$139 \quad \dot{A}(t) = \psi(t) A(t) - \eta(t) A^3(t), \quad (\text{S1.29})$$

$$\psi(t) = \frac{k^2 N_x^g(t)}{k^4 D_f^{\text{eff}}(t) + \frac{k}{2} \gamma_s E_s^{\text{eff}}(t)}, \quad \eta(t) = \frac{h k^6 E_f^{\text{eff}}(t) N_x^g(t)}{2 \left[k^4 D_f^{\text{eff}}(t) + \frac{k}{2} \gamma_s E_s^{\text{eff}}(t) \right]^2}, \quad (\text{S1.30})$$

where the linear growth rate $\psi(t)$ reflects the balance between the driving force and the resistance, and the nonlinear saturation coefficient $\eta(t)$ reflects the intensity of geometric hardening.

Then, in order to obtain the saturation amplitude and the amplitude saturation time, we separate the variables in the above equation, and have $dA/A(\psi - \eta A^2) = dt$. We can further get

$$\frac{1}{\psi} \ln \left| \frac{A}{\sqrt{\psi - \eta A^2}} \right| = t + C. \quad (\text{S1.31})$$

Taking the initial conditions $t = t_{\text{surf}}$, $A = A_0$ and $\ln \left| A_0 / \sqrt{\psi - \eta A_0^2} \right| = \psi(t_{\text{surf}} + C)$, the saturation amplitude and time can be obtained as $A_{\text{sat}} = \sqrt{\psi / \eta}$ and $A(t_{\text{sat}}) = \rho A_{\text{sat}}$ with $\rho \approx 0.95$, respectively. The solution is:

$$t_{\text{sat}} = t_c^{\text{surf}} + \frac{1}{\psi} \ln \left(\frac{\rho A_{\text{sat}}}{A_0} \frac{\sqrt{\psi - \eta A_0^2}}{\sqrt{\psi(1 - \rho^2)}} \right). \quad (\text{S1.32})$$

Then, the saturation time can be approximately calculated as

$$t_{\text{sat}} \approx t_c^{\text{surf}} + \frac{\ln(A_{\text{sat}}/A_0)}{\psi_{\text{sat}}}. \quad (\text{S1.33})$$

1.5 Finite element modeling for the mucosal epithelium and submucosa.

To explore surface morphological formation and evolution of viscoelastic soft tissues in growth, we perform numerical simulations using the finite element method with ABAQUS/STANDARD, as shown in Fig.S1. We discretize this nonlinear problem of finite growth using double-precision hybrid plane strain elements (CPE8H) for the mucosal epithelium and the submucosa, coupling single-precision reduced integration plane strain elements (CPE4R) for the substrate. The layered tissue (mucosal epithelium and the submucosa) and substrate are adopted as incompressible power-law viscoelastic material, which are characterized by the Prony series and more rigid elastic material,

respectively¹⁰. We apply thermal dilatation to model volumetric growth and gradually increase the growth rate (G_c and G_s) in time. To investigate the biologically relevant post-buckling behavior of the growing viscoelastic tissue, we adopt the pseudo-dynamic method by introducing a tiny dissipated energy fraction as 0.02%. Notably, the critical wavelength in our simulations are consistent with the theoretical prediction as shown in Fig. S2.

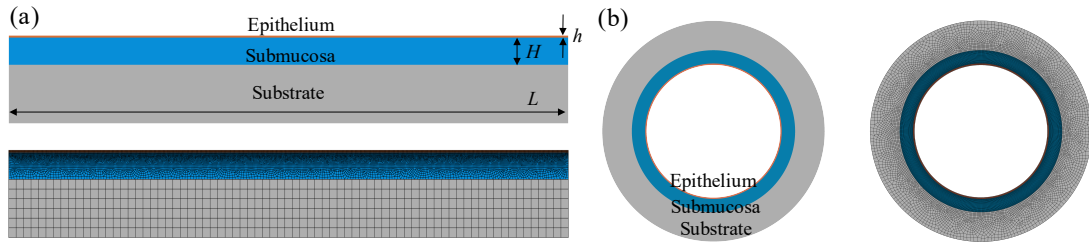


Fig. S1. The bilayer viscoelastic epithelial tissue model under planar constraints (a) and annular constraints (b). In which, the mucosal epithelium thickness is h , the submucosa thickness is H , and the tissue length is L , respectively.

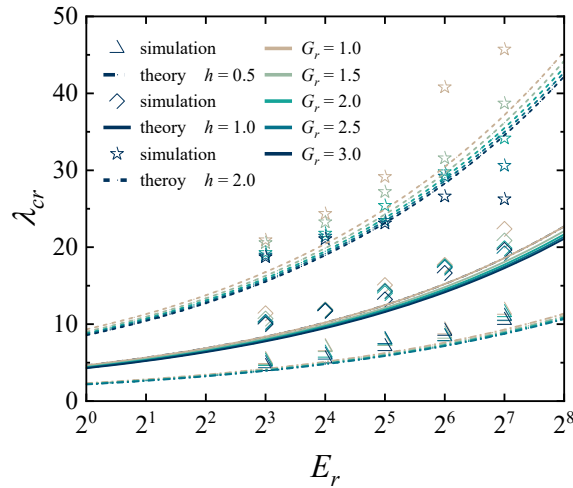


Fig. S2 Comparison of finite element simulation results with theoretical results.

Supplementary Note 2. Global buckling of the mucosal epithelium and submucosa.

Based on the Timoshenko beam theory in elastic theory framework, the bending critical force considering shear is expressed as,

$$P^{cr} = \frac{P_0^{cr}}{1 + mP_0^{cr}/(\mu A)}, \quad (S2.1)$$

where $P_0^{cr} = n^2 \pi^2 EI / l^2$ represents the critical bending force of an n-order Euler beam,

181 μ represents the shear modulus, $A_0 = A/m$ represents the corrected cross-sectional
 182 area, and the correction coefficient is taken as $m = 1.2$. The equivalent axial stiffness
 183 is $EA_{eq} = \bar{E}_c h + \bar{E}_s H$, the equivalent shear stiffness is
 184 $GA_{eq} = (h + H)\mu_c \mu_s / (\mu_c H + \mu_s h)$, and the equivalent bending stiffness is,

$$185 \quad EI_{eq} = \frac{(\bar{E}_c h^2 - \bar{E}_s H^2) + 4\bar{E}_c \bar{E}_s hH (h + H)^2}{12EA_{eq}}. \quad (S2.2)$$

186 Therefore, the critical force for global buckling can be expressed as,

$$187 \quad P^{cr} = \frac{P_0^{cr}}{1 + 1.2P_0^{cr}/GA_{eq}}. \quad (S2.3)$$

188 If we extend the above derivation to the theory of viscoelastic Timoshenko beams, there
 189 is,

$$190 \quad \int_0^t EI_{eq}(t - \tau) \frac{\partial^5 v}{\partial x^4 \partial \tau} d\tau + P_g(t) \frac{\partial^2 v}{\partial x^2} = 0, \quad (S2.4)$$

191 where the buckling mode is $v(x, t) = B(t) \sin(\pi x/L)$. The critical load can be obtained

$$192 \quad P_{cr}(t) = \frac{\pi^2 EI_{eq}(t)}{L^2}, \quad (S2.5)$$

193 and, considering the shear deformation within Timoshenko beam theory, Eq. (S2.5)
 194 becomes

$$195 \quad P_{cr}(t) = \frac{\pi^2 EI_{eq}(t)}{L^2 + \frac{1.2\pi^2 EI_{eq}(t)}{GA_{eq}(t)}} \quad (S2.6)$$

196 The lateral force caused by growth can be expressed as

$$197 \quad P_g(t) = b \frac{G_c h + G_s H}{h + H} \int_0^t [E_c(t - \tau)h + E_s(t - \tau)H] d\tau, \text{ and for power-law viscoelastic}$$

$$198 \quad \text{tissues, } P_g(t) = b \frac{G_c h + G_s H}{h + H} \left[\frac{E_{c0} h t^{1-\alpha}}{1-\alpha} + \frac{E_{s0} H t^{1-\beta}}{1-\beta} \right]. \text{ Then, the critical time equation for}$$

199 soft tissue buckling instability can be expressed as

$$P_g(t_c^{\text{global}}) = P_{cr}(t_c^{\text{global}}) \Leftrightarrow$$

$$b \frac{G_c h + G_s H}{h + H} \left[\frac{E_{c0} h (t_c^{\text{global}})^{1-\alpha}}{1-\alpha} + \frac{E_{s0} H (t_c^{\text{global}})^{1-\beta}}{1-\beta} \right] = \frac{\pi^2 EI_{eq}(t_c^{\text{global}})}{L^2 + \frac{1.2\pi^2 EI_{eq}(t_c^{\text{global}})}{GA_{eq}(t_c^{\text{global}})}}. \quad (\text{S2.7})$$

Supplementary Note 3. Hierarchical instability of the mucosal epithelium and submucosa: coupled evolution of surface wrinkling and global buckling.

The total deformation of hierarchical instability can be expressed as $w_{\text{total}}(x, t) = A(t) \cos(kx) + B(t) \sin(\pi x / L)$, and the potential energy can be given by,

$$\Pi[A, B] = \Pi_{\text{surf}} + \Pi_{\text{global}} + \Pi_{\text{couple}}. \quad (\text{S3.1})$$

In which, the potential energies of the surface wrinkling, global buckling and the geometric deformation coupling are expressed as, respectively,

$$\Pi_{\text{surf}}[A] = \frac{L}{4} \left[k^4 \int_0^t D_f(t-\tau) \dot{A}^2(\tau) d\tau + k^2 N_x^{\text{total}} A^2 \right], \quad (\text{S3.2})$$

$$\Pi_{\text{global}}[B] = \frac{1}{2} \left[\int_0^t EI_{eq}(t-\tau) \dot{B}^2(\tau) d\tau + \left(\frac{\pi^4 EI_{eq}(t)}{L^4} - \frac{\pi^2 P_g(t)}{L^2} \right) B^2 + \frac{\eta_B}{2} B^4 \right],$$

(S3.3)

$$\Pi_{\text{couple}}[A, B] = -\frac{1}{2} \int_0^L \kappa A^2 B^2 \phi^2(x) dx = -\frac{L}{4} \kappa A^2 B^2, \quad (\text{S3.4})$$

with the coupling coefficient $\kappa = \frac{k^2 \pi^2}{L^2} E_c(t)$ and $\phi(x) = \sin(\pi x / L)$. The coupled evolution equation can be derived by the variational formula, $\delta \Pi = 0$. In which, the amplitude evolution equation of the surface wrinkling and global buckling are expressed as, respectively,

$$\frac{d}{dt} \left(\frac{\partial \Pi}{\partial \dot{A}} \right) - \frac{\partial \Pi}{\partial A} = 0, \quad (\text{S3.5})$$

$$\frac{d}{dt} \left(\frac{\partial \Pi}{\partial \dot{B}} \right) - \frac{\partial \Pi}{\partial B} = 0. \quad (\text{S3.6})$$

Consider the quasi-static assumption, the coupled evolution equations are expressed as,

$$\begin{aligned}
& \left[k^4 D_f(t) + \frac{k}{2} \gamma_s E_s(t) - k^2 N_x^g(t) \right] A - \frac{hk^4}{4} E_f(t) A^3 + \frac{L}{2} \kappa A B^2 = 0 \\
& \left[\frac{\pi^4 E I_{eq}(t)}{L^4} - \frac{\pi^2 P_g(t)}{L^2} \right] B + \eta_B B^3 + \frac{L}{2} \kappa A^2 B = 0
\end{aligned} \quad (S3.7)$$

Then, the stability conditions of hierarchical instability can be defined as,
the first-order condition

$$\frac{\partial \Pi}{\partial A} = 0, \quad \frac{\partial \Pi}{\partial B} = 0, \quad (S3.8)$$

and second-order condition

$$\begin{cases} \frac{\partial^2 \Pi}{\partial A^2} > 0, \quad \frac{\partial^2 \Pi}{\partial B^2} > 0, \quad \frac{\partial^2 \Pi}{\partial A \partial B} < 0 \\ \det(H) = \frac{\partial^2 \Pi}{\partial A^2} \frac{\partial^2 \Pi}{\partial B^2} - \left(\frac{\partial^2 \Pi}{\partial A \partial B} \right)^2 > 0 \end{cases} \quad (S3.9)$$

The existence of hierarchical instability corresponds to a local minimum $(A, B) \neq (0, 0)$,
and it satisfies

$$\Pi_{\text{multi}}(A^*, B^*) < \min(\Pi_{\text{surf}}(A_{\text{sat}}, 0), \Pi_{\text{global}}(0, B_{\text{sat}})). \quad (S3.10)$$

In which, the potential energies of the pure surface wrinkling, the pure global buckling,
and the hierarchical instability are expressed as, respectively (for Eq. 5 in main text),

$$\Pi_{\text{surf}} = \frac{L}{4} \left[k^4 D_f A_{\text{sat}}^2 + k^2 N_x^g A_{\text{sat}}^2 + \frac{k}{2} \gamma_s E_s A_{\text{sat}}^2 - \frac{hk^4}{4} E_f A_{\text{sat}}^4 \right], \quad (S3.11)$$

$$\Pi_{\text{global}} = \frac{1}{2} \left[\frac{\pi^4 E I_{eq}}{L^4} B_{\text{sat}}^2 - \frac{\pi^2 P_g}{L^2} B_{\text{sat}}^2 + \frac{\eta_B}{2} B_{\text{sat}}^4 \right], \quad (S3.12)$$

$$\Pi_{\text{multi}} = \Pi_{\text{surf}}(A^*) + \Pi_{\text{global}}(B^*) - \frac{L}{4} \kappa A^{*2} B^{*2}. \quad (S3.13)$$

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