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Design and validation of a 3-DOF wrist rehabilitation trainer with BP-PID impedance control

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Abstract

Insufficient or delayed rehabilitation training following wrist joint injuries often leads to tissue adhesion and impaired neurological recovery. To better support the rehabilitation of patients with wrist joint injuries, this paper presents the design of a three-degree-of-freedom (3-DOF) wrist rehabilitation trainer that compensates for the misalignment between the trainer's center of rotation and that of the human wrist joint, enabling three combined motion patterns: flexion/extension, ulnar/radial deviation, and pronation/supination. The rationality of the trainer structure was validated through motion simulation and virtual prototype simulation. For motion control, a BP-PID controller combined with an impedance control strategy is proposed. Through Adams–MATLAB co-simulation and physical prototype testing, it is validated that compared with conventional PID control, the wrist rehabilitation trainer with the proposed BP-PID impedance control method satisfies the requirements of 3-DOF combined wrist motions, with the range of motion improved by 31.5°, 28.5°, and 12.4°, respectively, and torque control performance improved by 24.9%, 32.9%, and 25.5%, respectively. Surface electromyography (sEMG) signals from the wrist-related muscles of three participants increased by 425%, 266%, and 181%, respectively. These results verify the trainer's mechanical rationality and the control system's effectiveness, while also demonstrating its positive effect on promoting wrist rehabilitation in patients.

Keywords: wrist rehabilitation, rehabilitation trainer, mechanism design, BP neural network, PID control, impedance control

1. Introduction

1.1 Motivation

The number of stroke patients is steadily increasing, with affected individuals typically presenting with upper-limb muscle atrophy. The wrist joint serves as the pivotal articulation connecting the forearm and the hand. Clinical evidence has demonstrated that insufficient mobilization of the wrist joint following injury predisposes patients to sensory and motor sequelae in the wrist. Immobilization or loss of functional capacity in the wrist joint imposes a substantial burden not only on patients but also on their families and society as a whole^{1,2}. Timely and appropriate wrist exercises can effectively reduce tissue adhesion resulting from joint damage,

promote neural plasticity, and facilitate the remodeling of injured nerves in the affected region^{3,4}. Therefore, the development of a medical device that addresses the specific wrist rehabilitation needs of stroke patients is of significant importance for improving quality of life and restoring daily functional abilities.

1.2 Related Work

In recent years, numerous scholars have conducted extensive research on wrist rehabilitation mechanisms. Zhang et al.⁵ employed springs as the supporting elements in a parallel wrist mechanism to emulate the human carpal ligament complex, enabling multidirectional motion of the wrist joint in the frontal plane. However, the device's weight must be borne entirely by the patient, and the rehabilitation range is constrained by the spring characteristics. Xu et al.⁶ proposed a novel 6-DOF serial-parallel anthropomorphic wrist joint with three rotational degrees of freedom, which addresses the limitation of existing robotic wrist joints that can only achieve two directional rotations. However, the driving requirements and wearability issues of the rehabilitation mechanism in practical applications were not addressed. Zimmermann et al.⁷ developed an exoskeleton for upper-limb rehabilitation that covers all relevant degrees of freedom of the human arm. However, it lacks targeted training for the wrist joint and has a relatively large physical footprint. Mandeljc et al.⁸ developed an exoskeletal robotic device that enables coupled flexion/extension motion of the wrist and finger joints, facilitating simple donning and doffing by the user. However, its single-DOF design limits the user's other rehabilitation requirements. Jiang et al.⁹ designed a cable-driven parallel robot with flexible springs. Springs were employed to replace the joint bones in the wrist/shoulder joint to support the motion platform. The feasibility of the parallel robot prototype and its control scheme was verified through simulations and experiments. Nevertheless, the wrist joint's range of motion is limited by spring performance, and rehabilitation stability deteriorates as the workspace increases. Martinez et al.¹⁰ designed a 3-DOF exoskeletal gimbal for forearm and wrist rehabilitation. The gimbal provides three active degrees of freedom and tracks the path of wrist motion, accommodating the human wrist's range of motion. However, the pronation/supination joint cannot be driven in parallel with the other joints, thereby reducing the combined motion workspace. Pezent et al.¹¹ developed a novel wrist exoskeleton device. The first rotational joint drives forearm pronation/supination, while the second and third rotational joints actuate wrist flexion/extension and radial/ulnar deviation, respectively. The device demonstrates good performance in terms of range of motion, torque output, and position control. Nevertheless, its operational complexity renders it unsuitable for daily use. Zhang et al.¹² developed a pneumatically actuated parallel wrist rehabilitation robot with two rotational degrees of freedom. The components connecting the forearm and wrist feature an open design to enhance the wearable device's convenience, making it suitable for most patients. However, the workspace is insufficient, and pneumatic safety remains a concern. Mehrez et al.¹³ designed a 2-DOF wearable exoskeleton robot for wrist rehabilitation that achieves rehabilitation through actuation of pneumatic artificial muscles. Dynamic analysis of the mechanism was conducted, and a PD control algorithm was adopted for the control system. The device is lightweight and easy to wear. However, it provides only a 2-DOF motion pattern, and its rehabilitation stability requires further improvement.

Based on an analysis of related studies on wrist rehabilitation trainers, several limitations of existing devices are identified. First, current research and development efforts have primarily

focused on overall arm recovery, treating the wrist joint as merely an auxiliary component of the forearm and neglecting its importance in daily activities. Second, the use of linkage mechanisms increases the interdependence of actuating components, leading to structural complexity and additional spatial occupation. In rehabilitation applications, this can result in mechanical obstruction or reduced precision in motion control. Third, the human wrist joint is ellipsoidal and has three degrees of freedom, and its motion axes vary continuously during movement. However, a significant kinematic discrepancy exists between the workspace of existing rehabilitation robots and the actual motion trajectories of the human wrist joint¹⁴.

1.3 Contribution

To address the aforementioned limitations of existing wrist rehabilitation robots, this study proposes a compensation structure that treats the human wrist as an elastic member. By adjusting the relative distance between the gripping position and the center of rotation, the deviation between the device's center of rotation and that of the human wrist joint is compensated, thereby reducing the risk of secondary injury from axis misalignment. A wrist rehabilitation trainer model is developed, combined with a coaxial spherical mechanism, that enables independent or combined wrist motions in three degrees of freedom. Subsequently, a pose feedback control system is designed for the trainer, and a hybrid control algorithm combining BP-PID with joint impedance control is proposed. Adams–Simulink co-simulation demonstrates that, compared with conventional PID control, the torque-tracking performance of each joint is significantly improved, meeting the rehabilitation requirements of patients with wrist joint injuries at different stages while enhancing safety and stability throughout the rehabilitation process. Finally, a physical prototype of the wrist rehabilitation trainer is fabricated as a test platform, and experimental studies on assisted wrist rehabilitation training are conducted. The experimental results indicate that the trainer improves the assisted range of motion and torque control performance for each joint, thereby validating the structure's mechanical rationality and the effectiveness of the control system. Furthermore, the positive effect of the wrist rehabilitation trainer on promoting wrist rehabilitation in patients is substantiated through sEMG signals collected from participants.

1.4 Structure of the Paper

Section 2 analyzes the wrist joint's physiological motion characteristics and presents the design of the wrist rehabilitation trainer. Section 3 presents the kinematic analysis of the trainer structure. Section 4 describes the virtual prototype simulation of the trainer. Section 5 proposes the BP-PID control method combined with impedance control and validates its effectiveness through simulation. Section 6 presents the development of the feedback control system and the rehabilitation training experiments.

2. Mechanical Structure Design

2.1 Wrist Joint Motion Analysis

The human wrist comprises three principal joints: the distal radioulnar joint, which enables pronation/supination of the forearm; the radiocarpal joint, which enables ulnar/radial deviation of the hand; and the midcarpal joint, which enables flexion/extension of the hand. The three wrist motions are decomposed into three revolute joint motions whose axes intersect at a single point. For rehabilitation purposes, the trainer is required to provide three degrees of freedom to match

the working states of the wrist muscle groups, as illustrated in Fig. 1. The active muscles involved and the ranges of motion of each joint in daily activities are listed in Table 1, providing data reference for the design of the wrist rehabilitation mechanism^{15,16,17}.

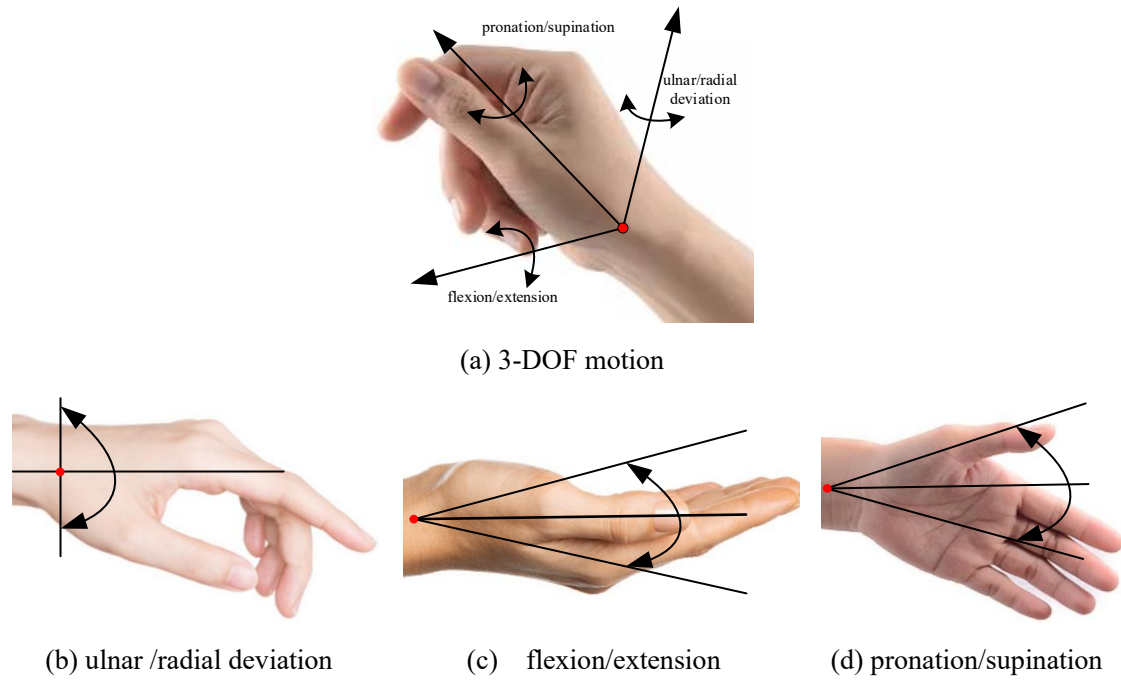


Fig. 1. Schematic diagram of wrist joint motion patterns

Table 1. Daily activity parameters of the wrist joint

Motion pattern	Range (°)	Torque	Muscles involved
Flexion	-60	2.5 N·m	Flexor carpi ulnaris, flexor carpi radialis, palmaris longus
Extension	65	2.5 N·m	Extensor carpi ulnaris, extensor carpi radialis
Ulnar deviation	-15	2.1 N·m	Flexor carpi radialis, extensor carpi radialis
Radial deviation	25	2.1 N·m	Flexor carpi ulnaris, extensor carpi ulnaris
Pronation	90	4 N·m	Extensor carpi radialis longus, brachioradialis
Supination	-75	4 N·m	Extensor carpi radialis brevis, flexor carpi radialis

2.2 Overall Configuration Design

The design of the wrist rehabilitation trainer shall comply with the principles of human biomechanics, ensuring that the end-effector trajectory aligns with the wrist's natural motion path and that the degrees of freedom and workspace match those of the wrist joint. To enable the wrist rehabilitation trainer to provide combined motions of flexion/extension, ulnar/radial deviation, and pronation/supination in three degrees of freedom, a coaxial spherical mechanism was adopted for the design¹⁸, fitting the ellipsoidal motion trajectory of the wrist joint, as shown in Fig. 2. In view of the motion axis deviation inherent in the human wrist joint, a configuration with adjustable deviation was implemented based on the coaxial spherical parallel mechanism, allowing compensation for the displacement of the wrist joint motion axes while providing 3-DOF motion, thereby achieving a biomimetic wrist joint structure¹⁹. The mechanism model of the wrist joint is shown in Fig. 3.

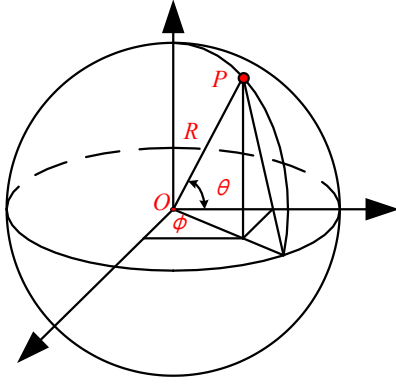


Fig. 2. Coaxial spherical parallel mechanism model.

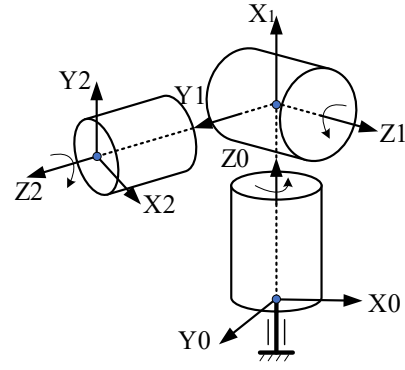


Fig. 3. Mechanism model of the wrist joint.

The mechanism connects the motion platforms for wrist flexion/extension, ulnar/radial deviation, and pronation/supination through three connecting links. The geometric configuration of the mechanism is determined by the trajectory positions of the wrist joint motion axes Z'_0 , Z'_1 and Z'_2 , while the intersection position of the joint axes is governed by the adjustable ellipsoidal workspace ranges d_x and L_y , as shown in Fig. 4.

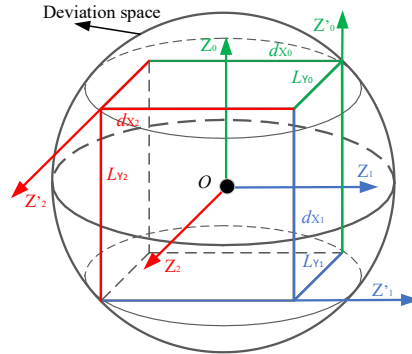


Fig. 4. Spatial range of axis displacement.

where:

$$\begin{aligned} Z'_0 &= Z_0 + d_{x0} + L_{y0} \\ Z'_1 &= Z_1 + d_{x1} + L_{y1} \\ Z'_2 &= Z_2 + d_{x2} + L_{y2} \end{aligned} \quad (1)$$

According to Grubler's formula for the degree of freedom of parallel mechanisms²⁰, the following is obtained:

$$M = d(n - g - 1) + \sum_{i=1}^g f_i + \nu - \zeta \quad (2)$$

where:

M : umber of degrees of freedom;

d : order of the mechanism;

n : number of links;

g : number of kinematic pairs;

f_i : number of degrees of freedom of a single kinematic pair;

ν : number of constraints;

ζ : number of redundant degrees of freedom (passive DOF).

The wrist rehabilitation trainer's mechanism design ensures that the motion axes of the three wrist motion patterns intersect at a single point. Therefore, with $d=3$, and both ν and ζ equal to zero, the degree of freedom of the mechanism is:

$$M = d(n - g - 1) + \sum_{i=1}^g f_i + \nu - \zeta = 3 \times (4 - 3 - 1) + 3 = 3 \quad (3)$$

2.3 Compensation structure design

There is a deviation between the rotational center of a typical articulated wrist device and the rotational center of the human wrist joint, which can easily lead to issues such as wrist muscle strain and soft tissue and bone wear during rehabilitation²⁰. As shown in Figure 5. To address this issue, a motion-compensation structure is designed in which the human wrist is modeled as a contractible elastic rod. By adjusting the distance from the grip position to the rotational center, the possibility of secondary injury to the wrist caused by deviation and displacement is reduced. The compensation mechanism is shown in Figure 6.

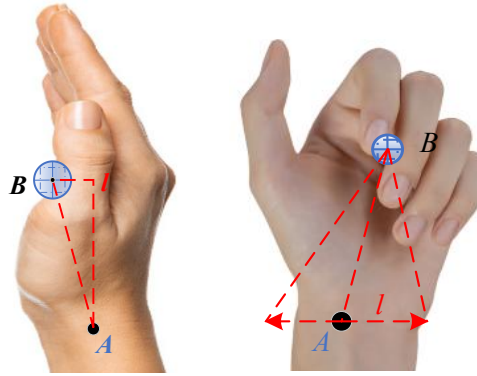


Fig. 5. Schematic diagram of wrist joint motion axis displacement.

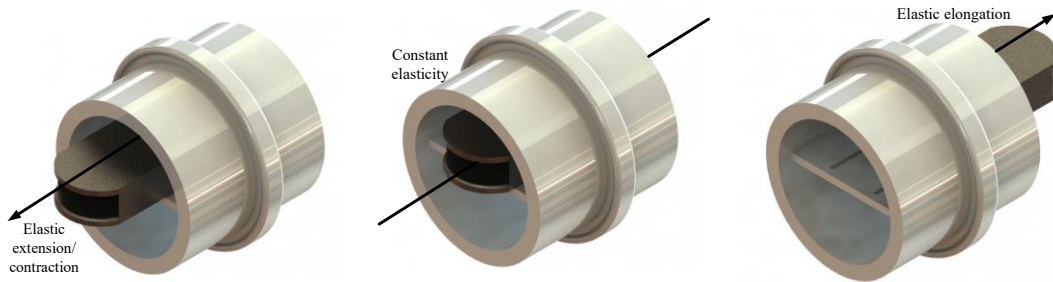


Fig. 6. Motion compensation structure of the wrist joint.

2.4 Overall Structure Design of the Trainer

To reduce friction between the mechanism and the wrist joint compensation structure, the wrist support section was designed as a fan-shaped symmetrical structure with rotary bearings embedded on the inner side. This configuration minimizes friction between the compensation structure and the frame, as well as environmental influences during the rehabilitation process, thereby ensuring alignment between the rehabilitation trainer and the human wrist's motion trajectory. Through the bearing assembly, the wrist motion compensation structure is connected to the motor platform. When the wrist is placed in the elastic interlayer within the compensation structure, the motor drives the compensation structure in rotational motion, while simultaneously compensating for any offset that may occur during wrist rehabilitation. The position of the elastic interlayer can be adjusted to accommodate the usage requirements of different patients.

The trainer employs three external drive sources to actuate the device, assisting the human wrist joint in pronation/supination, flexion/extension, and ulnar/radial deviation, respectively. With this configuration, the coaxial spherical parallel mechanism can be transformed into a single-axis spherical serial mechanism. The compensation structure and supporting structure together provide the spatial platform for the drive system of pronation/supination motion. The schematic diagram of the overall structure design is shown in Fig. 7.

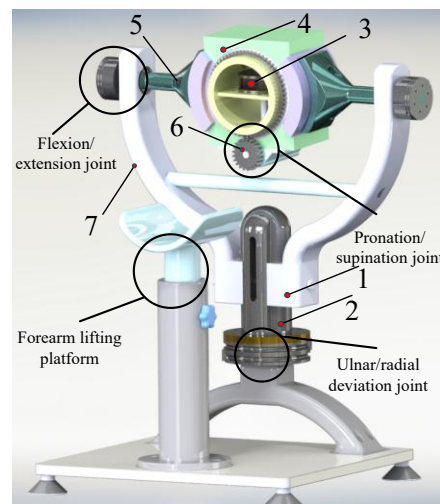


Fig. 7. Schematic diagram of the overall structure design

1-frame; 2-transmission shaft sleeve; 3-compensation structure; 4-drive platform;
5-transmission shaft; 6-transmission gear; 7-rib frame.

During operation, the rehabilitation trainer is worn on the human forearm and wrist as the carrier. Through external drive and mechanical assembly, the device's complexity is reduced, and its versatility for use in various scenarios is enhanced. The device fully simulates the wrist postures of the human wrist and meets the requirements of combined motions, including pronation/supination, ulnar/radial deviation, and flexion/extension. The height and angle of the forearm placement are adjustable through the sliding groove between the lifting base and the wrist joint support frame, as well as the sleeve between the forearm lifting platforms, thereby accommodating the upper-limb dimensions of different patients. The wrist joint position adjustment plate compensates for joint axis displacement during m

otion, ensuring greater comfort for patients throughout rehabilitation training. The three motion processes of the wrist trainer—flexion/extension, ulnar/radial deviation, and pronation/supination—are shown in Figs. 8, 9, and 10, respectively.

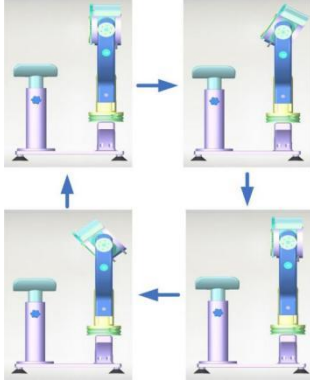


Fig. 8. Schematic diagram of flexion/extension motion.

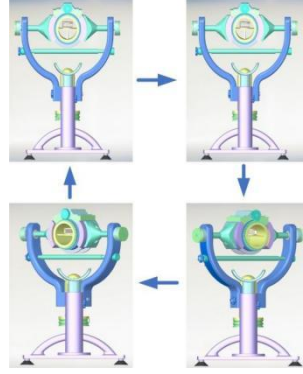


Fig. 9. Schematic diagram of ulnar deviation/radial deviation motion.

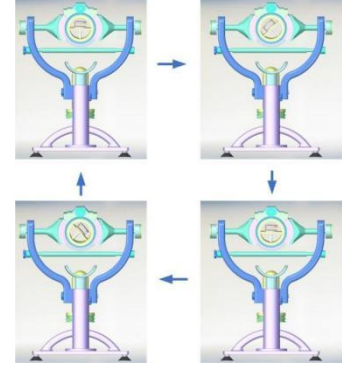
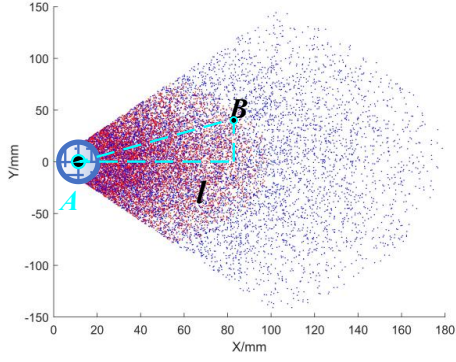


Fig. 10. Schematic diagram of pronation/supination motion.

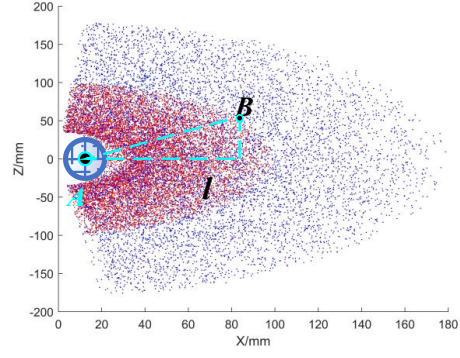
A kinematic model was established based on the structure of the wrist rehabilitation trainer to analyze the trainer's workspace. The joint angle variables θ_1 , θ_2 , and θ_3 of the wrist joint, corresponding to ulnar/radial deviation, flexion/extension, and pronation/supination, were substituted into the Monte Carlo equation. Random sampling was performed for each joint variable to analyze the effects of deviations in motion axes and axial displacement on wrist rehabilitation outcomes during human–robot interaction.

$$\theta_i = \theta_{i\min} + rand(1)(\theta_{i\max} - \theta_{i\min}) \quad (4)$$

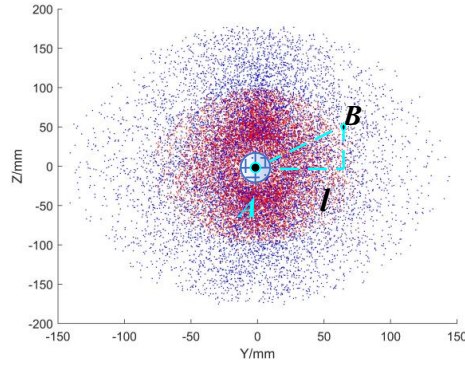
The randomly generated 10,000 joint variable values were substituted into the equation to obtain the workspace of the wrist rehabilitation trainer, considering the joint angle limits, as shown in Fig. 11. Owing to the symmetrical structural design of the device, the joint motion workspace is approximately ellipsoidal in shape. In the figure, the blue points represent the workspace limited by the joint angle constraints, while the red points represent the approximate deviation space corresponding to axial displacement of the motion axes during rehabilitation activities. Compared with conventional wrist rehabilitation structures, the proposed compensation structure improves the workspace of ulnar/radial deviation, flexion/extension, and pronation/supination by 36%, 45%, and 50%, respectively. This enhancement increases the alignment between the wrist joint's motion trajectories and the rehabilitation trainer's center of rotation, thereby compensating for motion errors, reducing the risk of secondary wrist injury, and improving rehabilitation outcomes.



(a) workspace of ulnar/radial deviation.



(b) workspace of flexion/extension.



(c) workspace of pronation/supination.

Fig. 11. Comparison of workspaces

3. Kinematic Analysis

3.1 Forward Kinematics Analysis

Kinematic analysis enables the determination of the position and orientation of the wrist rehabilitation trainer's end-effector, facilitating precise posture control and positioning by the control system^{21,22}. Let the pose matrix relationship of the i -th link relative to the $(i-1)$ -th link be denoted as T_i , and ${}^{i-1}T_i$ can be expressed as:

$${}^{i-1}T_i = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & a_{i-1} \\ S\theta_i C\alpha_{i-1} & C\theta_i C\alpha_{i-1} & -S\alpha_{i-1} & -S\alpha_{i-1}d_i \\ S\theta_i S\alpha_{i-1} & C\theta_i S\alpha_{i-1} & C\alpha_{i-1} & C\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

where, $S\theta_i = \sin \theta_i$; $C\theta_i = \cos \theta_i$; $S\alpha_i = \sin \alpha_i$; $C\alpha_i = \cos \alpha_i$.

The homogeneous transformation matrices of the trainer joints are multiplied sequentially, and the resulting matrix represents the pose relationship between the end-effector and the base:

$${}^0T_i = {}^0T_1 T_2 \dots {}^{i-2}T_{i-1} T_i \quad (6)$$

Based on physiological studies of the wrist joint, a deviation exists between the intersection point of the motion axes and that of the joint axes of the wrist rehabilitation mechanism. The three motions of the wrist joint are represented by three mechanical links, and the motion error value is denoted by L . Therefore, the wrist rehabilitation mechanism model can be simplified using the

Denavit–Hartenberg (D–H) convention, and a D–H coordinate system can be established. The resulting kinematic model of the wrist rehabilitation trainer is shown in Fig. 12, and the corresponding parameters are listed in Table 2.

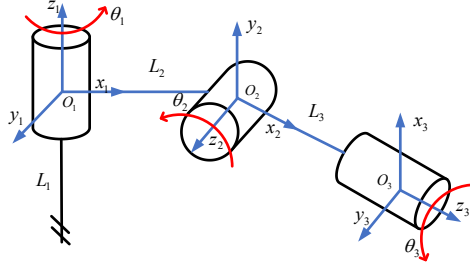


Fig. 12. Kinematic model of the wrist rehabilitation trainer.

Table 2. D–H parameters of the wrist rehabilitation trainer.

i	α_{i-1}	a_{i-1}	d_i	θ_i	Range
1	0	L_1	d_1	θ_1	$\pm 55^\circ$
2	$\pi / 2$	L_2	d_2	θ_2	$\pm 58^\circ$
3	$\pi/2$	L_3	d_3	θ_3	$\pm 90^\circ$

Substituting the parameters in Table 2 into Eq. (5), the transformation matrix of each link can be obtained as follows:

$${}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & a_0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^1_2T = \begin{bmatrix} c_2 & -s_2 & 0 & a_1 \\ 0 & 0 & -1 & d_2 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^2_3T = \begin{bmatrix} c_3 & -s_3 & 0 & a_2 \\ 0 & 0 & -1 & d_3 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

The kinematic equation is thus obtained in Eq. (8), which lays the theoretical foundation for the trainer's motion control and human–robot interaction simulation.

$${}^0_3T = {}^0_1A {}^1_2A {}^2_3A = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

Where: $\begin{bmatrix} n_x & s_x & a_x \\ n_y & s_y & a_y \\ n_z & s_z & a_z \end{bmatrix}$ denotes the orientation of the third joint of the wrist rehabilitation

device, $\begin{bmatrix} p_x & p_y & p_z \end{bmatrix}$ denotes the position of the end-effector relative to the base coordinate system.

3.2 Inverse Kinematics Analysis

Based on the forward kinematics pose results obtained above, the joint variables θ of the trainer are solved inversely. According to physiological studies of the human wrist joint and the analysis of the trainer model, the pose of the trainer has no direct relationship with the third joint

angle θ_3 . Therefore, the parameters a_i and d_i are treated as known quantities, while the third joint of the wrist rehabilitation trainer is excluded from the kinematic analysis, and the joint variables θ_1 and θ_2 are to be solved.

(1) Joint variable θ_1

$${}^0T^{-1} {}^0T = {}^1T {}^2T \quad (9)$$

$$\begin{bmatrix} c_1 & s_1 & 0 & -a_0 * c_1 \\ -s_1 & c_1 & 0 & a_0 * s_1 \\ 0 & 0 & 1 & -d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_2 * c_3 & -c_2 * c_3 & s_2 & s_2 + 1 \\ -s_3 & -c_3 & 0 & 0 \\ c_3 * s_2 & -s_2 * s_3 & -c_2 & -c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rearranging yields:

$$p_z - 1 = -c_2 \quad (10)$$

where p is a known quantity. The joint variable θ_1 can then be obtained as:

$$\theta_1 = \arccos(1 - p_z) \quad (11)$$

(2) Joint variable θ_2

$${}^1T^{-1} {}^0T^{-1} {}^0T = {}^2T \quad (12)$$

$$\begin{bmatrix} c_1 * c_2 & c_2 * s_1 & s_2 & -a_1 * c_2 - d_1 * s_2 - a_0 * c_1 * c_2 \\ -c_1 * s_2 & -s_1 * s_2 & c_2 & a_1 * s_2 - c_2 * d_1 + a_0 * c_1 * s_2 \\ s_1 / c_1^2 + s_1^2 & -c_1 & 0 & -d_2 - a_0 * s_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_3 & -s_3 & 0 & a_2 \\ 0 & 0 & -1 & d_3 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rearranging yields:

$$\begin{cases} p_z * s_2 - s_2 - c_2 + c_2 * p_x = a_2 \\ c_2 * p_z - c_2 + s_2 - p_x * s_2 = d_3 \end{cases} \quad (13)$$

where p is a known quantity. Combined with Eq. (13), the joint variable θ_2 can then be solved as:

$$\theta_2 = \arcsin\left(\frac{(1-p_x)d_3 + (p_z-1)a_2}{(1-p_x)^2 + (p_z-1)^2}\right) \quad (14)$$

3.3 Rehabilitation Trajectory Planning

Based on the study of human hip rehabilitation motion models, the rehabilitation motion patterns of flexion/extension, adduction/abduction, and internal/external rotation typically follow a velocity profile that starts from zero, gradually increases, remains constant after reaching a certain value, and then decelerates to a stop when the hip joint reaches the maximum motion angle, during which muscle stretching occurs²³. To observe the variations in position, velocity, and acceleration of the hip rehabilitation robot over time, the initial joint angle is set as θ_0 , the target angle as θ_f , and the start and end times as t_0 and t_f , respectively.

The cubic polynomial function for the joint motion angles of the hip rehabilitation trainer is then expressed as:

$$\theta_t = a_3 t^3 + a_2 t^2 + a_1 t + a_0 \quad (15)$$

The velocity and acceleration functions can be obtained by differentiation as follows:

$$\begin{aligned} \dot{\theta}_t &= 3a_3 t^2 + 2a_2 t + a_1 \\ \ddot{\theta}_t &= 6a_3 t + 2a_2 \end{aligned} \quad (16)$$

For the undetermined coefficients a_0, a_1, a_2, a_3 , based on the boundary conditions $\theta(t_0) = \theta_0$, $\theta(t_f) = \theta_f$, $\dot{\theta}(t_0) = 0$, the coefficients can be determined as:

$$\begin{cases} a_0 = \theta_0 \\ a_1 = 0 \\ a_2 = \frac{3(\theta_f - \theta_0)}{t_f^2} \\ a_3 = \frac{-2(\theta_f - \theta_0)}{t_f^3} \end{cases} \quad (17)$$

Substituting the coefficients from Eq. (17) into the following equations yields the displacement, angular velocity, and angular acceleration equations of the joint motion:

$$\theta_i = \frac{-2(\theta_f - \theta_0)}{t_f^3} t^3 + a_2 = \frac{3(\theta_f - \theta_0)}{t_f^2} t^2 + \theta_0 \quad (18)$$

$$\dot{\theta}_i = \frac{-6(\theta_f - \theta_0)}{t_f^3} t^2 + \frac{6(\theta_f - \theta_0)}{t_f^2} t \quad (19)$$

$$\ddot{\theta}_i = \frac{-12(\theta_f - \theta_0)}{t_f^3} t + \frac{6(\theta_f - \theta_0)}{t_f^2} \quad (20)$$

The displacement, angular velocity, and angular acceleration curves of the joint motion are shown in Fig. 13:

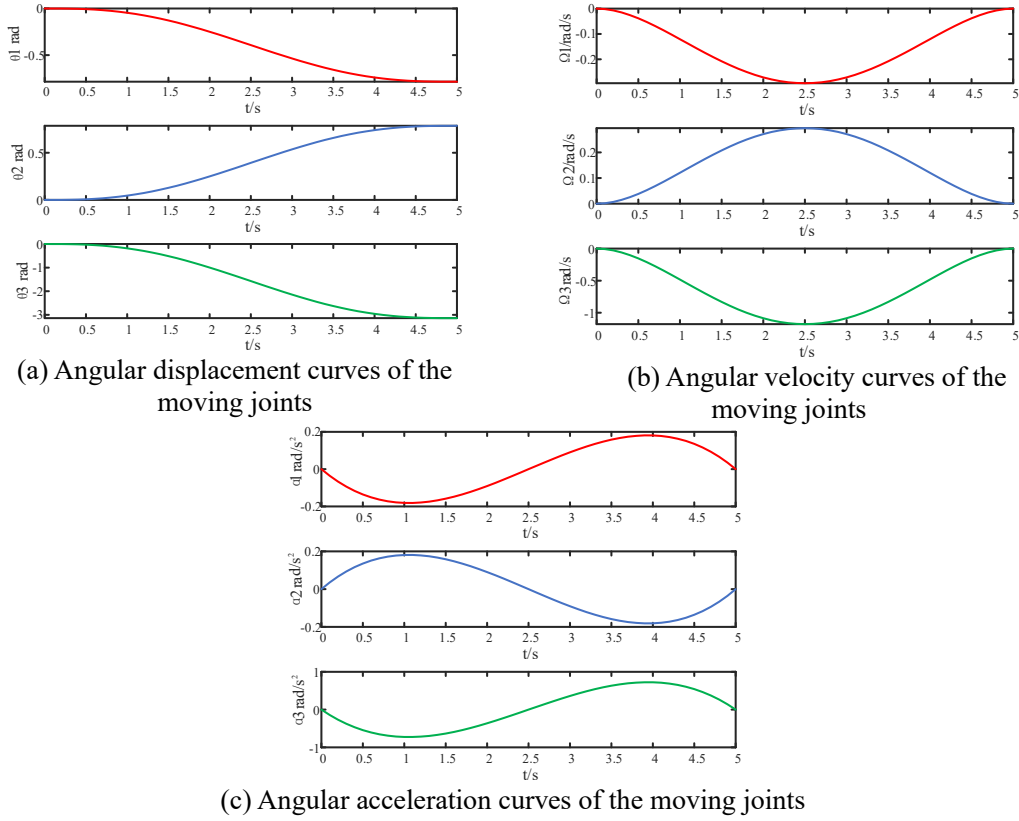


Fig. 13. Angular displacement, angular velocity, and angular acceleration curves of each joint variable in trajectory planning.

At the initial stage of motion, the displacement curves of the joints change smoothly without abrupt variations. The angular velocity curves of the joints are zero at both the start and the end of the motion, and the angular acceleration curves are also zero at the start and the end when the joints move along the predefined trajectories. In addition, all simulated rehabilitation trajectory curves of the device are continuous and smooth. The angular velocity and angular acceleration curves vary within ranges of 1.6 rad/s and 2.4 rad/s², respectively. The overall motion trend shows no abrupt changes, indicating that the rehabilitation trainer operates reliably and meets the requirements for stable wrist rehabilitation motion.

4 Virtual Prototype Simulation

To validate the rationality of the model, the prototype model of the wrist rehabilitation trainer was imported into Adams to obtain the trajectory, velocity, and torque parameters of the end-effector in practical applications²⁴. This further enables analysis of the kinematic performance and influencing factors of the trainer structure and the human wrist joint during human–robot interaction²⁵, thereby establishing a theoretical foundation for the motion-algorithm control and controller design of the trainer. Drive functions were added to the joints, as listed in Table 3, to enable the wrist joint model to perform predefined combined motions of flexion/extension, ulnar/radial deviation, and pronation/supination.

Table 3. Drive functions for joint motions

Drive joint	Motion Function
Joint1	IF(time-1 : -20cosd, 0, IF(time-2 : -30cosd, 0, 0))
Joint2	IF(time-1 : -25sind, 0, IF(time-1.5 : -30cosd, 0, 0))
Joint3	IF(time-1 : -20cosd, 0, IF(time-2 : -30cosd, 0, 0))

Marker points P_1 and P_2 were arbitrarily selected in the third joint coordinate system for trajectory simulation. To more intuitively illustrate the motion posture of the trainer end-effector in the workspace, the three-dimensional motion trajectories were further extracted, as shown in Fig. 15.

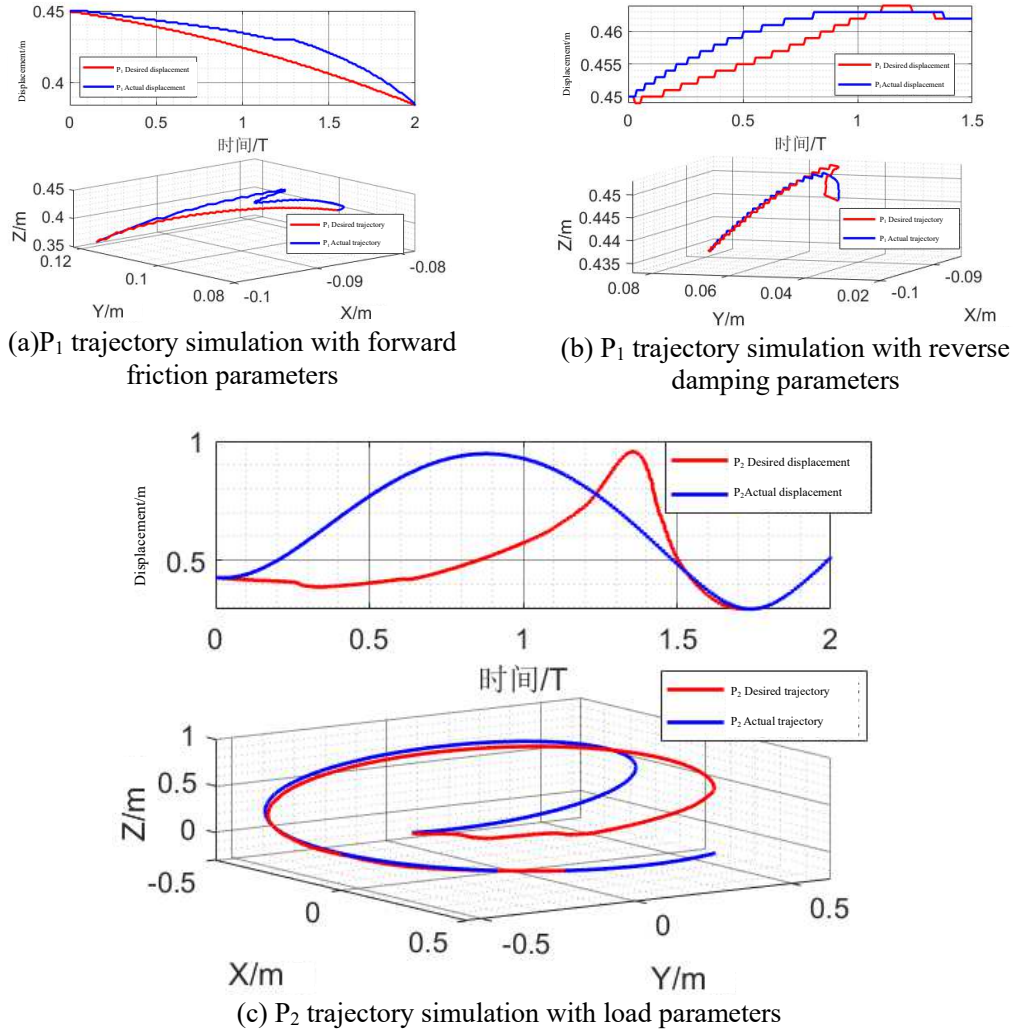


Fig. 14. Three-dimensional motion trajectories.

Figures 14(a) and (b) show the motion trajectory data of Marker point P_1 , while Fig. 14(c) shows the spatial trajectory data of Marker point P_2 . The differences among the data in Figs. 14(a), (b), and (c) result from different configurations of the dynamic model parameters. The three motion patterns of the marker points were assigned with friction, damping, and end-effector load properties, respectively. A comparative analysis of Figs. 14(a) and (b), which include friction and damping properties, reveal that the trajectory in (a) is smoother than that in (b) in terms of displacement and spatial curves, while the trajectory in (b) with damping properties is closer to the

desired values. Furthermore, with friction and damping parameters configured, the displacement curve in (c) begins to approach the theoretical values after 1.4 s, and the spatial trajectory in (c) exhibits a smooth spiral curve.

To further analyze the factors influencing torque during the rehabilitation motion, the center-of-mass positions and mass properties of each joint were computed using the physical model of the wrist joint. The torque output curves under different environmental parameters were observed, as shown in Fig. 15.

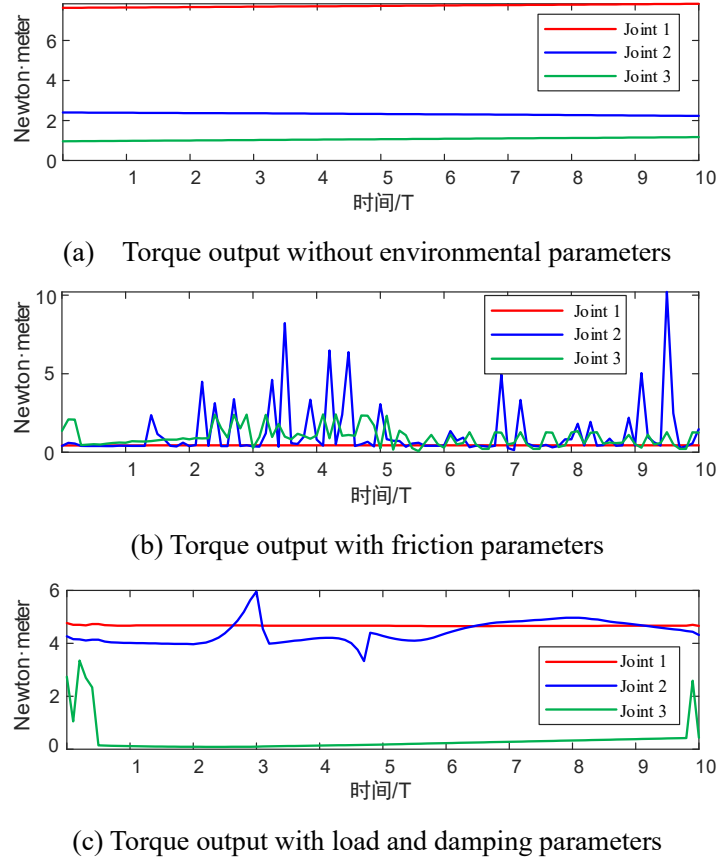


Fig. 15. Joint torque output curves.

Comparing the simulation results in Fig. 15(a), the torque of the pronation/supination joint remains unchanged under friction and load-damping conditions. This is because the joint is structurally unaffected by the actuation of other joints; thus, the torque is less influenced by environmental parameters and remains consistently at 0.47 N·m. In contrast, the ulnar/radial deviation joint and the flexion/extension joint are actuated based on the structure of the ulnar/radial deviation joint, and the output torque reaches a maximum of 10 N·m during the reversal phases of joint motion. Among these, the flexion/extension joint bears the primary load of the wrist mass and structurally supports the other joints, so its torque is increasingly affected as environmental parameters increase, as shown in Figs. 15(b) and 16(c). Therefore, during human–robot interaction in rehabilitation motion, the friction and damping between joints, together with the loads from the human wrist joint and the rehabilitation trainer itself, can reduce the stability of the trainer's torque output and consequently affect the rehabilitation outcome. These findings

provide a reference for the subsequent design of the virtual prototype control system and should be fully considered in the fabrication of the physical prototype.

5. Control System Design

To enhance the safety and control accuracy of the trainer, a pose feedback control system was designed by adopting an external drive to actuate joint rotations. The system is equipped with torque sensors, rotary encoders, and a motion control card, enabling multiple rehabilitation modes—including active, passive, and assist-as-needed training—to accommodate different user requirements, thereby improving rehabilitation efficiency and meeting the needs of patients at various recovery stages. The core module of the control system is the servo motor. Based on the maximum joint torque of approximately 10 N·m obtained from virtual prototype simulation, an appropriate servo motor was selected, equipped with a two-stage reducer with a reduction ratio of $i=10:1$, capable of delivering 12.7 N·m of torque to meet the wrist trainer's torque requirements.

5.1 Servo Control System Model

The motion of the wrist rehabilitation trainer is driven by the system's current input. The process involves the current passing through the armature to generate torque, which then drives the rotor to convert electrical energy into mechanical energy for actuation^{26,27}. The mathematical model of the servo motor is shown in Fig. 16.

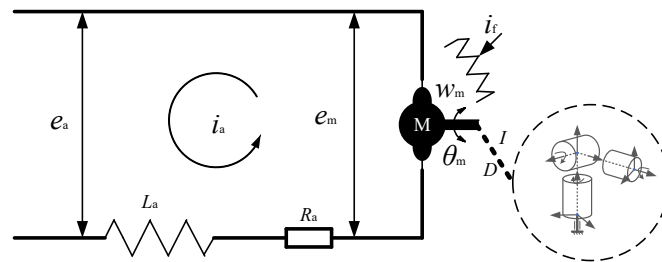


Fig. 16. Mathematical model of the servo motor.

Based on the mathematical modeling of the servo control system, drive stage, and reduction unit of the wrist rehabilitation trainer, together with the selection of practical parameters according to the actual rehabilitation process, the transfer function of the servo control system model is established as follows:

$$G(s) = \frac{53000}{1.426S^3 + 2815S^2} \quad (21)$$

5.2 Human–Robot Interaction System Model

During human rehabilitation training, the mechanical structure and mass parameters of the wrist joint govern the motion characteristics of the mechanical system and influence wrist motion. Therefore, a mechanical system model of the wrist joint was established in Adams, and a virtual prototype control model was developed in MATLAB via the control interface²⁸. Based on the virtual prototype simulation, the stiffness and damping coefficients of the human wrist mechanical

structure were incorporated, as listed in Table 4.

Table 4. Stiffness and damping coefficients of the mechanical structure.

Joint	Stiffness parameter range (N·m/rad)	Damping parameter range (N·m·s/rad)	Approximate stiffness parameter(N·m/rad)	Approximate damping parameter (N·m·s/rad)
1	[10,35]	0.5	20	0.5
2	[10,35]	0.5	20	0.5
3	[8,20]	0.5	15	0.5

Input and output variables were established in the Adams model of the wrist rehabilitation trainer. Three algebraic equations, named Motion1, Motion2, and Motion3, were created to define the motor input rotation angles, while the end-effector position of the trainer was set as the output value. At this stage, the input and output quantities in Adams are defined in MATLAB through the mechanical system for system-level definition. The specific information of the control model is shown in Fig. 17, where Motion serves as the input variable and is output as the end-effector position variable "VARIABLE_" through the Adams Plant interface.

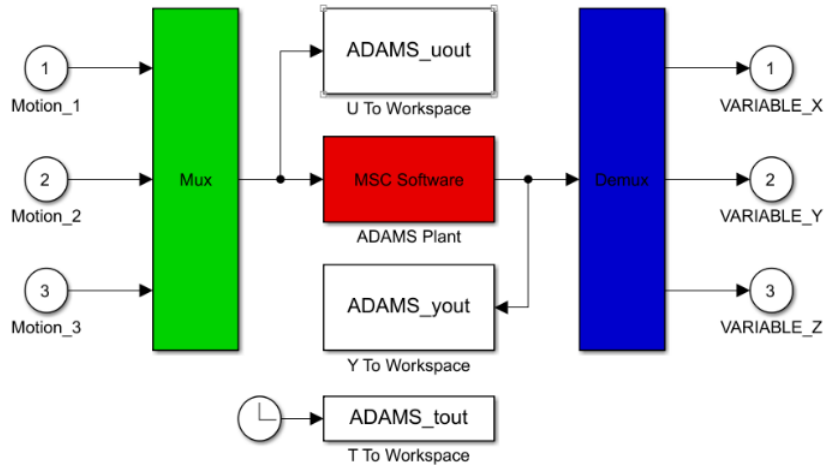


Fig. 17. Human-robot interaction model.

5.3 BP-PID Control Algorithm

The servo motor control system contains numerous nonlinear factors, such as friction, damping, and load variations, whose key parameters continuously change over time, temperature, and other influencing factors during actual operation. The parameter tuning of conventional PID controllers can hardly achieve optimal performance under such conditions. The BP-PID controller combines the self-learning capability of the BP neural network with the effectiveness of PID control. It can automatically adjust the PID parameters to accommodate dynamic changes in the system, thereby better adapting to nonlinear factors, improving the control system's adaptive capability, achieving faster response time and smaller steady-state error, and effectively reducing overshoot and oscillations.

By integrating the BP neural network with PID control and performing online tuning of the parameters through the BP neural network algorithm, the optimal PID parameters K_p , K_i , and K_d can be obtained during the error backpropagation training process²⁹. The model is shown

in Fig. 18.

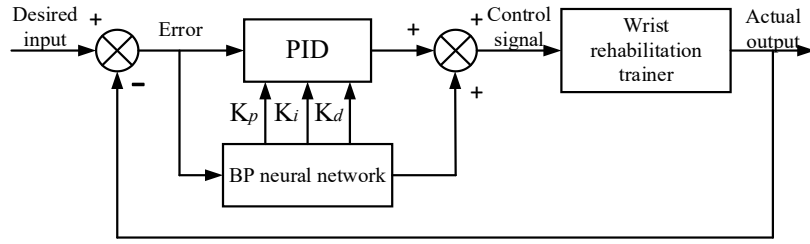


Fig. 18. Block diagram of the BP-PID control model.

The input nodes of the BP-PID algorithm model are $e(T)$, $e(T-1)$, and $e(T-2)$, respectively, and the output nodes are the PID parameters K_p , K_i , and K_d after tuning by the BP neural network algorithm.

Input nodes:

$$R_a = e(k - a) \quad (22)$$

The output control quantity of the PID controller is expressed as:

$$u(T) = u(T-1) + K_p(e(T) - e(T-1)) + K_i e(T) + K_d(e(T) - e(T-1) - e(T-2) + e(T-1))$$

Through linear weighted summation, the hidden layer input can be obtained as:

$$Net_{in} = \sum_{a=1}^3 W_{ac} R_a(T) \quad (23)$$

The hidden layer output is:

$$Net_{out} = \sum_{a=1}^3 W_{ad} R_c(T) \quad (24)$$

The BP-PID mathematical model is:

$$MSE = \frac{1}{2} [y_i(T+1) - y_o(T+1)]^2 \quad (25)$$

The mathematical model for error backpropagation using the gradient descent method is:

$$W_{dc}(T+1) = -\eta \frac{\partial MSE}{\partial y_o(T+1)} \frac{\partial y_o(T+1)}{\partial u(T)} \frac{\partial u(T)}{\partial O_i(T)} \frac{\partial O_i(T)}{\partial Net_{in}(T)} \frac{\partial Net_{in}(T)}{\partial W_{dc}} + \alpha W_{dc}(T) \quad (26)$$

The connection weights of the output layer are updated as:

$$W_{dc}(T+1) = \eta e(T+1) \text{sgn}\left(\frac{y_o(T+1)}{\partial u(T+1)}\right) \frac{\partial u(T)}{\partial O_d(T)} g'(Net_d(T)) O_c(T) + \alpha W_{dc}(T) \quad (4-37)$$

The connection weights of the hidden layer are updated as:

$$W_{cd}(T+1) = \eta f'(Net_c(T)) \sum_{c=1}^3 W_{dc} O_a(T) + \beta W_{ca}(T) \quad (27)$$

A servo control system model of the wrist rehabilitation trainer was established, as shown in Fig. 19.

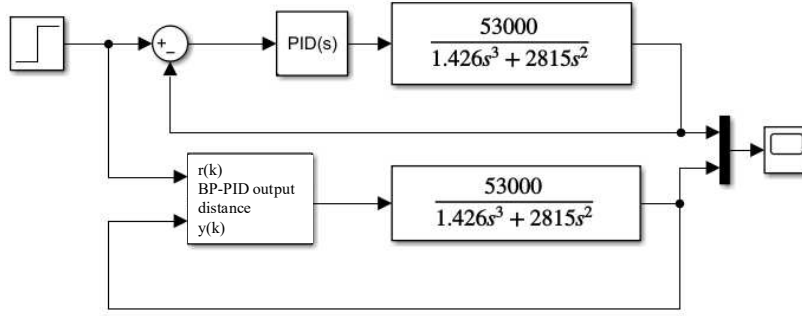


Fig. 19. Simulation model diagram.

The simulation results of the BP-PID self-learning process and conventional PID under unit step signal are shown in Fig. 20, while the K_p , K_i , and K_d parameters obtained after neural network training and tuning are shown in Fig. 21.

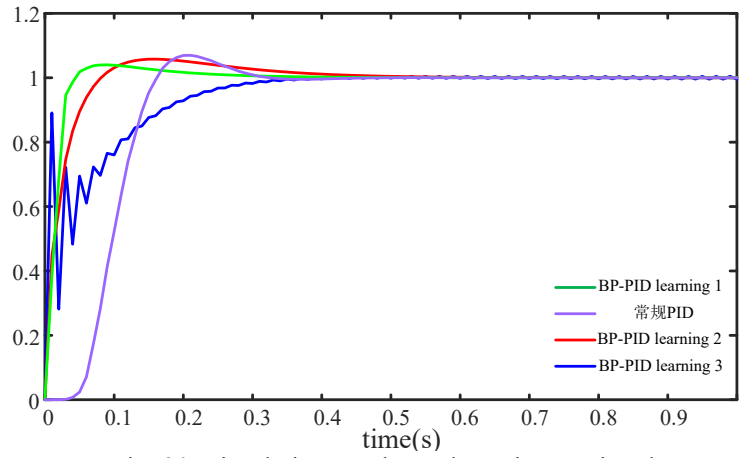


Fig. 20. Simulation results under unit step signal.

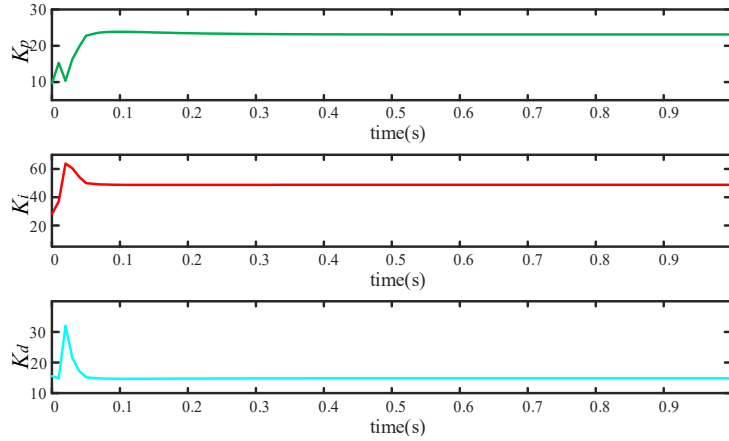


Fig. 21. On-line tuning parameter curves.

Through multiple iterations of self-learning and on-line tuning of the neural network, the final parameters were obtained as 23, 49, and 15, corresponding to K_p , K_i , and K_d , respectively. The results demonstrate that both the tuned BP-PID and conventional PID controllers can achieve stable control. Compared with conventional PID, the BP-PID controller improves stability by 29%, and reduces response time and overshoot by 44% and 42%, respectively. The PID controller optimized by the BP neural network algorithm exhibits smaller overshoot and a

shorter settling time, thereby better satisfying the stability requirements of the wrist rehabilitation trainer control system. A comparison of the dynamic performance indices is presented in Table 5.

Table 5. Dynamic performance indices

Control method	Settling time	Response time	Overshoot
PID	0.35s	0.16s	0.0698
BP-PID	0.26s	0.09s	0.04

5.4 Impedance Controller Design

During rehabilitation exercises assisted by the wrist rehabilitation trainer, the interactive forces between the trainer and the environment (including human wrist loads, friction, etc.) that need to be controlled are replaced by equivalent impedance forces. This approach reduces force–position deviations generated during rehabilitation within the trainer, ensuring continuous and stable rehabilitation motion for the patient. A physical model of the wrist mechanism was established, as shown in Fig. 22.

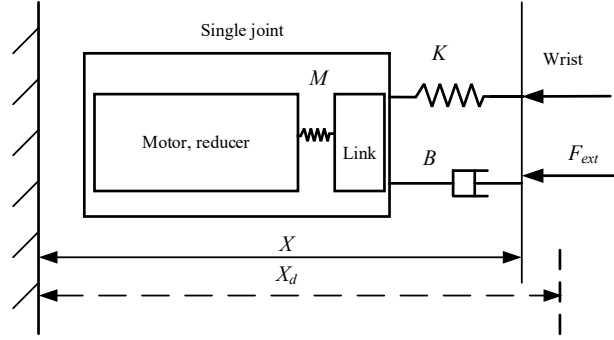


Fig. 22. Physical model of the wrist mechanism.

The contact between the human wrist and the trainer's end-effector is equivalent to an impedance control system characterized by the desired mass M , damping B , and stiffness K ³⁰. The desired input for the assisted motion is X_d , while the actual output (the actual joint displacement) is X , and the human–robot interaction force is F . A simulated mass–spring–damper physical system is employed to regulate the interaction force between the trainer and the environment, thereby achieving compliant control, as expressed in Eq. (28).

$$F = M(\ddot{X}_d - \ddot{X}) + B(\dot{X}_d - \dot{X}) + K(X_d - X) \quad (28)$$

Impedance control enables the determination of the required contact force through position deviation. The wrist trainer employs a closed-loop control structure comprising rotary encoders and torque sensors. In the wrist joint control system, the system's dynamic parameters tend to become unstable due to external environmental factors and actuation effects. During assisted rehabilitation, the trainer's rigidity tends to induce contact force errors. Therefore, a force-based closed-loop impedance control method is adopted, in which the trainer's joint motion is used as the desired motion and proportional–derivative (PD) feedback control is incorporated, as shown in Fig. 23.

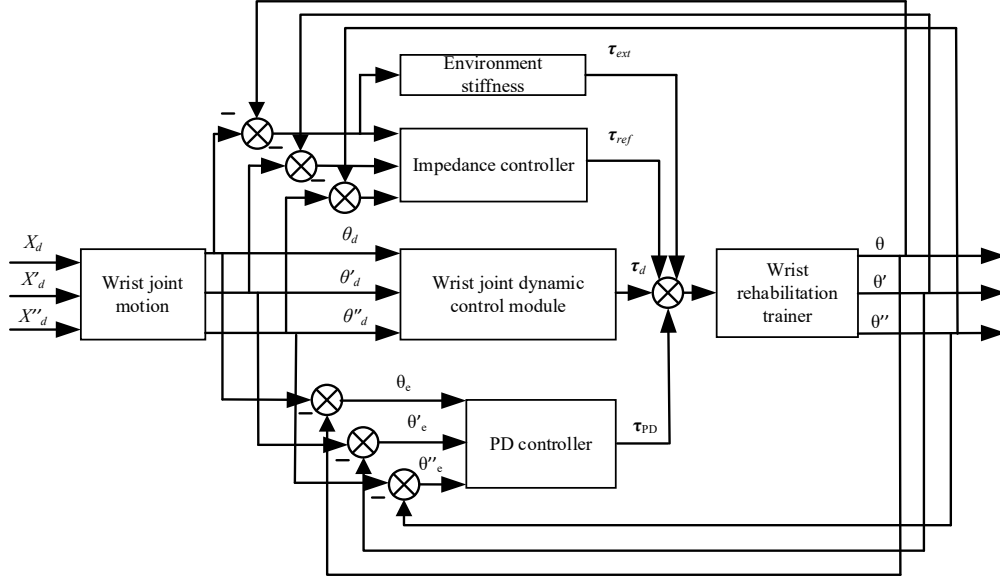


Fig. 23. Force-based impedance control model.

The dynamic model is given as:

$$\tau = \tau_{pd} + \tau_d + \tau_{ext} + \tau_{ref} = M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) \quad (29)$$

where τ_{pd} is the compensation torque from the PD feedback control law; τ_d is the gravity compensation torque and dynamic feedforward; τ_{ext} is the human-robot interaction torque; and τ_{ref} is the torque compensated by the impedance controller for motion errors. The parameters M , B , and K denote the inertia, damping, and gravitational parameters of the mechanism, respectively. The torque components are calculated as follows:

$$\tau_{pd} = P\theta_e + D\dot{\theta}_e \quad (30)$$

$$\tau_d = \tilde{M}(\theta_d)\ddot{\theta}_d + \tilde{C}(\theta_d, \dot{\theta}_d)\dot{\theta}_d + \tilde{G}(\theta_d) \quad (31)$$

$$\tau_{ext} = -J^T(\theta)F_{ext} = -J^T(\theta)K_{ext}\Delta X \quad (32)$$

$$\tau_{ref} = -J^T(\theta)F_{ref} \quad (33)$$

where $\tilde{M}$, $\tilde{C}$, and $\tilde{G}$ represent the inertia, damping, and gravitational parameter matrices in the control model; F_{ext} is the external contact force between the robot end-effector and the human; F_{ref} is the virtual impedance force; and $\theta_e = \theta_d - \theta$. The joint torque is calculated as:

$$\begin{aligned} \tau = & \tilde{M}(\theta_d)\ddot{\theta}_d + \tilde{C}(\theta_d, \dot{\theta}_d)\dot{\theta}_d + \tilde{G}(\theta_d) + P\theta_e + D\dot{\theta}_e \\ & - J^T(\theta)(M\Delta\ddot{X} + B\Delta\dot{X} + K\Delta X) \end{aligned} \quad (34)$$

For a single joint, $\Delta\theta_{ref} = \Delta X$, and the desired joint position is:

$$\tilde{\theta}_d = \theta_d - \Delta\theta_{ref} \quad (35)$$

Let $\theta_e = \tilde{\theta}_d - \theta$. The position control method is then computed as:

$$\tau = \tilde{M}(\theta_d)\ddot{\tilde{\theta}}_d + \tilde{C}(\tilde{\theta}_d, \dot{\tilde{\theta}}_d)\dot{\tilde{\theta}}_d + \tilde{G}(\tilde{\theta}_d) + P\theta_e + D\dot{\theta}_e \quad (36)$$

5.5 Co-simulation Analysis of BP-PID Impedance Control

To reduce the end-effector position error and the motor angular error, a closed-loop control strategy was employed. The position of the end-effector on the moving platform was transmitted to the controller, where it was compared with the desired position. Meanwhile, the feedback torque in the dynamic model was adjusted based on the dynamic solution and the position impedance equation to determine the motor angular parameters for driving joint motions³¹.

The co-simulation control system for the wrist rehabilitation trainer, established in Simulink, is shown in Fig. 24, and the BP-PID subsystem is shown in Fig. 25. General motion torque data for the human wrist were collected as the desired input. The output torques of the trainer's joint motors under different control strategies were measured, and the simulation results for the two control methods are shown in Fig. 26.

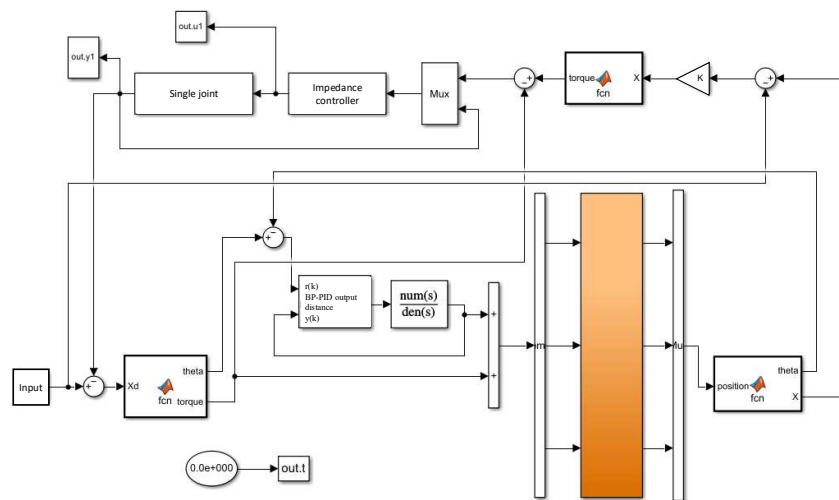


Fig. 24. Co-simulation control system.

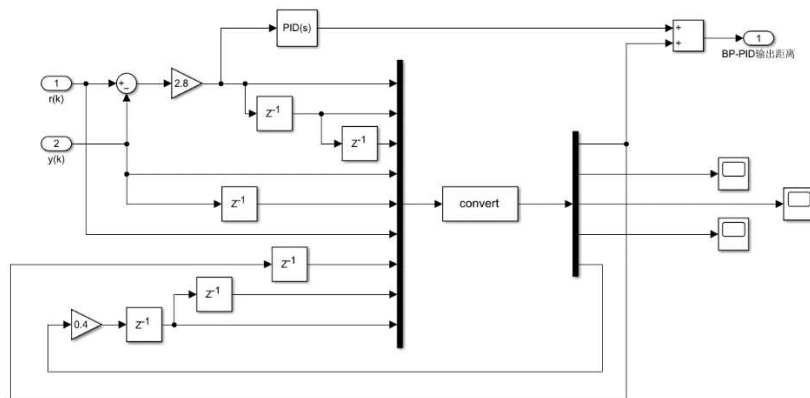
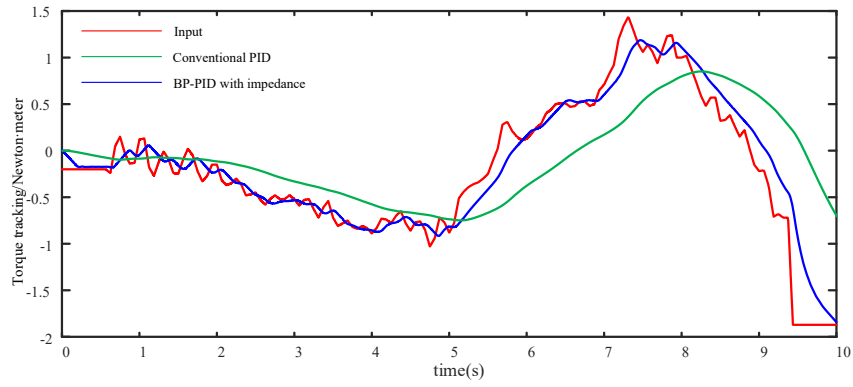
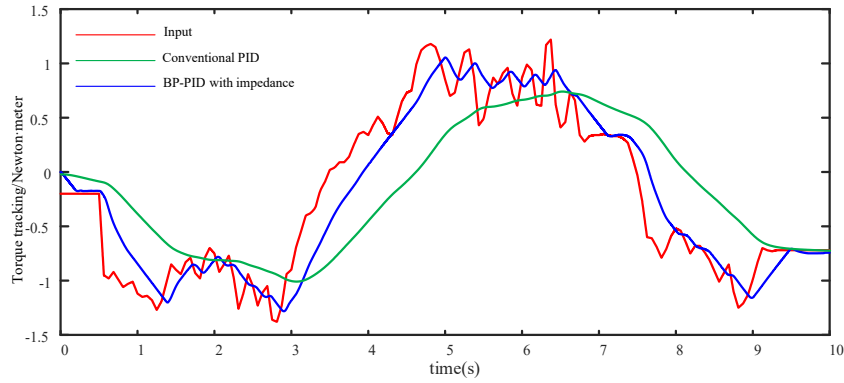


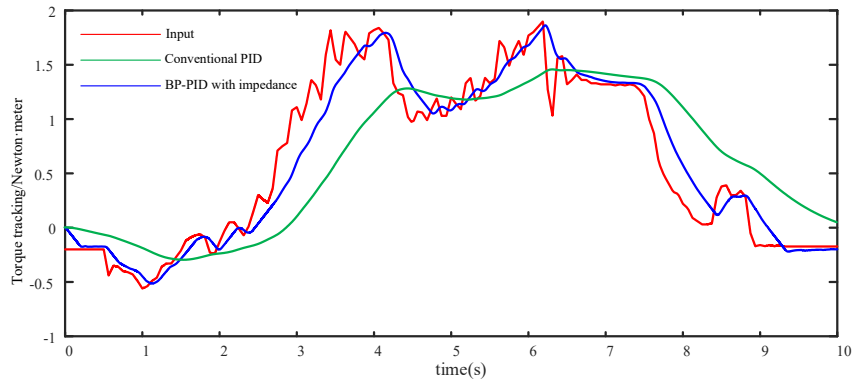
Fig. 25. Subsystem of the integrated BP-PID control.



(a) ulnar/radial deviation joint



(b) flexion/extension joint



(c) pronation/supination joint.

Fig. 26. Simulation results of torque tracking

Table 6. Comparison of simulation results

Motion pattern	Control method	Pearson correlationPearson	RMSE	Mean error
Ulnar/radial deviation	Conventional PID	0.72916	0.4013	0.3152
	BP-PID with impedance	0.97787	0.1068	0.0835
Flexion/extension	Conventional PID	0.74075	0.5285	0.4413
	BP-PID with impedance	0.96842	0.1795	0.1336
Pronation/supination	Conventional PID	0.77761	0.4839	0.3792
	BP-PID with impedance	0.97418	0.1465	0.1116

The simulation results, as presented in Table 6, demonstrate that, with the BP-PID impedance control strategy, the goodness of fit improves by 34.1%, 30.7%, and 24.3%, respectively, compared with conventional PID control. The overall error is reduced by 0.2317 N·m, 0.3077 N·m, and 0.2676 N·m, respectively, while the maximum error is decreased by 56.3%, 31.9%, and

50.4%, respectively. In terms of overall settling time, the BP-PID system with impedance control exhibits a shorter settling time and faster response, enabling more effective assistance for patient rehabilitation training in practical applications and thereby improving rehabilitation efficiency.

6. Experimental Validation

Based on the above analysis, the mechanical structure of the three-degree-of-freedom wrist rehabilitation trainer can theoretically achieve assisted rehabilitation motion with good wearability and can satisfy the kinematic requirements of rehabilitation training. The relevant curves obtained from motion simulation have demonstrated the rationality of the structural design. However, the simulation results were obtained under ideal conditions. To verify the feasibility of the wrist rehabilitation trainer in actual rehabilitation practice, participants were recruited to conduct rehabilitation experiments with the physical prototype of the trainer, and the results were used to validate the trainer's stability and effectiveness.

6.1 Experimental Setup and Data Collection

The control system of the wrist rehabilitation trainer consists of a host computer and a drive control unit. The joint actuation system includes encoders, torque sensors, harmonic reducers, and servo motors, which together with the motion control card and data acquisition board of the control system form an integrated drive control module. The joint sensors transmit information, such as displacement errors and torques generated during human–robot interaction, to the control unit for processing, and the processed signals are then fed back to the joint drive units. The components of the system are shown in Fig. 27, and the physical prototype of the wrist rehabilitation trainer is shown in Fig. 28.

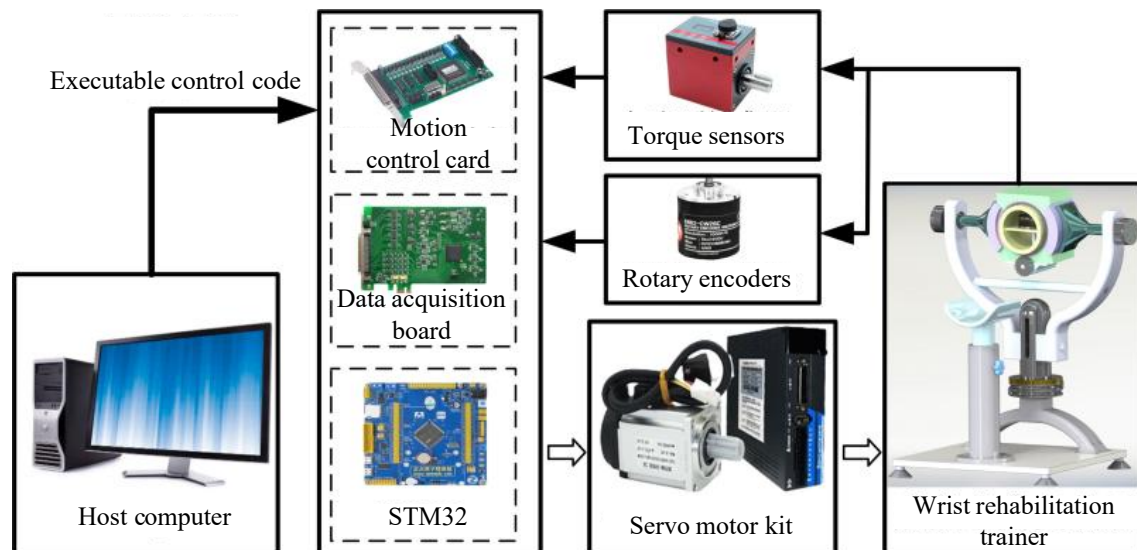
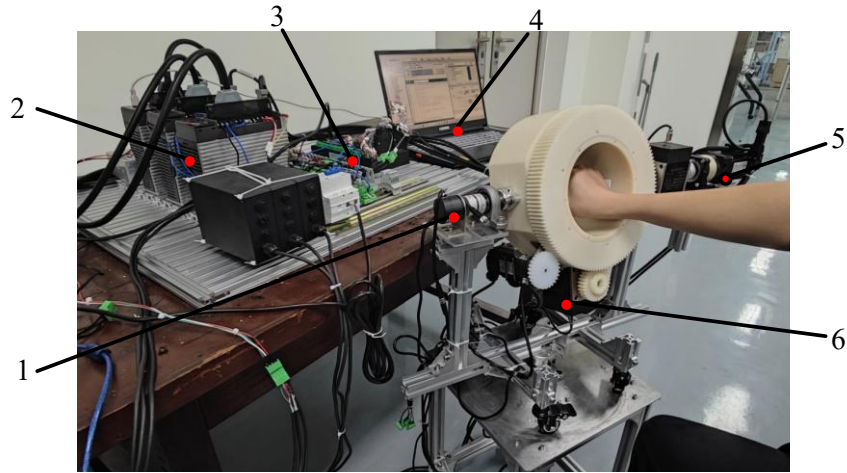


Fig. 27. Structure diagram of the control system



1-rotary encoder; 2-servo controller; 3-motion control card; 4-host computer; 5-drive motor; 6-torque sensor.

Fig. 28. Physical prototype of the wrist rehabilitation trainer.

To validate the rationality of the structural design and the effectiveness of the wrist rehabilitation trainer's control system, motion data of the wrist joint during daily activities were collected from three participants. The three volunteers were healthy adult males and females, and their detailed information is presented in Table 7.

Table 7. Participant information

Volunteer	Gender	Age	Body weight	Height
A	Female	18	50kg	165cm
B	Male	25	65kg	170cm
C	Male	32	75kg	175cm

The acquisition devices were a single-turn absolute rotary encoder and a dynamic torque sensor. The encoder has a linearity accuracy of 0.1% and a maximum rotational speed of 8000 RPM, and is capable of recording the displacement, velocity, and acceleration of the participant's wrist joint motion. The torque sensor has a measuring range of 0–10 N·m, a non-linearity accuracy of 0.1%, and a rated speed of 4000 r/min. sEMG sensors were securely attached to the skin over the wrist flexor/extensor muscles and the ulnar/radial side muscles with straps to collect sEMG signals, as shown in Fig. 29.

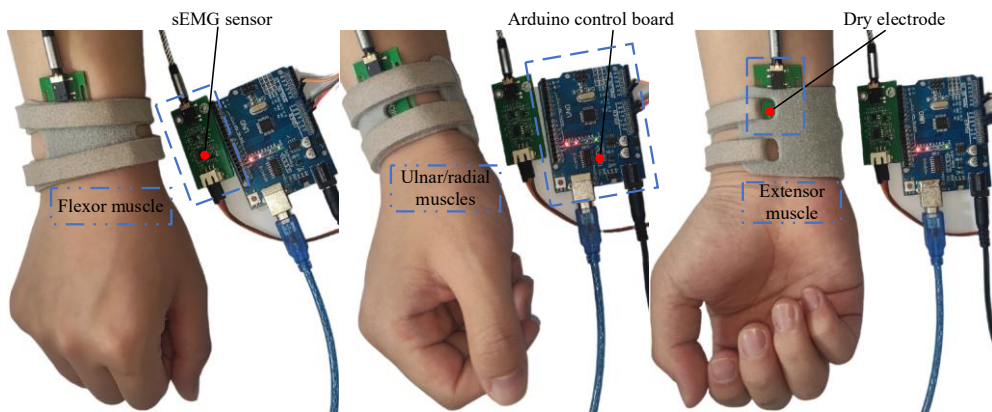


Fig. 29. Experimental setup for sEMG signal acquisition.

First, the participant aligned the wrist joint's center of motion with the end-effector's center of rotation and grasped the end-effector's internal crossbar to maintain the wrist joint and forearm in a horizontal position. This posture was maintained for five minutes to allow the muscles and tissues around the wrist joint to relax. Next, the wrist rehabilitation trainer's servo motion control system was activated, and an appropriate rehabilitation mode was selected based on the participant's rehabilitation needs. Finally, a preset rehabilitation motion trajectory was executed, during which the participant moved the wrist to the extreme positions of flexion/extension, ulnar/radial deviation, and pronation/supination, and performed reciprocating motions along each wrist motion pattern. During this process, sEMG signals from the participant's wrist area were recorded by sEMG sensors, while motion space and torque data were collected via the host computer.

6.2 Experimental Results and Analysis

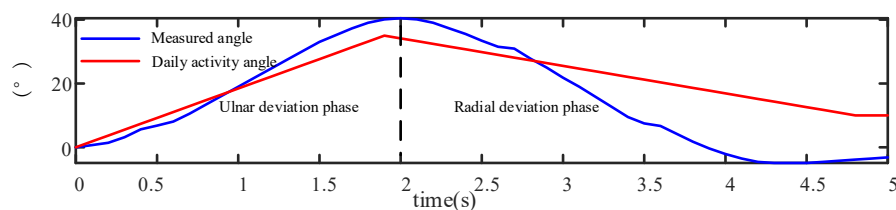
The experimental results were analyzed from the following three aspects:

- (1) whether the workspace of the wrist rehabilitation trainer mechanism meets the requirements of the three-degree-of-freedom combined motion patterns;
- (2) whether the pose feedback control system of the wrist rehabilitation trainer can reduce the force–position interaction effects;
- (3) whether the wrist rehabilitation trainer has a positive rehabilitation effect when assisting rehabilitation motions.

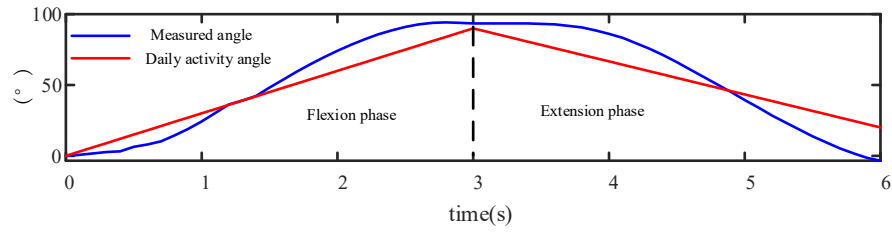
With the pose feedback control system activated, the participants performed rehabilitation motions using the trainer. The measured joint motion angles were compared with the preset clinical daily activity data for the wrist joint, as presented in Table 8, and the results are shown in Fig. 30.

Table 8. Wrist joint motion data

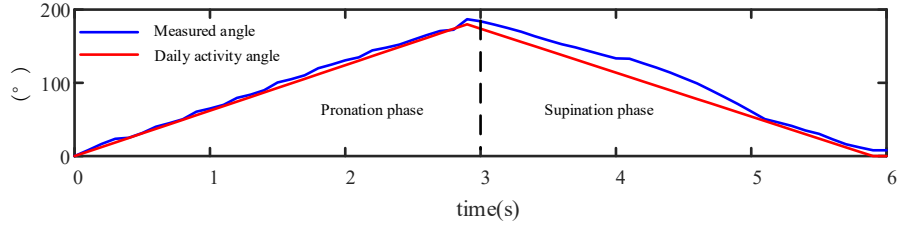
Motion pattern	Range of motion (°)		Torque (N·m)	
	Daily activity	Measured	Daily activity	Measured
Flexion	-70	0~93.7	2.5	-3.05
Extension	90	93.7~4.1	2.5	2.51
Ulnar deviation	25	0~41.7	2.1	-2.76
Radial deviation	35	41.7~5.1	2.1	2.87
Pronation	90	0~186.9	4	5.19
Supination	-90	186.9~5.5	4	-4.09



(a) ulnar/radial deviation



(b) flexion/extension



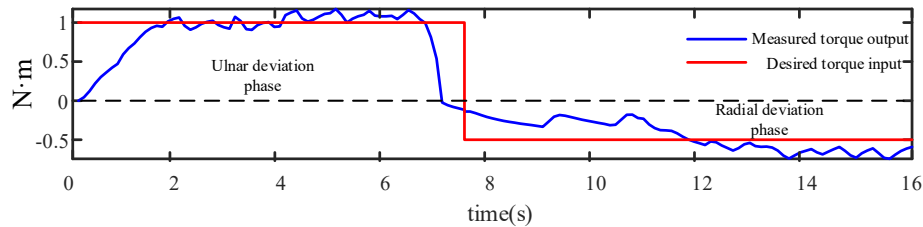
(c) pronation/supination.

Fig. 30. Wrist joint motion angles

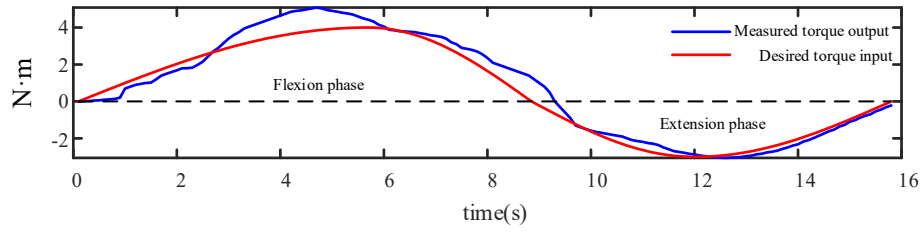
Table 9. Joint position error metrics

Motion pattern	Pearson correlation coefficient	RMSE	Mean error (deg)
Ulnar/radial deviation	0.8730	12.9013	11.1446
Flexion/extension	0.9718	10.1283	8.6639
Pronation/supination	0.99272	11.1347	7.879

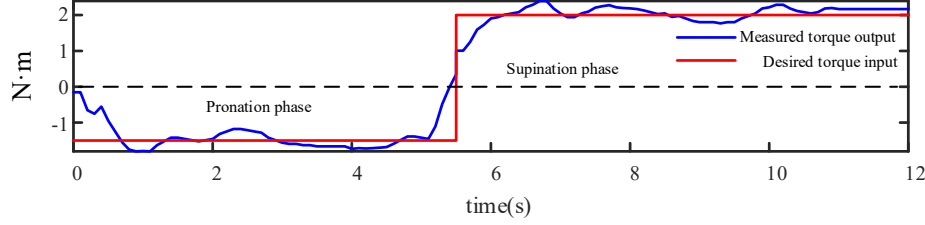
During the experiments, the measured wrist joint motion angles exceeded the preset ranges (based on anatomical and clinical data). The motion angles of the participants for flexion/extension, ulnar/radial deviation, and pronation/supination exceeded the preset ranges by 31.5° , 28.5° , and 12.4° , respectively, and the final return angles were greater than the initial angles. This indicates that the rehabilitation trainer can assist joint motion beyond the wrist joint's normal range, thereby meeting the workspace requirements for wrist rehabilitation in practical conditions. During the assisted motion, the wrist joint position errors of the participants were 7.9° , 8.7° , and 11° , respectively, and the trajectory tracking correlation coefficients were 0.87, 0.97, and 0.99, which represent improvements of 19.7%, 31.2%, and 27.7%, respectively, compared with conventional PID control, as shown in Table 9. The joint torque data during the rehabilitation motion with the trainer were recorded and analyzed, as shown in Fig. 31, and the torque error metrics are presented in Table 10.



(a) ulnar/radial deviation



(b) flexion/extension



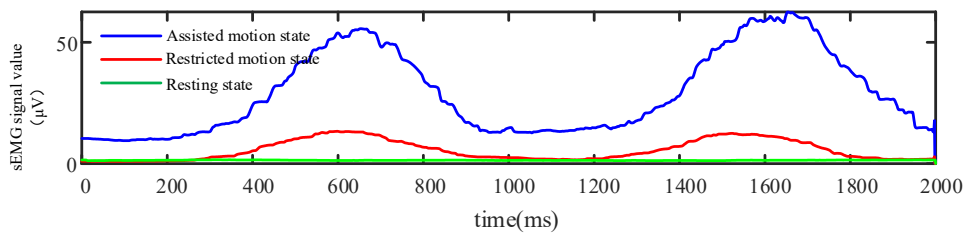
(c) pronation/supination.

Fig. 31. Wrist joint motion torque data

Table 10. Joint torque error metrics

Motion pattern	Pearson correlation coefficient	RMSE	Mean error (N·m)
Ulnar/radial deviation	0.91131	0.3126	0.207
Flexion/extension	0.9844	0.5689	0.45
Pronation/supination	0.97566	0.3936	0.2461

During the experiments, the joint torques reached the desired values at 1.6 s, 1.1 s, and 0.65 s, respectively, and the motion directions changed at 7.5 s, 7 s, and 5.5 s, respectively, with the corresponding settling times of 2.1 s, 1.2 s, and 0.9 s. Owing to the assembly methods and material properties of the trainer components, the friction and inertia forces between joints were not constant. Consequently, the overall error metrics of the experimental results were inferior to those of the neural network control simulation: the overall RMSE and mean error of the joint torques were 0.425 and 0.301, respectively, both higher than the theoretical simulation results of 0.144 and 0.11. Among the three joints, the ulnar/radial deviation joint exhibited a Pearson correlation coefficient of 0.91131, with relatively lower response speed and stability. This is because the ulnar/radial deviation joint serves as the base of the trainer, supporting the flexion/extension and pronation/supination joints, and its additional friction and inertia forces are significantly affected by the motions of the other joints. Nevertheless, the overall error of the experimental results remained lower than that of the conventional PID control simulation, with torque control performance improved by 24.9%, 32.9%, and 25.5%, respectively, demonstrating the effectiveness of the BP-PID feedback control system with the impedance control strategy.



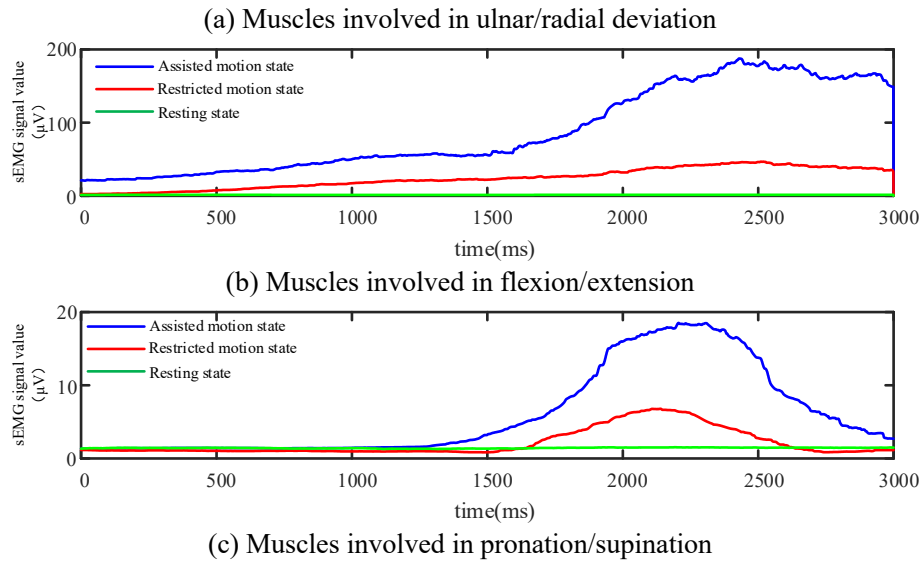


Fig. 32. sEMG signals during wrist joint rehabilitation exercises.

To validate the actual rehabilitation effectiveness of the wrist rehabilitation trainer, sEMG signals were collected from the participants under three conditions: resting state, wrist joint motion under restricted conditions, and assisted rehabilitation motion using the wrist rehabilitation trainer, as shown in Fig. 32. Under different wrist joint motion conditions, the related muscle groups generate varying levels of electrical signals. By comparing the training group across different muscle strength levels with the non-training group using the Fugl-Meyer Assessment (FMA)³², the rehabilitation needs of patients with varying requirements can be effectively targeted, thereby improving rehabilitation efficiency. The assessment criteria and the rehabilitation feedback scores of the volunteers are presented in Table 11.

Table 11. Selected items of the Fugl-Meyer Assessment (FMA) for wrist rehabilitation

Region	Motor function test	Scoring criteria	Score A	Score B	Score C
Wrist	(1) Shoulder stationary, elbow at 90°, wrist flexion	0 points: flexion range < 15° 1 point: completes flexion without resistance 2 points: completes flexion under certain resistance	2	1	2
	(2) Shoulder stationary, elbow at 90°, wrist extension	0 points: difficulty in movement 1 point: limited range of motion 2 points: normal movement within the joint ROM	2	2	2
	(3) Elbow stationary, shoulder at 30°, wrist flexion	Same scoring criteria as (1)	2	1	1
	(4) Elbow at 0°, shoulder at 30°, wrist extension	Same scoring criteria as (2)	2	2	1
	(5) Wrist circumduction	0 points: unable to perform 1 point: movement is effortful or incomplete	1	2	1

Region	Motor function test	Scoring criteria	Score A	Score B	Score C
		2 points: performs normally			

According to the Fugl-Meyer Assessment scoring criteria for wrist motor function, the participants achieved an average score of 8, which is close to the value of healthy wrist joint motor function. Compared with the sEMG signals in the resting state and the motion state under restricted conditions, the maximum sEMG signal values during assisted rehabilitation motion using the wrist rehabilitation trainer were 62, 187, and 18, respectively, representing overall signal amplitude improvements of 425%, 266%, and 181% over the impaired motion state. These results indicate that the wrist rehabilitation trainer can stimulate the corresponding wrist joint muscles during rehabilitation, thereby promoting wrist rehabilitation in users and further validating its effectiveness for wrist joint rehabilitation training.

7. Conclusions

This study developed a three-degree-of-freedom wrist rehabilitation trainer for patients with wrist injuries. The main research contents are as follows: First, the structural design of the rehabilitation trainer was completed, and a kinematic analysis of the trainer model was conducted. The results show that the overall motion trend exhibits no abrupt changes, confirming the correctness and feasibility of the joint model. A virtual prototype simulation platform was established, and it was found that friction, damping, and human–robot contact contribute to torque errors in the actuated joints, providing a reference for the subsequent control system design of the prototype. Subsequently, a method combining BP-PID with impedance control was proposed and validated through Adams–MATLAB co-simulation. Compared with conventional PID control, this method improves stability by 29%, increases response speed by 44%, and reduces overshoot by 42%, demonstrating the effectiveness of the BP-PID impedance control approach. Finally, experiments on assisted wrist rehabilitation training were conducted. The experimental results show that the assisted range of motion for each joint increased by 31.5°, 28.5°, and 12.4°, respectively, and the torque control performance improved by 24.9%, 32.9%, and 25.5%, respectively, thereby validating the mechanical rationality and control effectiveness of the system. During the experiments, the participants' sEMG signal values increased by 425%, 266%, and 181%, respectively, and the average Fugl-Meyer Assessment score was 8, confirming that the wrist rehabilitation trainer positively promotes functional recovery of the wrist joint in patients.

Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Institutional Review Board Statement

All subjects gave their informed consent for inclusion before they participated in the study. This study was conducted in accordance with the declaration of Helsinki, and the protocol was

approved by the Science and Technology Department of Anhui Institute of Information Technology.

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Author contributions

Conceptualization, Y.K.,C.G and Y.C.; methodology, Y.K.,C.G and Y.C.; resources, Y.K.,W.D and L.C.; investigation, Y.K.,C.G,Y.C.,W.D and X.H; Structural design and analysis,Y.K.,C.G and Y.C.; Control system design,Y.K.,C.G,W.D and Y.C.; validation, Y.K.,C.G and Y.C.; data curation, Y.K. and C.G.; writing—original draft preparation, Y.K.,C.G,Y.C.,W.D.,X.H and Y.C.; visualization, Y.K. and C.G.; supervision,Y.C.,X.H.; project administration, L.C.,X.H.; funding acquisition, Y.K.,X.H and Y.C. All authors have read and agreed to the published version of the manuscript.

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Declarations

Competing interests

The authors declare no competing interests.