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Effects of Corrugated Bed Roughness on the Flow Dynamics of Submerged Hydraulic Jumps: A Comparative Study Using Large Eddy Simulation

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Abstract

This study offers a comparative analysis of the flow characteristics of a submerged hydraulic jump formed by a wall jet emerging from a sluice gate on smooth and sinusoidal corrugated bed geometries. The submerged jump phenomenon was numerically simulated for various bed configurations using a three-dimensional Large Eddy Simulation (LES) model combined with the volume of fluid (VOF) method for free-surface tracking. The numerical simulation results were validated against previously reported experimental data. Three different sinusoidal macro-roughnesses with different wavelengths and crest heights were considered, along with the smooth bed, to study the effects of these roughened surfaces on flow parameters, particularly the free-surface profile, mean velocity field, pressure distribution and turbulence characteristics. The simulated flow patterns reveal that the wall jet emerging from the sluice opening grazes the wall boundary in the smooth-bed case, whereas the presence of corrugated roughness causes the submerged jet to lift away from the bed towards the free surface, leading to differences in the evolution of the shear layer and a shorter submerged roller length. The overall flow characteristics of the submerged jump are significantly influenced by the geometry of the corrugations. Corrugated beds with high wave steepness exhibit the shortest submerged rollers and the rapid decay of streamwise velocity of the wall jet. In contrast, the corrugated bed with the smallest wavelength yields turbulence characteristics closer to those of the smooth bed within the jet layer. Furthermore, the energy dissipation of the wall jet within the submerged jump occurs over a shorter streamwise distance in the presence of the corrugated aprons, suggesting a potential for designing cost-effective stilling basins.

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Statements and Declaration

The authors have no competing interests to declare that are relevant to the content of this article.

1. Introduction

Hydraulic structures, such as sluice gates and stilling basins in channels, which are designed to operate at low heads, often operate under submerged conditions due to higher downstream flow depth, leading to the formation of submerged jumps [1]. A submerged hydraulic jump is formed when the tailwater depth of the free hydraulic jump exceeds its subcritical sequent depth [2]. This flow phenomenon has been characterized as a two-dimensional plane turbulent wall jet under an adverse pressure gradient by Rajaratnam [3]. Subsequently, Wu and Rajaratnam [4] classified the submerged jump as a transition between the free jump and the classical plane wall jet.

The experimental studies by Long et al. [5], Ead and Rajaratnam [6], and De Dios et al. [7] provide details on the velocity and turbulence characteristics of submerged hydraulic jumps, offering comprehensive insight into this flow phenomenon. Alongside experimental studies, numerical simulations of the submerged hydraulic jump in a two-dimensional (2D) numerical model have been conducted by solving the Reynolds Averaged Navier-Stokes (RANS) equations, primarily using eddy viscosity models for turbulence closure [8-12]. However, relatively recent three-dimensional (3D) simulations of submerged sluice gate flows using the Detached Eddy Simulation (DES) by Jesudhas et al. [13] have studied the large-scale turbulence structures and instantaneous flow features under fixed submergence conditions.

While the majority of the previously mentioned studies were conducted for submerged jumps forming over smooth beds, the study of the submerged wall jet over rough beds is equally important [14]. Experimental studies by Aamir and Ahmad [15] and Chaudhary et al. [16] have conclusively shown that apron bed roughness positively influences the local scour downstream in the unprotected mobile bed due to submerged jumps. Ead and Rajaratnam [17] investigated the impact of macro roughness in the form of sinusoidal corrugated beds on plane turbulent wall jets. Dey and Sarkar [18] experimentally examined the hydraulic and physical properties of a submerged hydraulic jump forming over a sand-ingrained bed. Dey et al. [14] conducted a comparative experimental study of turbulent flow characteristics of submerged wall jets over smooth and fully rough beds surfaced with gravel particles having varying mean sizes. Nasrabadi et al. [19] demonstrated the effect of suspended

sediments on the characteristics of submerged jumps over smooth beds and beds with sand-grain roughness. Wang and Diao [20] extended the work of Ead and Rajaratnam [17] by varying the inlet and submergence conditions for the same sinusoidal corrugation geometry of the apron. Chowdhury et al. [21] and Chowdhury [22] studied the characteristics of the submerged horizontal wall jet over a bed roughened with an array of hemispherical macro-rough elements. Ghaderi et al. [23, 24] investigated the impact of various macro-roughness element shapes and their arrangement on the characteristics of submerged jumps using RANS simulations.

The flow pattern of a submerged jump forming immediately downstream of a sluice gate located over a horizontal smooth apron is shown schematically in Figs. 1(a and b), where y_1 is the gate opening or the initial jet thickness, y_t is the tailwater depth, and U_1 is the inlet jet velocity. The submergence ratio (S) and the inlet jet Froude number (Fr) can be defined as $(y_t - y_2)/y_2$ and $U_1/(gy_1)^{0.5}$, respectively, where y_2 is the subcritical sequent depth calculated as $y_2 = 0.5y_1(-1 + \sqrt{1 + 8Fr^2})$, and g is the acceleration due to gravity. When a submerged jump forms over a bed with either micro or macro-scale roughness, the flow within the jump is characterized by that roughness in addition to Fr and S . In both physical model studies and numerical experimentations, submergence is guided by the experimentally controlled level, y_t , while Fr is regulated by the inflow discharge and y_1 . As the flow evolves downstream of the submerged sluice gate, it may be demarcated into three distinct regions along the streamwise direction as the developing zone, the fully developed zone, and the recovering zone, as depicted in Fig. 1(a) [5, 14]. The flow region of primary interest in this study is the fully developed zone, of which the typical time-averaged streamwise velocity ($\langle u \rangle$) profile of the submerged wall jet is presented in Fig. 1(b), along with the different layers defined according to the flow behaviour [14, 21]. The first layer is the inner jet layer, which extends from the bed to the point where $\langle u \rangle$ reaches its maximum value ($\langle u \rangle_m$) at a vertical distance δ_1 from the bed, and is similar to a boundary layer. Beyond this is the outer jet layer which extends from δ_1 to the outer edge of the jet, which is marked off by the inflection point of the $\langle u \rangle$ distribution ($\partial^2 \langle u \rangle / \partial y^2 = 0$). Moving further outward, there exists the circulatory flow layer, formed due to the interaction of the wall jet with the ambient fluid. The circulatory flow layer

may again be subdivided into inner and outer layers, in which the distribution of $\langle u \rangle$ is positive in the former, while in the outer layer it becomes negative, indicating a reverse flow. The vertical distance δ_2 from the bed, where the velocity becomes half of $\langle u \rangle_m$ is termed as the half-width of the submerged wall jet and serves as an important parameter to scale the velocity distribution.

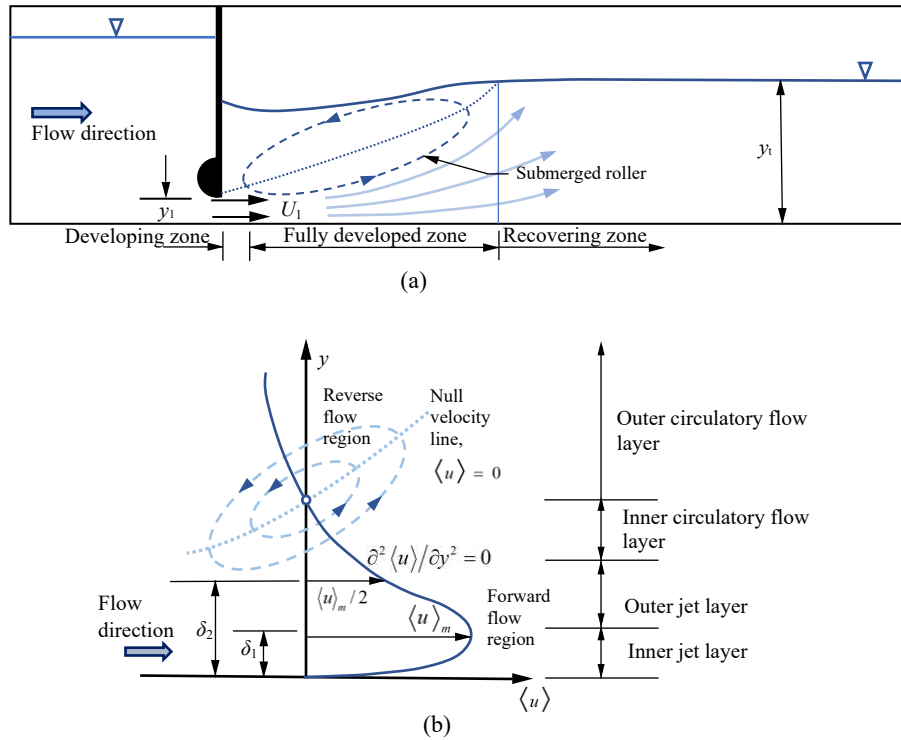


Fig. 1 (a) Schematic diagram of a submerged hydraulic jump on a smooth bed; (b) typical time-averaged streamwise velocity distribution in the fully developed zone

Despite the large number of studies reported on submerged jumps over smooth and rough beds, there remains a limited body of research specifically investigating the submerged hydraulic jump or submerged wall jet flows over a macro-rough boundary [17, 20-24]. Ead and Rajaratnam [17] and Wang and Diao [20] primarily addressed the velocity characteristics and length scales associated with the submerged jump phenomenon. Chowdhury et al. [21] and Chowdhury [22] provided a comprehensive analysis of the velocity and turbulence fields associated with a submerged wall jet over a horizontal boundary fitted with hemispherical roughness elements, reporting three-dimensional macro-roughness configurations in contrast to largely two-dimensional macro-roughness setups. However, in these studies, the geometry of the hemispherical elements has been held constant, and so has been the submergence condition. Furthermore, the studies have predominantly been analyzed as two-

dimensional, with all results being presented along the central vertical plane. Ghaderi et al. [23-24], apparently, stand out among the few numerical studies to simulate the submerged hydraulic jump over a macro-rough bed. While the simulations were conducted in 3D, the findings were largely confined to the central vertical plane and focused specifically on the velocity distribution and free-surface profiles. Apparently, therefore, there seems to be a lack of an in-depth understanding of the submerged wall-jet hydraulics over a macro-roughness flow boundary. The present study aims at investigating the effects of aprons with different sinusoidal corrugation geometries on the physical and hydraulic characteristics of submerged hydraulic jumps downstream of a sluice gate. Working with computational fluid dynamics simulations using an LES model, pre-validated against experimental data, comparative analyses of the mean velocity and turbulence characteristics of the submerged wall jet are presented, along with the evolution of the shear layer and pressure distribution for different roughness boundary profiles within the 3D numerical framework.

2. Computational Modeling and Experimentation

In this study, the open-source CFD software package OpenFOAM-v2012 is utilized to develop the numerical model simulating a submerged hydraulic jump over a sinusoidal corrugated bed. The model solves the filtered 3D Navier-Stokes equations, which govern the resolved quantities in LES. The filtered governing equations for conservation of mass and momentum [25] are given by:

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\nu \frac{\partial \bar{u}_i}{\partial x_j} \right) - \frac{\partial \tau_{ij}^{SGS}}{\partial x_j} \quad (2)$$

In the above equations, the overbar ($\bar{}$) denotes the filtered or resolved quantities and is used henceforth for results of the current LES model. The variables u_i are the velocity components along the Cartesian coordinate x_i ($i = 1, 2, 3$ corresponding to the x , y , and z direction, respectively), t is the time, p is the pressure field, ρ is the fluid density, ν is the kinematic viscosity, and $\tau_{ij}^{SGS} = \left(\overline{u_i u_j} - \bar{u}_i \bar{u}_j \right)$ is the subgrid-

scale (SGS) stress tensor which represents the effect of the unresolved fluctuations on the resolved motion. The SGS stress tensor can be split into isotropic and anisotropic components, and using the Eddy viscosity theory, the anisotropic component can be modelled with:

$$\tau_{ij}^{SGS} = -2\nu_t \bar{S}_{ij} + \frac{1}{3} \delta_{ij} \tau_{kk}^{SGS} \quad (3)$$

where ν_t is the kinematic eddy viscosity, $\bar{S}_{ij} (= 1/2 (\partial \bar{u}_i / \partial x_j + \partial \bar{u}_j / \partial x_i))$ is the filtered strain rate tensor, δ_{ij} is the Kronecker delta function, and τ_{kk}^{SGS} is the sum of the SGS normal stresses which is twice the kinetic energy of the SGS fluctuations.

In the present study, the wall-adapting local eddy viscosity (WALE) model proposed by Nicoud and Ducros [26] was employed as the explicit SGS model for the unresolved subgrid-scale stresses. The WALE model offers notable advantages, including its ability to account for near-wall effects without the requirement of wall-damping functions and its applicability to complex geometries, since it does not require explicit filtering. The WALE model uses the traceless symmetric part of the square of the resolved velocity-gradient tensor $g_{ij} (= \partial \bar{u}_i / \partial x_j)$ to calculate the eddy viscosity as:

$$\nu_t = (C_w \Delta^2) \frac{\left| (G_{ij}^a) \right|^{6/2}}{(\bar{S}_{ij} \bar{S}_{ij})^{5/2} + \left| (G_{ij}^a) \right|^{5/2}} \quad (4)$$

where C_w is the WALE model constant, Δ is the filter width, which is equal to the grid size, and G_{ij}^a is the traceless part of $G_{ij} (= 1/2 (g_{ik} g_{kj} + g_{jk} g_{ki}))$.

The interface of the two-phase flow problem is modeled using the *interIsoFoam* solver [27] included in the OpenFOAM-v2012 package. This solver incorporates the VOF scheme [28], utilizing the *isoAdvector* algorithm for interface capturing. No specific boundary condition is required at the free surface since the VOF method used in the solver treats air and water as a combined flow, as discussed by Bombardelli et al. [29]. The solver performs numerical integration of the 3D filtered Navier-Stokes

and VOF equations by implementing the velocity-pressure coupling using the PIMPLE algorithm, a combination of two widely used algorithms: PISO [30] and SIMPLE [31]. The simulations are conducted on a high-performance computing facility at the Indian Institute of Technology Kharagpur.

A general schematic of the computational domain considered in this study, along with the initial conditions, is presented in Figs. 2(a-c), which shows the sinusoidal wavy corrugations on the apron just downstream of the sluice gate. The corrugated roughness elements act as depressions on the bed rather than protrusions, and the crest level of the corrugations was set at the same level as the bed. For the corrugated beds, the height of the crest from the trough is given by a , and the wavelength by λ . A small gap of length x_1 was kept between the sluice gate and the first corrugation in order not to affect the initial jet velocity at the sluice opening. The depth of the tailwater downstream was regulated by a tail gate whose height was adjusted according to the head-discharge curve for a sharp-crested weir as proposed by Bijankhan et al. [32]. The dimensions of the 3D flow domain were set to a length and height of 6.6m and 0.55m, respectively, with the tail gate located 6.4m from the sluice opening. The width of the domain was fixed at 0.446m, matching the channel width reported by Ead and Rajaratnam [17]. The numerical domain comprised of a uniform mesh with refinements near the walls and the sluice inlet. The top boundary patch was set at a symmetry condition. At the inlet boundary, a constant velocity condition equal to the specified depth-averaged velocity of the initial jet (U_1) was implemented for the water phase, with the inlet patch depth equal to the sluice opening. The inlet condition for air was set to zero. At the outlet boundary, the total pressure was specified to be zero. The velocity was extrapolated at the outlet boundary using a Neumann condition with zero backflow, thereby ensuring a physically stable outflow. The wall boundaries of the bed, sidewalls, sluice gate, and tail gate were considered stationary, where the no-slip condition was specified for the velocity field, and Neumann conditions were set for pressure and volume fraction (α).

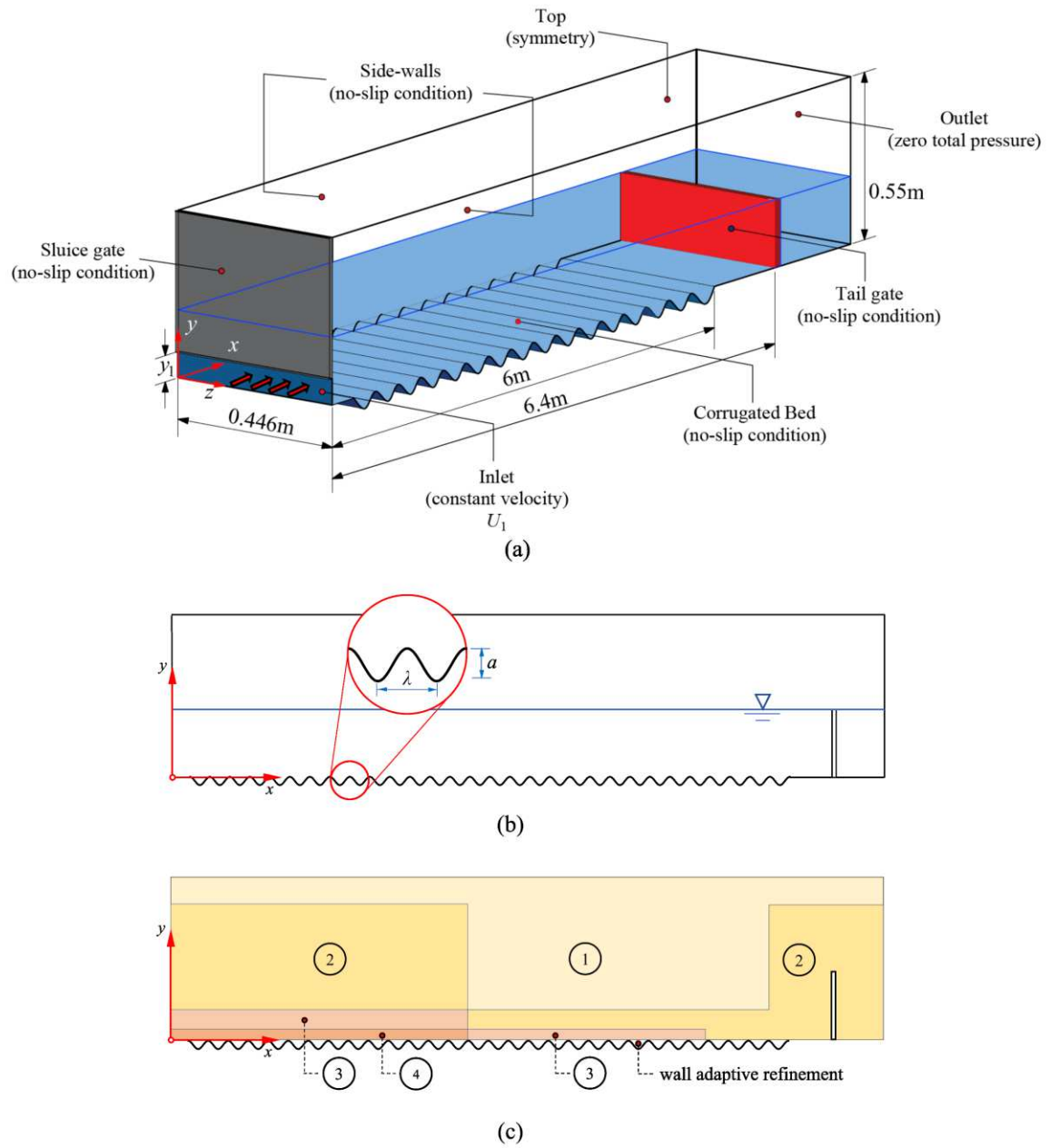


Fig. 2 (a) 3D schematic diagram of the numerical domain with initial conditions for LES of submerged hydraulic jump over corrugated bed, (b) orientation of the sinusoidal corrugations within the computation domain, and (c) mesh refinement zones of the numerical domain for Mesh 2 with different grid sizes and the details are as: (1) 6.7mm, (2) 3.35mm, (3) 1.26mm, (4) 0.63mm (figures are only for representation purposes and not according to scale)

The transient terms in the governing equations were discretized using the first-order implicit Euler scheme. The spatial advection terms were discretized with the total variation diminishing (TVD) scheme, a blend of the first-order upwind and second-order central-differencing scheme, while the diffusion terms were handled with the second-order accurate Gauss linear corrected scheme a

combination that has been shown to yield stable and fairly accurate results [33]. An adaptive time-step method was used with a maximum Courant-Friedrichs-Lewy (CFL) number fixed at 0.8, and an initial time step of 10^{-4} s.

The flow was initialized with the water level set to match the tail gate height (as shown in Fig. 2a), with both the air and water phases having no initial velocity. With the passage of time, the simulation progresses to increase the tailwater depth till it reaches the experimental tailwater depth specified in the experimental data [17], leading to a quasi-steady state. As may be observed from Fig. 3 this state is achieved after approximately 20s. Time-averaged mean quantities and turbulence statistics were calculated from 25s onward, at every second time step, for at least 30s.

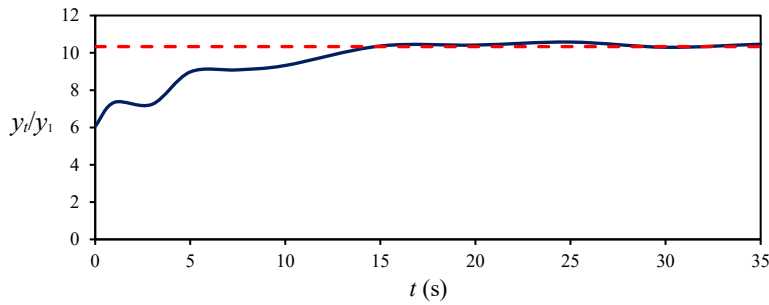


Fig. 3 Comparison of the temporal variation of the tailwater depth obtained from the simulation with the experimental tailwater depth reported by Ead and Rajaratnam [17]

A sensitivity analysis was conducted to assess the dependence of the simulation results on mesh size and grid resolution. The details of the grid and the spatially averaged non-dimensional length scale y^+ ($y^+ = yu_* / \nu$, where u_* is the shear velocity and ν is the kinematic viscosity) are given in Table 1. Figure 4 compares the simulation results of normalized streamwise velocities obtained with the three different mesh sizes alongside the experimental data from Ead and Rajaratnam [17]. To assess accuracy, the Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) for the parameters of Fig. 4 were calculated as given below:

$$MAE = \frac{1}{N} \sum_{i=1}^N \left| \langle \bar{u} \rangle_{LES} - \langle u \rangle_{exp} \right| \quad (5)$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N \left(\langle \bar{u} \rangle_{LES} - \langle u \rangle_{exp} \right)^2} \quad (6)$$

where $\langle \bar{u} \rangle_{LES}$ and $\langle u \rangle_{exp}$ are the time-averaged filtered streamwise velocity obtained from the simulations and the mean streamwise velocity reported in the experimental data, and N is the number of data points. The MAE and RMSE values for all three mesh sizes are shown in Table 1. The turbulent viscosity (ν_t) measures the dissipative effects of modeled small-scale fluctuations in high-fidelity simulations, such as LES [34]. When ν_t is too high, it can suppress flow unsteadiness, leading to a smooth flow field with reduced unsteady fluctuations. Consequently, Table 1 also lists the maximum and average turbulent viscosity ratios (ν_t/ν) for each mesh resolution. Figure 4, along with the MAE values in Table 1, clearly indicate that the accuracy of the numerical simulations does not improve significantly when the mesh size is refined further beyond that of Mesh 2. Thus, to maintain an optimal balance between accuracy and computational expense, the grid resolution of Mesh 2 is deemed sufficient, and the same mesh size is used for all subsequent simulations.

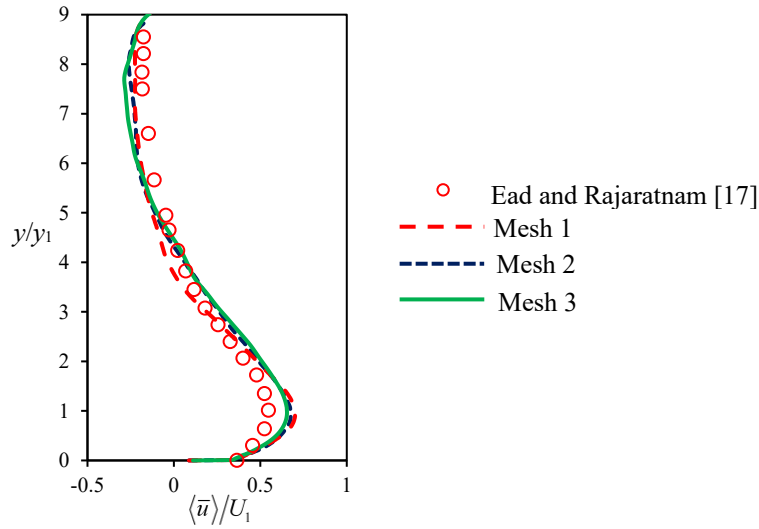


Fig. 4 Comparison of the numerical results obtained from the three different sizes of mesh with the experimental data of Ead and Rajaratnam [17] in terms of normalized mean streamwise velocity ($\langle \bar{u} \rangle / U_1$) within the fully developed zone at streamwise distance $x/y_1 = 16.06$ along the vertical central plane

Table 1. Grid Resolution details for different mesh sizes and the mean absolute errors of the simulated results with respect to different parameters

Mesh	Total number of cells ($\times 10^6$)	Spatially averaged y^+ (near bed)	Average ν_t/ν	Maximum ν_t/ν	MAE for Fig. 4 (m/s)	RMSE for Fig. 4 (m/s)
Mesh 1 (coarse)	5.85	5.71	10.62	103.82	0.198	0.245
Mesh 2 (intermediate)	8.95	4.83	8.51	73.71	0.185	0.211
Mesh 3 (fine)	13.78	3.48	6.84	69.75	0.181	0.205

2.1. Validation

The experimental data of Ead and Rajaratnam [17] for submerged hydraulic jumps over sinusoidal corrugated roughness elements were used to validate the current numerical model. In the numerical simulation, the flow parameters were configured with $Fr = 4$ and $S = 1$, while the bed roughness parameters were set to $a = 21.8\text{mm}$ and $\lambda = 68\text{mm}$, matching those with the experimental data. A comparison between the observed data and the simulated results for the mean velocity and free surface profiles generated by the 3D LES model is presented in Fig. 5. The free surface profile downstream of the sluice gate, as obtained from surface tracking within the simulation using $\alpha = 0.5$, appears to agree well with the experimental measurements, despite some minor discrepancies. Additionally, the comparison of the normalized mean streamwise velocity ($\langle \bar{u} \rangle / U_1$) profiles at different locations indicates that the present numerical model produces reasonably satisfactory results, although there are some inconsistencies in the velocity profiles, particularly in the proximity of the point of maximum velocity. Unfortunately, the results of the turbulence parameters obtained from the numerical model could not be compared due to the unavailability of experimental data. However, the overall agreement of the experimental data and numerical results indicates that the present 3D LES numerical model is capable of adequately predicting the flow characteristics of submerged jumps over sinusoidal corrugated beds.

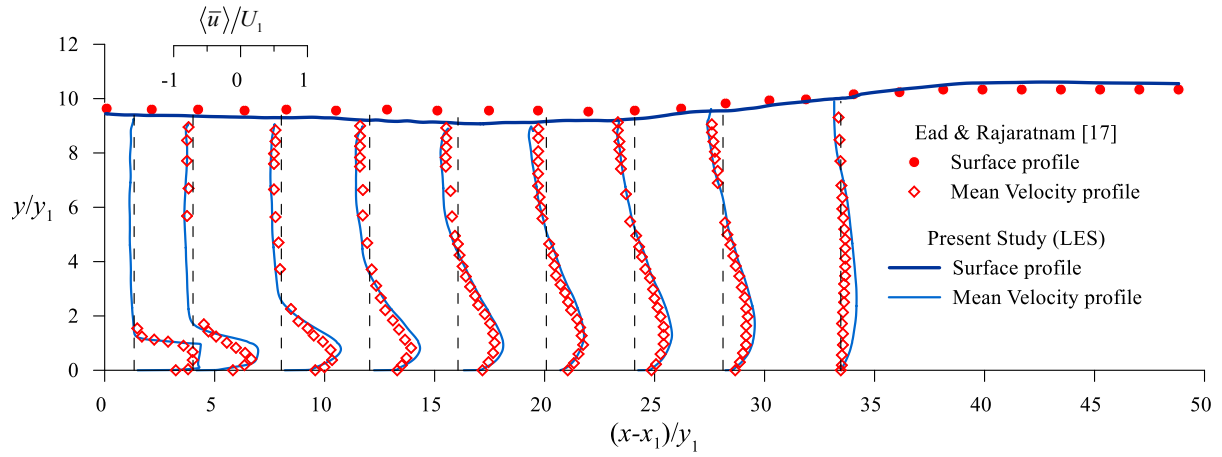


Fig. 5 Comparison of the normalized mean streamwise velocity ($\langle \bar{u} \rangle / U_1$) and free surface profile of the overall domain at the central vertical plane

3. Analysis of Results

With the 3D LES numerical model successfully validated, additional simulations are undertaken with a constant height of tail gate to maintain the tailwater depth for evaluating the effect of the corrugated roughness elements on the flow characteristics. As outlined in Table 2, four different bed conditions are considered with different combinations of corrugation wavelength (λ) and crest height (a). The comparison of these results highlights how the bed roughness conditions of the submerged hydraulic jumps influence the flow physics. The bed conditions are also characterized by the wave steepness ratio (a/λ) calculated by the bed geometry conditions.

Table 2. Flow conditions for the numerical simulations used in the present study.

Run	Bed Type	Froude No. Fr	Initial jet thickness y_1 (mm)	Tailwater depth y_t (m)	Initial jet velocity U_1 (m/s)	Corrugation wavelength λ (mm)	Corrugation crest height a (mm)	Wave Steepness ratio a/λ	Submergence ratio $S = (y_t - y_2)/y_2$
S	Smooth	7.35	15	0.301	2.82	0	0	0	1.044
C1	Corrugated	7.35	15	0.305	2.82	50	10	0.2	1.051
C2	Corrugated	7.35	15	0.305	2.82	50	20	0.4	1.052
C3	Corrugated	7.35	15	0.304	2.82	25	10	0.4	1.048

3.1. General Flow Properties

The mean free surface profiles and the locus of points with zero streamwise velocity ($\langle \bar{u} \rangle = 0$) for all the conducted simulations along the central vertical plane of the domain are presented in Fig. 6. The tailwater depths of all the simulated cases are consistent, yet there is a distinct variation in the transition of the surface profile while moving from the sluice gate to the tail end. It is evident that the free surface profile exhibits a more gradual increase in the case of the smooth bed, transitioning to the tailwater surface level at a greater streamwise distance compared to those of the corrugated beds. Among the corrugated bed conditions, the mean surface profiles of C2 and C3 nearly overlap and reach the tailwater level within the shortest distance. However, the free surface profile of C1 transitions into the tailwater depth further downstream than C2 and C3, although the distance is still less than that observed in the smooth bed condition. The locus of points where the mean streamwise velocity is zero, referred to here as the null velocity line, acts as the boundary between the forward and reverse flow within the submerged roller. The mean streamwise velocity is negative above these lines and positive below them. Consequently, the null velocity lines define the longitudinal extent of the submerged roller of the subsequent cases. The submerged jump with the smooth-bed conditions exhibits the largest roller length. In contrast, the introduction of corrugated bed roughness reduces the roller length, as reported by Ghaderi et al. [23]. Among the various corrugated bed conditions, the null velocity lines for C2 and C3 are in close proximity, similar to the mean free surface and form the shortest roller lengths. Conversely, the roller length of the run C1 is greater than the other corrugated bed conditions, though still significantly less than that observed in the smooth bed condition. Here, it can be inferred that the roller length and the overall jump length in runs C2 and C3 are reduced more due to their geometric configuration, which has a higher wave steepness with $a/\lambda = 0.4$.

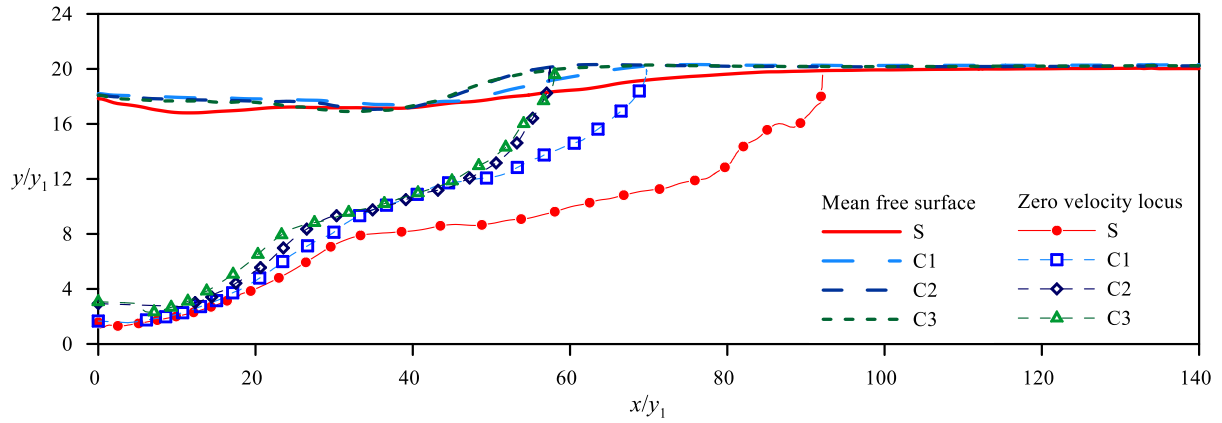


Fig. 6 Surface profiles and the null velocity lines at the central vertical plane of the submerged hydraulic jumps for different bed roughness conditions

3.2. Mean Flow Fields

The mean free surfaces of the simulated submerged jumps over different bed configurations are presented in Figs. 7(a-d), superimposed with the contours of normalized mean streamwise velocity ($\langle \bar{u} \rangle / U_1$). The negative velocity contours over the flow surface before reaching the tailwater depth indicate reverse flow in the roller region. As the flow depth reaches the tailwater level, the flow velocity remains uniformly positive, signifying the flow recovering to be uniform downstream. For submerged jumps over corrugated beds, the peak negative velocity occurs within a narrow band in the region of the abrupt change in the flow surface, while the reverse flow upstream is relatively insignificant. Conversely, in the submerged jump over a smooth bed, the reverse or negative velocity tends to spread across the fully developed region, and the peak value at the surface is lower than in the corrugated bed scenario. The streamwise velocity contours over the flow surface reveal that the phenomenon of the “climb of the wall jet”, first reported by Long et al. [5], is less pronounced in the submerged jumps over the corrugated roughness as opposed to that of the smooth bed case. In the case of smooth bed, strong forward velocity regions exist near the sidewalls immediately downstream of the sluice gate; however, these velocities are noticeably weak in the corrugated bed cases. This indicates that the vertical counter-rotating vortices near the sluice gate are weaker over the corrugated beds than on the smooth bed.

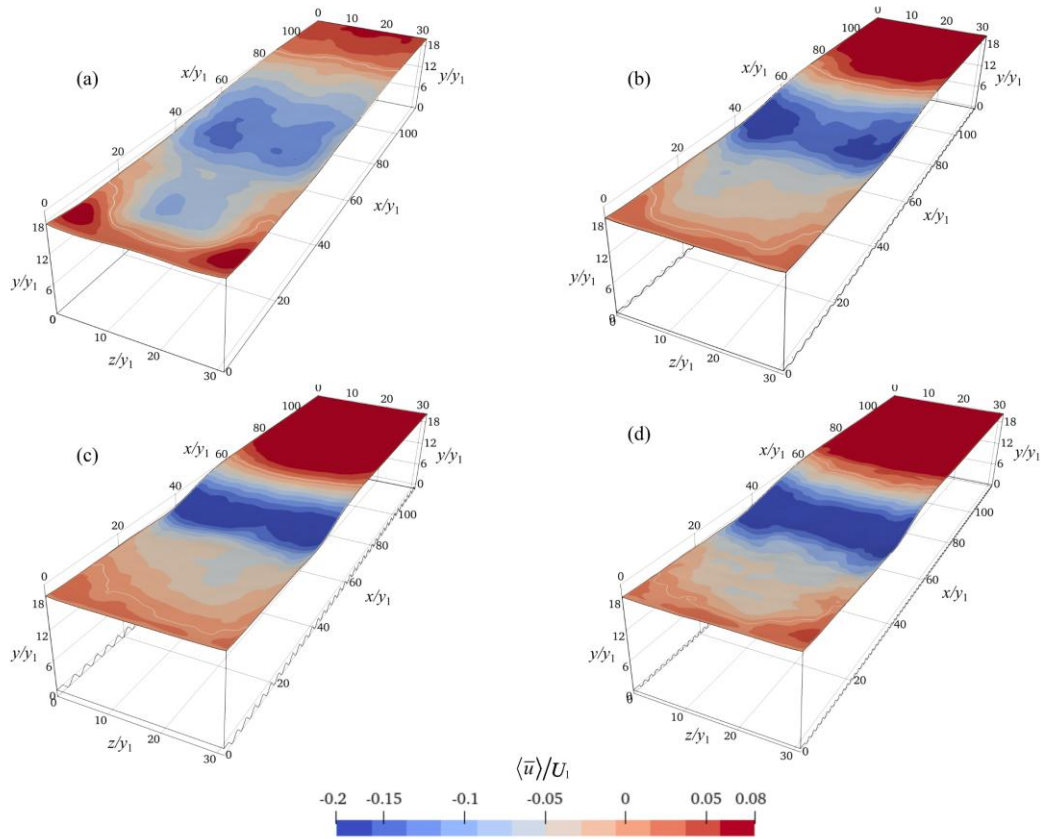


Fig. 7 Normalized mean streamwise velocity ($\langle \bar{u} \rangle / U_1$) contours plotted over the mean surface profile of the submerged jumps with bed roughness conditions: (a) S, (b) C1, (c) C2, and (d) C3. Views looking from upstream (near end) to downstream (far end)

Figures 8(a-h) present the distribution of the normalized mean streamwise velocity of the submerged jumps over different bed conditions across the central vertical plane of the domain. Among these figures, Figs. 8(a-d) portray the overall velocity distribution, whereas Figs. 8(e-h) show the enlarged views of the region close to the sluice opening, with null velocity line also depicted. In the case of the smooth bed (Figs. 8a and e), the wall jet emerging out of the sluice opening grazes along the smooth bed, and its velocity dissipates along the streamwise distance. However, when the wall jet passes over a rough, corrugated bed, a separation of the flow occurs as the jet passes over the crest of the corrugations, creating a circulatory flow within the grooves of the corrugations. Figures 8(f-h) show clearly the extent of recirculation within the grooves, as the flow separates at the wake of the crests of the roughness elements and reattaches to the bed at a point on the windward side just before the subsequent crest. These points, referred to as the point of reattachment, are indicated by arrows in Figs. 8(f-h), with their significance further explained in the subsequent sections. Additionally, in the wake of

the corrugations' crests, a shear layer is created that separates the inner circulatory flow within the groove from the outer wall jet, as previously studied for flow over sinusoidal wavy surfaces [35-37]. For submerged jumps over corrugated beds in the final phase of the fully developed region, the wall jet begins to lift up and away from the bed, moving towards the free surface, rather than continuing to graze the bed as seen in smooth-bed conditions (Figs. 8b-d), which effectively reduces the length of the submerged roller. Additionally, for corrugated beds, the magnitude of the reverse flow within the submerged roller is less significant than for the smooth bed condition. However, the forward flow submerged roller is less significant than for the smooth bed condition. However, the forward flow velocity near the surface downstream of the submerged roller in corrugated beds is observed to be higher.

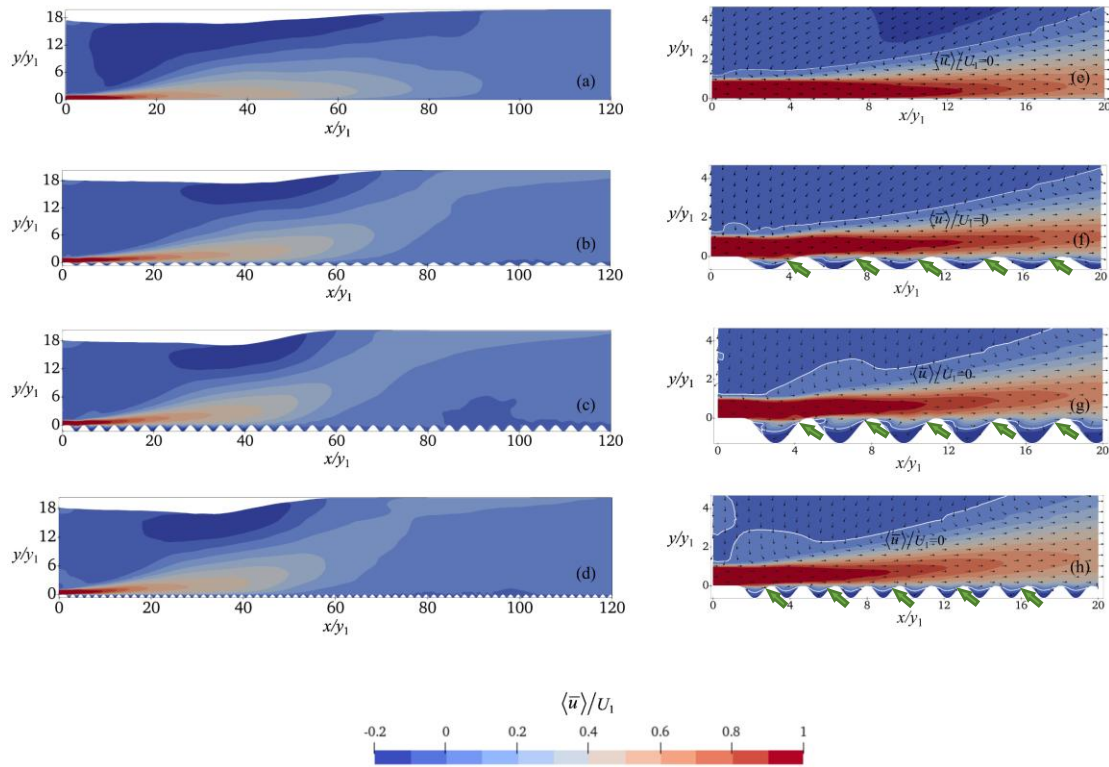


Fig. 8 Distribution of normalized mean streamwise velocity ($\langle \bar{u} \rangle / U_i$) across longitudinal central vertical plane for bed roughness conditions: (a) S, (b) C1, (c) C2, and (d) C3 along with the enlarged views for (e) S, (f) C1, (g) C2, and (h) C3 (The arrows shown in the figures point to the reattachment points)

Figures 9(a-d) illustrate the normalized mean streamwise velocity across the horizontal plane at $y/y_t = 0.5$ of the submerged jumps for different bed conditions. In the case of a smooth bed (Fig. 9a), a prominent reverse flow with negative velocity is observed to emanate from both sides and progress

towards the sluice gate through the centre as the flow experiences boundary layer separation at both sidewalls. As the flow approaches the sluice gate, the velocity of the reverse flow decreases, and a net flow is observed from the centre towards the sidewalls, resulting in patches of positive velocity adjacent to the side walls. This phenomenon leads to the formation of counter-rotating vortices downstream of the sluice gate [7]. However, for submerged jumps over corrugated roughnesses, the mass transfer system in the horizontal plane exhibits some differences. The visualization of the velocity fields in Figs. 9(b-d) indicates that the reverse flow velocity is less than the smooth bed condition, and the counter-rotating vertical vortices are rather weak. Nonetheless, the local peak in positive velocity at the centre of the horizontal plane, away from the sluice gate, indicates the rising jet moving away from the bed, as observed in Figs. 8(b-d).

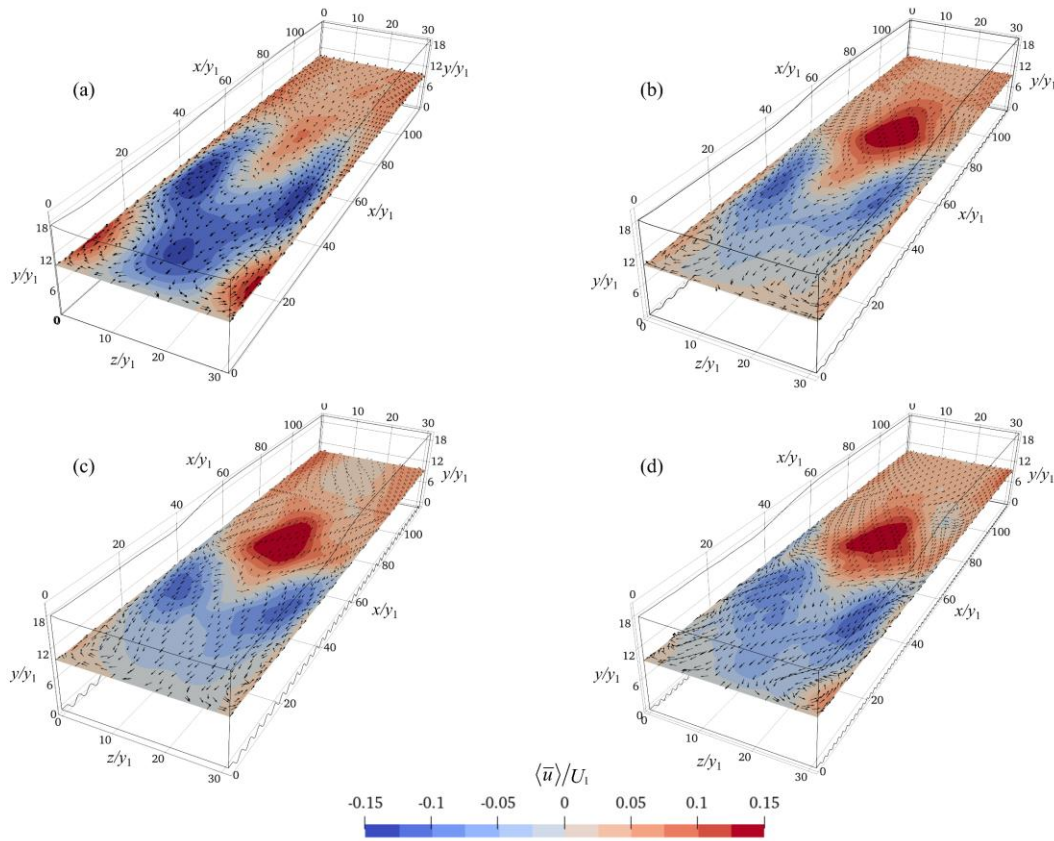


Fig. 9 Distribution of normalized mean streamwise velocity $\langle \bar{u} \rangle / U_1$ across horizontal plane at a distance of $y/y_1 = 0.5$ for bed roughness conditions: (a) S, (b) C1, (c) C2, and (d) C3

The individual mean streamwise velocity profiles of the submerged jump exhibit unique similarity characteristics in the fully developed zone when normalized using the velocity scale, the maximum value of the streamwise velocity at a particular section, and length scale $y = \delta_2$, where

330 $\langle \bar{u} \rangle_{(y=\delta_2)} = \langle \bar{u} \rangle_m / 2$ and $\partial \langle \bar{u} \rangle / \partial y < 0$, termed as half-width of the jet [4-7, 14]. Figures 10(a-d) show the

331 consolidated plot of the normalized mean velocity $(\langle \bar{u} \rangle / \langle \bar{u} \rangle_m)$ profiles against the normalized vertical

332 distance (y/δ_2) for the sections in the fully developed region $(x/y_1 > 12)$ of the submerged wall jets for

333 different bed conditions. These plots demonstrate that the length scale δ_2 effectively merges the

334 individual profiles into a single band in the nondimensional space, supporting the idea of potential

335 similarity in the streamwise velocity profiles for submerged wall jets over both smooth and corrugated

336 beds. Figure 10(a) shows the velocity profile of the submerged jet over a smooth bed, and it follows the

337 plane wall jet curve [4] in the inner and outer layers of the wall jet and thereafter diverges beyond y/δ_2

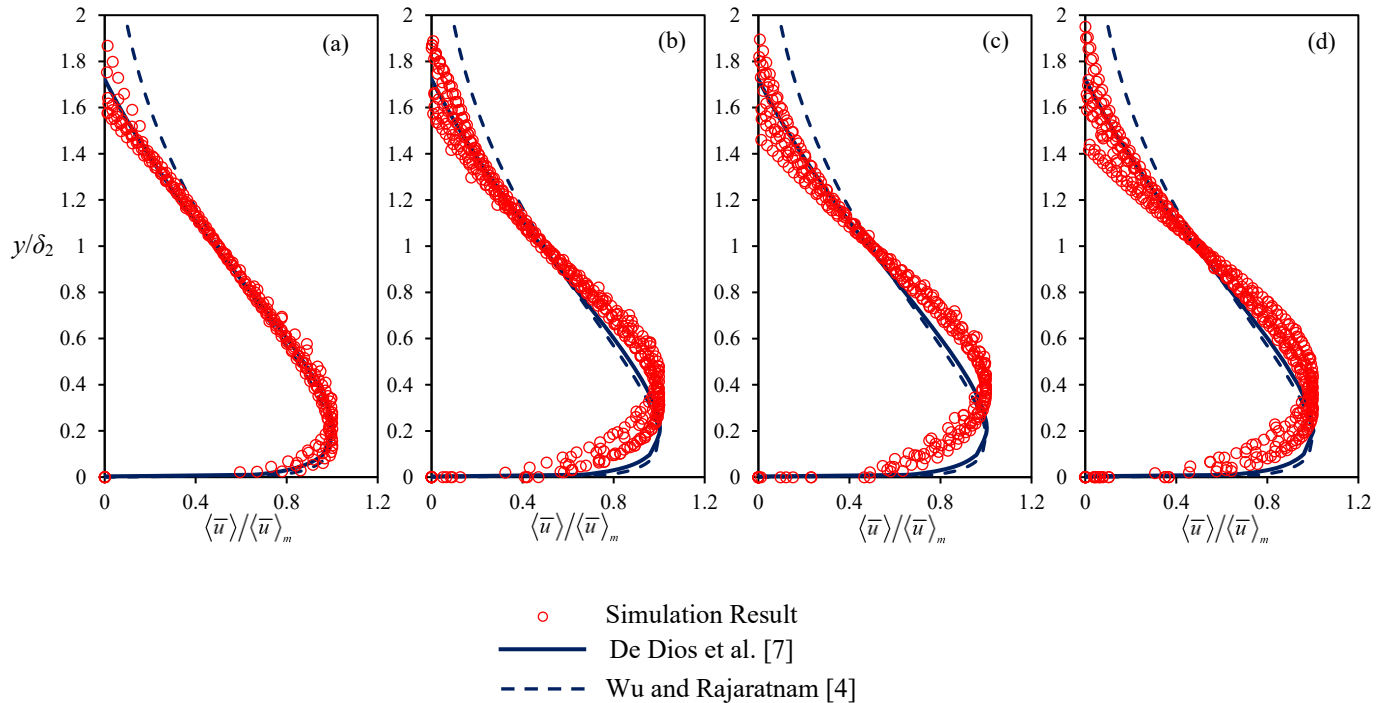
338 $= 1.1$ but closely follows the self-similarity curve for submerged wall jets given by De Dios et al. [7]

339 throughout. Figures 10(b-d), depicting the cases of submerged wall jets over corrugated roughness,

340 exhibit a stark difference from the smooth bed case. These velocity trends illustrate the impact of bed

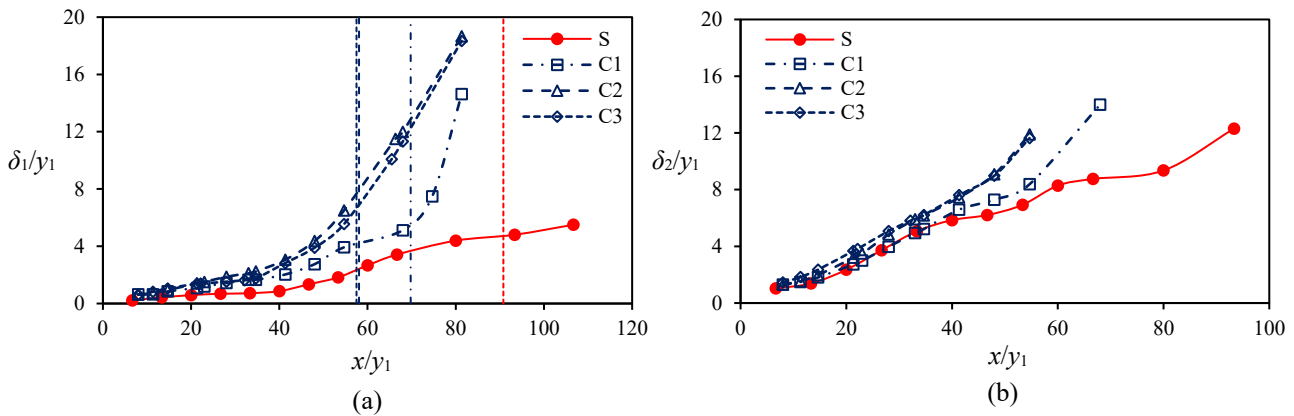
341 roughness on the jet layer, as the local maxima shift away from the wall and follow the same trend as

342 in the smooth-bed condition in the outer recirculatory layer.



343 **Fig. 10** Variation of the normalised longitudinal mean velocity of submerged jump with normalised
344 distance from the bed (y/b) for bed roughness conditions: (a) S, (b) C1, (c) C2, and (d) C3

345 To examine the effect of roughness elements on the jet layer, the variation of the length scales δ_1
346 and δ_2 along the streamwise direction are analyzed. Figure 11(a) presents the normalized inner jet layer
347 thickness (δ_1/y_1) plotted against the normalized streamwise distance. Initially, the δ_1 scales are
348 comparable across all cases, but as the streamwise distance exceeds $x/y_1 = 25$, a notable difference
349 emerges. The extent of the submerged roller for each flow case is also illustrated in Fig. 11(a) with
350 vertical dotted lines. For corrugated bed cases, it may be observed that the vertical scale δ_1 representing
351 the thickness of the inner jet layer increases sharply after the submerged roller, which aligns with the
352 findings presented in Figs. 8(b-d), showing that the wall jet rises steadily towards the flow surface. The
353 variations of the normalized half-width (δ_2/y_1) of the submerged jets for all cases (Fig. 11b) along the
354 streamwise direction are seen to be nearly identical up to around $x/y_1 = 40$, regardless of the bed
355 roughness conditions. However, beyond this point, the curves corresponding to the corrugated beds
356 exhibit a sharper rise. The growth of the vertical scales, δ_1 and δ_2 , is comparable between Runs C2 and
357 C3, but is considerably higher than in the other cases.



358 **Fig. 11** Variation of the normalized vertical length scales (a) δ_1/y_1 , and (b) δ_2/y_1 along the streamwise
359 direction

360 To gain a comprehensive understanding of the submerged jump over the corrugated roughness,
361 the decay of the streamwise velocity is examined and is illustrated in Figs. 12(a and b). Figure 12(a)
362 presents the plots of the normalized maximum mean streamwise velocity ($\langle \bar{u} \rangle_m / U_1$) of different
363 sections along the streamwise distance normalized with the sluice gate opening (x/y_1). It is evident that
364 in the cases C2 and C3, the velocity of the submerged jumps decays at a faster rate than in the smooth
365 bed case after $x/y_1 = 40$, with the decay trends showing notable similarity. On the other hand, the velocity

decay trend for case C1 is comparable to that of the smooth bed condition. In Figure 12(b) the streamwise distance is normalized with the length scale L , which represents the distance from the sluice gate to the point where $\langle \bar{u} \rangle_m = U_1/2$. The use of L for normalization of the streamwise distance leads to the consolidation of all data into a consistent trend [4-5,14]. Figure 12(b) shows that the velocity decay trend of all the submerged jump cases almost follows a similar pattern, which remains within the velocity decay curves of free jump and plane wall jet. The deviation between the smooth bed case and the corrugated bed cases occurs approximately after $x/L = 1.5$, where the velocity of the submerged jumps over the corrugated beds decays at a faster rate.

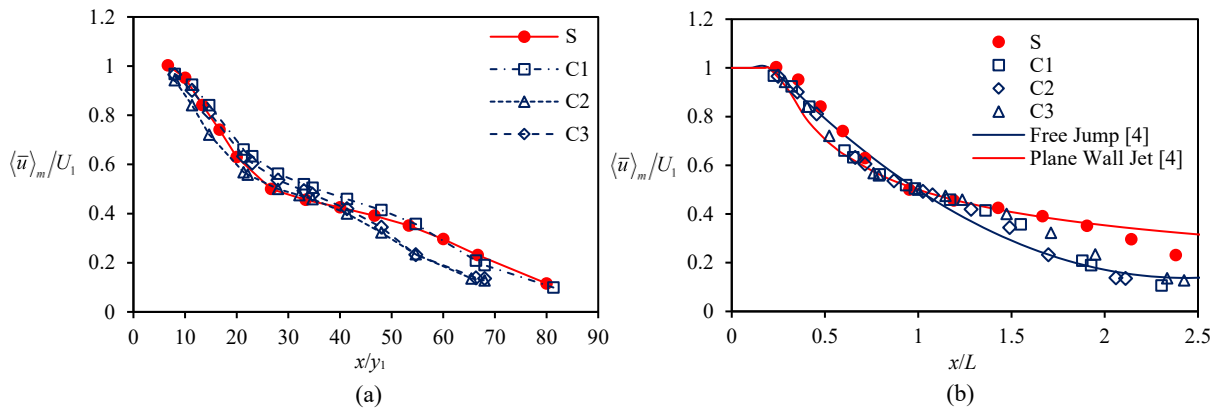


Fig. 12 Variation of the normalized maximum mean streamwise velocity ($\langle \bar{u} \rangle_m / U_1$) along the streamwise direction normalized with the (a) initial jet thickness (y_1) and (b) length scale L

Observations made thus far on the flow features of the submerged hydraulic jump, including the submerged roller length, the growth of δ_1 and δ_2 , and the streamwise velocity decay, suggest that the overall mean flow characteristics are severely affected by the wave steepness ratio (a/λ) of the corrugated beds. Notably, the results for cases C2 and C3, which have the highest a/λ values among the corrugated geometries studied, exhibit the most significant deviations from the smooth bed condition. Thus, it can be inferred that the wave steepness ratio of the corrugated bed most significantly influences the overall mean flow characteristics of the submerged jump.

3.3. Shear layer development

Figure 13 displays the instantaneous vorticity fields alongside the velocity vectors for the case of a submerged jump occurring over the corrugated bed C1, depicting the dynamics of the wall jet as it emerges from the sluice opening and flows over the roughness elements. On the outer edge of the emerging jet, which is in contact with the overlain fluid, shear layer instability is seen to be initiated by Kelvin-Helmholtz-type waves. These waves grow and culminate in the formation of discrete anti-clockwise vortical structures characterized by positive z -vorticity (as shown in the enlarged view in Fig. 13). Conversely, the inner edge of the wall jet generates a clockwise recirculatory motion within the grooves of the corrugations, with the sizes of these vortices being influenced by the geometry of the roughness elements.

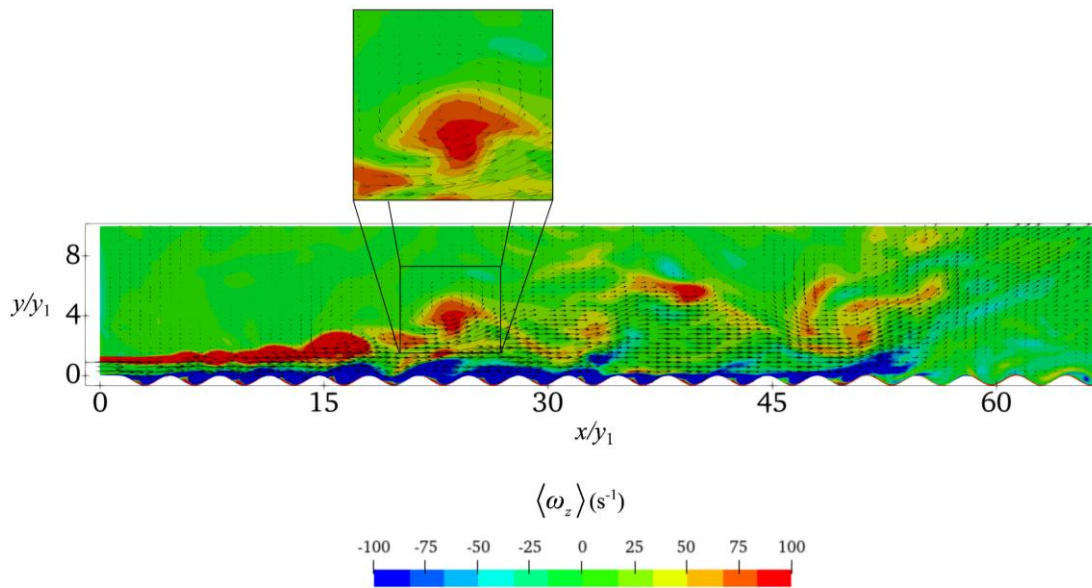


Fig. 13 Instantaneous spanwise vorticity (ω_z) fields along the central vertical plane, along with the velocity vectors of the emerging wall jet over corrugated bed C1

Dey et al. [14] suggested that the velocity and turbulence features of submerged wall jets are characterized by a two-layer shear flow influenced by the wall and the overlying recirculating flow. To visualize the shear layer evolution, the isosurfaces representing the mean vorticity magnitude of 7 s^{-1} are illustrated in Figs. 14(a-h). In these figures, the fluid-bound shear layer is characterised by positive mean z -vorticity, while the wall-bound shear layer produces a negative mean z -vorticity. In the case of

submerged jumps over smooth bed conditions (Fig. 14a), the thickness of the fluid-bound shear layer is observed to increase with distance away from the sluice opening, but the wall-bounded shear layer shows minimal increase in thickness. Conversely, for submerged jumps over corrugated aprons (Figs. 14 b-d), the wall-bounded shear layer exhibits a significant increase in thickness in the fully developed zone. Additionally, the fluid-bound shear layer appears to be displaced away from the bed as the streamwise distance increases, while the wall-bounded shear layer thickness decreases abruptly in the streamwise direction, indicating that the wall jet is rising away from the bed level as observed in Figs. 8(b-d). Figures 14(e-h) illustrate the formation of the 3D shear layer in the spanwise direction, coloured with the contours of mean y -vorticity. A notable difference between the smooth bed and corrugated beds is the presence of a pair of strong counter-rotating vertical vortices just downstream of the sluice gate in the smooth bed scenario (also observed in Figs. 7 and 9), extending from the free surface to the shear layer. The absence of strong vertical vortices under corrugated bed conditions can be advantageous from a structural design perspective, reducing the possibility of structural vibrations and sidewall erosion [38].

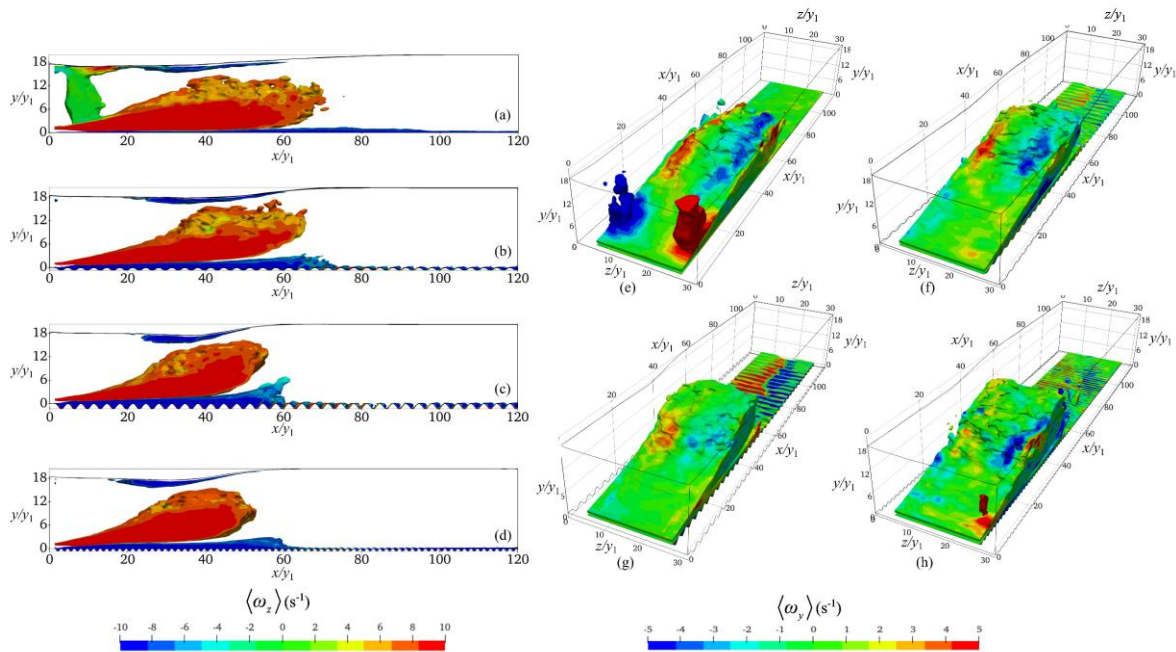


Fig. 14 Evolution of the shear layer visualized by the isosurface with mean vorticity magnitude 7 s^{-1} in 2D view for different bed roughness conditions: (a) S, (b) C1, (c) C2, and (d) C3; 3D view of the shear layer for different bed roughness conditions: (a) S, (b) C1, (c) C2, and (d) C3

3.4. Pressure distribution within the corrugation roughness

Previous studies of Buckles et al. [35] and Ganju et al. [37] have highlighted a significant aspect of the mean pressure variation within the corrugation grooves by pointing out that, in the vicinity of the reattachment points on the windward face of the roughness elements, the mean pressure distribution shows a region of high pressure, while a region of low pressure is observed at the crest. The findings from the present study also demonstrate characteristics similar to the mean pressure distribution. The elevated pressure near the reattachment points on the windward face of the roughness elements occurs because in this region the flow produces a stagnation point, with impingement on the surface at an oblique angle [35]. Furthermore, pressure fluctuations serve as indicators of turbulence, local vibrations, and instabilities, which may be significant from a design perspective for hydraulic structures [39-40]. To analyse this point further, the normalized mean pressure and the root-mean-square (RMS) of pressure fluctuations at the bed surface along the central vertical plane are examined for different bed conditions. The RMS values are calculated as follows:

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (\bar{p}_i - \langle \bar{p} \rangle)^2}{N}} \quad (7)$$

where σ is the RMS of pressure fluctuations, $\bar{p}_i$ is the instantaneous pressure values obtained from the simulations, $\langle \bar{p} \rangle$ is the time-averaged pressure, and N is the number of pressure sample data.

Figures 15(a-f) demonstrate how $\langle \bar{p} \rangle$ and σ , normalized with the maximum mean pressure ($\langle \bar{p} \rangle_m$) and the maximum RMS value of the pressure fluctuations (σ_m), vary along the corrugated surfaces. Both parameters exhibit an overall trend along the streamwise direction, accompanied by a periodic variation that is significantly more pronounced than the changes observed per wavelength of the corrugation elements. It is evident that both the mean pressure and RMS of pressure fluctuations form local maxima near the points of reattachment (demarcated by the vertical dotted lines in the enlarged view in Figs. 15d-f) and local minima just downstream of the crests of corrugation roughness

elements, where the flow separates from the corrugated surface. Apart from the first corrugation groove, the mean pressure follows a consistent trend, barring the periodic fluctuations. The magnitudes of the fluctuations in the mean pressure distribution around the reattachment points diminish moving downstream. This reduction occurs due to the decay of the wall jet, weakening the recirculation within the corrugation grooves, thereby reducing the impact of the flow on impingement on the bed surface. Furthermore, as the wall jet rises away from the bed, the mean pressure distribution begins to align closely with the profile of the corrugated roughness at the bed, without any intermediate fluctuations across the wavelengths (Figs. 8b-d). The pressure fluctuation RMS follows a general trend characterized by an initial increase to a global maximum, followed by a gradual decline with streamwise distance, ultimately approaching negligible values. Notably, in the instances of corrugations C2 and C3, a distinct secondary peak appears as the RMS value increases from its local minimum to its local maximum within each corrugation wavelength, whereas this is not significant in C1, attributed to the lower a/λ value in this case. Additionally, difference between the local maxima and local minima within the wavelength in the case of C3 is much less than that observed in C1 and C2, given that λ in C3 is smaller than that of the other two. In conclusion, both the mean pressure and pressure fluctuation RMS exhibit a reduction in the periodic fluctuations with increasing streamwise distance post the fully developed zone, ultimately vanishing as the wall jet rises away from the bed level, and the fluctuations of the RMS values are also dependent on the geometry of the corrugations.

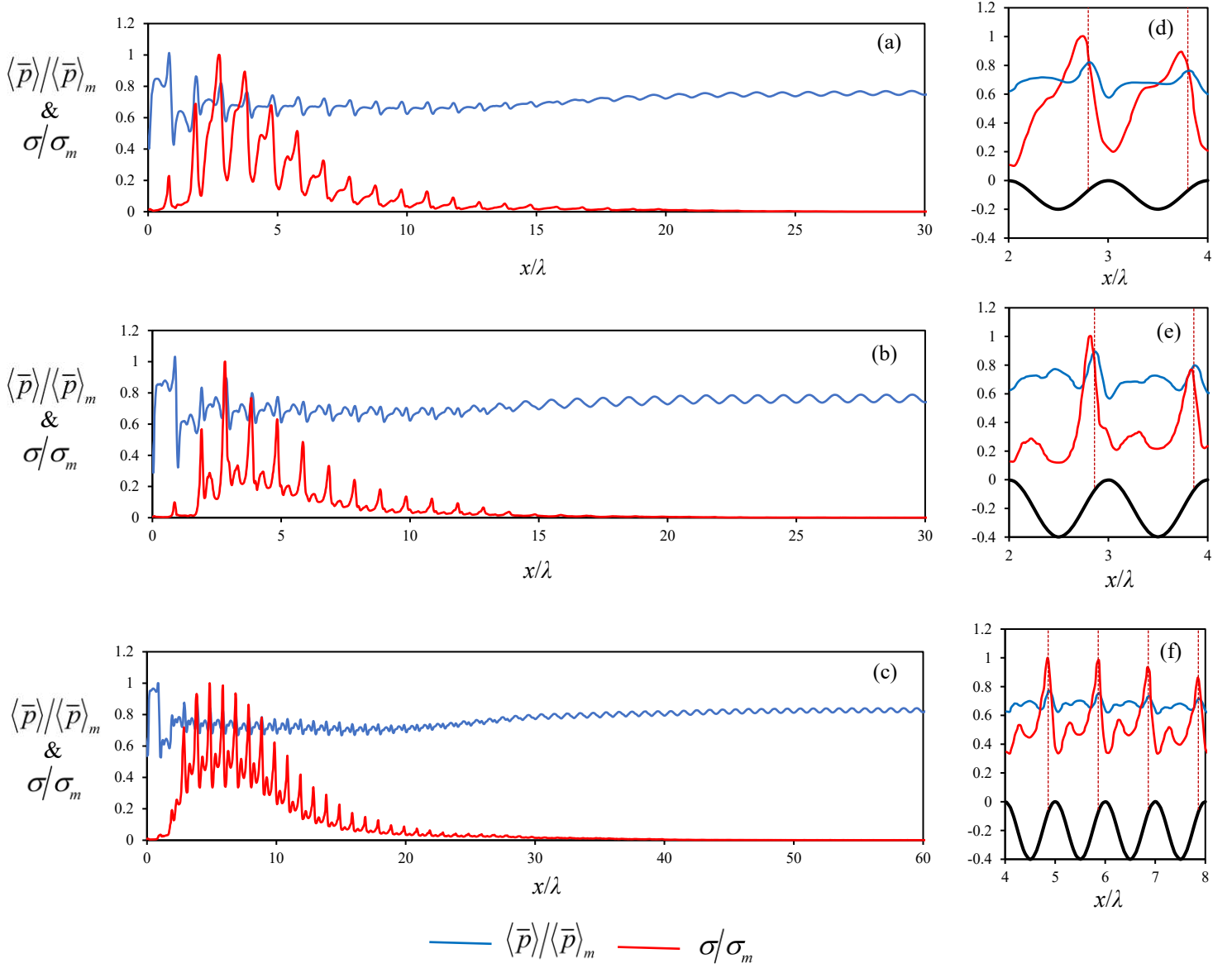


Fig. 15 Variation of normalized mean pressure ($\langle \bar{p} \rangle / \langle \bar{p} \rangle_m$) and pressure fluctuation RMS (σ / σ_m) along the streamwise distance at the bed level for (a) C1, (b) C2, and (c) C3; enlarged view showing the variation with the curvature of the bed surface for (d) C1, (e) C2, and (f) C3

3.5. Turbulence Characteristics

3.5.1. Reynolds Shear Stresses

The second-order turbulent statistics, specifically the Reynolds normal and shear stresses, are examined at the central vertical plane of the submerged jumps. Figures 16(a-h), 17(a-h), and 18(a-h) illustrate the distribution of the normalized streamwise ($\langle \bar{u}'\bar{u}' \rangle / U_1^2$), wall-normal ($\langle \bar{v}'\bar{v}' \rangle / U_1^2$) and shear stresses ($\langle \bar{u}'\bar{v}' \rangle / U_1^2$) from the current LES across different bed roughness conditions. The

$\langle \bar{u}'\bar{u}' \rangle$ distribution (Figs. 16a-d) reveals elevated values along both the boundary layer and outer shear layer, with comparatively lower values in between, indicating the presence of the jet core within the fully developed zone. The $\langle \bar{u}'\bar{u}' \rangle$ distributions are observed to produce localized peaks above the crests of the roughness elements for bed types C1 and C2 (highlighted in Figs. 16e-h), where the flow detaches to create recirculation within the corrugation grooves. Conversely, for bed type C3, due to its smaller λ value, similar isolated peaks are not observed. Instead, a band of high $\langle \bar{u}'\bar{u}' \rangle$ value is found in the boundary layer, similar to the smooth bed condition. Figures 17(a-d) reveal that the $\langle \bar{v}'\bar{v}' \rangle$ distribution peaks in the fully developed zone, similar to $\langle \bar{u}'\bar{u}' \rangle$, and then decays as the flow recovers to be uniform downstream. Here, the local-peak magnitudes of $\langle \bar{v}'\bar{v}' \rangle$ lie along the outer shear layer of the wall jets, primarily due to the vertical fluid mixing induced by the flow entrainment caused by the jet. For the corrugated beds C1 and C2, notable spikes in the $\langle \bar{v}'\bar{v}' \rangle$ values can be observed on the leeward side of the roughness elements at the onset of the fully developed zone (illustrated in the enlarged views in Figs. 17e-h), these spikes diminish along the streamwise distance. However, such features are absent in the case of C3, likely due to the smaller λ value compared to the other corrugated beds. Another interesting observation is that for C1, the magnitude of $\langle \bar{v}'\bar{v}' \rangle$ along the shear layer is significantly lower than the other two corrugated beds, which may be attributed to the lower a/λ value of C1. As for in Figs. 18(a-d) the $\langle \bar{u}'\bar{v}' \rangle$ distribution is observed to have positive peak values along the shear layer present at the outer edge of the wall jets. The enlarged views of the corrugation (Fig. 18e-h) reveal that the $\langle \bar{u}'\bar{v}' \rangle$ is positive in the proximity of the reattachment point and negative in the wake of the corrugation crests.

A comparison of the Reynolds stress distributions reveals that submerged hydraulic jumps experience vigorous turbulent mixing in the fully developed zone. However, the more rapid decay of the Reynolds stresses in the presence of corrugated roughness indicates a dampened fluid mixing effect in the downstream region, particularly near the bed, when compared to the smooth bed condition. The

corrugated roughness elements, characterized by larger λ , reduce the $\langle \bar{u}'\bar{u}' \rangle$ values along the boundary layer, thereby inhibiting the fluid mixing in the streamwise direction in that zone. In contrast, the dominance of the $\langle \bar{v}'\bar{v}' \rangle$ in the shear layer at the outer edge of the wall jet for corrugated beds with a higher a/λ suggests an enhanced vertical fluid mixing in the region. To further investigate the turbulent events contributing to the Reynolds stresses, quadrant decomposition is performed at the region corresponding to high Reynolds stresses in the subsequent section. This analysis facilitates the quantification of the relative contributions of different events to turbulent momentum transfer.

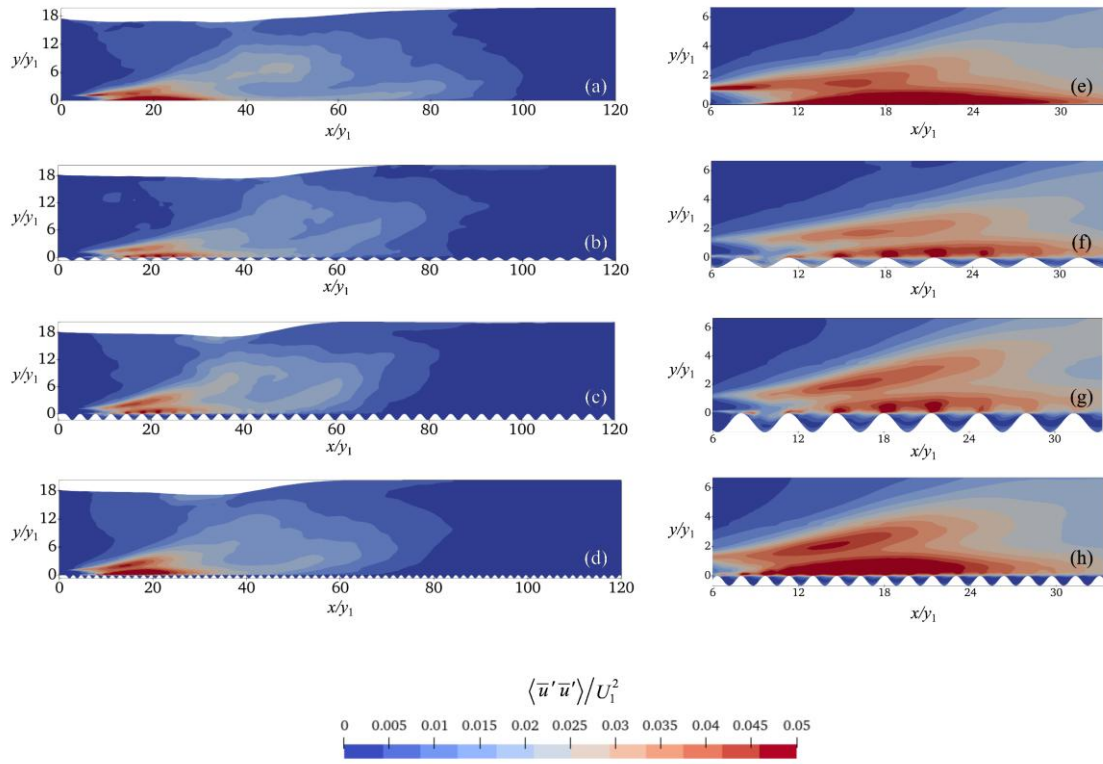
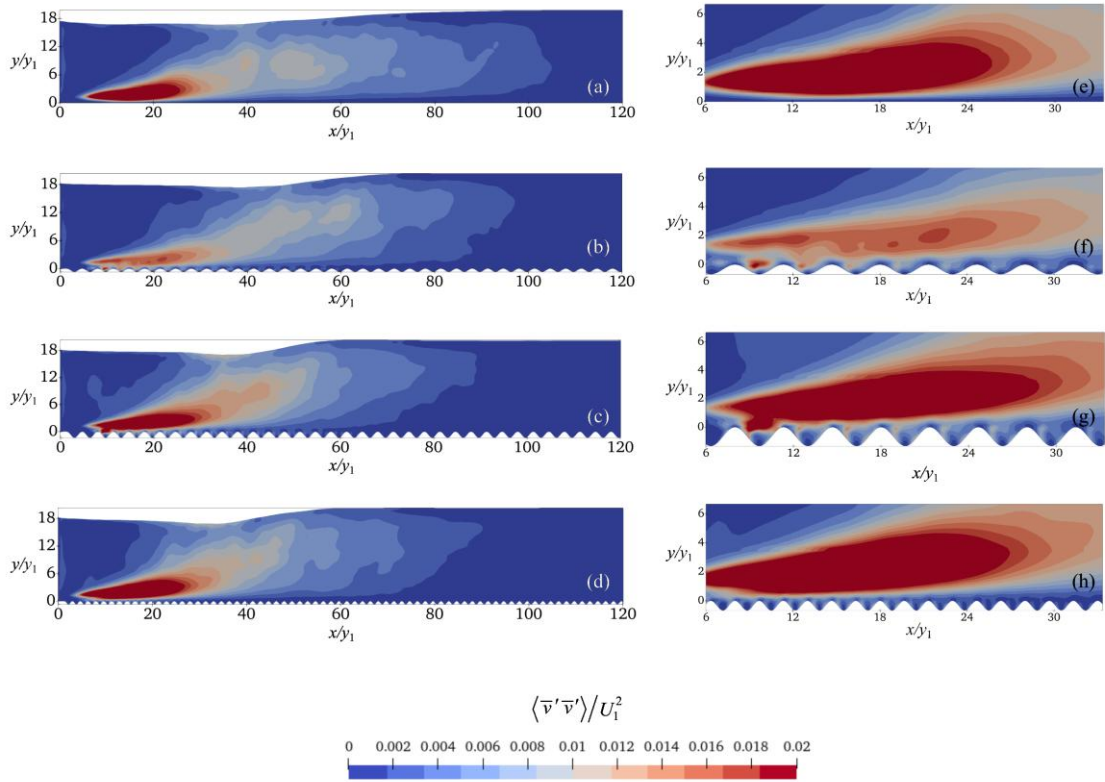


Fig. 16 Distribution of the normalized streamwise Reynolds normal stress $\langle \bar{u}'\bar{u}' \rangle / U_1^2$ along the central vertical plane: (a) S, (b) C1, (c) C2, and (d) C3; along with the enlarged views for (e) S, (f) C1, (g) C2, and (h) C3



501 **Fig. 17** Distribution of the normalized wall normal Reynolds normal stress ($\langle \bar{v}'\bar{v}' \rangle / U_1^2$) along the
 502 central vertical plane: (a) S, (b) C1, (c) C2, and (d) C3; along with the enlarged views for (e) S, (f) C1,
 503 (g) C2, and (h) C3

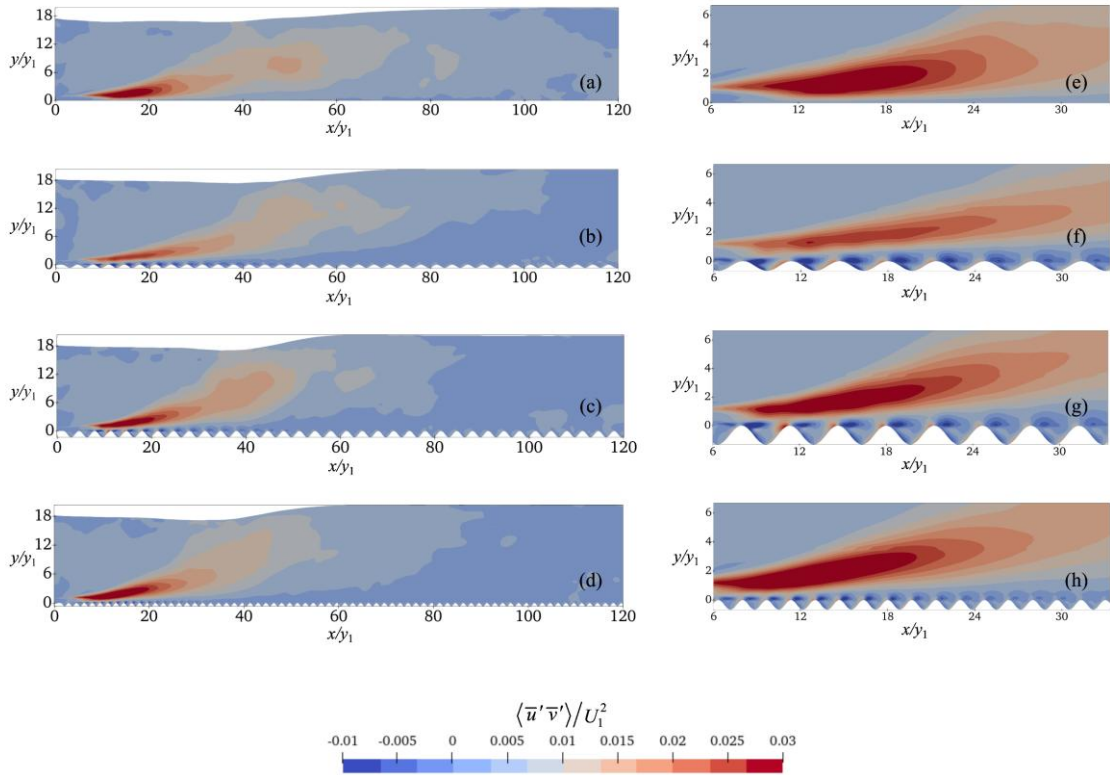


Fig. 18 Distribution of the normalized Reynolds shear stress $(\langle \bar{u}'\bar{v}' \rangle / U_1^2)$ along the central vertical plane: (a) S, (b) C1, (c) C2, and (d) C3; along with the enlarged views for (e) S, (f) C1, (g) C2, and (h) C3

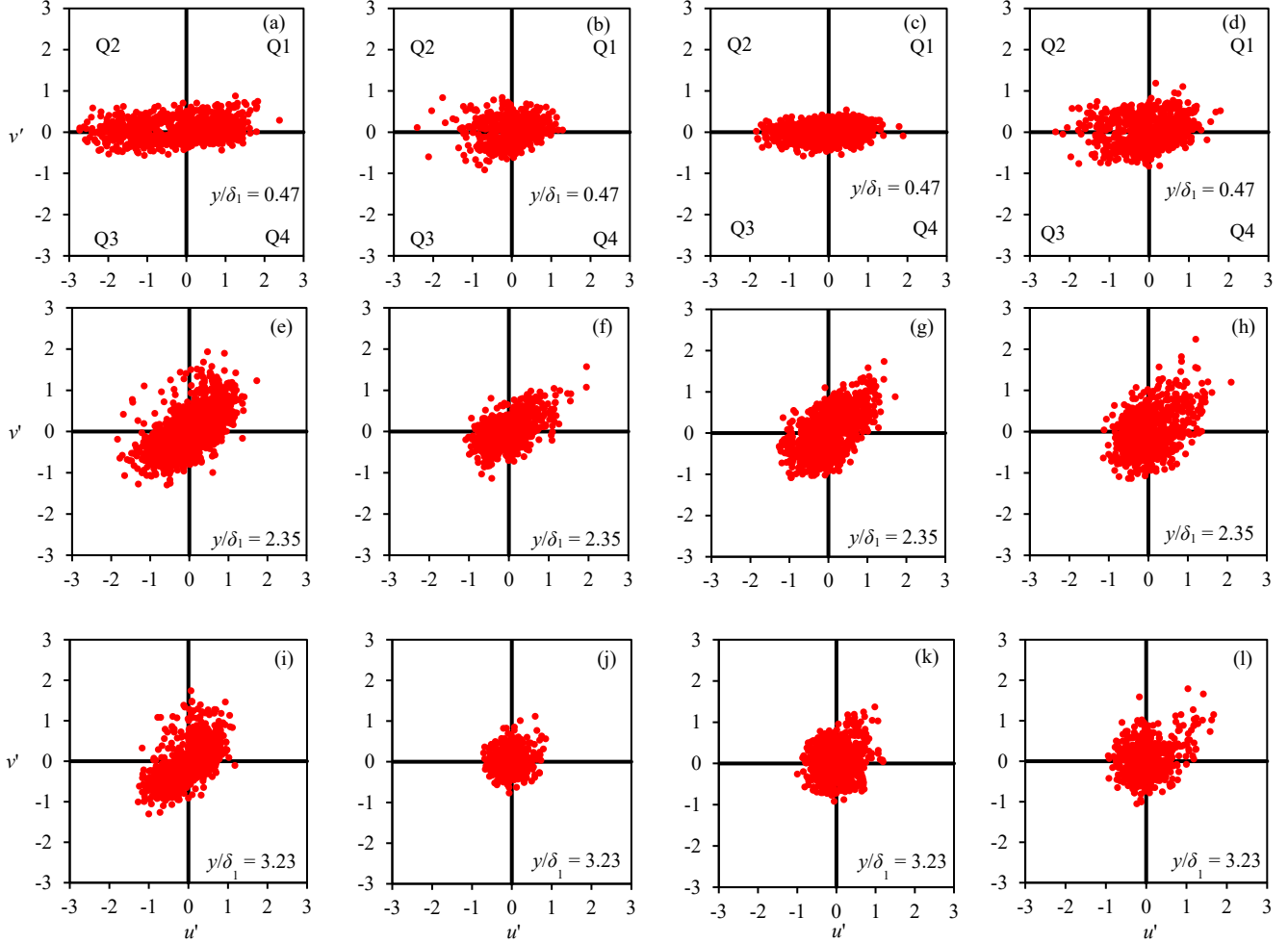
3.5.2. Quadrant Decomposition

The turbulent bursting events can be quantified by conditionally decomposing fluctuating components of the velocity in the streamwise and vertical directions, $\bar{u}'$ and $\bar{v}'$, into four quadrants. This conditional decomposition can effectively highlight the contributions of turbulent bursting events from each quadrant to the overall Reynolds shear stress by plotting the velocity fluctuations according to quadrants in a $\bar{u}'\bar{v}'$ plane [41]. The turbulent events can be categorized as outward interactions Q1 ($\bar{u}' > 0$ and $\bar{v}' > 0$), ejections Q2 ($\bar{u}' < 0$ and $\bar{v}' > 0$), inward interactions Q3 ($\bar{u}' < 0$ and $\bar{v}' < 0$), and sweeps Q4 ($\bar{u}' > 0$ and $\bar{v}' < 0$). To differentiate between the strong from the weak events, a hyperbolic curve is utilized, which is defined as

$$\langle \bar{u}'\bar{v}' \rangle = H \left(\langle \bar{u}'\bar{u}' \rangle \right)^{0.5} \left(\langle \bar{v}'\bar{v}' \rangle \right)^{0.5} \quad (8)$$

where the parameter H is called the hole size. By employing this methodology, significant contributions to the Reynolds shear stress from each quadrant can be isolated, thereby facilitating the extraction of larger contributions, leaving the smaller fluctuations during more quiescent intervals. Figures 19(a-l) illustrate the quadrant distributions of the velocity fluctuations at various depths at a streamwise distance of $x/y_1 = 14.67$ for different bed roughness conditions. At depth $y/\delta_1 = 0.47$, situated within the inner-jet layer, the Q2 and Q4 events are observed to be more dominant than those at other vertical locations, signifying the prevalence of ejection and sweeps in the near-bed flow. Conversely, in the outer-jet layer at a depth $y/\delta_1 = 2.35$, events Q1 and Q3 emerge as the dominant occurrences, regardless of the bed conditions. This change in the characteristics of the velocity fluctuations can be explained by the fact that, in the outer-jet layer, the direction of the velocity gradient reverses from that in the inner-jet region. Thus, in both the inner and outer jet layers, the high-momentum fluid is swept from the jet, and the low-momentum fluid is ejected towards the jet [42]. These ejections and sweeps are responsible for mass and heat transfer between the jet and the ambient fluid, eventually decaying the jet. Notably, Q1 events are more significant than the Q3 events in the presence of the bed roughness elements at y/δ_1

531 $= 2.35$. As one moves further away from the bed to $y/\delta_1 = 3.23$, the velocity fluctuations predominantly
 532 cluster around the origin, with some significant Q1 events in rough bed conditions, while smooth bed
 533 conditions exhibit an almost even distribution of Q1 and Q3 events.



534 **Fig. 19** Quadrant decomposition of the turbulent fluctuations at depth $y/\delta_1 = 0.47$ for (a) S, (b) C1, (c)
 535 C2, (d) C3; $y/\delta_1 = 2.35$ for (e) S, (f) C1, (g) C2, (h) C3; $y/\delta_1 = 3.23$ for (i) S, (j) C1, (k) C2, (l) C3

536 To further analyze the turbulent events, the contributions from each quadrant at each point are
 537 calculated. At any point, the contribution from the quadrant i outside of the hyperbolic hole region of
 538 size H to the total Reynolds shear stress is given by

539
$$\langle \bar{u}' \bar{v}' \rangle_{i,H} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \bar{u}'(t) \bar{v}'(t) I(t) dt \quad (9)$$

540 where T is the sampling duration and $I(t)$ is a detection function, defined as

$$I(t) = \begin{cases} 1, & \text{if } (\bar{u}'\bar{v}') \text{ is in quadrant } i \text{ and if } |\bar{u}'\bar{v}'| \geq H \left(\langle \bar{u}'\bar{u}' \rangle \right)^{0.5} \left(\langle \bar{v}'\bar{v}' \rangle \right)^{0.5} \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

In this study, the quadrant contributions to the Reynolds shear stress are calculated by filtering the background turbulence with hole size $H = 1$, normalized by $\left(\langle \bar{u}'\bar{u}' \rangle \right)^{0.5} \left(\langle \bar{v}'\bar{v}' \rangle \right)^{0.5}$, and presented in Figs. 20(a-d) at the section of streamwise distance of $x/y_1 = 14.67$ for different bed roughness conditions. Within the inner-jet layer close to the bed, Q2 and Q4 events show a general trend of attaining their maximum negative values, indicating the dominance of ejections and sweeps in that region. The contributions from Q2 and Q4 events become almost zero above $y/\delta_1 = 1$, then again increase slightly upon approaching the null velocity line, and again become zero in the reverse flow region. Conversely, the contributions from the Q1 and Q3 events increase in the inner-jet layer to a maximum just above $y/\delta_1 = 1$, then decrease again as they approach the reverse flow region. Therefore, between the point of maximum velocity and the point of null velocity within the fully developed zone of a submerged wall jet, the Q1 and Q3 type events are seen to be the maximum contributors to the Reynolds shear stress. This indicates that, in this region, the flow is predominantly influenced by the shear layers. A notable distinction between the smooth and corrugated bed conditions within the jet layer is that, for smooth bed conditions, the Q1 and Q3 events are comparable in the inner-jet layer, but above $y/\delta_1 = 1$, the Q1 contribution exceeds that of Q3. In contrast, in the case of corrugated bed conditions within the inner-jet layer, the Q3 contributions are significantly greater than Q1, while above $y/\delta_1 = 1$, Q1 again exceeds Q3. Additionally, for corrugated conditions the ejection and sweep events are nearly balanced close to the bed, unlike in smooth bed conditions, where the ejection events clearly dominate. Among the corrugated beds, bed type C3 has characteristics most closely mimicking smooth bed conditions, due to its low value of λ .

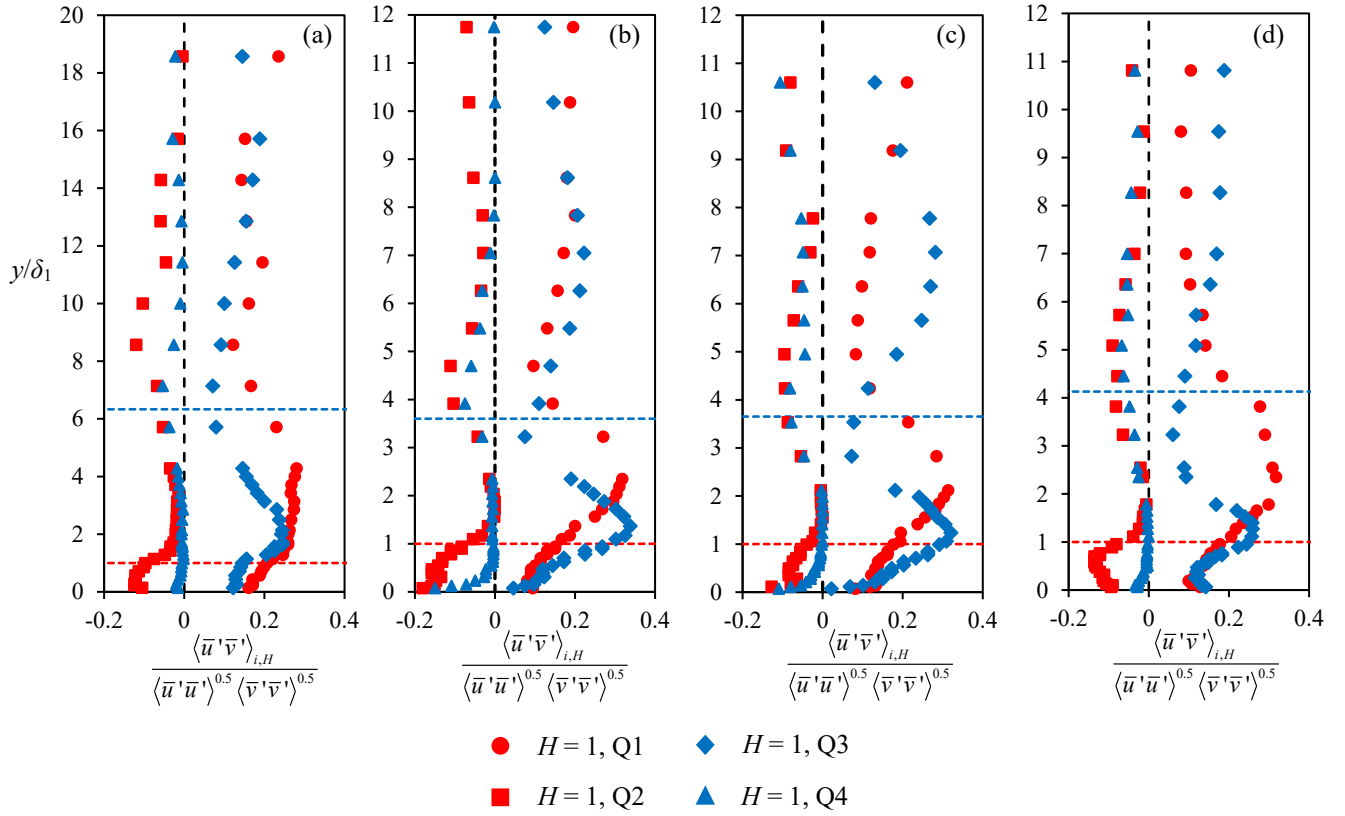


Fig. 20 Quadrant contributions to the Reynolds shear stress calculated at a section of streamwise distance $x/y_1 = 14.67$ along the central vertical plane for different bed roughness conditions: (a) S, (b) C1, (c) C2, and (d) C3, with the dotted horizontal lines depicting the inner-jet layer thickness and the null velocity line

3.6. Energy loss within the submerged jump

The energy of the wall jet upon entry through the sluice opening is converted into turbulent energy, contained by the eddies and vortices within the fully developed zone [43-44]. This turbulent energy ultimately dissipates in the recovering zone under the action of viscosity. As the kinetic energy of the wall jet is converted into the turbulent energy, the turbulent kinetic energy (TKE), defined as $k = 1/2(\langle \bar{u}'^2 \rangle + \langle \bar{v}'^2 \rangle + \langle \bar{w}'^2 \rangle)$, peaks within the fully developed zone as evident from the distribution of Reynolds stress in Figs. 16 and 17. To analyze the energy loss mechanism, the depth-integrated TKE ($\bar{k}$) at the central vertical plane for all the simulated cases are plotted along the streamwise direction in Fig. 21. The general trend shows that $\bar{k}$ rises initially as the kinetic energy of the wall jet is converted into turbulence and then decays as the turbulent energy is passed down the energy cascade and finally dissipated. In case of the smooth bed, $\bar{k}$ rises fast and decays over an extended distance,

while for the corrugated bed configurations, the increase is slower but with a faster decay within a shorter streamwise distance. Notably, the rising limb of the curve for case C3 is the closest to that of the smooth bed for up to $x/y_1 = 20$, which can be attributed to its smaller λ value. Furthermore, the falling limbs of the curves for C2 and C3 are closely comparable due to their higher a/λ values, with $\bar{k}$ values in these cases decaying at a faster rate than in C1 and the smooth bed conditions. It may, therefore, be inferred from Fig. 21 that corrugated beds with higher values of a/λ dissipate energy over shorter streamwise distances

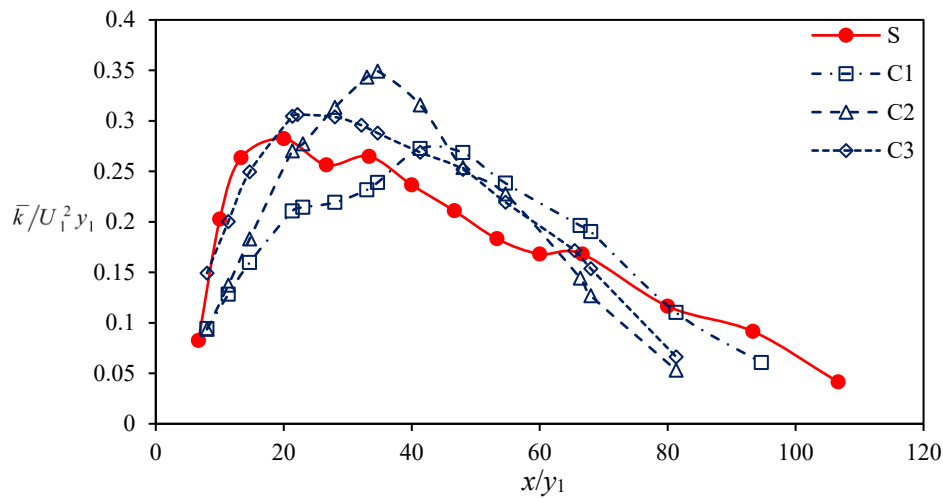


Fig. 21 Variation of the normalized depth-integrated TKE ($\bar{k}/U_1^2 y_1$) along the streamwise distance for different bed conditions

4. Conclusions

This study analyzes the influence of macro-roughness, specifically in the form of sinusoidal corrugations, on the submerged hydraulic jump using numerical simulations carried out with an LES-VOF model. A validated computational model was executed on a high-performance computing system to replicate the physical experiments. The submerged hydraulic jump phenomenon downstream of a sluice gate was simulated over different bed configurations, including a smooth bed and three types of corrugated beds with varying combinations of wavelengths and crest heights. The analysis looks into the overall flow characteristics, mean velocity distribution, shear layer development, pressure distribution, turbulence characteristics and energy loss within the flow domain, which are summarized below:

The overall physical properties of the submerged jump indicate a reduction in the submerged roller length due to the presence of corrugated roughnesses. The mean streamwise velocity distribution indicates that reverse flow within the submerged roller of the jump is relatively weak in the presence of corrugated roughness and, unlike the smooth bed conditions, the counter-rotating pair of vertical vortices downstream of the sluice gate is not as well developed. Additionally, the mean velocity distribution along the central vertical plane of the domain shows that at the end of the fully developed zone, the wall jet rises away from the bed due to the roughness elements towards the free surface, effectively shortening the submerged roller length. Within the fully developed zone, the similarity of the streamwise velocity distribution can be established using the vertical length scale of the jet half-width. Notably, the similarity trends of the velocity distributions of smooth beds and corrugated beds differ in the inner and outer jet layers. When comparing vertical length scales, such as the inner-jet layer thickness and the jet half-width, it is evident that these values for corrugated beds increase more rapidly in the streamwise direction. The mean flow characteristics of the submerged jump were found to be predominantly affected by the steepness ratio of the corrugations, as the cases with the higher wave steepness exhibit the shortest submerged roller, accelerated growth of the vertical length scales, and a faster decay of the streamwise velocity of the wall jet.

The flow characteristics of the submerged wall jet are significantly influenced by both the wall-bound and fluid-bound shear layers as it emerges from the sluice opening. The inner edge of the wall jet is characterized by roughness elements while on the outer edge the shear layer undergoes instabilities that lead to its thickening. As the wall jet traverses over the corrugations, flow separation occurs at the crest of the roughness elements, leading to a recirculating flow within the grooves. This results in an increased thickness of the wall-bounded shear layer. The thickness of the wall-bounded layer decreases abruptly at the end of the fully developed zone in case of corrugated beds, indicating the lifting of the jet away from the wall. In all cases, the thickness of the fluid-bound shear layer increases with distance from the sluice opening. Notably, the shear layer for corrugated bed conditions appears to be positioned farther from the bed than in the smooth bed case.

The distributions of the pressure field indicate that the flow recirculation within the corrugation grooves weakens with distance from the sluice opening, reducing the impact of the flow on impingement on the bed surface upon reattachment. The variations in the mean and RMS distribution along the bed surface within the grooves are characterized by the geometry of the sinusoidal corrugations, specifically the crest height, wavelength, and steepness ratio.

The distributions of Reynolds stresses demonstrate that submerged hydraulic jumps experience significant turbulent mixing in the fully developed zone. Notably, there is a faster decay of turbulence along the streamwise direction in the presence of corrugated roughness, particularly near the bed. The quadrant decomposition of the velocity fluctuations exhibits variations within the jet layer under roughness conditions. In the region of high turbulence within the jet layer of the submerged jumps, the corrugated bed with a smaller wavelength yields results close to those observed in smooth bed conditions. The submerged jump over the corrugated beds dissipates the kinetic energy of the wall jet within a shorter streamwise distance. These findings may contribute to the development and design of corrugated apron stilling basins as a potentially cost-effective approach for energy dissipation.

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6. Data Availability Statement

Data sets generated during the current study are available from the corresponding author upon reasonable request.

7. Funding Declaration

This research did not receive any funding.

8. Notation List

The following symbols are used in this paper:

$\langle \bar{u} \rangle$ = time-averaged streamwise velocity [LT^{-1}];

648	$\bar{u}_i$	=	filtered velocity components [LT ⁻¹];
649	$\langle p \rangle_m$	=	maximum mean pressure [ML ⁻¹ T ⁻²];
650	$\langle \bar{u} \rangle_m$	=	maximum mean streamwise velocity at a vertical section [LT ⁻¹];
651	$\langle p \rangle$	=	mean pressure field [ML ⁻¹ T ⁻²];
652	$\langle \bar{u}'\bar{v}' \rangle$	=	Reynolds shear stress [L ² T ⁻²];
653	$\langle \bar{u}'\bar{u}' \rangle$	=	streamwise Reynolds normal stress [L ² T ⁻²];
654	$\langle \omega_x \rangle, \langle \omega_y \rangle$	=	time-averaged vorticity in the x and y direction [T ⁻¹];
655	$\langle \bar{v}'\bar{v}' \rangle$	=	wall normal Reynolds normal stress [L ² T ⁻²];
656	$\bar{u}', \bar{v}', \bar{w}'$	=	Fluctuations in the velocity corresponding to x, y, and z directions [LT ⁻¹];
657	τ_{ij}^{SGS}	=	subgrid-scale (SGS) stress tensor [ML ⁻¹ T ⁻²];
658	a	=	height from trough to crest of the corrugations [L];
659	a/λ	=	wave steepness ratio [-];
660	C_w	=	WALE model constant [-];
661	Fr	=	supercritical Froude Number or the Froude number of the jet [-];
662	g	=	acceleration due to gravity [LT ⁻²];
663	$\bar{k}$	=	depth-integrated TKE [L ³ T ⁻²];
664	k	=	turbulent kinetic energy (TKE) [L ² T ⁻²];
665	k	=	turbulent kinetic energy (TKE) [L ² T ⁻²];
666	L	=	longitudinal length scale [L];
667	S	=	submergence ratio [-];
668	t	=	time [T];
669	U_1	=	supercritical velocity of the jet below the sluice gate [LT ⁻¹];
670	x_i	=	Cartesian coordinate ($i = 1, 2, 3$ corresponding to the x, y, and z direction
671			respectively) [L];

672	y^+	=	non-dimensional wall coordinate [-];
673	y_1	=	sluice gate opening [L];
674	y_2	=	subcritical sequent depth to y_1 [L];
675	y_t	=	tailwater depth [L];
676	α	=	volume fraction [-];
677	Δ	=	grid spacing or mesh size [L];
678	δ_1	=	inner jet layer thickness [L];
679	δ_2	=	half-width of jet [L];
680	λ	=	wavelength of the corrugations [L];
681	ν	=	kinematic viscosity [L^2T^{-1}];
682	ν_t	=	turbulent or eddy viscosity [L^2T^{-1}];
683	ρ	=	density of fluid [ML^{-3}];
684	σ	=	root mean square (RMS) of the pressure fluctuations [$ML^{-1}T^{-2}$];
685	σ_m	=	maximum root mean square (RMS) of the pressure fluctuations [$ML^{-1}T^{-2}$];
686	ω_z	=	instantaneous z-vorticity [T^{-1}];

687 9. References

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